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GEOMETRY OF SPONTANEOUSLY BROKEN LOCAL N = 1 SUPERSYMMETRY IN SUPERSPACE

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1 Introduction

Supersymmetric field theories are adequately formulated in superspace via unconstrained superfields. Unconstrained superspace formulations make manifest the invariance properties of a given theory (both on classical and quantum levels) and bring to light its intrinsic geometry. One of central problems in supersymmetry consists in finding out the group-theoretic and geometric structures inherent in the theories of interest (such as super-Yang-Mills, supergravity or superstring theories) and selecting superspaces where these structures reveal themselves in a most unambiguous way. For supergravity (SG in what follows) that program is now completed in the cases of $N = 1$ [1-5] and $N = 2$ [6].

Since supersymmetry in Nature is believed to be spontaneously broken, it is of importance to understand in full how this breakdown is inscribed in the superspace geometric picture of supersymmetric theories.

An appropriate framework for analyzing theories with spontaneously broken symmetry is provided by nonlinear realizations (group realizations in coset manifolds) [7].The main advantage of the nonlinear realization method is that it allows one to reveal the geometric model-independent content of spontaneous breakdown by identifying the relevant Goldstone fields with the coordinates of a coset manifold where the group of spontaneously broken symmetry acts as left shifts. Given an invariant action with spontaneously broken symmetry, one can always rewrite it, by means of an equivalence field redefinition, in terms of fields having standard transformation properties with respect to the corresponding nonlinear realization. In this parametrization, the minimal self-interaction of Goldstone fields is described by a unique effective Lagrangian whatever the initial action is.The pure consequences of spontaneous breakdown (low-energy theorems, Higgs effect, etc.) turn out to be separated from those connected with the specific mechanism of this breaking.In fact, the range of applications of nonlinear realizations is not limited to conventional spontaneously broken symmetries. For instance, gauge theories (including gravity) can be interpreted as nonlinear realizations of certain (infinite-dimensional) symmetries [8]. Recently, it has been pointed out that the theories of current interest, such as those of strings and membranes, can also be understood as nonlinear realizations [9-12].

For rigid $N = 1$ Poincaré supersymmetry the nonlinear realization has been constructed in the pioneering papers by Volkov and Akulov [13] (see also [14]) with employing standard techniques of refs. [7]. These techniques equally apply to other rigid supersymmetries (see, e.g., [15]). The relationship between the nonlinear realization of $N = 1$ supersymmetry and linear realizations of the latter in superspace has been investigated in detail in our papers [16, 17] and in [18].

The standard nonlinear realization method as it was described in [7] ideally suits to rigid supersymmetries but ceases to be too useful when trying to construct nonlinear realizations of spontaneously broken local supersymmetries in the context of superspace geometric formulations of SG theories. The origin of difficulties lies in that the underlying gauge groups of SG's are infinite-dimensional and, as a rule, cannot be obtained via a naive gauging of corresponding rigid supergroups [1,6]. Thus there emerges the problem of how to set up nonlinear realizations of such nontrivial groups with preserving the original group-theoretic and geometric structure.

Attempts to extend the geometric set-up of spontaneously broken rigid $N = 1$ supersymmetry [13, 16-18] to the case of SG have been undertaken in [19-21]. In the letter [21] we argued that this can be done most naturally with taking advantage of the unconstrained superspace formulation of $N = 1$ SG given by Ogievetsky and Sokatchev (OS henceforth)[1,2]. We have shown how to formulate a nonlinear realization of the $N = 1$ SG group consistently with the intrinsic geometry of unbroken theory.

Our consideration in [21] was rather schematic and it concerned mainly the case of conformal $N = 1$ SG. Now we find it timely to return to this theme and to give a more detailed exposition of our approach in application both to conformal and Einstein $N = 1$ SG's. Our motivation is two-fold. First, for the last years there was a considerable growth of interest in phenomenological models based on $N = 1$ SG, especially in connection with the study of the point-like limit of superstring theories (see, e.g., [22]). Spontaneous breakdown of supersymmetry is an important ingredient of these models, so it is desirable to have a clear understanding of its intrinsic nature. As a second reason, we wish to point out that the methods we use to describe spontaneously broken $N = 1$ supersymmetry are in fact more universal and can be applied for a model-independent treatment of spontaneous breakdown of any symmetry realized by coordinate transformations. This regards higher N SG's, the theories of extended objects, etc. In particular, these techniques may hopefully be used for analyzing the phenomenon of partial supersymmetry breaking in the superstring and supermembrane theories along the lines of refs.[10-12].

The modified approach to nonlinear realizations of spontaneously broken supersymmetries we have applied first in [21] and which we follow here has the advantage of being equally suited for treating the rigid and curved cases. It proceeds from the realization of corresponding unbroken supergroup in an appropriate superspace and goes straightforwardly once a realization of that sort is known. Surprisingly, it opens up a way to construct the nonlinear realization covariants without resorting to the customary formalism of Cartan's forms (at least, in the examples we are considering here). One more attractive feature of this approach is that it immediately yields the relations between linear and nonlinear realizations of underlying symmetry.

The paper is planned as follows. In Sect.2 we illustrate the basic features of our approach by the hand-book example of the Volkov-Akulov nonlinear realization. In Sect.3 we construct the minimal nonlinear realization of superspace group of conformal $N = 1$ SG containing only one extra field, goldstino $\lambda^\mu(x)$, in addition to the fields of SG multiplet and establish the

relation of this realization to the OS geometric picture of conformal $N = 1$ SG. Our consideration preserves manifest invariance and does not require any gauge-fixing. Sect.4 treats, along the same lines, the case of minimal Einstein $N = 1$ SG in the formulation with a chiral compensator. In Sect.5 we extend to curved space the basic ingredients of the relationship between the linear and nonlinear realizations of rigid $N = 1$ supersymmetry [16, 17] and discuss the flat space limit of our formulas. Sect.6 collects concluding remarks and outlines perspectives of applying our methods in some theories of similar nature.

2 Superspace view on Volkov-Akulov nonlinear realization

To fix the basic ideas of our approach, it is instructive to begin with reformulating the nonlinear realization of rigid $N = 1$ Poincaré supersymmetry [13, 14].

2.1 Superspace genesis of N=1 goldstino

What we need to proceed is the familiar transformation law of $N = 1$ supersymmetry in chiral $N = 1$ superspace $\mathbf{C}^{4|2} = \{x^m_L, \theta^\mu_L\} \equiv \{\zeta^M_L\}$ [1]

$$
\begin{aligned}
x^{m'}_L = G^m(\zeta_L) &= x^m_L + a^m + 2i\theta_L\sigma^m\bar\epsilon + i\epsilon\sigma^m\bar\epsilon \\
\theta^{\mu'}_L = G^\mu(\zeta_L) &= \theta^\mu_L + \epsilon^\mu,
\end{aligned}
\tag{2.1}
$$

a^m, ϵ^μ being the parameters of ordinary and spinor translations. A direct way to get the nonlinear realization is to restrict ζ^M_L in (2.1) to the 4-dimensional hypersurface

$$
x^m_L = y^m_L, \theta^\mu_L = \kappa^\mu(y_L) \tag{2.2}
$$

This way, the Zumino's version of nonlinear realization of $N = 1$ supersymmetry [23] emerges. However, looking at the transformation laws of $y^m_L, \kappa^\mu(y_L)$ it is difficult to immediately figure out how to construct the relevant covariant quantities.

Another, more suggestive possibility we shall follow is based on viewing (2.1) as a finite element of $N = 1$ Poincaré supergroup (modulo Lorentz transformations) parametrized by $a^m, \epsilon^\mu, \bar\epsilon^{\dot\mu}$. Successive transformations of

[1] We use the standard two-component spinor formalism with the conventions

$$
(\sigma_n)_{\alpha\dot\beta} = (1, \vec\sigma)_{\alpha\dot\beta}, \quad (\bar\sigma_n)^{\dot\beta\alpha} = \epsilon^{\dot\beta\dot\rho}\epsilon^{\alpha\beta}(\sigma_n)_{\beta\dot\rho} = (1, -\vec\sigma)^{\dot\beta\alpha}
$$

$$
\epsilon^{12} = \epsilon_{21} = 1, \quad \eta^{mn} = \text{diag}(1, -1, -1, -1), \quad \epsilon^{0123} = 1.
$$

ζ_L^M generate the left action of $N=1$ supergroup in the space of its parameters $\{a^m, \epsilon^\mu, \bar{\epsilon}^{\dot{\mu}}\}$:

$$G_1^M(G(\zeta_L)) = G^{M'}(\zeta_L) \Rightarrow \tag{2.3}$$
$$a^{m'} = a^m + a_1^m + i\epsilon\sigma^m\bar{\epsilon}_1 - i\epsilon_1\sigma^m\bar{\epsilon}$$
$$\epsilon^{\mu'} = \epsilon^\mu + \epsilon_1^\mu, \bar{\epsilon}^{\dot{\mu}'} = \bar{\epsilon}^{\dot{\mu}} + \bar{\epsilon}_1^{\dot{\mu}}$$

Here the primes refer to the parameters of the resulting transformation rather than to the superspace coordinates ζ_L^M which are regarded to be unaffected.

In this language, the transformations of special form

$$\begin{aligned} Y^m(\zeta_L) &= x_L^m + 2i\theta_L\sigma^m\bar{\epsilon} + i\epsilon\sigma^m\bar{\epsilon} \\ Y^\mu(\zeta_L) &= \theta_L^\mu + \epsilon^\mu \end{aligned} \tag{2.4}$$

represent the left cosets of $N=1$ supergroup over its Poincaré subgroup. Clearly, the whole supergroup can be realized on these restricted elements. The supertranslations act as

$$\begin{aligned} G_1^M(Y(\zeta_L)) &= Y^{M'}(G_0(\zeta_L)) \\ \epsilon^{\mu'} = \epsilon^\mu &+ \epsilon_1^\mu \\ G_0^m(\zeta_L) &\equiv x_L^{m'} = x_L^m + \tilde{a}^m(\epsilon, \epsilon_1) = x_L^m + i\epsilon\sigma^m\bar{\epsilon}_1 - i\epsilon_1\sigma^m\bar{\epsilon} \\ G_0^\mu(\zeta_L) &\equiv \theta_L^{\mu'} = \theta_L^\mu \end{aligned} \tag{2.5}$$

A difference as compared with (2.3) is that in (2.5) there appears an induced 4-translation with the composite parameter $\tilde{a}^m(\epsilon, \epsilon_1) = i\epsilon\sigma^m\bar{\epsilon}_1 - i\epsilon_1\sigma^m\bar{\epsilon}$. This phenomenon is typical for group realizations in coset spaces [7]. The left action of some group G on the coset elements G/H induces a "gauge" right transformation belonging to the stability subgroup H, with the parameters properly composed out of the original coset coordinates and the parameters of the group transformation

$$g_1 \cdot g = g'(g, g_1) \cdot h(g, g_1), \quad g_1 \in G; \quad g, g' \in G/H; \quad h \in H \tag{2.6}$$

To construct a genuine nonlinear realization of given group in some coset space, one has to regard the coset parameters as fields defined on some manifold. For consistency, coordinates of this manifold should either be inert under the action of the group (this occurs in the case of internal symmetries) or transform through themselves and/or via the coset parameters.

In the case at hand one meets just the second possibility. Let us change the constant parameters $\epsilon, \bar{\epsilon}$ in (2.4) to the fields $\lambda(\tilde{x}_L), \bar{\lambda}(\tilde{x}_L) \equiv (\lambda(\tilde{x}_L^\dagger))^\dagger$

$$\begin{aligned} \tilde{Y}^m(\tilde{\zeta}_L) &= \tilde{x}_L^m + 2i\tilde{\theta}_L\sigma^m\bar{\lambda}(\tilde{x}_L) + i\lambda(\tilde{x}_L)\sigma^m\bar{\lambda}(\tilde{x}_L) \\ \tilde{Y}^\mu(\tilde{\zeta}_L) &= \tilde{\theta}_L^\mu + \lambda^\mu(\tilde{x}_L) \end{aligned} \tag{2.7}$$

and keep for $\tilde{Y}^M(\tilde{\zeta}_L)$ the same transformation law (2.5)

$$G_1^M(\tilde{Y}(\tilde{\zeta}_L)) = \tilde{Y}^{M'}(\tilde{G}_0(\tilde{\zeta}_L)) \tag{2.5'}$$

Here we have substituted $\tilde{\zeta}_L^M$ for ζ_L^M because the newly introduced coordinates $\tilde{\zeta}_L^M$ behave differently under $N=1$ supersymmetry. One gets from (2.5')

$$\begin{aligned} \lambda^{\mu'}(\tilde{x}'_L) &= \lambda^\mu(\tilde{x}_L) + \epsilon_1^\mu \\ \tilde{x}_L^{m'} &= \tilde{x}_L^m + \tilde{a}^m(\lambda(\tilde{x}_L), \epsilon_1) = \tilde{x}_L^m + i\lambda(\tilde{x}_L)\sigma^m\bar{\epsilon}_1 - i\epsilon_1\sigma^m\bar{\lambda}(\tilde{x}_L) \\ \tilde{\theta}_L^{\mu'} &= \tilde{\theta}_L^\mu \end{aligned} \tag{2.8}$$

The pair $\{\tilde{x}_L^m, \lambda^\mu(\tilde{x}_L)\}$ is easily recognized to constitute the Volkov-Akulov nonlinear realization [13][2] while the coordinate $\tilde{\theta}_L^\mu$ turns out to be inert with respect to $N=1$ supersymmetry. The fact that $\tilde{x}_L^m$ is complex whereas in the original Volkov-Akuliov approach the space-time coordinate is real should not lead to confusion because the transformation of λ^μ at a fixed point is given by

$$\delta^* \lambda^\mu(x) = \epsilon^\mu - \tilde{a}^m(\lambda(x), \epsilon_1)\partial_m\lambda^\mu(x) \tag{2.8'}$$

irrespective of whether x^m is real or complex.Nevertheless, the complexity of $\tilde{x}_L^m$ turns out to be important for deducing the covariants of nonlinear realization in the present approach.

