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ELECTROWEAK RADIATIVE CORRECTIONS TO DEEP INELASTIC SCATTERING AT HERA. CHARGED CURRENT SCATTERING

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1. Introduction

Deep inelastic electron-proton scattering will be investigated at the storage ring HERA /1/ in a new energy range. Physics at HERA is governed by electroweak interactions due to the exchange of the corresponding vector bosons - photon and Z- boson in the neutral current reactions, and W- boson in the charged current case. We have studied the neutral current deep inelastic reaction at HERA in an earlier article /2/. Here, we present analogue results for the charged current reaction,

$$e^{\pm} + p \longrightarrow \overset{(-)}{\nu}_{e} + X, \tag{1.1}$$

which is accompanied by the bremsstrahlung process,

$$e^{\pm} + p \longrightarrow \overset{(-)}{\nu}_{e} + \gamma + X. \tag{1.2}$$

Electroweak radiative corrections (EWRC) for reaction (1.1) at an energy scale comparable to the W-boson mass have been first published in /3/. Corrections including also soft bremsstrahlung and several selected parts of hard bremsstrahlung in a Monte-Carlo treatment are presented in /4/. A discussion of the present status may be found in the contributions /5,6/ to /7/ and refs. therein.

In this article, we present explicit analytic results obtained in the unitary gauge on both EWRC to (1.1) and on the soft and hard bremsstrahlung process (1.2) without cuts. In sect. 2 we introduce the notation and propose a procedure which allows to separate what one could call genuine QED contributions from weak corrections. This separation can be made unambigously, though not in a one-to-one correspondence to certain classes of Feynman diagrams. Weak loop corrections will be contained in a form factor ρ_C. The remaining contributions show the general features of QED terms: logarithmic fermionic mass singularities due to the massless photon and some apparatus dependence connected with more or less complete photon observation. In sect. 3 these QED contributions are presented under the assumption that the bremsstrahlung photon is totally inclusive (i.e. not observed). Sect. 4 contains a discussion of our results and of the present status. Some more technical details are collected in the Appendices.

2. Cross section formulae and weak loop corrections

The charged current deep inelastic cross section for the scattering of electrons or positrons with polarization degree λ off protons can be denoted very compact:

$$\frac{d^2\sigma}{dxdy} = \frac{G_\mu^2 Sx}{\pi}\left[\frac{M_W^2}{Q^2+M_W^2}\right]^2\left[\frac{1+\lambda Q_e}{2}\right]\sum_{b=o,e,i,q}\;\sum_{Q,\bar{Q}}\Theta(-Q_eQ_Q)\;c_b$$

$$\cdot\left[\rho_c(p)\right]^2\left[\frac{1+p}{2}R_b+\frac{1-p}{2}\bar{R}_b\right]. \qquad (2.1)$$

The notations are those already introduced in /2/:

$$Q_e = \{+1,\,-1\} \qquad \text{for } e^+,\, e^-, \qquad (2.2)$$

$$Q_Q = \{+\tfrac{2}{3},\, -\tfrac{2}{3},\, -\tfrac{1}{3},\, +\tfrac{1}{3}\} \qquad \text{for } Q_Q = U,\bar{U},D,\bar{D}, \qquad (2.3)$$

$$p = p_e \cdot p_Q, \qquad (2.4)$$

$$p_{e(Q)} = \{+1,\,-1\} \text{ for particles, antiparticles.} \qquad (2.5)$$

The step function $\Theta(-Q_eQ_Q)$ ensures charge conservation. The index b stands for scattering without photon production (b = o) or with single-photon bremsstrahlung:

$$c_b = \left\{1,\; Q_e^2,\; Q_eQ_Q,\; Q_Q^2\right\}, \quad b = o,e,i,q. \qquad (2.6)$$

The quark distributions are contained in the dynamical functions $\overset{(-)}{R_b}$. For reaction (1.1) they are:

$$R_o = f_Q(x,Q^2), \qquad \bar{R}_o = y_1^2 f_Q(x,Q^2). \qquad (2.7)$$

For the numerical results to be presented below the quark distribution functions $f_Q(x,Q^2)$ have been taken from /8/ with $Q_o^2 = 4$ GeV^2 and $\Lambda_{QCD} = 200$ MeV. Further, we use the common scaling variables: $S = 4E_eE_p$, $Q^2 = S\,x\,y$, $U = Q^2 - S$, $y_1 = 1-y$, where E_e and E_p are the electron and proton energies in the laboratory system. The y may be defined by lepton energy E_ν and scattering angle θ_ν with respect to the proton direction,

$$y = 1 - \frac{E_\nu}{E_e}\sin^2\frac{\theta_\nu}{2}. \qquad (2.8)$$

Problems connected with the experimental determination of Q^2, x, y for (1.1,2) are discussed in /7/.

