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STRUCTURAL POINT OBJECTS AND THE CLASSICAL MODEL OF SPIN

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The investigation of the question about possibilities of a classical model of spin has begun even in the 30ies with the appearance of the work[1] by E.Schrödinger, who showed that a free Dirac particle possessed some rapidly oscillating movement (zitterbewegung) of a "hidden" coordinate related to spin, and of the work[2] by Ya. Frenkel, who demonstrated that under certain suppositions the spin of an electron could be considered as the inherent mechanical moment arizing due to some distributions of mass and charge "hidden" from the direct observations. The ideas of works[1] and[2] have led to the appearance of different classical (and correspondingly quantum) models for the spin of elementary particles, for example: bilocal rotator[3], relativistic oscillator[4], stochastic oscillator[5], dequantized spin-particle[6], extended objects[7-9]. To the mentioned above constructions one has to add the classical models considering spin as a function of some "hidden" variables which obey to the Grassmann algebra (see for example[10,11]).

In the present paper one more classical model of spin is discussed. This model is based on the introduced in[12] notion of a structural point object and continues in fact the Frenkel idea about the hidden distributions.

It is proposed[12] that the structural point object should be understood as a cluster of point particles with masses m_κ, charges e_κ, coordinates $\vec{q}_\kappa$ and kinetic momenta $\vec{p}_\kappa = m_\kappa \dot{\vec{q}}_\kappa$ which motion, by force of some specific "confinement" interaction between its point particles, is such that

$$| \vec{q}_\kappa(t) - \vec{q}_n(t) | \le l_0 \qquad \text{for any } \kappa,\ n \text{ and } t\,, \tag{1}$$

where l_0 is a small inaccessible for the experimental observation interval.

The considered cluster of point particles from the point of view of experimental observations is a point object itself (the coordinates $\vec{q}_\kappa$ of separate particles in accordance with (1) are experimentally indistinguishable). That gives rise to the introducing of the term "structural point object".

In the presence of external stationary gravitational, electric and magnetic fields with the potentials $\Phi(\vec{r})$, $\varphi(\vec{r})$, $\vec{A}(\vec{r})$ and correspondingly the intensities

$$\vec{G} = -\nabla\Phi, \qquad \vec{E} = -\nabla\varphi, \qquad \vec{H} = [\nabla \times \vec{A}], \tag{2}$$

the confinement condition (1) can be explained[12] by the existence of some hypothetic potential W of the "confinement" interaction. Supposing that W depends only on the mutual positions of the point particles forming the cluster for its energy we have:

$$\mathcal{H} = \sum_\kappa \left(\frac{1}{2m_\kappa}\vec{p}_\kappa^{\,2} + m_\kappa \Phi(\vec{q}_\kappa) + e_\kappa \varphi(\vec{q}_\kappa) \right) + W(\ldots, \vec{q}_\kappa - \vec{q}_n, \ldots). \tag{3}$$

The evolution of the considered cluster can be written down in the Hamilton formalism with the help of the Poisson brackets explicitly depending on the magnetic field intensity (for more details see[12]):

$$\dot{X} = \{\mathcal{H}, X\} \qquad \text{for any} \quad X = X(\ldots, \vec{q}_\kappa, \vec{p}_\kappa, \ldots), \tag{4}$$

$$\{X, Y\} = \sum_\kappa \left(\frac{\partial X}{\partial p_{\kappa\alpha}} \frac{\partial Y}{\partial q_{\kappa\alpha}} - \frac{\partial X}{\partial q_{\kappa\alpha}} \frac{\partial Y}{\partial p_{\kappa\alpha}} - \frac{e_\kappa}{c} \varepsilon_{\alpha\beta\gamma} H_\alpha(\vec{q}_\kappa) \frac{\partial X}{\partial p_{\kappa\beta}} \frac{\partial Y}{\partial p_{\kappa\gamma}} \right). \tag{5}$$

As the observable characteristics of a structural point object one has to mention the quantities characterizing it as a mass point (mass m, charge e, coordinate $\vec{q}$ of the center of mass, momentum $\vec{p}$),

$$m = \sum_\kappa m_\kappa, \qquad e = \sum_\kappa e_\kappa, \qquad \vec{q} = \frac{1}{m}\sum_\kappa m_\kappa \vec{q}_\kappa, \qquad \vec{p} = \sum_\kappa \vec{p}_\kappa, \tag{6}$$

as well as the integral (structural) quantities reflecting its "hidden" from the observations nonpointness (for example, the inherent dipole electric $\vec{d}$, mechanical $\vec{s}$ and magnetic $\vec{\mu}$ moments),

$$\vec{d} = \sum_\kappa e_\kappa \vec{\xi}_\kappa, \qquad \vec{s} = \sum_\kappa m_\kappa [\vec{\xi}_\kappa \times \dot{\vec{\xi}}_\kappa], \qquad \vec{\mu} = \sum_\kappa \frac{e_\kappa}{2c} [\vec{\xi}_\kappa \times \dot{\vec{\xi}}_\kappa]. \tag{7}$$