2.2 Covariants from an axial vector superfield

The coordinates $\tilde{\zeta}_L^M = (\tilde{x}_L^m, \tilde{\theta}_L^\mu)$ are adequate to spontaneously broken supersymmetry as they transform according to its nonlinear realization. On the other hand, the transformation law (2.5') implies that the coset space representatives $\tilde{Y}^M(\tilde{\zeta}_L)$ transform under $N=1$ supersymmetry as the original superspace coordinates ζ_L^M (c.f. (2.5') and (2.1)) and so can be identified with them

$$\zeta_L^M = \tilde{Y}^M(\tilde{\zeta}_L) \tag{2.9}$$

This relation is just the one derived by us ten years ago [16]. With its help any superspace action with spontaneously broken $N=1$ supersymmetry can be expressed in terms of fields of the nonlinear realization.

In [16, 17] we did not to full extent exploit the property that $\tilde{x}_L^m$ is complex. One may take advantage of this property to reproduce the basic covariants of the nonlinear realization within the present framework.

To this end, we first recall the well-known relation between the flat chiral and real $N=1$ superspaces $\mathbf{C}^{4|2} = \{x_L^m, \theta_L^\mu\}$ and $\mathbf{R}^{4|4} = \{x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}}\} \equiv \{z^M\}$:

$$x_L^m = x^m + i\theta\sigma^m\bar{\theta}, \quad \theta_L^\mu = \theta^\mu, \overline{(\theta_L^\mu)} = \bar{\theta}^{\dot{\mu}} \tag{2.10}$$

[2] This realization is related to (2.2) via changing variables as $y_L^m = \tilde{x}_L^m + i\lambda(\tilde{x}_L)\sigma^m\bar{\lambda}(\tilde{x}_L)$, $\kappa^\mu(y_L(\tilde{x}_L)) = \lambda^\mu(\tilde{x}_L)$.

Likewise, one may single out in $\tilde{\mathbf{C}}^{4|2} = \{\tilde{x}^m_L, \tilde{\theta}^\mu_L\} \equiv \{\tilde{\zeta}^M_L\}$ a real 4 | 4 dimensional subspace $\tilde{\mathbf{R}}^{4|4} = \{\tilde{x}^m, \tilde{\theta}^\mu, \bar{\tilde{\theta}}^{\dot{\mu}}\} \equiv \{\tilde{z}^M\}$:

$$\tilde{x}^m_L = \tilde{x}^m + i\tilde{H}^m(\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}}), \quad \tilde{\theta}^\mu_L = \tilde{\theta}^\mu, \quad \overline{(\tilde{\theta}^\mu_L)} = \bar{\tilde{\theta}}^{\dot{\mu}} \tag{2.11}$$

and,taking into account (2.9) and (2.10),

$$x^m = \tilde{x}^m + i\tilde{\theta}\sigma^m\bar{\lambda}(\tilde{x} + i\tilde{H}) - i\lambda(\tilde{x} - i\tilde{H})\sigma^m\bar{\tilde{\theta}}$$

$$+\tfrac{i}{2}\lambda(\tilde{x} + i\tilde{H})\sigma^m\bar{\lambda}(\tilde{x} + i\tilde{H}) - \tfrac{i}{2}\lambda(\tilde{x} - i\tilde{H})\sigma^m\bar{\lambda}(\tilde{x} - i\tilde{H}) \tag{2.12a}$$

$$\theta^\mu = \tilde{\theta}^\mu + \lambda^\mu(\tilde{x} + i\tilde{H}) \;\; , \;\; \bar{\theta}^{\dot{\mu}} = \bar{\tilde{\theta}}^{\dot{\mu}} + \bar{\lambda}^{\dot{\mu}}(\tilde{x} - i\tilde{H}) \tag{2.12b}$$

$$\theta\sigma^m\bar{\theta} = \tilde{H}^m(\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}}) + \tfrac{1}{2}\lambda(\tilde{x} + i\tilde{H})\sigma^m\bar{\lambda}(\tilde{x} + i\tilde{H}) + \tfrac{1}{2}\lambda(\tilde{x} - i\tilde{H})\sigma^m\bar{\lambda}(\tilde{x} - i\tilde{H})$$

$$+\tilde{\theta}\sigma^m\bar{\lambda}(\tilde{x} + i\tilde{H}) + \lambda(\tilde{x} - i\tilde{H})\sigma^m\bar{\tilde{\theta}} \tag{2.12c}$$

It is a simple exercise to evaluate $\tilde{H}^m(\tilde{z})$ using (2.12 b,c)

$$\begin{aligned} \tilde{H}^m(\tilde{z}) \;\; = \;\; & (T^{-1})^m_n \Big[\tilde{\theta}\sigma^n\bar{\tilde{\theta}} - \tilde{\theta}\tilde{\theta}\left(\bar{\tilde{\theta}}\bar{\sigma}^k\sigma^n\nabla_k\bar{\lambda}\right) \\ & +\bar{\tilde{\theta}}\bar{\tilde{\theta}}\left(\nabla_k\lambda\sigma^n\bar{\sigma}^k\tilde{\theta}\right) + \tilde{\theta}\tilde{\theta}\,\bar{\tilde{\theta}}\bar{\tilde{\theta}}\left(\nabla^k\lambda\sigma^n\nabla_k\bar{\lambda}\right)\Big] \end{aligned} \tag{2.13}$$

where

$$T^m_n = \delta^m_n + i\lambda\sigma^m\partial_n\bar{\lambda} - i\partial_n\lambda\sigma^m\bar{\lambda} \tag{2.14}$$

$$\nabla_m = (T^{-1})^n_m\partial_n, \;\; (T^{-1})^n_m = \delta^n_m - i\lambda\sigma^n\nabla_m\bar{\lambda} + i\nabla_m\lambda\sigma^n\bar{\lambda} \tag{2.15}$$

So, the axial superfield $\tilde{H}^m$ collects two basic entities of the Volkov-Akulov nonlinear realization: the vierbein T^a_n and the covariant derivative of Goldstone fermion $\nabla_m\lambda^\mu(x)$.In the conventional approach [13] the same objects come out as the coefficients of Cartan's one-forms associated, respectively, with the 4-translation and supertranslation generators. Notice a close resemblance at this point to the OS formulation of $N = 1$ SG where the primary geometric object is the axial vector prepotential $H^m(x, \theta, \bar{\theta})$ replacing the flat quantity $\theta\sigma^m\bar{\theta}$ [1,2]. In the next Sect. we shall see that this similarity is not accidental; the intrinsic geometry of spontaneously broken $N = 1$ SG in superspace is formulated most elegantly via an axial vector superfield $\tilde{H}^m(\tilde{z})$ the flat limit of which is just (2.13). In fact, all the formulas of the superfield formalism in the "splitting" basis [16] can be compactly rewritten via $\tilde{H}^m(\tilde{z})$ (2.13) (with taking account of the remark below). Thus, this superfield proves to be the basic geometric object of the Volkov-Akulov nonlinear realization re-examined within the superspace context.

Before closing this Sect. we remark that one could choose as a starting point, instead of (2.1), the realization of $N = 1$ supersymmetry in real $N = 1$

superspace $\mathbf{R}^{4|4} = \{x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}}\}$. Proceeding as before, one again arrives at the Volkov-Akulov realization. However, the relations (2.12a,c) are replaced by the following ones [16]

$$x^m = \hat{x}^m + i\hat{\theta}\sigma^m\bar{\lambda}(\hat{x}) - i\lambda(\hat{x})\sigma^m\bar{\hat{\theta}}, \quad \hat{x}^{m\dagger} = \hat{x}^m$$

$$\theta^\mu = \hat{\theta}^\mu + \lambda^\mu(\hat{x}), \;\; \bar{\theta}^{\dot{\mu}} = \bar{\hat{\theta}}^{\dot{\mu}} + \bar{\lambda}^{\dot{\mu}}(\hat{x}) \tag{2.16}$$

$$\lambda^{\mu\prime}(\hat{x}') = \lambda^\mu(\hat{x}) + \epsilon^\mu_1, \;\; \hat{x}^{m\prime} = \hat{x}^m + i\lambda(\hat{x})\sigma^m\bar{\epsilon}_1 - i\epsilon_1\sigma^m\bar{\lambda}(\hat{x}) \tag{2.17}$$

$$\hat{\theta}^{\mu\prime} = \hat{\theta}^\mu$$

Both changes of variables,although looking quite different at first sight, are related to each other by the equivalence redefinition of the coordinates involved

$$\begin{aligned} \hat{x}^m \;\; &= \;\; \tilde{x}^m + \frac{i}{4}\tilde{\theta}\tilde{\theta}\,\bar{\tilde{\theta}}\bar{\tilde{\theta}}(\nabla^k\nabla_k\lambda\sigma^m\bar{\lambda} - \lambda\sigma^m\nabla^k\nabla_k\bar{\lambda}) \\ & \quad - F^p(\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}})(T^{-1})^m_p \\ \hat{\theta}^\mu \;\; &= \;\; \tilde{\theta}^\mu + F^p\nabla_p\lambda^\mu + i\tilde{H}^n\partial_n\lambda^\mu - \tilde{H}^n\partial_n(\tilde{H}^t\partial_t\lambda^\mu) \\ F^p(\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}}) \;\; &= \;\; \frac{1}{2}\Big[\tilde{\theta}\tilde{\theta}\left(\bar{\tilde{\theta}}\bar{\sigma}^k\sigma^p\nabla_k\bar{\lambda}\right) + \bar{\tilde{\theta}}\bar{\tilde{\theta}}\left(\nabla_k\lambda\sigma^p\bar{\sigma}^k\tilde{\theta}\right) \\ & \quad + i\tilde{\theta}\tilde{\theta}\,\bar{\tilde{\theta}}\bar{\tilde{\theta}}(\nabla_k\lambda\sigma^k\nabla^p\bar{\lambda} - \nabla^p\lambda\sigma^k\nabla_k\bar{\lambda})\Big] \end{aligned} \tag{2.18}$$

The transformation properties of $\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}}$ following from (2.8), (2.11) imply for $\hat{x}, \hat{\theta}, \bar{\hat{\theta}}$ the transformation laws (2.17).

To summarize, the main lesson one draws from the above consideration is that nonlinear realizations of rigid supersymmetries can be constructed in an algorithmic way, if the coordinate realizations of these supersymmetries in some appropriate superspaces ($\mathbf{C}^{4|2}$ in the $N = 1$ case) are known. We shall demonstrate in Sect.3 and 4 that the same procedure, with minor modifications, works in the case of spontaneously broken $N = 1$ SG's. With its help it becomes possible to define nonlinear realizations of the superspace SG gauge groups consistently with the intrinsic geometries of these theories.

3 Model-independent description of spontaneously broken conformal N=1 supergravity

3.1 Geometric basics of N=1 supergravity

To generalize the consideration of previous Sect. to the SG case we need first to recall the basic facts about the geometric description of $N = 1$ SG in superspace.