Now we will consider in more detail the cross-section normalization and the weak form factor $\rho_C(p)$ introduced in (2.1):

$$\rho_C(+1) = \rho_C(S\cdot x,\ Q^2,\ U\cdot x),$$
$$\rho_C(-1) = \rho_C(U\cdot x,\ Q^2,\ S\cdot x). \qquad (2.9)$$

As input parameters, we use the fine-structure constant α , muon decay constant G_μ , and Z- boson mass M_Z . The mass of W- boson and weak mixing angle are derived quantities:

$$M_W = M_Z\left\{ \frac{1}{2} + \frac{1}{2}\left[1 - \left(\frac{74.562}{M_Z} \right)^2 (1 - \delta r)^{-1} \right]^{1/2} \right\}^{1/2}, \qquad (2.10)$$

$$\sin^2\theta_W = 1 - M_W^2 \ / \ M_Z^2 \ . \qquad (2.11)$$

The role of δr has been discussed in /2/ and refs. quoted therein. The connection of (2.1) with a notation using the input parameters (α, M_Z , M_W) may be established as follows:

$$\frac{G_\mu^2}{\pi} M_W^4\ \rho_C^2 + O(\alpha^2) = \frac{2}{\pi} \left(\frac{g}{8}\right)^2 \rho_W^2 \equiv \frac{2\pi\alpha^2}{(2\sin^2\theta_W)^2}\ \rho_W^2 \ , \qquad (2.12)$$

$$\rho_C = (1 - \delta r)\ \rho_W \ , \qquad (2.13)$$

where ρ_W would be the corresponding form factor in the other scheme. The main advantage of using G_μ in (2.1) is a reduction of radiative corrections by about 14 % /3,6/. The ρ_C is explicitly given in Appendix A. An exact definition deserves several comments and will be based on the following ansatz:

$$\rho_C = (1 - \delta r)\cdot(\ \mathcal{F}_C - \delta\rho_{QED}\ ), \qquad (2.14)$$

$$\delta\rho_{QED} = \frac{\alpha}{4\pi}\ [\ 2\ A(S,Q^2,U;m_f)\ P_{IR} + B(m_f;M_W) + L_W C(S,Q^2,U;M_W^2)\ ]\ , \qquad (2.15)$$

$$L_W = \ln\frac{Q^2}{M_W^2}\ . \qquad (2.16)$$

In (2.14), the gauge-invariant quantity $\mathcal{F}_C$ represents the sum of all Born plus one-loop contributions to (1.1) /9/. This includes also the diagrams with additional internal photon exchange. As is well- known, the W- boson has both weak and electrodynamic interactions which as a whole cannot be uniquely separated. But one can uniquely isolate the infra-red divergency:

$$A(S,Q^2,U;m_f) = Q_e^2 + Q_d^2 + Q_u^2 + 2|Q_u Q_d|\ln\frac{Q^2}{m_d m_u} \qquad (2.17)$$

$$- 2|Q_eQ_u|\ln\frac{S}{m_em_u} - 2|Q_eQ_d|\ln\frac{|U|}{m_em_d} \ ,$$

$$P_{IR} = \frac{1}{n-4} + \frac{1}{2\gamma} + \ln\frac{M_W}{2\sqrt{\pi}\eta} \equiv \ln\frac{M_W}{\lambda} \ , \qquad (2.18)$$

where λ is a finite, small photon mass used as IR-regulator e.g. in /4/. One can also uniquely isolate the logarithmic fermion mass singularities due to the internal photon exchange:

$$B(m_f;M_W) = Q_e^2\ln\frac{m_e^2}{M_W^2} + Q_u^2\ln\frac{m_u^2}{M_W^2} + Q_d^2\ln\frac{m_d^2}{M_W^2} + b(e,u) + b(e,d) - b(u,d),$$

$$b(e,u) = -\frac{1}{2}|Q_eQ_u|\cdot\left[\ln^2\frac{m_e^2}{M_W^2} + \ln^2\frac{m_u^2}{M_W^2} - 6\ln\frac{m_em_u}{M_W^2}\right] . \qquad (2.19)$$

It is quite natural to combine (2.17, 2.19) with corresponding bremsstrahlung terms since they are genuine QED corrections and the IR-divergency will be cancelled subsequently. Finally we decided to exclude from the weak form factor the following terms in (2.15):

$$C(S,Q^2,U;\ M_W^2) = |Q_uQ_d|(L_W - 3) - |Q_eQ_u|(L_W - 2\ln\frac{Q^2}{S}) -$$
$$- |Q_eQ_d|(L_W - 2\ln\frac{Q^2}{|U|}). \qquad (2.20)$$

These terms are potentially large at small values of Q^2,S.|U| and will be compensated for by corresponding terms from the bremsstrahlung part so that their combined contribution to the cross-section vanishes. We decided not to include them artifically into all the partial corrections.