Here and in what follows we denote:

$$\vec{\xi}_\kappa = \vec{q}_\kappa - \vec{q}, \qquad \dot{\vec{\xi}}_\kappa = \dot{\vec{q}}_\kappa - \dot{\vec{q}} = \frac{1}{m_\kappa}\vec{p}_\kappa - \frac{1}{m}\vec{p}. \tag{8}$$

To formulate the theory of a structural point object in terms of the observables like (6) and (7) it is natural to use the smallness of parameter ℓ_0 since by force of (1) and (8) $|\xi_{\kappa\alpha}(t)| \leq \ell_0$ for any κ, α and t. With this, different approximations with respect to the smallness of ℓ_0 are possible.

In particular in the 2nd order approximation for energy $\mathcal{H}$ and for the derivatives of $\vec{q}$, $\vec{p}$, $\vec{d}$, $\vec{s}$ with respect to t from the relations (2)-(8) after the multipole development we have:

$$\mathcal{H} = \frac{1}{2m} p_\alpha p_\alpha + \tau + m\Phi + e\varphi - A_{\alpha\beta} G_{\beta,\alpha} - d_\alpha E_\alpha - B_{\alpha\beta} E_{\beta,\alpha} + W, \tag{9}$$

$$\dot{q}_\alpha = \frac{1}{m} p_\alpha, \tag{10a}$$

$$\begin{aligned} \dot{p}_\alpha = {} & m G_\alpha + e E_\alpha + d_\beta E_{\beta,\alpha} + \mu_\beta H_{\beta,\alpha} + A_{\beta\gamma} G_{\alpha,\beta,\gamma} + \\ & + B_{\beta\gamma} E_{\alpha,\beta,\gamma} + \varepsilon_{\alpha\beta\gamma} \Big[\frac{e}{mc} p_\beta H_\gamma + \frac{1}{c} f_\beta H_\gamma + \\ & + \Big(\frac{1}{mc} p_\beta d_\lambda + C_{\beta\lambda}\Big) H_{\gamma,\lambda} + \Big(\frac{1}{mc} p_\beta B_{\lambda\nu} + D_{\lambda\nu\beta}\Big) H_{\gamma,\lambda,\nu} \Big], \end{aligned} \tag{10b}$$

$$\dot{d}_\alpha = f_\alpha, \tag{10c}$$

$$\begin{aligned} \dot{f}_\alpha = {} & d_\beta G_{\beta,\alpha} + \Big(B_{\beta\gamma} - \frac{e}{m} A_{\beta\gamma}\Big) G_{\alpha,\beta,\gamma} + \ell c^2 E_\alpha + d_\beta^{(1)} E_{\beta,\alpha} + \\ & + B_{\beta\gamma}^{(1)} E_{\alpha,\beta,\gamma} + \varepsilon_{\alpha\beta\gamma} \Big[\frac{1}{mc} p_\beta \Big(\ell c^2 H_\gamma + d_\lambda^{(1)} H_{\gamma,\lambda} + B_{\lambda\nu}^{(1)} H_{\gamma,\lambda,\nu} \Big) + \\ & + \frac{1}{c} f_\beta^{(1)} H_\gamma + E_{\lambda\beta}^{(1)} H_{\gamma,\lambda} + D_{\lambda\nu\beta}^{(1)} H_{\gamma,\lambda,\nu} \Big] - M_\alpha, \end{aligned} \tag{10d}$$

$$\begin{aligned} \dot{s}_\alpha = {} & \varepsilon_{\alpha\beta\gamma} \Big[2A_{\beta\lambda} G_{\gamma,\lambda} + d_\beta E_\gamma + 2B_{\beta\lambda} E_{\gamma,\lambda} - N_{\beta\gamma} + \\ & + \varepsilon_{\gamma\lambda\nu} \Big(\frac{1}{mc} d_\beta p_\lambda H_\nu + \frac{2}{mc} B_{\beta\rho} p_\lambda H_{\nu,\rho} + E_{\beta\lambda} H_\nu + 2D_{\beta\rho\lambda} H_{\nu,\rho} \Big) \Big]. \end{aligned} \tag{10e}$$

Here and in what follows all the fields are written down as functions of $\vec{q}$, index $,\alpha$ denotes the partial derivative with respect to q_α .