The most elegant superspace formulation of $N = 1$ SG is that due to Ogievetsky and Sokatchev(OS)[1,2]. They have shown that the fundamental gauge group G of $N = 1$ SG has an adequate realization as the group of analytic diffeomorphisms of complex chiral $N = 1$ superspace $\mathbf{C}^{4|2} = \{x_L^m, \theta_L^\mu\} \equiv \{\zeta_L^M\}$

$$x_L^{m'} \equiv G^m(\zeta_L) = x_L^m + a^m(x_L) + ib^m(x_L) + \theta_L^\mu \varphi_\mu^m(x_L) + \theta_L\theta_L s^m(x_L)$$

$$\theta_L^{\mu'} \equiv G^\mu(\zeta_L) = \theta_L^\mu + \epsilon^\mu(x_L) + \theta_L^\nu \omega_\nu^\mu(x_L) + \frac{1}{4}\theta_L\theta_L\eta^\mu(x_L) \quad (3.1)$$

(factor 1/4 is introduced in (3.1) for further convenience). The basic geometric object of $N = 1$ SG is the axial vector gauge superfield $H^m(x,\theta,\bar\theta)$ appearing as the imaginary part of x_L^m

$$Im\ x_L^m = H^m(x,\theta,\bar\theta)$$

$$Re\ x_L^m = x^m,\ \theta_L^\mu = \theta^\mu,\ \overline{(\theta_L^\mu)} = \bar\theta^{\dot\mu} \quad (3.2)$$

$$x^{m'} = \tfrac{1}{2}\left[G^m(x+iH,\theta) + \overline{G}^m(x-iH,\bar\theta)\right] \quad (3.3a)$$

$$H^{m'}(x',\theta',\bar\theta') = \tfrac{1}{2i}\left[G^m(x+iH,\theta) - \overline{G}^m(x-iH,\bar\theta)\right] \quad (3.3b)$$

The role of the conditions (3.2) is to single out in $\mathbf{C}^{4|2}$ the real $N = 1$ superspace $\mathbf{R}^{4|4} = \{x^m, \theta^\mu, \bar\theta^{\dot\mu}\} \equiv \{z^M\}$ as a 4 | 4-dimensional hypersurface. As it follows from (3.1), (3.3), $H^m(z)$ and the coordinates of $\mathbf{R}^{4|4}$ are transformed nonlinearly and nonpolynomially in $H^m(z)$.

The group (3.1), (3.3) with unconstrained parameters corresponds to conformal $N = 1$ SG. The relevant gauge multiplet is comprised by $H^m(z)$. For further use, we quote the θ-decomposition of the latter

$$\begin{aligned} H^m(z) &= B^m(x) + \theta^\mu\chi_\mu^m(x) + \bar\theta_{\dot\mu}\bar\chi^{\dot\mu m}(x) + \theta\theta F^m(x) + \bar\theta\bar\theta\bar F^m(x) \\ &+\theta\sigma^a\bar\theta e_a^m(x) + \bar\theta\bar\theta\ \theta^\mu\psi_\mu^m(x) + \theta\theta\ \bar\theta_{\dot\mu}\bar\psi^{m\dot\mu}(x) \\ &+\theta\theta\ \bar\theta\bar\theta\left(A^m(x) - \frac{1}{4}\epsilon^{mnks}e_{an}\partial_k e_s^a\right) \end{aligned} \quad (3.4)$$

The components $B^m(x), \chi_\mu^m(x), F^m(x)$ represent pure gauge degrees of freedom while $e_a^m(x), \psi_\mu^m(x), \bar\psi_\mu^m(x)$, and $A^m(x)$ are, respectively, the fields of graviton (vierbein) and gravitino and the $U(1)$ gauge field (canonical dimensions for the fields $\psi^m(x), A^m(x)$ are achieved by extracting from them the Einstein constant κ; we shall not worry here about this). All the superspace geometric objects of conformal $N = 1$ SG (curvatures, torsions...) have an adequate representation in terms of $H^m(z)$ [24].

To pass to Einstein $N = 1$ SG one should either constrain the group G (3.1) by the condition of preserving the "volume" of $\mathbf{C}^{4|2}$ [1]

$$Ber\left(\frac{\partial G^N(\zeta)}{\partial\zeta^M}\right) = 1 \quad (3.5)$$

or add to $H^m(x,\theta,\bar\theta)$ a properly chosen compensating superfield [3,4] with maintaining the original group structure. In this way, one arrives at the "old minimal" version of Einstein $N = 1$ SG. The other known versions can be given a similar geometric description [3-5].

3.2 Nonlinear realization of N=1 conformal SG group

In this Section we consider the conformal case.[3] A nonlinear realization adequate to spontaneous breaking of local supersymmetry in minimal Einstein $N = 1$ SG will be constructed in the next Sect. Hereafter, we shall refer to the standard unbroken realization of $N = 1$ SG group G as the linear one (despite nonlinearities in $H(z)$) to distinguish it from the genuine nonlinear realization of G involving a Goldstone fermion in addition to the fields of SG gauge multiplet.

In constructing a nonlinear realization of G we shall closely follow the lines of previous Sect. Let both local supersymmetries present in (3.1) (parameters $\epsilon^\mu(x), \eta^\mu(x)$) be spontaneously broken by some mechanism the precise nature of which is of no interest for us here (other patterns of spontaneous breaking are also admitted, see the end of this Sect.). How to describe this particular situation in a model-independent way consistent with the underlying superspace geometry of $N = 1$ SG ? The strategy is prompted by the rigid case. One has to define the stability subgroup G_0, to construct the coset space G/G_0 and to implement G as left shifts of the coset elements. Among the coset parameters one may then expect to find the corresponding Goldstone fermions with the transformation laws completely specified by the constructed coset realization of group G.

A specific feature of "linear realization" of $N = 1$ SG group consists in that the symmetries associated with the parameters $b^m(x_L), \varphi_\mu^m(x_L), s^m(x_L)$ in (3.1) are broken from the very beginning. Indeed, the group variations of pure gauge components of $H^m(z)$ start with these parameters

$$\begin{aligned} \delta B^m(x) &= b^m(x) + \ldots \\ \delta\chi_\mu^m(x) &= -\frac{i}{2}\left[\varphi_\mu^m(x) - 2i\left(\sigma^m\bar\epsilon(x)\right)_\mu\right] + \ldots \\ \delta F^m(x) &= -\frac{i}{2}s^m(x) + \ldots \end{aligned} \quad (3.6)$$

indicating that the above symmetries are spontaneously broken and $B^m(x)$, $\chi_\mu^m(x), F^m(x)$ are corresponding Goldstone fields. When local supersymmetries are also assumed to be broken, we are left with the ordinary general covariance transformations (parameters $a^m(x)$) and the tangent space $L(2,C)$ rotations (parameters $\omega_\mu^\nu(x)$) as the only unbroken symmetries

$$x_L^{m'} \equiv G_0^m(\zeta_L) = x_L^m + a^m(x_L)$$

[3] A brief account of this case has been already given in our letter [21].

$$\theta_L^{\mu'} \equiv G_0^\mu(\zeta_L) = \theta_L^\mu + \omega_\nu^\mu(x_L)\theta_L^\nu \tag{3.7}$$

The next step is to find an appropriate representation for the elements of the coset G/G_0. One may proceed by observing that an arbitrary element (3.1) of the group G can be uniquely decomposed as

$$\begin{aligned} G^M(\zeta_L) &= Y^M(G_0(\zeta_L)) \qquad (3.8)\\ Y^m(\zeta_L) &= x_L^m + i\hat{b}^m(x_L) + \theta_L^\mu \hat{\varphi}_\mu^m(x_L) + \theta_L\theta_L \hat{s}^m(x_L) \\ Y^\mu(\zeta_L) &= \theta_L^\mu + \hat{\epsilon}^\mu(x_L) + \frac{1}{4}\theta_L\theta_L\hat{\eta}^\mu(x_L) \qquad (3.9)\end{aligned}$$

where $G_0^M(\zeta_L)$ is given by (3.7) and the parameters with a hat are related to the initial ones via an evident redefinition. The group elements (3.9) collect all the parameters of the coset G/G_0 and hence can be taken as the representatives of the latter. These are the true curved space analogs of $Y^M(\zeta_L)$ (2.4). Doing as in Sect.2, one may now construct a nonlinear realization of G in the coset space G/G_0 by identifying the group parameters in (3.9) as the Goldstone fields

$$\begin{aligned} \tilde{Y}^m(\tilde{\zeta}_L) &= \tilde{x}_L^m + i\tilde{B}^m(\tilde{x}_L) + \tilde{\theta}_L^\mu \tilde{\chi}_\mu^m(\tilde{x}_L) + \tilde{\theta}_L\tilde{\theta}_L\tilde{F}^m(\tilde{x}_L) \\ \tilde{Y}^\mu(\tilde{\zeta}_L) &= \tilde{\theta}_L^\mu + \lambda^\mu(\tilde{x}_L) + \frac{1}{4}\tilde{\theta}_L\tilde{\theta}_L q^\mu(\tilde{x}_L) \qquad (3.10)\end{aligned}$$

and postulating the following transformation law for them (c.f. (2.5'))

$$G^M(\tilde{Y}(\tilde{\zeta}_L)) = \tilde{Y}^{M'}(\tilde{G}_0(\tilde{\zeta}_L)) \tag{3.11}$$

$$\begin{aligned} \tilde{G}_0^m(\tilde{\zeta}_L)) &= \tilde{x}_L^{m'} = \tilde{x}_L^m + \tilde{a}^m(G^M, \tilde{Y}) \\ \tilde{G}_0^\mu(\tilde{\zeta}_L) &= \tilde{\theta}_L^\mu = \tilde{\theta}_L^{\mu'} + \tilde{\omega}_\nu^\mu(G^M, \tilde{Y})\tilde{\theta}^\nu \qquad (3.12)\end{aligned}$$

The induced general covariance and $L(2, \mathbf{C})$ transformation parameters appearing in (3.12) are composed out of the group parameters entering into $G^M(\zeta_L)$ and of the G/G_0 fields. They can be read off from the explicit form of the transformations of $\tilde{x}_L^m$ and $\tilde{\theta}_L^\mu$

$$\tilde{x}_L^{m'} = \frac{1}{2}\left[G^m(\tilde{x}_L + i\tilde{B}(\tilde{x}_L), \lambda(\tilde{x}_L)) + \bar{G}^m(\tilde{x}_L - i\tilde{B}(\tilde{x}_L), \bar{\lambda}(\tilde{x}_L))\right] \tag{3.13}$$

$$\tilde{\theta}_L^{\mu'} = \tilde{\theta}_L^\nu \tilde{\mathcal{D}}_\nu G^\mu(\tilde{x}_L + i\tilde{B}(\tilde{x}_L), \tilde{\theta}_L)\Big|_{\tilde{\theta}=\lambda(\tilde{x}_L)} \equiv \tilde{\varphi}_\nu^\mu(\tilde{x}_L)\tilde{\theta}_L^\nu \tag{3.14}$$

where

$$\begin{aligned} \tilde{\mathcal{D}}_\nu &= \tilde{\partial}_\nu + \tilde{\chi}_\nu^m(\tilde{x}_L)(A^{-1})_m^n\tilde{\partial}_n, \; \tilde{\partial}_M \equiv \frac{\partial}{\partial\tilde{\zeta}_L^M} \\ A_m^n &= \delta_m^n + i\tilde{\partial}_m\tilde{B}^n(\tilde{x}_L) \qquad (3.15)\end{aligned}$$

We quote the transformation laws of several coset fields

$$\tilde{B}^{m'}(\tilde{x}_L') = \frac{1}{2i}\left[G^m(\tilde{x}_L + i\tilde{B}(\tilde{x}_L), \lambda(\tilde{x}_L)) - \bar{G}^m(\tilde{x}_L - i\tilde{B}(\tilde{x}_L), \bar{\lambda}(\tilde{x}_L))\right] \tag{3.16}$$

$$\tilde{\chi}_\mu^{m'}(\tilde{x}_L') = 2i\tilde{\mathcal{D}}_\mu G^m(\tilde{x}_L + i\tilde{B}(\tilde{x}_L), \tilde{\theta}_L)\Big|_{\tilde{\theta}_L=\lambda(\tilde{x}_L)} \tag{3.17}$$

$$\lambda^{\nu'}(\tilde{x}_L') = G^\nu(\tilde{x}_L + i\tilde{B}(\tilde{x}_L), \lambda(\tilde{x}_L)) \tag{3.18}$$

The group transformations of the remaining fields can also be explicitly written (we give below the transformation law of q^μ, eq.(3.37)) but these do not look too enlightening. As was expected, all these coset fields in their transformations involve inhomogeneous pieces typical for the Goldstone fields.

One may directly check that (3.13)–(3.18) indeed possess the group structure inherent in transformations (3.1) we started with. It is worth mentioning that these transformations involve nonlinearities in the coset fields even upon restriction to the stability subgroup G_0. Nevertheless, the latter preserves the origin in the manifold of Goldstone fields, in full agreement with the general definition of the stability subgroup in the theory of nonlinear realizations [7][4].

3.3 Relation to initial superspace formulation

At this stage, the nonlinear realization constructed bears no direct relation to the geometry of unbroken theory. One deals with the superspace $\tilde{\mathbf{C}}^{4|2} = \{\tilde{x}_L^m, \tilde{\theta}_L^\mu\} \equiv \{\tilde{\theta}_L^M\}$ and the superfields of special form $\tilde{Y}^M(\tilde{\zeta}_L)$ given on it. Put together, these constitute a closed nonlinear representation of the group G in their own right.