As indicated in (2.14), the three parts of (2.15) are combined with bremsstrahlung. The rest of the weak loops together with δr is called here one-loop corrections. They are shown in Fig. 1 as $\delta^{\pm}_{1L}(W)$ for positron (electron) scattering off protons,

$$\delta^{\pm}_{1L}(W) = \left(\frac{d^2\sigma_{1L}}{dx\ dy}\Big/\frac{d^2\sigma_0}{dx\ dy} - 1\right)\cdot 100\ \% \ . \qquad (2.21)$$

The $d^2\sigma_{1L}$ is defined from (2.1) taking into account only the case b = 0. The $d^2\sigma_0$ is the Born cross section where additionally $\rho_C(p)=1$.

Parameters chosen are:

$$M_Z = 93\ \text{GeV},\quad m_t = 60\ \text{GeV},\quad M_H = 100\ \text{GeV}, \qquad (2.22)$$

where m_t and M_H are the masses of t-quark and Higgs boson, resp. As one should expect from the above discussion, the one-loop corrections are smooth functions of x and y over the full kinematic region. They do not exceed the order of 5 % over a wide range of x and y and are

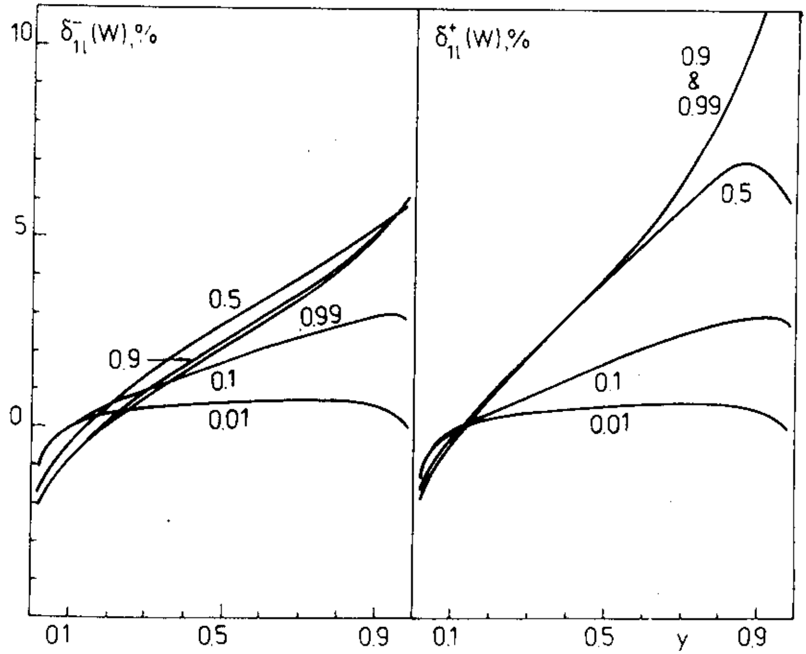

Fig. 1: The one-loop electroweak corrections $\delta^{\pm}_{1L}(W)$ of (2.21) to the charged current deep inelastic $e^{\pm}p$ cross-sections for $S=10^5 GeV^2$ and $\lambda=0$.

especially small at small x and intermediate y. The valence quark part to positron scattering is proportional to $(1-y)^2$ in Born approximation. This explains the steep rize of $\delta^{+}_{1L}(W)$ at large y and large x where the naturally dominating valence quarks are suppressed and thus the denominator of (2.21) becomes extremely small. At smaller x, the sea quark part becomes also more important. In Born approximation it is proportional not to $(1-y)^2$ but to 1. This regulates the behaviour of $\delta^{+}_{1L}(W)$ at large y, small x. Our Fig. 1 is in crude agreement with Fig. 7 of /4/ if one takes into account the different calculational scheme and the slightly different treatment of QED.

3. The bremsstrahlung corrections

The bremsstrahlung cross-section arises from the coherent sum over radiation from the lepton leg, initial and final quark legs and from the exchanged W- boson. In contrast to the neutral current case, there are no gauge invariant separated contributions due to lepton leg radiation, quark leg radiation, and their interference. Nevertheless, the full, pure bremsstrahlung contribution is a gauge-invariant quantity. Additionally, it is also infrared-finite after combination with the corresponding divergent terms (2.17). We will not repeat here the technical remarks concerning the integration over the photon degrees of freedom /2/. The tables of bremsstrahlung integrals which have been quoted there have to be completed for the charged current case by the bremsstrahlung integrals given in Appendix C. The cross-section depends on the charges of the lepton Q_e, the initial and final quarks Q_Q , $Q_{Q'}$, and of the W- boson Q_W. Due to charge conservation, only two of them are independent while the others may be eliminated: $Q_{Q'} = Q_Q + Q_W$, $Q_W = Q_e$. As a consequence of this choice, we are faced with three types of bremsstrahlung contributions as is indicated in (2.1): the terms for b = e are proportional to Q_e^2 , for b = q to Q_Q^2 , and for b = i to $Q_e Q_Q$. The above remarks explain that in a strict sense one should not call them lepton and quark leg radiation and interference of them though they have exactly the behaviour one would expect. In combining the $\delta\rho_{QED}$ of (2.15) due to loops with bremsstrahlung, one also has to rewrite it in terms of the charges Q_e and Q_Q only.