As an addition to (6) and (7) there appeared in the relations (9)-(10) a positive constant

$$\ell = \frac{1}{c^2}\sum_{\varkappa} m_\varkappa \left(\frac{e_\varkappa}{m_\varkappa} - \frac{e}{m}\right)^2 > 0 \tag{11}$$

and a set of structural characteristics: the internal kinetic $\mathcal{T}$ and potential W energies, the speed $\vec{f}$ of the dipole moment changement,

$$\mathcal{T} = \sum_{\varkappa} \frac{1}{2m_\varkappa}\dot{\vec{\xi}}_\varkappa^{\,2}, \quad W = W_0 + \sum_{\varkappa,n} \omega^{\varkappa n}_{\alpha\beta}\,\xi_{\varkappa\alpha}\,\xi_{n\beta}, \quad \vec{f} = \sum_{\varkappa} e_\varkappa \dot{\vec{\xi}}_\varkappa, \tag{12}$$

quantities $A_{\alpha\beta}$, $B_{\alpha\beta}$, $C_{\alpha\beta}$, $D_{\alpha\beta\gamma}$,.... , related to the inherent quadrupolar and toroidal (see for example[13]) moments. The expressions for these quantities in terms of variables (8) are given in paper[15].

The closing of the system of equations (10) demands that the analogous equations for μ_α , $A_{\alpha\beta}$, $B_{\alpha\beta}$, ... should be written down as well. The equations for these quantities can be easily written out with the help of their definitions and of the relations (2)-(8). But with this there appeares a new set of structural characteristics, and so on.

Thus, a structural point object even in the 2nd order approximation possesses, together with its point characteristics m, e , $\vec{q}$, $\vec{p}$, an infinite chain of structural characteristics ℓ , $\vec{d}$, $\vec{s}$, $\vec{\mu}$, $\mathcal{T}$, W, ... The corresponding system of the equations of motion is also infinite.

That is why to construct a close theory of a structural point object in terms of the observables, with a finite number of independent variables and correspondingly with a finite number of equations of the motion, one needs a selfconsistent and reasonable breaking of the chain of structural characteristics.

One of the possible methods of such a breaking, introduced in paper[12], is represented by an analysis of the system of the Poisson brackets (5), calculated pairwisely for all the structural characteristics, with the supposition about a smallness of the structural constant ℓ (11). In the 2nd order approximations with respect to ℓ_0 and ℓ simultaneously one can choose the set

$$a = (a_i\,,\; i \in \overline{1,15}) = (\vec{q}, \vec{p}, \vec{d}, \vec{f}, \vec{s}\,). \tag{13}$$

as the multitude of independent variables while all the other quantities can be written down as the following functions of a (the detailed analysis and the corresponding rather long calculations are given in works[14,15]):

$$\mu_\alpha = g S_\alpha + \frac{e}{2m\ell c^3}\varepsilon_{\alpha\beta\gamma} d_\beta f_\gamma\,, \qquad A_{\alpha\beta} = \frac{1}{2\ell c^2} d_\alpha d_\beta\,, \tag{14a}$$

$$B_{\alpha\beta} = \frac{e}{2m\ell c^2} d_\alpha d_\beta\,, \qquad C_{\alpha\beta} = \frac{e}{2m\ell c^3}\left(d_\alpha f_\beta + d_\beta f_\alpha\right), \tag{14b}$$

$$D_{\alpha\beta\gamma} = \frac{g}{\ell c}\left(g - \frac{e}{2mc}\right)\left(d_\alpha \varepsilon_{\beta\gamma\lambda} + d_\beta \varepsilon_{\alpha\gamma\lambda}\right) S_\lambda\,, \qquad d^{(1)}_\alpha = \frac{e}{m} d_\alpha\,, \tag{14c}$$

$$D^{(1)}_{\alpha\beta\gamma} = 0\,, \qquad f^{(1)}_\alpha = \frac{e}{m} f_\alpha\,, \qquad E_{\alpha\beta} = g\varepsilon_{\alpha\beta\gamma} S_\gamma + \frac{e}{m\ell c^3} d_\alpha f_\beta\,, \tag{14d}$$