Recall, however, that the fields possessing inhomogeneous transformation laws similar to those of the above G/G_0 coset fields (except for goldstinos) are already contained in the linear realization gauge superfield $H^m(z)$ (eqs.(3.6)). To avoid the doubling of degrees of freedom we are then led to relate both sets of fields by an equivalence transformation. This can be done after establishing a link with the linear realization of $N = 1$ SG group.

A key step in revealing the relationship between the two realizations of G is to get sight of the fact that $\tilde{Y}^M(\tilde{\zeta}_L)$ transform under G in precisely the same manner as the original coordinates ζ_L^M of $\mathbf{C}^{4|2}$ and therefore can be identified with them

$$\zeta_L^M = \tilde{Y}^M(\tilde{\zeta}_L) \tag{3.19}$$

This relation generalizes (2.9) to curved space.

Further, one may single out in $\tilde{\mathbf{C}}^{4|2} = \{\tilde{\zeta}_L^M\}$ a 4 | 4–dimensional real hypersurface $\tilde{\mathbf{R}}^{4|4} = \{\tilde{x}^m, \tilde{\theta}^\mu, \bar{\tilde{\theta}}^{\dot{\mu}}\} \equiv \{\tilde{z}^M\}$ by the embedding conditions analogous to (2.11)

$$\begin{aligned} Im\, \tilde{x}_L^n &= \tilde{H}^n(\tilde{x}, \tilde{\theta}, \bar{\tilde{\theta}}) \\ Re\, \tilde{x}_L^n &= \tilde{x}^n, \; \tilde{\theta}_L^\mu = \tilde{\theta}^\mu, \; \overline{(\tilde{\theta}_L^\mu)} = \bar{\tilde{\theta}}^{\dot{\mu}} \qquad (3.20)\end{aligned}$$

[4] According to the linearization lemma [7], the transformations of this type can always be made linear by a field redefinition.

Putting together eqs.(3.2), (3.19) and (3.20) yields the relations between the coordinates of superspaces $\mathbf{R}^{4|4}$ and $\tilde{\mathbf{R}}^{4|4}$

$$x^m = \frac{1}{2}\left[\tilde{Y}^m(\tilde{x}+i\tilde{H},\tilde{\theta}) + \overline{\tilde{Y}}^m(\tilde{x}-i\tilde{H},\bar{\tilde{\theta}})\right]$$

$$\theta^\mu = \tilde{Y}^\mu(\tilde{x}+i\tilde{H},\tilde{\theta}),\ \bar{\theta}^{\dot{\mu}} = \overline{\tilde{Y}}^{\dot{\mu}}(\tilde{x}-i\tilde{H},\bar{\tilde{\theta}}) \qquad (3.21)$$

$$H^m(x,\theta,\bar{\theta}) = \frac{1}{2i}\left[\tilde{Y}^m(\tilde{x}+i\tilde{H},\tilde{\theta}) - \overline{\tilde{Y}}^m(\tilde{x}-i\tilde{H},\bar{\tilde{\theta}})\right] \qquad (3.22)$$

The G transformation properties of $\tilde{x}^n$ and $\tilde{H}^m(\tilde{x},\tilde{\theta},\bar{\tilde{\theta}})$ are as follows

$$\tilde{x}^{m\prime} = \tilde{x}^m + \tfrac{1}{2}\left[\tilde{a}^m(\tilde{x}+i\tilde{H}) + \tilde{a}^m(\tilde{x}-i\tilde{H})\right] \qquad (3.23a)$$

$$\tilde{H}^{m\prime}(x',\tilde{\theta}',\bar{\tilde{\theta}}') = \tilde{H}^m(\tilde{x},\tilde{\theta},\bar{\tilde{\theta}}) + \tfrac{1}{2i}\left[\tilde{a}^m(\tilde{x}+i\tilde{H}) - \tilde{a}^m(\tilde{x}-i\tilde{H})\right] \qquad (3.23b)$$

By inspecting the transformation laws (3.23) one concludes that the lower-dimensional components of $\tilde{H}^m(\tilde{z})$, in contrast to those of $H^m(z)$ in (3.3b), transform homogeneously, with no field-independent gauge shifts. Moreover, their set is closed under the action of G because the components of higher dimension do not enter into transformations of lower dimension. Thus one can put

$$\tilde{H}^m(\tilde{z})\big|_{\tilde{\theta}=0} = \tilde{\partial}_\mu\tilde{H}^m(\tilde{z})\big|_{\tilde{\theta}=0} = \tilde{\partial}_\mu\tilde{\partial}_\nu\tilde{H}^m(\tilde{z})\big|_{\tilde{\theta}=0} = 0 \qquad (3.24)$$

without conflicting with G covariance. These constraints settle the sought equivalence relation between the Goldstone fields $\tilde{B}^m, \tilde{\chi}^m_\mu, \tilde{F}^m$ and the pure gauge components of $H^m(z)$

$$\begin{aligned}
\tilde{B}^m(\tilde{x}_L) &= H^m(\tilde{x}_L,\lambda(\tilde{x}_L),\bar{\lambda}(\tilde{x}_L)) \\
\tilde{\chi}^m_\mu(\tilde{x}_L) &= 2i\Delta_\mu H^m(x,\theta,\bar{\theta})\big|_{\tilde{x}_L} \\
\tilde{F}^m(\tilde{x}_L) &= -\frac{i}{2}\left[\Delta^\mu\Delta_\mu H^m(x,\theta,\bar{\theta}) - q^\mu(x)\Delta_\mu H^m(x,\theta,\bar{\theta})\right]\Big|_{\tilde{x}_L}
\end{aligned} \qquad (3.25)$$

where [2]

$$\Delta_\mu = \partial_\mu + i\Delta_\mu H^n\partial_n$$

$$\Delta_\mu H^m = (1-i\mathcal{H})^{-1n}_{m}\partial_\mu H^m,\ \mathcal{H}^n_m \equiv \partial_m H^n \qquad (3.26)$$

and the symbol $|_y$ hereafter means restriction to the four-dimensional hypersurface

$$x^m = y^m,\ \ \theta^\mu = \lambda^\mu(y),\ \ \bar{\theta}^{\dot{\mu}} = \bar{\lambda}^{\dot{\mu}}(y)\,.$$

Thus we succeeded in formulating the nonlinear realization of G in the coset space G/G_0 via gauge multiplet of conformal $N=1$ SG and two extra goldstinos $\lambda^\mu(x), q^\mu(x)$. Substituting the expression (3.25) for $\tilde{B}^m(\tilde{x}_L)$ into (3.13) yields a model-independent transformation law of the first goldstino in $N=1$ SG group. This field transforms through itself and the components of $H^m(z)$.

3.4 Eliminating second goldstino

It turns out that the above set of fields is not yet minimal. It can be further reduced by eliminating $q^\mu(x)$ at the expense of the remaining fields.

To see this, we need to plunge into the structure of $\tilde{H}^m(\tilde{z})$. After imposing constraints (3.24) it takes the form highly resembling $H^m(z)$ in the WZ gauge

$$\begin{aligned}
\tilde{H}^m(\tilde{z},\tilde{\theta},\bar{\tilde{\theta}}) &= \tilde{\theta}\sigma^a\bar{\tilde{\theta}}\tilde{e}^m_a(\tilde{x}) + \bar{\tilde{\theta}}\bar{\tilde{\theta}}\,\tilde{\theta}^\mu\tilde{\psi}^m_\mu(\tilde{x}) + \tilde{\theta}\tilde{\theta}\,\bar{\tilde{\theta}}_{\dot{\mu}}\bar{\tilde{\psi}}^{m\dot{\mu}} \\
&+ \tilde{\theta}\tilde{\theta}\,\bar{\tilde{\theta}}\bar{\tilde{\theta}}\left(\tilde{A}^m(\tilde{x}) - \frac{1}{4}\epsilon^{mnks}\tilde{e}_{an}(\tilde{x})\tilde{\partial}_k\tilde{e}^a_s(\tilde{x})\right)
\end{aligned} \qquad (3.27)$$

However, in contradistinction to the WZ gauge for $H^m(z)$, this form of $\tilde{H}^m(\tilde{z})$ is retained under the action of full group G

$$\begin{aligned}
\delta^*\tilde{e}^{ma} &= \tilde{e}^{na}\tilde{\partial}_n\delta\tilde{a}^m - \delta\tilde{a}^n\tilde{\partial}_n\tilde{e}^{ma} - \delta\tilde{\omega}^{ac}\tilde{e}^m_c \\
\delta^*\tilde{\psi}^m_\mu &= \tilde{\psi}^n_\mu\tilde{\partial}_n\tilde{a}^m - \delta\tilde{a}^n\tilde{\partial}_n\tilde{\psi}^m_\mu - \delta\tilde{\omega}^\nu_\mu\tilde{\psi}^m_\nu - \delta\bar{\tilde{\omega}}^{\dot{\mu}}_{\dot{\mu}}\tilde{\psi}^m_\mu
\end{aligned} \qquad (3.28)$$

$$\delta^*\tilde{A}^m = \tilde{A}^n\tilde{\partial}_n\tilde{a}^m - \delta\tilde{a}^n\tilde{\partial}_n\tilde{A}^m - \frac{1}{2}\delta\tilde{\omega}^a_a\tilde{A} - \frac{i}{4}\tilde{\partial}^m(\delta\tilde{\omega}^\mu_\mu - \delta\bar{\tilde{\omega}}^{\dot{\mu}}_{\dot{\mu}}) \qquad (3.29)$$

where $\delta\tilde{\omega}^{ac} = 1/2(\bar{\sigma}^a\sigma^c\delta\bar{\tilde{\omega}}) + 1/2(\sigma^c\bar{\sigma}^a\delta\tilde{\omega})$ and $\delta\tilde{\omega}^\mu_\nu, \delta\bar{\tilde{\omega}}^{\dot{\mu}}_{\dot{\nu}} \equiv (\delta\tilde{\omega}^\mu_\nu)^\dagger, \delta\tilde{a}^m$ are infinitesimal parameters of transformations (3.12). We see that $\tilde{e}^m_a$ and $\tilde{A}^m$ have the transformation properties characteristic of the vierbein and the $U(1)$ gauge field. It is easy to check that they are related via an equivalence transformation to their counterparts in $H^m(z)$.

$$\tilde{e}^{am}(\tilde{x}) = e^{am}(\tilde{x}) + \dots$$

$$\tilde{A}^m(\tilde{x}) = A^m(\tilde{x}) + \dots$$

where dots stand for the terms of higher order in the involved fields. An important point is that $\tilde{\psi}^{m\mu}, \bar{\tilde{\psi}}^{m\dot{\mu}}$ transform homogeneously and so carry degrees of freedom inherent in a massive spin 3/2 field. The linearized structure of $\tilde{\psi}^{m\mu}$ is as follows

$$\tilde{\psi}^m = \psi^m + \delta^m_a\delta^n_b(\partial_n\lambda\sigma^a\bar{\sigma}^b) + \delta^m_a(\sigma^a\bar{q}) + \dots, \qquad (3.30)$$

thus indicating that $\tilde{\psi}^m$ is a covariant combination of massless spin 3/2 gravitino $\psi^{m\mu}$ and goldstinos λ^μ, q^μ. It is worthwhile to mention that $e^m_a, \tilde{\psi}^{m\mu}$ are the obvious gauge-covariantization of the flat space objects $T^{ma}, \nabla_m\lambda^\mu$ (2.14), (2.15). In the flat space limit $\tilde{H}^m$ (3.27) goes over to (2.13)(see Sect.5).