The general structure of the QED functions in (2.1) is:

$$\overset{(-)}{R}_C = \frac{\alpha}{\pi} \left\{ \overset{(-)}{S}_C f_Q(x,Q^2) + \int_1^{1/x} dz \left[\overset{(-)}{T}_C \frac{r_C \cdot f_Q(zx,Q^2) - f_Q(x,Q^2)}{z-1} + \right.\right.$$

$$\left.\left. + \overset{(-)}{U}_C \frac{1}{z} f_Q(zx,Q^2) \right]\right\} , \quad r_C = \{ z,1,1 \}, \quad c = e,\ i,\ q. \tag{3.1}$$

As an immediate consequence of the present handling, the quark-type QED functions $\overset{(-)}{R}_q$ are essentially the same as those of the neutral current case:

$$S_q = S_q(1,y_1) , \qquad \bar{S}_q = S_q(-y_1, -1) , \tag{3.2}$$

$$S_q(a,b) = S_q(Z;a,b) + a^2(2- \frac{\pi^2}{6}), \tag{3.3}$$

$$T_q = T_q(Z;1,y_1) ; \quad \bar{T}_q = T_q(Z;-y_1, -1), \tag{3.4}$$

$$U_q = U_q(Z;1,y_1) ; \quad \bar{U}_q = U_q(Z;-y_1, -1), \tag{3.5}$$

where the neutral current functions in (3.3-5) are given in Appendix C of /2/. The constant deviation of $S_q(a,b)$ from $S_q(Z;a,b)$ is due to the fact that the corresponding loops with virtual photon exchange have been treated slightly differently. The logarithmic quark mass singularities are due to initial state radiation. They depend only on $\ln(Q^2/m_Q^2)$ containing the initial state quark mass, but not on $\ln(Q^2/m_{Q'}^2)$.

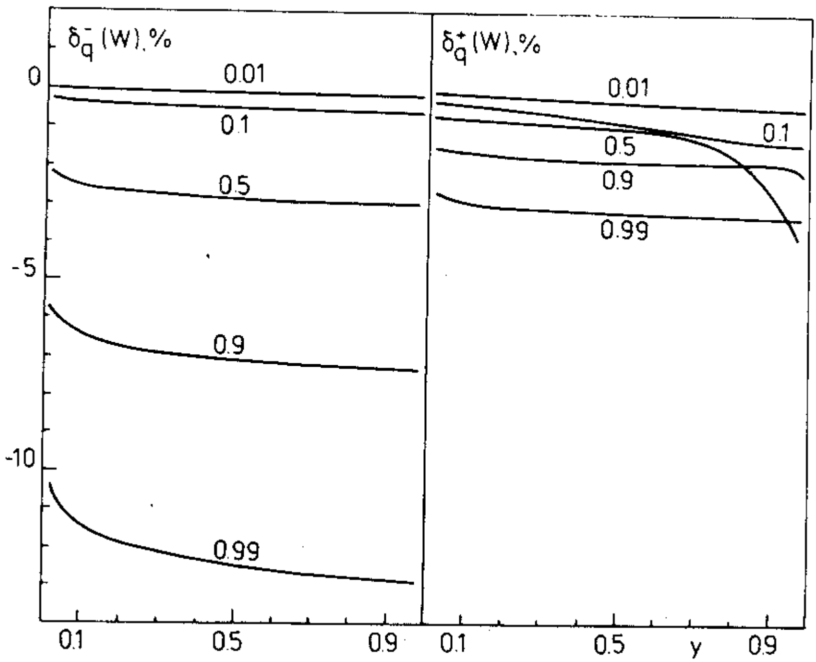

Fig. 2: The quark-type bremsstrahlung corrections $\delta_q^{\pm}(W)$ of (3.6). Parameters as in Fig.1.