$$E^{(1)}_{\alpha\beta} = 2cg\left(g - \frac{e}{2mc}\right)\varepsilon_{\alpha\beta\gamma} S_\gamma\,, \qquad N_{\alpha\beta} = \frac{\omega^2}{\ell c^2} d_\alpha d_\beta\,, \tag{14e}$$

$$B^{(1)}_{\alpha\beta} = 0\,, \qquad M_\alpha = \omega^2 d_\alpha\,, \qquad W = W_0 + \frac{\omega^2}{2\ell c^2} d_\alpha d_\alpha\,, \tag{14f}$$

$$\mathcal{T} = \frac{1}{2\ell c^2} f_\alpha f_\alpha - g S_\alpha H_\alpha - \frac{2g}{\ell c}\left(g - \frac{e}{2mc}\right) d_\alpha S_\beta H_{\beta,\alpha} + \mathcal{T}_0\,. \tag{14g}$$

In the relations (14) we have denoted:

$$S_\alpha = s_\alpha - \frac{1}{\ell c^2}\varepsilon_{\alpha\beta\gamma} d_\beta f_\gamma\,. \tag{15}$$

The constants $\ell > 0$, g , $\omega > 0$, $\mathcal{T}_0$ and W_0 , having the dimensions of length, gyromagnetic factor, frequency and energy correspondingly, in the considered approximation stay, as well as m and e , indetermined.

The bonds (14) close the system (10) in that sence that it contains now only the variables of the set (13). Now the relations (10) represent the system of equations of the motion of a structural point object in external stationary fields, allowing one to determine uniquely the trajectory $\vec{q}(t)$ and the values of all other physical quantities for any instant $t > t_0$ if the values of the independent variables (13) are known for instant t_0 .

Differentiating expression (9) with respect to time with the help of relations (2), equations of the motion (10) and bonds (14) we have:

$$\dot{\mathcal{H}} \equiv 0 , \qquad \mathcal{H} = Const. , \tag{16}$$

i.e. the energy of a structural point object is an integral of the motion in any stationary fields.

Let us consider now the quantity $\vec{S}$,defined as function (15) of the independent variables $\vec{d}$, $\vec{f}$, $\vec{s}$ and having the dimension of mechanical moment. Differentiating S_α by force of equations (10), bonds (14), we obtain:

$$\dot{S}_\alpha = g\varepsilon_{\alpha\beta\gamma} S_\beta \left[H_\gamma + \frac{2}{\ell c}\left(g - \frac{e}{2mc}\right) d_\lambda H_{\gamma,\lambda} \right] . \tag{17}$$

This leads to one more conservation law:

$$\dot{\vec{S}}^2 = 2\vec{S}\dot{\vec{S}} \equiv 0 , \qquad \vec{S}^2 = Const. . \tag{18}$$

Besides it follows from bonds (14) that the addition of a homogeneous magnetic field to the energy (9) is $g\vec{S}\vec{H}$. At last in the considered approximation from (5) we have:

$$\{S_\alpha , a_i\} = 0 , \quad a_i \in (\vec{q}, \vec{p}, \vec{d}, \vec{f}) ; \qquad \{S_\alpha , S_\beta\} = -\varepsilon_{\alpha\beta\gamma} S_\gamma , \tag{19}$$

i.e. $\vec{S}$ forms an independent algebra of a moment.

Hence, quantity $\vec{S}$ is namely that physical characteristic of a structural point object which in accordence with the system of the notions of the modern physics has to be identified with the spin.

Thus in the 2nd order approximation we have constructed a closed classical description (independent variables (13), equations of the motion (10) and bonds (14)) of the motion of a structural point object in stationary fields and at the same time we have discovered a new classical model of spin (definition (15) with unconditional conservation law (18)).

Here, in contrast to all known before classical models /2-11/, the inherent mechanical moment $\vec{s}$, the inherent magnetic moment $\vec{\mu}$ and spin $\vec{S}$ are different nonproportional in a general case quantities,

$$\vec{s} = \vec{S} + \frac{1}{\ell c^2}[\vec{d}\times\vec{f}] , \qquad \vec{\mu} = g\vec{S} + \frac{e}{2m\ell c^3}[\vec{d}\times\vec{f}] , \tag{20}$$

because $\vec{d} \neq 0$ (if $\vec{d} \equiv 0$, it follows from (14e) that $W =$ Const. and the confinement condition (1) is not fulfilled).

The physical meaning of different characteristics of a structural point object and the questions of possible applications are considered in the work /15/.

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