In fact, it is straightforward to find the explicit expressions for $\tilde{e}^{am}, \tilde{\psi}^{m\mu}, \tilde{A}^m$ in terms of H^m and λ^μ, q^μ. For our purpose it suffices to know $\tilde{e}^{am}, \bar{\tilde{\psi}}^{m\dot{\mu}}$

$$\tilde{e}^{am}(\tilde{x}) = (T^{-1})^m_n e^{an}\big|_{\tilde{x}} \qquad (3.31)$$

$$\bar{\tilde{\psi}}^{\alpha\dot{\sigma}}_{\dot{\mu}} \equiv \tilde{e}^a_m(\bar{\sigma}_a)^{\dot{\sigma}\alpha}\bar{\tilde{\psi}}^m_{\dot{\mu}} = \frac{1}{2}r^{\alpha\dot{\sigma}}_m\left(\Delta^\beta r^m_{\beta\dot{\mu}}\right)\Big|_{\tilde{x}} - 2i(\sigma^a)^\alpha_{\dot{\mu}}\tilde{e}^n_a\tilde{\partial}_n\bar{\lambda}^{\dot{\sigma}} - \frac{1}{2}q^\alpha\delta^{\dot{\sigma}}_{\dot{\mu}} \qquad (3.32)$$

$$r^m_{\beta\dot\beta} = \Delta_\beta \bar\Delta_{\dot\beta} H^m, \quad c^m_a = \frac{1}{4}(\Delta\sigma_a\bar\Delta - \bar\Delta\bar\sigma_a\Delta)H^m \tag{3.33}$$

$$T^m_n = \delta^m_n - i\partial_n\lambda^\alpha\Delta_\alpha H^m\Big|_{\tilde x} - i\partial_n\bar\lambda^{\dot\alpha}\bar\Delta_{\dot\alpha}H^m\Big|_{\tilde x} \tag{3.34}$$

Here $r^m_{\beta\dot\beta}$ and c^m_a are well-known building blocks of the differential geometry formalism of $N = 1$ SG in the OS approach [2] (the vertical line indicates as before that all these objects are placed on the hypersurface $x^m = \tilde x^m, \theta^\mu = \lambda^\mu(\tilde x), \bar\theta^{\dot\mu} = \bar\lambda^{\dot\mu}(\tilde x)$). The matrix (3.34) is the genuine curved space generalization of the Volkov-Akulov vierbein (2.14) while (3.32) gauge-covariantizes the flat space covariant derivative $\nabla_n\bar\lambda^{\dot\mu}$. Thus these quantities can be regarded as the coefficients of Cartan's forms associated with the nonlinear realization of $N = 1$ SG group we have constructed.

The transformation property (3.28) and the explicit structure of $\tilde\psi^{m\mu}$ suggest that $q^\mu(x)$ can be covariantly eliminated by imposing the constraint

$$\bar{\tilde\psi}^{\alpha\dot\beta}_{\dot\beta} = 0 \quad \text{and h.c.} \tag{3.35}$$

whence

$$q^\alpha(x) = \frac{1}{2} r^{\alpha\dot\alpha}_m \Delta^\beta r^m_{\beta\dot\alpha}\Big|_x - 2i\tilde e^m_a(\partial_m\bar\lambda\bar\sigma^a)^\alpha \tag{3.36}$$

Eq. (3.35) is manifestly covariant with respect to the group G which guarantees that $q^\mu(x)$ (3.36) possesses correct transformation properties.

An important property to be taken into account when checking the last statement is that the $L(2,\mathbf{C})$ matrix $\tilde\varphi^{-1\mu}_\nu(\tilde x_L)$ in (3.14) with $\tilde\chi^m_\mu(\tilde x_L)$ defined from eqs.(3.25) coincides with the linear realization $L(2,\mathbf{C})$ transformation matrix [2, 24] restricted to the hypersurface $x^m = \tilde x^m_L, \theta^\mu = \lambda^\mu(\tilde x_L), \bar\theta^{\dot\mu} = \bar\lambda^{\dot\mu}(\tilde x_L)$

$$\begin{aligned}\tilde\varphi^\mu_\nu(\tilde x_L) &= \tilde{\mathcal D}_\nu G^\mu(\tilde x_L + i\tilde B(\tilde x_L), \tilde\theta_L)\Big|_{\tilde\theta=\lambda(\tilde x_L)} = \\ &= \Delta_\nu G^\mu(x + iH(x,\theta,\bar\theta),\theta)\,|_{\tilde x_L} \equiv \varphi^\mu_\nu(z)|_{\tilde x_L}\end{aligned} \tag{3.37}$$

where Δ_ν is defined in (3.26). Note that the transformation law of q^μ is essentially simplified after substitution of the explicit expressions (3.25) for the G/G_0 coset fields

$$\begin{aligned} q^{\mu'}(\tilde x'_L) &= \tilde\varphi^{-1}(\tilde x_L)\tilde\varphi^\mu_\nu(\tilde x_L)q^\nu(\tilde x_L) - \tilde\varphi^{-1}(\tilde x_L)\Delta^\nu\varphi^\mu_\nu(z)\Big|_{\tilde x_L} \\ \tilde\varphi(\tilde x_L) &= det\, \tilde\varphi^\nu_\mu(\tilde x_L) \end{aligned} \tag{3.38}$$

One more remark concerns the uniqueness of constraint (3.35) and hence of the nonlinear realization constructed. Generally speaking, one might equate $\bar{\tilde\psi}^{\alpha\dot\beta}_{\dot\beta}$ to any extra spinor $\tilde\chi^\alpha$ having the same transformation properties under G

$$\bar{\tilde\psi}^{\alpha\dot\beta}_{\dot\beta}(\tilde x) = \tfrac{1}{4}\tilde\chi^\alpha(\tilde x) \tag{3.35'}$$

$$\delta^*\tilde\chi^\alpha(\tilde x) = -\delta\tilde a^n\tilde\partial_n\tilde\chi^\alpha(\tilde x) - \delta\tilde\omega^\mu_\mu\tilde\chi^\alpha(\tilde x) + \delta\tilde\omega^\alpha_\lambda\tilde\chi^\lambda(\tilde x) \tag{3.39}$$

Although there is no appropriate $\tilde\chi$ in the pure SG sector, a field of this kind may be present among the components of matter or Yang-Mills superfields written in the nonlinear realization superspace bases. The explicit expression for q^μ deduced on the ground of (3.35′) differs from (3.36) by a linear term proportional to $\tilde\chi^\alpha(x)$, so the expressions for the G/G_0 coset superfields (3.10) with the coefficients (3.25) are modified in the $\tilde\theta_L\tilde\theta_L$ terms. However, one returns to the previous case after the analytic replacement

$$\tilde\theta^\mu_L \to \tilde\theta^\mu_L - \tilde\theta_L\tilde\theta_L\tilde\chi^\mu(\tilde x_L) \tag{3.40}$$

preserving the transformation property (3.14). This change of Grassmann variable cancells all the extra terms in q^μ and $\tilde F$ and also redefines $\tilde\psi^{m\mu}, \bar{\tilde\psi}^{m\dot\mu}$

$$\bar{\tilde\psi}^{m\dot\mu} \to \bar{\tilde\psi}^{m\dot\mu} - \tilde\chi^\mu(\tilde\sigma^b)^{\dot\mu}_\mu\tilde e^m_b$$

so that they satisfy the previous constraint (3.35). Thus the latter is most general and the nonlinear realization we are considering is unique.

3.5 Resume

We have constructed the nonlinear realization of the conformal $N = 1$ SG group G in the coset space G/G_0, G_0 being the subgroup of G consisting of the general covariance transformations in $\mathbf{R}^4$ and the tangent space $L(2,\mathbf{C})$ rotations. This realization is compatible by construction with the superspace geometry of conformal $N = 1$ SG and is minimal in the sense that it contains only one extra field, goldstino $\lambda^\mu(x)$, in addition to the fields of SG gauge multiplet (8 + 8 fields off shell in the WZ gauge). In Sect.5 we shall demonstrate how to represent the superfield actions with spontaneously broken local supersymmetry in manifestly geometric terms of the nonlinear realization (in application to the case of Einstein $N = 1$ SG).

Before ending this Sect. let us remark that other choices of G_0 are in principle admissible. All these can be treated along similar lines. For instance, it is a conceivable option to include into G_0 the local supersymmetry with parameter $\eta^\mu(x)$

$$\begin{aligned} G^m_0(\tilde\zeta_L) &= \tilde x^m_L + a^m(\tilde x_L) \\ G^\mu_0(\tilde\zeta_L) &= \tilde\theta^\mu_L + \omega^\mu_\nu(\tilde x_L)\tilde\theta^\nu_L + \frac{1}{4}\tilde\theta_L\tilde\theta_L\eta^\mu(\tilde x_L) \end{aligned} \tag{3.41}$$

These transformations are easily checked to close. The corresponding coset representatives $\tilde Y^M(\tilde\zeta_L)$ can be got from (3.10) by putting there $q^\mu(\tilde x_L) = 0$. A formal difference with the case already considered is that the relevant $\tilde\psi^{m\mu}(\tilde x)$ and $\bar{\tilde\psi}^{m\dot\mu}(\tilde x)$ transform inhomogeneously under η-supersymmetry. However, one may completely fix this gauge freedom by imposing the gauge condition coinciding by form with constraint (3.35). Under this choice of gauge the theory in question coincides with the previous one taken in the gauge $q^{\mu}(x) = 0$.

Finally, we wish to point out that it is a matter of the choice of a concrete model which pattern of spontaneous breakdown $G \rightarrow G_0$ really comes about. However, once that pattern is fixed, it can be further treated in a model-independent geometric way within the scheme presented here.

4 Geometric structure of spontaneous supersymmetry breakdown in Einstein N=1 supergravity

For simplicity, we confine our consideration in this paper to the old minimal version of Einstein $N = 1$ SG.Other versions (nonminimal and new minimal ones) can be treated analogously, based upon their geometric superspace formulations [5].

4.1 Sketch of unbroken case

Like in the preceding Sect. we start with a brief account of geometric basics of the unbroken theory.

For our purpose most suitable is the formulation of Einstein $N = 1$ SG via a chiral compensator [3,4]. In this formulation, the underlying gauge group is the same as in the conformal case and it is given by transformations (3.1).The crucial new feature is the presence of a compensating chiral superfield $S(\zeta_L)$ which transforms under the group G according to

$$S'(\zeta'_L) = I^{-1}(\zeta_L)S(\zeta_L), \quad I(\zeta_L) = Ber\left(\frac{\partial\zeta_L^{M'}}{\partial\zeta_L^N}\right) \tag{4.1}$$

This superfield is pure gauge and it is assumed to start with a unity. So it can be made equal to unity by choosing an appropriate gauge. The subgroup of G which preserves this gauge is just the subgroup singled out by constraint (3.5).This subgroup does not contain local conformal supersymmetry and local $L(2,\mathrm{C})/SL(2,\mathrm{C})$ transformations which thus turn out to be completely compensated. Correspondingly, some components of the axial-vector prepotential $H^m(z)$ lose their status of pure gauge degrees of freedom and become auxiliary fields. This concerns the field $A^m(x)$ and the longitudinal part of $F^m(x)$ $(\partial_m F^m(x))$ in the decomposition (3.4). Besides, the gauge group acting on the vierbein and gravitino degrees of freedom is reduced by one local bosonic and four local fermionic parameters. As a result, the Einstein $N = 1$ SG multiplet contains 12 + 12 fields off shell (in the WZ gauge for $H^m(z)$) as distinct from 8 + 8 off-shell fields of conformal SG. In any other gauge, the component fields of the SG multiplet are distributed between the superfields $H^m(z)$ and $S(\zeta_L)$. Thus these superfields are the primary geometric objects

of minimal Einstein $N = 1$ SG. In what follows, we shall not fix their gauge with respect to the group G in order to have manifest G covariance at each step.

From the geometric point of view, the distinction between conformal and Einstein $N = 1$ SG's roots in the different choice of the tangent space group. The latter is $L(2,\mathrm{C})$ in conformal and $SL(2,\mathrm{C})$ in Einstein cases. Having at our disposal compensators $S(\zeta_L), \bar{S}(\bar{\zeta}_L)$, we may define the densities $F, \bar{F}$ [2,3] transforming in G as

$$F' = \varphi^{\frac{1}{2}}F, \quad \bar{F}' = \bar{\varphi}^{\frac{1}{2}}\bar{F} \tag{4.2}$$

where

$$\varphi = det\, \varphi_{\mu}^{\nu}(z), \quad \varphi_{\mu}^{\nu} = \Delta_{\mu}G^{\nu}(z) \tag{4.3}$$

and construct $SL(2,\mathrm{C})$ covariant spinor derivatives

$$\nabla_{\mu} = F\Delta_{\mu}, \; \bar{\nabla}_{\dot{\mu}} = \bar{F}\Delta_{\dot{\mu}}, \; \nabla'_{\mu} = \varphi^{\frac{1}{2}}(z)\varphi_{\mu}^{-1\nu}(z)\nabla_{\nu} \equiv \gamma_{\mu}^{-1\nu}(z)\nabla_{\nu} \tag{4.4}$$

The objects $\nabla_{\mu}, \bar{\nabla}_{\mu}, F$ and $\bar{F}$ are the basic building blocks of the differential geometry formalism of Einstein $N = 1$ SG [2,3]. The explicit expression of $F, \bar{F}$ through superfields $H^m(z), S(\zeta_L)$ is as follows

$$F = 2^{\frac{2}{3}}r^{-\frac{1}{3}}l^{\frac{1}{6}}\bar{S}^{-\frac{1}{3}}S^{\frac{1}{6}}, \; \bar{F} = (F)^{\dagger} = 2^{\frac{2}{3}}l^{-\frac{1}{3}}r^{\frac{1}{6}}S^{-\frac{1}{3}}\bar{S}^{\frac{1}{6}} \tag{4.5}$$

$$r = (l)^{\dagger} = det\, r_a^m, \quad r_a^m = r_{\alpha\dot{\alpha}}^m(\bar{\sigma}_n)^{\dot{\alpha}\alpha}$$

where the matrix $r_{\alpha\dot{\alpha}}^m$ has been already defined in (3.33).