The bremsstrahlung corrections for positron (electron) scattering are defined as follows:

$$\delta_b^{\pm}(W) = \left(\frac{d^2\sigma_b}{dx\,dy} \Big/ \frac{d^2\sigma_o}{dx\,dy} \right) \cdot 100\ \% \ , \ b = e,i,q. \qquad (3.6)$$

These corrections are shown for b = q in Fig. 2. The process $e^- + U(\bar{D}) \longrightarrow \nu_e + D(\bar{U}) + \gamma$ has corrections which equal those known from $\delta_q^{\pm}(Z)$ as shown in /2/. In fact, the formulae are essentially the same, see (3.1-5). The deviating features of charged current positron scattering, $e^+ + D \longrightarrow \bar{\nu}_e + U + \gamma$, $e^+ + \bar{U} \longrightarrow \bar{\nu}_e + \bar{D} + \gamma$, have two-fold origin. The dominating contribution at not too large y is due to valence quark scattering. As a consequence, the additional charge factor Q_Q^2 in the numerator of (3.6) is for $\delta_q^+(W)$ only one fourth of that in $\delta_q^-(W)$. If y approaches 1, the factor y_1^2 vanishes in front of the D-quark distributions. The factor 1 which scales the $\bar{U}$-quark distributions survives. As a result, in this limit the $\delta_q^+(W)$ approaches finally the $\delta_q^-(W)$. At smaller x where sea-quarks tend to dominate this tendency sets in earlier.

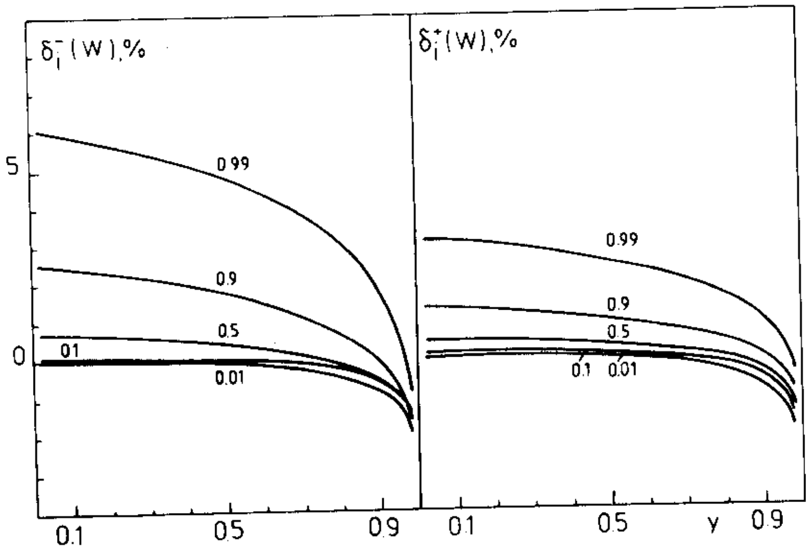

Fig. 3: The lepton-quark interference type bremsstrahlung corrections $\delta_i^{\pm}$ (W) of (3.6). Parameters as in Fig. 1.

The interference type QED functions, b = i, are given in App. B. They have no mass singularities. The corrections $\delta_i^{\pm}(W)$ are shown in Fig.3. Their behaviour is smooth for small y and arbitrary x while there is some kinematic divergency at y = 1. For y <0.98, the $\delta_i^{\pm}(W)$ are of the order of 6% or smaller. Again, there is some similarity with the neutral current correction $\delta_i(Z)$ in /2/. The difference is mainly due to a shift of $\delta_i^{\pm}(W)$ compared to $\delta_i(Z)$ of several percent which shows some dependence on x and, less significant, on y. Similarly to the quark type corrections, the $\delta_i^-(W)$ is about twice as large as $\delta^{+}_{l}(W)$ due to the charges of the dominating valence quark contributions.

The contribution from bremsstrahlung of the electron leg type, b = e, yields the numerically largest correction. The corresponding formulae may be found in App. B. The fermion mass singularities of these terms are only due to the electron mass. The numerical behaviour of the corrections is shown in Fig.4. Since the most

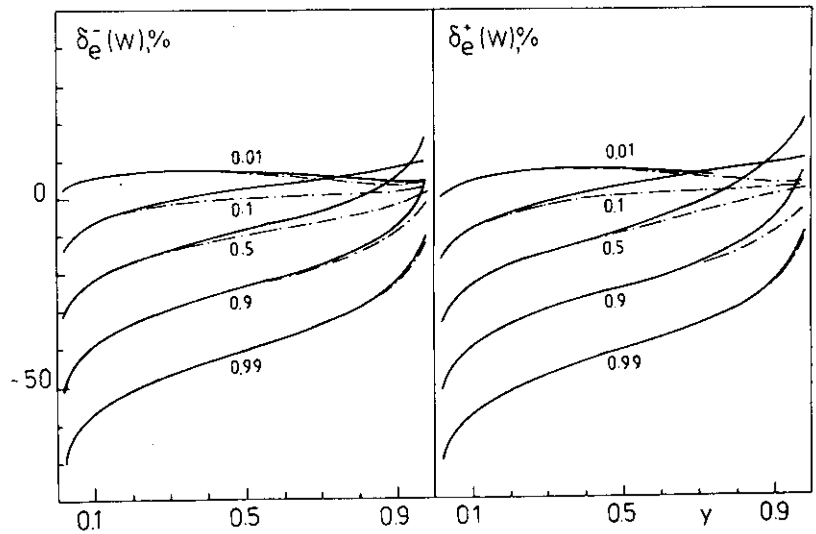

Fig. 4: The lepton-type bremsstrahlung corrections $\delta^{\pm}_{l}(W)$ of (3.6). Parameters as in Fig.1.