4.2 Nonlinear realization of N=1 SG group in Einstein case

After these introductory remarks we are prepared to turn to our task, i.e. to constructing a nonlinear realization of G adequate to spontaneously broken Einstein $N = 1$ SG.

As before, we begin with specifying the stability subgroup $G_{E(0)}$. In the present case it is natural to choose it to consist of general covariance transformations of x_L^m and the tangent space $SL(2,\mathrm{C})$ rotations of θ^{μ}

$$\begin{aligned} G_{E(0)}^m(\zeta_L) &\equiv x_L^{m'} = x_L^m + a^m(x_L) \\ G_{E(0)}^{\mu}(\zeta_L) &\equiv \theta_L^{\mu'} = \varphi^{-\frac{1}{2}}(x_L)\varphi_{\nu}^{\mu}(x_L)\theta_L^{\nu} \equiv \gamma_{\nu}^{\mu}(x_L)\theta_L^{\nu} \end{aligned} \tag{4.6}$$

Respectively, the relevant coset representatives $\tilde{Y}_E^M(\tilde{\zeta}_L)$ should incorporate the $L(2,\mathrm{C})/SL(2,\mathrm{C})$ parameters which have dropped from the stability subgroup. It is convenient to represent $\tilde{Y}_E^M(\tilde{\zeta}_L)$ as a result of the change of variables in (3.10)

$$\tilde{x}_L^m \rightarrow \tilde{x}_L^m, \; \tilde{\theta}_L^{\mu} \rightarrow \tilde{\theta}_L^{\mu}e^{\phi(\tilde{x}_L)}; \; \tilde{Y}^M \rightarrow \tilde{Y}_E^M \tag{4.7}$$

whence

$$
\begin{aligned}
x_L^m = \tilde{Y}_E^m(\zeta_L) &= \tilde{x}_L^m + i\tilde{B}^m(\tilde{x}_L) + \tilde{\theta}_L^\mu \tilde{\chi}_\mu^m(\tilde{x}_L)e^{\phi(\tilde{x}_L)} + \tilde{\theta}_L\tilde{\theta}_L\tilde{F}^m(\tilde{x}_L)e^{2\phi(\tilde{x}_L)} \\
\theta_L^\mu = \tilde{Y}_E^\mu(\zeta_L) &= \tilde{\theta}_L^\mu e^{\phi(\tilde{x}_L)} + \lambda^\mu(\tilde{x}_L) + \frac{1}{4}\tilde{\theta}_L\tilde{\theta}_L q^\mu(\tilde{x}_L)e^{2\phi(\tilde{x}_L)} \qquad (4.8)
\end{aligned}
$$

The additional complex field $\phi(\tilde{x}_L)$ is the $L(2,\mathbf{C})/SL(2,\mathbf{C})$ coset coordinate. All the remaining Goldstone fields are the same as in (3.10).

The transformation law of $\tilde{Y}_E^M$ is given by the generic formula (3.11) with the obvious replacement $\tilde{G}_0 \to \tilde{G}_{E(0)}$. One may easily check that the $\tilde{\mathbf{R}}^4$ coordinate $\tilde{x}_L^m$ and the old coset fields have the same laws as in the conformal case. The quantities $\tilde{\theta}_L^\mu$ and $\phi(\tilde{x}_L)$ are transformed according to

$$
\tilde{\theta}_L^{\mu'} = \varphi^{-\frac{1}{2}}(\tilde{x}_L)\varphi_\nu^\mu(\tilde{x}_L)\tilde{\theta}_L^\nu \equiv \tilde{\gamma}_\nu^\mu(\tilde{x}_L)\tilde{\theta}_L^\nu, \; e^{\phi'(\tilde{x}_L')} = \varphi^{\frac{1}{2}}(\tilde{x}_L)e^{\phi(\tilde{x}_L)} \qquad (4.9)
$$

$$
\varphi_\nu^\mu(\tilde{x}_L) = \varphi_\nu^\mu(z)|_{\tilde{x}_L}
$$

It follows from this consideration that constraints (3.24) and (3.35) remain covariant in the case in question, therefore expressions (3.25) and (3.36) for the fields present in (4.8) do not change. The components of the nonlinear realization prepotential $\tilde{H}^m(\tilde{z})$ are slightly modified as a result of substitution (4.7)

$$
\begin{aligned}
\tilde{e}_E^{am}(\tilde{x}) &= e^{\phi(\tilde{x})}e^{\bar{\phi}(\tilde{x})}\tilde{e}^{am}(\tilde{x}) \\
\tilde{\psi}_E^{m\mu}(\tilde{x}) &= e^{2\bar{\phi}(\tilde{x})}e^{\phi(\tilde{x})}\tilde{\psi}^{m\mu}(\tilde{x}) \\
\tilde{A}_E^m(\tilde{x}) &= e^{2\phi(\tilde{x})}e^{2\bar{\phi}(\tilde{x})}\left[\tilde{A}^m(\tilde{x}) + \frac{i}{2}\tilde{\partial}^m(\phi - \bar{\phi})\right] \qquad (4.10)
\end{aligned}
$$

The meaning of this modification is transparent. The objects in the l.h.s.of (4.10) undergo only induced general covariance and $SL(2,\mathbf{C})$ transformations while the G transformations of the old quantities $\tilde{e}^{am}, \tilde{\psi}^{m\mu}, \tilde{A}^m$ involve in addition the induced $L(2,\mathbf{C})/SL(2,\mathbf{C})$ terms (recall (3.28) and (3.29)). These terms are now compensated by the transformation of $\phi(\tilde{x})$.

4.3 Passing to irreducible set of fields

The last question to be answered is how to relate $\phi(\tilde{x})$ to the fields of linear realization. Indeed, like in the preceding Sect. we aim at having the minimally possible set of fields in the nonlinear realization of G, that is 12 + 12 components of SG multiplet plus four fermionic degrees of freedom associated with goldstino $\lambda^\mu(x)$.

We proceed by rewriting the compensator $S(\zeta)$ in the nonlinear realization superspace basis $\{\tilde{\zeta}_L\}$

$$
S(\zeta_L) = Ber^{-1}\left(\frac{\partial\zeta_L^N}{\partial\tilde{\zeta}_L^M}\right)\cdot\tilde{S}(\tilde{\zeta}_L) \qquad (4.11)
$$

$$
Ber\left(\frac{\partial\zeta_L^{N'}}{\partial\tilde{\zeta}_L^{M'}}\right) = I(\zeta_L)\cdot Ber\left(\frac{\partial\zeta_L^N}{\partial\tilde{\zeta}_L^M}\right)\cdot\tilde{I}^{-1}(\tilde{\zeta}_L) \qquad (4.12)
$$

$$
\tilde{S}'(\tilde{\zeta}_L') = \tilde{I}^{-1}(\tilde{\zeta}_L)\tilde{S}(\tilde{\zeta}_L) \qquad (4.13)
$$

$$
\tilde{I}(\tilde{\zeta}_L) = Ber\left(\frac{\partial\tilde{\zeta}_L^{M'}}{\partial\tilde{\zeta}_L^N}\right) = det\left(\frac{\partial\tilde{x}_L^{m'}}{\partial\tilde{x}_L^n}\right) = det(\delta_n^m + \tilde{\partial}_n\tilde{a}^m(\tilde{x}_L)) \qquad (4.14)
$$

where ζ_L^N and $\tilde{\zeta}_L^M$ are related by (4.8).The conformal density $F(z)$ (4.5) is redefined as

$$
\begin{aligned}
F(z) &= \tilde{\omega}^{\frac{1}{3}}(\tilde{z})\cdot\tilde{F}(\tilde{z}) \qquad (4.15) \\
\tilde{\omega}(\tilde{z}) &= det(\tilde{\Delta}_\mu\theta^\nu(\tilde{z})) \\
\tilde{F}(\tilde{z}) &= 2^{\frac{2}{3}}\tilde{r}^{-\frac{1}{3}}\tilde{l}^{\frac{1}{6}}\tilde{S}^{\frac{1}{6}}\bar{\tilde{S}}^{-\frac{1}{3}} \qquad (4.16)
\end{aligned}
$$

$$
\tilde{\omega}'(\tilde{z}') = \varphi(z(\tilde{z}))\cdot\tilde{\omega}(\tilde{z})\cdot\varphi^{-1}(\tilde{z}), \quad \tilde{F}'(\tilde{z}') = \varphi^{\frac{1}{3}}(\tilde{z})\tilde{F}(\tilde{z}) \qquad (4.17)
$$

where

$$
\varphi(\tilde{z}) = det\;\varphi_\mu^\nu(\tilde{z}), \quad \varphi_\mu^\nu(\tilde{z}) = \tilde{\Delta}_\mu\tilde{\theta}^{\nu'} = \tilde{\gamma}_\mu^\nu(\tilde{x}_L) + \left(\tilde{\Delta}_\mu\tilde{\gamma}_\rho^\nu\right)\tilde{\theta}^\rho \qquad (4.18)
$$

and $\tilde{\Delta}_\mu, \tilde{r}, \tilde{l}$ are defined by eqs. (3.26), (3.33) and (4.5) with $H^m(z)$ replaced by $\tilde{H}_E^m(\tilde{z})$.

Keeping in mind (4.16), the transformation law (4.17) of $\tilde{F}(\tilde{z})$ and the property $det\;\tilde{\gamma}_\nu^\mu = 1$ one reveals that the lowest component of $\tilde{F}(\tilde{z})$ starts with a constant (which can be put equal to unity without loss of generality) and does not transform under the group G. So, the constraint

$$
\tilde{F}(\tilde{z})\big|_{\tilde{\theta}=0} = 1 \qquad (4.19)
$$

is manifestly G covariant.Taking into account that

$$
\tilde{\Delta}_\mu\theta^\nu(\tilde{z})\big|_{\tilde{\theta}=0} = \delta_\mu^\nu e^{\phi(\tilde{x})} \qquad (4.20)
$$

one readily deduces from (4.15) and (4.19)

$$
e^{\phi(\tilde{x})} = F(z)\big|_{\tilde{x}} = e^{-\frac{1}{6}}S_0^{\frac{1}{6}}(\tilde{x})\bar{S}_0^{-\frac{1}{3}}(\tilde{x}) + \mathcal{O}(\lambda,\bar{\lambda}) \qquad (4.21)
$$

$$
e = det\;e_a^m(\tilde{x})
$$

where $S_0(\tilde{x}), \bar{S}_0(\tilde{x})$ are the lowest components of $S, \bar{S}$ and dots denote the terms of higher order in fields.

Thus all the originally introduced coset fields proved to be expressed as functions of the SG fields and goldstino $\lambda^\mu(\tilde{x})$. This shows that the nonlinear realization of spontaneously broken Einstein $N = 1$ SG group we have constructed is indeed minimal (for the given choice of the stability subgroup $G_{E(0)}$).

From (4.16) and (4.19) it follows that

$$\tilde{S}\Big|_{\tilde{\theta}=0} = \tilde{e}^{-1} \equiv (det\ \tilde{e}_E^{ma}(\tilde{x}))^{-1} \tag{4.22}$$

thus indicating that $\tilde{S}(\tilde{\zeta}_L)$ has the form

$$\tilde{S}(\tilde{\zeta}_L) = \tilde{e}^{-1}(\tilde{x}_L)\left[1 + \tilde{\theta}^{\mu}_L\tilde{\xi}_{\mu}(\tilde{x}_L) + \tilde{\theta}_L\tilde{\theta}_L\tilde{M}(\tilde{x}_L)\right] \tag{4.23}$$

The component $\tilde{M}(\tilde{x}_L)$ is a scalar of G while $\tilde{\xi}_{\mu}(\tilde{x}_L)$ undergoes also the induced $SL(2,\mathbf{C})$ rotations

$$G: \quad \tilde{\xi}'_{\mu}(\tilde{x}'_L) = \tilde{\gamma}_{\mu}^{-1\nu}(\tilde{x}_L)\tilde{\xi}_{\nu}(\tilde{x}_L), \ \ \tilde{M}'(\tilde{x}'_L) = \tilde{M}(\tilde{x}_L) \tag{4.24}$$

Of course, these fields are related by an equivalence transformation to their counterparts from the SG multiplet.

The ultimate result of our study is that the basic geometric objects of spontaneously broken Einstein $N=1$ SG are superfields $\tilde{H}^m_E(\tilde{z}), \tilde{S}(\tilde{\zeta}_L)$ accomodating the set of fields

$$\{\tilde{e}_E^{ma}(\tilde{x}),\ \tilde{\psi}_E^{m\mu}(\tilde{x}),\ \overline{\tilde{\psi}}_E^{m\dot{\mu}}(\tilde{x}),\ \tilde{A}^m_E(\tilde{x}),\ \tilde{\xi}^{\mu}(\tilde{x}),\ \tilde{M}(\tilde{x})\} \tag{4.25}$$

Note that the Goldstone fermion $\lambda^{\mu}(\tilde{x})$ proves to be completely hidden inside these objects which can be regarded, like in the conformal case, as an appropriate generalization of Cartan's forms of the nonlinear realization of rigid $N=1$ supersymmetry. One can be easily convinced that they carry just $12+16$ essential degrees of freedom. This is verified with taking account of constraint (3.35) [5] and the fact that the group G is realized on these fields by induced general covariance and local $SL(2,\mathbf{C})$ transformations. Though goldstino $\lambda^{\mu}(x)$ does not appear explicitly in the set (4.25), it essentially enters into the induced transformation parameters. One should keep in mind this property when analyzing the Lie bracket structure of these transformations.

Finally, we mention that another way to construct the nonlinear realization adequate to the Einstein case is to proceed from the supervolume-preserving subgroup of G singled out by constraint (3.5) [21]. It can be shown that the arising theory is related, via an equivalence redefinition of superspace coordinates, to the $S(\zeta_L)=1$ gauge of the theory constructed here. In terms of the nonlinear realization objects this gauge is implemented as (see eq.(4.11))

$$\tilde{S}(\tilde{\zeta}_L) = Ber\left(\frac{\partial\zeta^N_L}{\partial\tilde{\zeta}^M_L}\right)$$

[5] According to the remark in the end of Subsect.(3.4), one is free to choose equivalent forms for this constraint, e.g. to set

$$\overline{\tilde{\psi}}_{\dot{\beta}}^{\mu\dot{\beta}}(\tilde{x}) = \tilde{\xi}^{\mu}(\tilde{x})$$

All these options are related by the analytic changes of $\tilde{\theta}^{\mu}$ of type (3.40).

5 Discussion and comments

Having defined the adequate nonlinear realization of $N=1$ SG group and being aware of how it is related to the conventional superspace formulation of $N=1$ SG we are ready to promote to curved space all the considerations carried out earlier in rigid $N=1$ supersymmetry [16-18]: to rewrite the actions with spontaneously broken supersymmetry in terms of nonlinear realization, to construct superfields from the nonlinear realization fields, etc. Here we briefly concern all these issues with emphasis on the peculiarities brought about by local supersymmetry. We shall restrict ourselves to the case of Einstein SG.

5.1 SG actions in terms of nonlinear realization

Given any G invariant model exhibiting the spontaneous breakdown $G \Rightarrow G_{E(0)}$, one can equivalently rewrite its superfield action in the superspaces $\tilde{\mathbf{C}}^{4|2}=\{\tilde{\zeta}^M_L\}$ and / or $\tilde{\mathbf{R}}^{4|4}=\{\tilde{z}\}$ possessing standard transformation properties relative to the nonlinear realization $G/G_{E(0)}$ constructed in Sect.4. To do this, one has to make the change of variables $\zeta^M_L \Rightarrow \tilde{\zeta}^M_L, z^M \Rightarrow \tilde{z}^M$ in the action (with taking account of (3.25) and (3.36)). A crucial difference from the analogous procedure in the rigid case [16, 17] lies in that these changes have the form of original gauge transformations (3.1), (3.3) and (4.1), though with the field-dependent parameters.So their net effect on the action is reduced to replacing elsewhere $H^m(z), S(\zeta_L)$ by $\tilde{H}^m_E(\tilde{z}), \tilde{S}(\tilde{\zeta}_L)$. In this "splitting" basis all the other superfields (the matter or Yang-Mills ones) are represented by components which are transformed under the whole SG group G according to its stability subgroup $G_{E(0)}$, with the induced parameters $\tilde{a}^m(\tilde{x}), \tilde{\gamma}^{\mu}_{\nu}(\tilde{x})$. To match the numbers of independent degrees of freedom in the initial and new parametrizations, one also needs, like in the rigid case [17], to set

$$\tilde{\chi}^{\alpha}(x) = 0 \tag{5.1}$$

where $\tilde{\chi}^{\alpha}(x)$ is composed out of the spinor fields of the nonlinear realization in precisely the same fashion as the goldstino of the linear realization out of the initial spinor fields. This manifestly covariant constraint gives the equivalence relation between the goldstinos of the linear and nonlinear realizations.

To be a bit more specific, let us give the generic form of the nonlinearly parametrized action of $N=1$ SG in the chiral representation

$$I = -\frac{3}{\kappa^2}\int d^6\tilde{\zeta}_L\tilde{S}(\tilde{\zeta}_L)\tilde{R}(\tilde{H}_E) + \int d^6\tilde{\zeta}_L\tilde{S}(\tilde{\zeta}_L)\tilde{\mathcal{L}}(\tilde{\zeta}_L) + \text{h.c.} \tag{5.2}$$

where κ is the Einstein constant, $\tilde{S}$ and $\tilde{H}_E$ are defined by eqs.(3.27),(4.10) and (4.23), $\tilde{\mathcal{L}}(\tilde{\zeta}_L)$ is the Lagrangian density of matter and Yang-Mills superfields brought into the splitting basis and

$$\tilde{R}(\tilde{H}_E) = R(H) = \bar{\Delta}^{\dot{\mu}}\bar{\Delta}_{\dot{\mu}}\bar{F}^2 = \overline{\tilde{\Delta}}^{\dot{\mu}}\overline{\tilde{\Delta}}_{\dot{\mu}}\overline{\tilde{F}}^2 \tag{5.3}$$

After performing $\tilde{\theta}$-integration the first piece in sum (5.2) takes the familiar form of component Einstein $N = 1$ SG action in one of WZ gauges employed in [3,4], with the SG fields replaced by the corresponding quantities from the set (4.25).[6] Recall that the latter objects as distinct from the former ones carry a representation of the full group G which closes on them with the same (field-independent) bracket parameters as in the initial superspace realization (3.1).

If supersymmetry is spontaneously broken, $\tilde{\mathcal{L}}$ necessarily starts with a field-independent term

$$\tilde{\mathcal{L}}(\tilde{\zeta}_L) = m_{sb}^4(\tilde{\theta}_L\tilde{\theta}_L) + \ldots$$

where m_{sb} is a mass characterizing the scale of spontaneous breaking. In view of the structure of $\tilde{S}(\tilde{\zeta}_L)$, (4.23), this term produces the self-interaction of goldstino $\lambda^\mu(x)$

$$\int d^6\tilde{\zeta}_L \tilde{S}(\tilde{\zeta}_L)\tilde{\theta}_L\tilde{\theta}_L \Rightarrow \int d^4\tilde{x}_L det\tilde{e}_m^a(\tilde{x}_L) = \int d^4\tilde{x}_L det T_m^a(\tilde{x}_L) + \ldots \quad (5.4)$$

where $detT_m^a$ is the familiar Volkov-Akulov Lagrangian density and so it generates an induced cosmological term proportional to m_{sb}^4 [25, 26]. If there is also an independent supersymmetric cosmological term in the pure SG sector (it amounts to the presence of θ-independent constant piece in $\tilde{\mathcal{L}}$ [3]), the component $\tilde{M}(\tilde{x})$ in $\tilde{S}(\tilde{\zeta}_L)$ (4.23) acquires a non-zero vacuum expectation value, $\tilde{M}(\tilde{x}) = const + \ldots$, and, as a result, additional contributions to (5.4) appear both from the first and second pieces in (5.2). The effective cosmological constant can be then made equal to zero by adjusting parameters (simultaneously this fixes the mass of a gravitino). This mechanism of annulling the cosmological constant by the super-Higgs effect has been discovered in [26, 27]. The nonlinear realization form of $N = 1$ SG action permits one to see how this mechanism works without entering into details of $\tilde{\mathcal{L}}(\tilde{\zeta}_L)$.

Let us now dwell on the relation with the pioneering paper by Volkov and Soroka [25] where spontaneously broken local $N = 1$ supersymmetry has been implemented as a direct gauging of nonlinearly realized rigid supersymmetry in ordinary space-time and where the super-Higgs effect has been discussed for the first time. The basic objects of this theory are also some gauge-covariantized Cartan's forms $\tilde{e}_V^{am}(x), \tilde{\psi}_V^{m\mu}(x)$ built up from the goldstino and the gauge fields of spins 2 and 3/2. Local supersymmetry is realized as induced general coordinate transformations of these objects, with the parameters composed in a proper way out of the goldstino and spinor gauge functions. However, the explicit expression of these objects in terms

[6] For entire coincidence with the minimal Einstein SG component action as it is given in [3], it is convenient to identify $\xi^\mu, \bar{\xi}^{\dot{\mu}}$ with the spinorial parts of $\bar{\tilde{\psi}}_E^{\dot{\mu}m}, \tilde{\psi}_E^{\mu m}$ using the freedom mentioned in the previous footnote.

of the goldstino and gauge fields essentially differs from that of the quantities $\tilde{e}_E^{am}, \tilde{\psi}_E^{m\mu}$ given above. Besides, the Lie bracket structure associated with the invariance group of this theory does not coincide with that of $N = 1$ SG group and, respectively, the transformation laws of the goldstino are different in both theories. Nevertheless, despite these distinctions, any action composed of the Volkov-Soroka Cartan's forms in the invariant way with respect to general covariance and local $SL(2, C)$ transformations will be automatically invariant under $N = 1$ SG group. Indeed, one may identify both sets of forms "by hand" and re-express the Volkov-Soroka gauge fields in terms of fields of $N = 1$ SG multiplet and the relevant goldstino. By this procedure, one implements $N = 1$ SG group on the objects of the Volkov-Soroka theory. Invariance of the action follows from the fact that this group is again realized as general coordinate and local $SL(2, C)$ transformations. In other words, both the Volkov-Soroka and standard $N = 1$ SG gauge groups can be realized on the same set of objects and, as a matter of fact, these groups coincide modulo some field-dependent general coordinate transformations and local $SL(2, C)$ rotations. From that standpoint, the component form of the first integral in (5.2) (with the nonpropagating fields $\tilde{M}, \tilde{A}^m$ eliminated[7]) can be interpreted as a special choice of the Volkov-Soroka action containing a non-minimal term which is quartic in $\tilde{\psi}_V^{m\mu}, \bar{\tilde{\psi}}_V^{m\dot{\mu}}$ and enters with a fixed coupling constant.

It is noteworthy that any G invariant action in ordinary space-time constructed from $\tilde{e}_E^{am}, \tilde{\psi}_E^{m\mu}, \bar{\tilde{\psi}}_E^{m\dot{\mu}}$ can be equivalently rewritten in terms of gauge superfields of the linear realization $S(\zeta_L), H^m(z)$ and goldstino $\lambda^\mu(x)$. To this end, one should simply complete the integrals over $\tilde{\mathrm{R}}^4$ in these actions to those over $\tilde{\mathrm{R}}^{4|4}$ or $\tilde{\mathrm{C}}^{4|2}$ by using the identities

$$1 = \int d^4\tilde{\theta}\,(\tilde{\theta})^2(\bar{\tilde{\theta}})^2 \quad \text{or} \quad 1 = \int d^2\tilde{\theta}_L(\tilde{\theta}_L)^2$$

and then perform the change of variables $\tilde{\zeta}_L^M \to \zeta_L^M$ and/or $\tilde{z}^M \to z^M$. Extra terms appearing in the SG action exhibit in general an explicit dependence on $\lambda^\mu(x)$ which disappears only in the unitary gauge $\lambda^\mu(x) = 0$.

5.2 Superfields from nonlinear realization fields

The main advantage of the present approach should be seen in its manifestly geometric character that allowed us, among other things, to find the explicit relation between original $N = 1$ SG superspaces $\mathrm{C}^{4|2}, \mathrm{R}^{4|4}$ and the nonlinear realization ones $\tilde{\mathrm{C}}^{4|2}, \tilde{\mathrm{R}}^{4|4}$. The knowledge of this relation makes it possible, as in the rigid case, to work out simple general recipes for constructing

[7] As distinct from the case of linearly realized G, elimination of nonpropagating degrees of freedom in the nonlinear realization does not destroy the off-shell closing of G transformations on the rest of fields because all the nonlinear realization fields transform independently of each other.

superfields of the linear realization from the quantities of the nonlinear realization, viz. goldstino $\lambda^{\mu}(x)$ and the fields of $N = 1$ SG gauge multiplet. To have such recipes is important, e.g., for the model building along the lines of refs.[28].

The simplest exercise is to build superfields having no external $SL(2,\mathrm{C})$ indices. One way to do this is as follows

$$\varphi(\zeta_L) = \tilde{\theta}^{\mu}(\zeta_L)\tilde{\theta}_{\mu}(\zeta_L) \tag{5.5}$$

where $\tilde{\theta}^{\mu}$ are to be expressed through ζ_L^M from eqs. (4.8) (this can be done by iterations).A more complicated problem is to set up superfields possessing nontrivial transformation properties with respect to local $SL(2,\mathrm{C})$. The origin of difficulty lies in that the $SL(2,\mathrm{C})$ frames of the linear and nonlinear realizations do not coincide and one needs "bridges" relating these frames to each other.