important difference to the neutral current case, radiation from the W- boson, does not produce additional singularities, there is great similarity to the analogue corrections due to Z- boson exchange $\delta_e(Z)$ in /2/. In Fig.4 we also show the low Q^2 limit of the lepton type bremsstrahlung corrections which has been obtained analytically from $\langle\bar{R}\rangle_e$ of (3.1). This is a rather instructive limit due the presence of positive powers of $R_W = M_W^2/Q^2$ which must cancel exactly for $Q^2 << M_W^2$. In the figure, the low Q^2 result is shown for all x and y. It becomes adequate for small x where Q^2 is small, but it is also very good at small y and at large x values. This behaviour one expects from the fact that the small y , large x region is dominated by soft photon radiation where corrections as defined in (3.6) should only weakly depend on Q^2. As has already been discussed in connection with Figs.1,2, due to the specific properties of the charged current scattering processes, there appear some peculiarities of corrections at large y. For $\delta_e^{\pm}(W)$, it is only the hard bremsstrahlung functions $U_e^{(-)}$ which are not Born-like connected with the kinematic factors 1, $(1-y)^2$. For e^+d and $e^-\bar{d}$ scattering, in the small x, large y limit the contribution of U_e to $\delta_q^{\pm}(W)$ is rising proportional to $1/(1-y)$. For $e^+\bar{u}$, e^-u scattering, the corresponding contribution due to U_e is falling in this region. The resulting mismatch leads to the somewhat tricky behaviour at large y in Fig. 4. This behaviour is also expected to be dependent on the special choice of quark distribution functions.

4. Discussion

The resulting total electroweak corrections $\delta^{\pm}(W)$ to reactions (1.1),

$$\delta^{\pm}(W) = \left(\frac{d^2\sigma^{\pm}}{dx\, dy} \bigg/ \frac{d^2\sigma_o^{\pm}}{dx\, dy} - 1 \right) \cdot 100\ \% , \qquad (4.1)$$

are gauge-independent and therefore the most certain basis of comparison. They are shown in Fig.5. They are dominated by lepton leg type radiation and show the same general behaviour as is seen in Fig.4. While the corrections are smooth and small at small x, they become steep functions of y at large x. The corrections are numerically large at small y, large x reaching there negative values of several dozens percent. In that region, it is recommended to exponentiate the soft photon part of the lepton leg type radiation as was done in /2/ for the neutral current case. In significant difference to the neutral current cross section, there is no steep rise of corrections at large y, small x which was due to the photon exchange diagrams there. The net corrections for electron and positron scattering have quite similar general behaviour. At large y, the corrections for different values of x approach each other more or less and may develop at small x some instability due to rising dominance of sea quark contributions. In this respect, but also in the overall behaviour, charged current electron (positron) scattering off protons is in close analogy to neutrino scattering as may be seen from a complete calculation /10/ or also from leading log results /11/. The quantitative differences at x —>0, y —>1 are not only due to the obvious fact that the dominating lepton leg type radiation is either from the initial state (ep-scattering) or from the final state (νp-scattering). They also trace back, e.g., to the different energy scale and use of different quark distributions to which $\delta^{\pm}(W)$ is very sensitive in this region.

A comparison of the net corrections (4.1) with independent results is not possible so far because there exist no published figures corresponding to our Fig.5; see /4,5/. Fig.6 in /4/ shows corrections due to leading logarithmic leptonic (Fig.6b) and hadronic (Fig.6c) contributions to (4.1). They look quite different from our Figs.2 and 4 in size (quark type) and also in the general behaviour (lepton type) though one should be careful in drawing definite conclusions. A more detailed comparison of the existing results is highly desirable in view of the importance of the reaction under consideration.