To construct the bridges, we first introduce the objects

$$a_{\mu}^{N} \equiv \tilde{\partial}_{\mu}\zeta_L^N, \quad a_N^{-1m} = \partial_N \tilde{x}_L^m \tag{5.6}$$

transforming as (the coordinates with prime are defined by (3.1), (3.12) and (4.9))

$$a_{\mu}^{N'} = \partial_M \zeta_L^{N'} \cdot a_{\nu}^{M} \cdot \tilde{\gamma}_{\mu}^{-1\nu}(\tilde{x}_L), \quad a_N^{-1m'} = \partial'_N \zeta_L^M \cdot a_M^{-1n} \cdot \tilde{\partial}_n \tilde{x}_L^{m'}$$

Using them we may convert any covariant world tensor or $SL(2,\mathrm{C})$ spinor of the nonlinear realization (e.g., in the left basis) into appropriate chiral world tensors of the linear realization (contravariant world tensors of the nonlinear realization can always be transformed into covariant ones by means of contraction with the metric $\tilde{g}_{mn} = \tilde{e}^{a}_{Em}\tilde{e}_{Ean}$). These objects can further be converted into $SL(2,\mathrm{C})$ tensors of the linear realization by contracting them with proper components of the left or right supervielbeins $l_A^M, r_A^M = (l_A^M)^{\dagger}$ [2] and of their inverses. For our purpose, it is sufficient to have the components $l_{\alpha}^{M}, l_N^{\alpha}$:

$$\begin{aligned} l_{\alpha}^{M'}(z') &= \gamma_{\alpha}^{-1\beta}(z) \cdot l_{\beta}^{N}(z) \cdot \partial_N \zeta_L^{M'} \\ l_N^{\alpha'}(z') &= \gamma_{\beta}^{\alpha}(z) \cdot l_M^{\beta}(z) \cdot \partial'_N \zeta_L^M \end{aligned} \tag{5.7}$$

where $\gamma_{\beta}^{\alpha}(z)$ are defined in (4.4). The explicit form of these objects can be found, e.g., in [2] and we do not give it here. As a simple example of applying this procedure we quote a spinor $N = 1$ superfield whose components are functions of the goldstino and the components of SG multiplet

$$\varphi^{\alpha}(z) = l_N^{\alpha}(z) a_{\mu}^{N}(\zeta_L)\tilde{\theta}^{\mu}(\zeta_L)$$

It possesses normal transformation properties under the linear realization of Einstein $N = 1$ SG group. Note that from the standpoint of the linear

realization the superfields of that kind are singled out by the nonlinear constraints of the type considered in [20]. The latter look rather complicated and, like in the rigid case, it is difficult to indicate any general principle of how to seek them.

It is worth mentioning that in the local case there are much more possibilities to construct the superfields of this sort compared to the rigid case. The reason is that in the nonlinear realization of SG one may form the quantities with the tensor type transformation laws not only from $\tilde{\theta}^{\mu}$ (as in the rigid case) but also with making use of fields (4.25) and the covariants constructed from them by standard rules of Riemann geometry.

Surprisingly, the SG multiplet, in its own right, can be covariantly constructed entirely from the goldstino, the linear realization vierbein and pure gauge components of $N = 1$ SG multiplet. All the other components (the gravitino field and auxiliary fields) can be expressed as nonlinear functions of these entities by equating to zero all the fields in the set (4.25) except for the vierbein field that is a manifestly G covariant procedure. Of course, such a composite SG multiplet, though possessing correct transformation properties under SG group, at the classical level does not give rise to any new dynamics beyond that connected with ordinary gravity (in the gauge $\lambda^{\mu}(x) = 0$ and a WZ gauge for the SG multiplet we are left with the single vierbein field). However, it could generate nontrivial supergravity interactions after quantization, as it was suggested, e.g., in a similar context in [29].

5.3 Flat-space limit

Finally, we discuss the flat Minkowski space limit of formulas obtained in Sect.3,4. This limit can be achieved by letting $\kappa \to 0$ elsewhere or, equivalently, by equating to zero all the members of the SG multiplet except for the vierbein which reduces to its flat Minkowski part $e^{am} = \eta^{am}$. We have

$$\begin{aligned} \tilde{H}_E^m(\tilde{z}) &\Rightarrow \tilde{H}^m(\tilde{z}) \ (2.13) \\ \phi(\tilde{x}) &\Rightarrow 0 \\ \tilde{B}^m &\Rightarrow \lambda\sigma^m\bar{\lambda}, \ \tilde{\chi}_{\alpha}^m \Rightarrow 2i(\sigma^m\bar{\lambda})_{\alpha} \\ \tilde{F}^m &\Rightarrow \frac{i}{2}q\sigma^m\bar{\lambda}, \ q^{\mu} \Rightarrow 2i(\sigma^m\nabla_m\bar{\lambda})^{\mu} \\ \tilde{S}(\tilde{\zeta}_L) &\Rightarrow Ber\left(\frac{\partial\zeta_L^N}{\partial\tilde{\zeta}_L^M}\right) \end{aligned} \tag{5.8}$$

Correspondingly, relations (4.8) go over to

$$x_L^m = \tilde{x}_L^m + i\lambda(\tilde{x}_L)\sigma^m\bar{\lambda}(\tilde{x}_L) + 2i\tilde{\theta}_L\sigma^m\bar{\lambda}(\tilde{x}_L) + \frac{i}{2}\tilde{\theta}_L\tilde{\theta}_L q(\tilde{x}_L)\sigma^m\bar{\lambda}(\tilde{x}_L)$$

$$\theta_L^{\mu} = \tilde{\theta}_L^{\mu} + \lambda^{\mu}(\tilde{x}_L) + \frac{i}{2}\tilde{\theta}_L\tilde{\theta}_L(\sigma^m\nabla_m\bar{\lambda})^{\mu}$$

After the additional replacement of Grassmann variable

$$\tilde{\theta}^{\mu}_{L} \rightarrow \tilde{\theta}^{\mu}_{L} - \frac{i}{2}\tilde{\theta}_{L}\bar{\theta}_{L}(\sigma^{m}\nabla_{m}\bar{\lambda})^{\mu}$$

one arrives at the familiar relations of the rigid case quoted in Sect.2.Note that in the flat Minkowski space limit the Lagrangian density of pure SG in (5.2) reduces to a full divergence and hence the relevant part of the whole action vanishes. The Volkov-Akulov low-energy Lagrangian may appear in this case only from the matter and Yang-Mills sectors.

6 Summary and outlook

In this paper we have constructed the nonlinear realizations adequate to conformal and minimal Einstein SG's with spontaneously broken local supersymmetry. In our study we proceeded from the manifestly invariant geometric description of $N = 1$ SG's in superspace via unconstrained superfield prepotentials. We have defined the superspaces where these nonlinear realizations are formulated in a most natural way and established their relation to conventional $N = 1$ superspaces. Our consideration directly generalizes the analogous one for rigid $N = 1$ supersymmetry [16, 17] and makes transparent the underlying geometry of spontaneous supersymmetry breakdown in the $N = 1$ SG theories.

Note that many questions we addressed here were earlier treated within the component [19] and constrained superfield [20] formalisms. Like in the case of unbroken theory, the group-theoretic and geometric foundations of spontaneously broken SG remain implicit when employing these approaches. In particular, none of the previous authors succeeded in extending to curved space the change of variables which relates ordinary $N = 1$ superspaces $\mathbf{R}^{4|4}, \mathbf{C}^{4|2}$ to their nonlinear realization counterparts $\tilde{\mathbf{R}}^{4|4}, \tilde{\mathbf{C}}^{4|2}$. (eqs.(3.19), (3.21), (3.22) and (4.8)). This relation lies in the basis of our consideration and it is of key importance for understanding the geometric structure of spontaneous supersymmetry breakdown in $N = 1$ SG. It should be pointed out that all the main conclusions of refs. [19, 20] can be reproduced without difficulties within the present framework. For instance, the transformation law of the goldstino found in [20] follows from ours, (3.18), upon passing to a new basis

$$\tilde{\tilde{x}}^{m}_{L} = \tilde{x}^{m}_{L} + iH^{m}(\tilde{x}_{L}, \lambda(\tilde{x}_{L}), \bar{\lambda}(\tilde{x}_{L}))$$
$$\tilde{\lambda}^{\alpha}(\tilde{\tilde{x}}_{L}) = \lambda^{\alpha}(\tilde{x}_{L})$$

and choosing a proper gauge with respect to the group G.

The methods we have applied throughout the paper are greatly general and can readily be adapted for other versions of $N = 1$ SG and for higher N SG's in unconstrained superspace formulations. What one really needs to know is the realization of relevant unbroken SG group in some appropriate superspace.

Perhaps, the most perspective new area of possible applications of our machinery is provided by recent developments in the string and membrane theories. It has been suggested [10-12] that these theories (in a fixed gauge) could be understood as an effective approximation of appropriate theories in higher dimensions which describes self-coupling of the Goldstone modes associated with spontaneous breaking of some continuous symmetries or supersymmetries. It seems that the methods worked out in [16, 17, 21] and in the present paper may turn out helpful for singling out such Goldstone modes in a manifestly covariant manner and for revealing relations between linear and nonlinear realizations of underlying spontaneously broken symmetries (like it has been done by us for spontaneously broken supersymmetry). We end the paper with a bit more detailed discussion of how this could work.

As a matter of fact, the generic transformation formula (2.5) we permanently relied upon applies to any situation where

i.There is a group (or supergroup) G acting as coordinate transformations on some manifold $\{\eta^{M}\}$ (it may involve both bosonic and fermionic variables)

$$G: \quad \eta^{M'} = G^{M}(\eta) \tag{6.1}$$

and

ii.There occurs a spontaneous breaking of G down to symmetry with respect to some subgroup G_0 leaving invariant a subspace $\{\eta^{m}\} \subset \{\eta^{M}\}$

$$G_0: \quad \eta^{m'} = G^{m}_{0}(\eta^{n}) \quad \eta^{\mu'} = \eta^{\nu}G^{\mu}_{\nu}(\eta^{\nu}, \eta^{m}) \tag{6.2}$$

where we have split indices as $\{M\} \Rightarrow \{m, \mu\}$. Such breaking may arise, e.g., due to some classical solution displaying no dependence on η^{m}.

To describe this situation irrespective of the choice of a concrete model, we may proceed by analogy with the supersymmetry case worked out in this paper. First, we decompose $G^{M}(\eta)$ in a series in η^{μ} (in the cases we dealt with before η^{μ} was a Grassmann coordinate) and further define the G/G_0 coset representatives $Y^{M}(\eta^{m}, \eta^{m})$ by setting to zero all those coefficients in this series which parametrize the stability subgroup G_0. The remaining coefficients are then the G/G_0 coset parameters and they can be identified with the relevant Goldstone fields. Their behaviour under G is fixed by postulating for the G/G_0 coset elements the generic transformation law (2.5)

$$G^{M}(\tilde{Y}) = \tilde{Y}^{M'}(\tilde{G}_0) = \tilde{Y}^{M'}(\tilde{G}^{m}_{0}(\tilde{\eta}^{n}), \tilde{\eta}^{\mu'}) \tag{6.3}$$

The Goldstone fields are defined to live on the manifold $\{\tilde{\eta}^{m}\}$ and they transform under G through themselves and the coordinates $\tilde{\eta}^{m}$ while the latter transform under the whole G according to the induced G_0 transformations (6.2), with the parameters explicitly involving the Goldstone fields. Hence these fields together with coordinates $\tilde{\eta}^{m}$ constitute a closed nonlinear realization of group G in the coset space G/G_0. The remaining coordinates $\tilde{\eta}^{\mu}$ parametrize the quotient of $\{\tilde{\eta}^{M}\}$ over $\{\tilde{\eta}^{m}\}$ and transform homogeneously, also according to the induced G_0 transformation law (6.2). As before, the relation to the original "linear" realization of G is immediately revealed by identifying

$$\eta^M = \tilde{Y}^M(\tilde{\eta}^m, \tilde{\eta}^\mu) \qquad (6.4)$$

and further by imposing the constraints of type (3.24), (3.35) and (5.1). The Jacobian of the change (6.4), $J = det\ \left(\frac{\partial\eta^N}{\partial\tilde{\eta}^M}\right)$ (Ber in the case of graded manifold), being expanded in powers of homogeneously transforming coordinates $\tilde{\eta}^\mu$, yields in the lowest order the minimal Lagrangian density of Goldstone modes. The examples presented in [10-12] naturally fit in this general scheme.

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