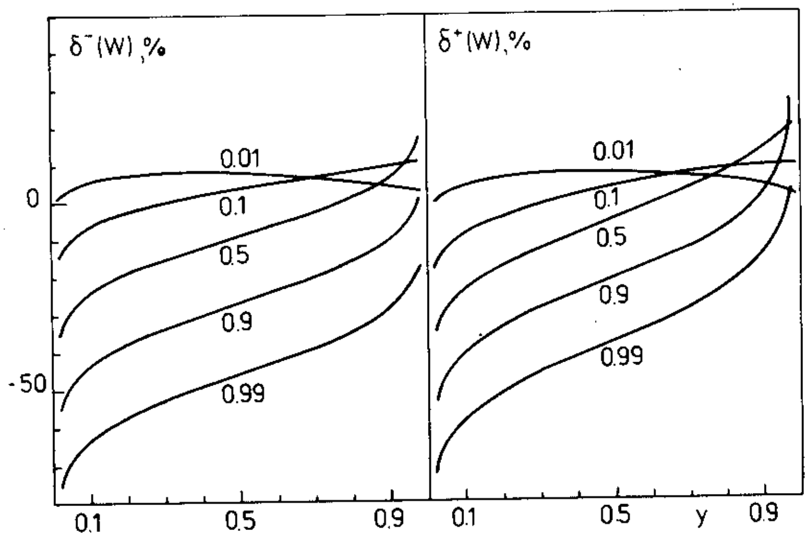

Fig. 5: The resulting total electroweak corrections $\delta^{\pm}(W)$ of (4.1). Paramters as in Fig. 1.

Acknowledgement

We would like to thank A.A.Akhundov and W.Hollik for useful discussions and comments.

Appendix A. The weak form factor ρ

After separation of infra-red divergent terms and of logarithmic mass singularities, the weak form factor $\rho_W(S,Q^2,U)$ for the reaction

$$f_D(Q_j) + f_U(Q_l) \longrightarrow f_{U'}(Q_i) + f_{D'}(Q_k) \tag{A.1}$$

is:

$$\rho_W(S,Q^2,U) = 1 + \delta\rho_W + \delta\rho_W^{QED} , \tag{A.2}$$

$$\begin{aligned}\delta\rho_W(S,Q^2,U) = \frac{\alpha}{4\pi\,\sin^2\theta_W}\Bigg\{ & \bar{D}_W\left(\frac{Q^2}{M_W^2}\right) - W(0) + W(-1) + \frac{5}{8}R(R+1) - \frac{11}{2} \\ & - \frac{9}{4}\frac{R}{1-R}\ln R + \left[-1 + \frac{1}{2R} - \frac{(1-R)^2}{R}\left(|Q_iQ_j|+|Q_kQ_l|\right)\right]\cdot\left[V_1(Q^2,M_Z^2) + \frac{3}{2}\right] \\ & +2R\left[\bar{V}_2(Q^2,M_W^2,M_Z^2) + \frac{3}{2}\right] - (1-R)\left[\left(1+\frac{M_W^2}{Q^2}\right)\cdot\ln\left|1+\frac{Q^2}{M_W^2}\right| + 2\bar{u}(Q^2,M_W^2)\right] \\ & +\left[2 - \frac{1}{R} + 2\frac{(1-R)^2}{R}\left(|Q_iQ_k|+|Q_jQ_l|\right)\right]\cdot S\cdot(Q^2+M_W^2)\cdot B(Q^2,S,M_W^2,M_Z^2) + \\ & +\left[2 - \frac{1}{R} + 2\frac{(1-R)^2}{R}\left(|Q_iQ_l|+|Q_jQ_k|\right)\right]\cdot(Q^2+M_W^2)\cdot\Big[U\cdot B(Q^2,U,M_W^2,M_Z^2) - \\ & - A(Q^2,U,S,M_W^2,M_Z^2)\Big] - \overline{R(Q^2+M_W^2)\cdot\omega(Q^2,M_W^2,M_Z^2)}\Bigg\} .\end{aligned} \tag{A.3}$$

The definitions may be found in /9/. For the problem studied here, it is $Q_i = 0$, $Q_j = Q_e$, $Q_k = Q_d$, $Q_l = Q_u$. In the low energy limit one gets

$$\delta\rho_W(S,Q^2,U) \longrightarrow -\frac{3}{4}\frac{\alpha}{\pi}\left(|Q_jQ_l| + |Q_iQ_k|\right)\cdot\ln R , \qquad S,Q^2,U \ll M_W^2 . \tag{A.4}$$

In this Appendix,

$$R = M_W^2 / M_Z^2 .$$

From the diagrams with additional internal photon exchange we get in the unitary gauge the following contribution to ρ_W:

$$\begin{aligned}\delta\rho_W^{QED}(S,Q^2,U) = \frac{\alpha}{4\pi}\Bigg\{ & \left(|Q_iQ_j|+|Q_kQ_l|\right)\cdot\left[4 - 2\,\mathrm{Li}_2(1) - \pi^2\Theta(-Q^2)\right] \\ & - \left(|Q_iQ_k|+|Q_jQ_l|\right)\cdot\left[\ln^2\frac{|S|}{Q^2} - 2\mathrm{Li}_2(1) - \pi^2\Theta(S)\right] \\ & - \left(|Q_iQ_l|+|Q_jQ_k|\right)\cdot\left[\ln^2\frac{|U|}{Q^2} - 2\mathrm{Li}_2(1) - \pi^2\Theta(U)\right] \\ & - \frac{1}{2} + 2\mathrm{Li}_2(1) - 2\mathrm{Li}_2\left(1+\frac{S}{M_W^2}\right) - 4\mathrm{Li}_2\left(-\frac{Q^2}{M_W^2}\right) - 4\ln\left|1+\frac{Q^2}{M_W^2}\right|\cdot\ln\frac{|S|}{M_W^2}\end{aligned}$$

$$-2\left(|Q_i Q_l|+|Q_j Q_k|\right)\left[Li_2\left(1+\frac{U}{M_W^2}\right) - Li_2\left(1+\frac{S}{M_W^2}\right) + 2\ln\left|1+\frac{Q^2}{M_W^2}\right|\cdot\ln\frac{|U|}{|S|}\right.$$

$$\left. + (Q^2+M_W^2)\cdot A_o(Q^2,U,S,M_W^2)\right]\Bigg\} . \qquad (A.5)$$

The A_o may be found in /9/.

Appendix B. The QED functions

The lepton-quark interference type QED functions are:

$$\overset{(-)}{S}_l = \overset{(-)}{a}{}^2\left[\frac{3}{2} - \frac{\pi^2}{2} - \frac{1}{2}\ln^2 y_1 + \ln\frac{x_1}{x}\left(2\ln y_1 - \ln\frac{x_1}{x} - \frac{3}{2}\right)\right], \qquad (B.1)$$

$$a = 1, \quad \bar{a} = y_1, \qquad (B.2)$$

$$\overset{(-)}{T}_l = \overset{(-)}{a}{}^2\left[2\ln\frac{y_1}{z-1} - \frac{3}{2}\right], \qquad (B.3)$$

$$U_l = \frac{1}{2}y^2\left(1-\frac{1}{z}\right) + 3y_1 + \frac{y}{z} + \frac{z}{z_1}y^2(1+R) + R\left(-2y - \frac{1}{z} - \frac{3}{z^2} + 2\frac{y}{z^2}\right)\cdot L_z$$

$$+ R\left[-\frac{3}{z} + 6\frac{y}{z} + 6\frac{y_1}{z^2} + y^2\cdot\frac{(z-1)^2}{z^2}\right]\cdot\left[\left(1+\frac{R}{z}\right)\cdot L_z - 1\right] + 2\frac{z}{z-1}\cdot\ln\frac{z_2}{y_1(R+z)}$$

$$+ \frac{1+R}{z-1}\left[y^2R + y(y-2)z\right]\cdot\ln\frac{z_2}{z_1(R+z)} + 2\frac{zR}{R+z}\cdot\ln\frac{z_2}{(z-1)R}, \qquad (B.4)$$

$$\bar{U}_l = \frac{3}{2}(1+y_1^2) - \frac{3}{2}\frac{1}{z}(1-y_1^2) + \frac{R}{z}\left[-1+\frac{1}{z}(y^2-3y_1^2)\right]\cdot L_z$$

$$+ \frac{R}{z}\left[-1-2y_1+\frac{1}{z}\left(y^2+6y_1\right)\right]\cdot\left[\left(1+\frac{R}{z}\right)\cdot L_z - 1\right] + \left(1 - y_1^2 - \frac{y^2}{z}\right)\cdot$$

$$\cdot\left[z\,\frac{R+1}{R+z}\cdot\ln\frac{z_2}{(z-1)R} - L_z\right] + y_1^2\cdot\left[2\frac{z}{z-1}\cdot\ln\frac{z_2}{y_1(R+z)} + 2\frac{zR}{R+z}\cdot\ln\frac{z_2}{(z-1)R}\right], \qquad (B.5)$$

where

$$z_1 = z - y, \quad z_2 = z\cdot y_1 + z_1\cdot R .$$

and

$$L_z = \ln\left(1+\frac{z}{R}\right) . \qquad (B.6)$$

In this Appendix,

$$R = M_W^2/Q^2 .$$

The electron type bremsstrahlung may be expressed by

$$\overset{(-)}{S}_e = \overset{(-)}{a}{}^2\left[\frac{3}{4} - \frac{\pi^2}{3} + \ln\frac{y_1}{y}\cdot\left(\ln\frac{yx_1^2}{x^2y_1} + 1\right) - \frac{1}{2}\ln\frac{x_1}{x}\left(\ln\frac{x_1}{x} + \frac{7}{2}\right) - \right.$$

$$\left. - \frac{3}{2}L_W + \left(\ln\frac{yx_1}{xy_1} + \frac{3}{4}\right)\cdot L_e\right], \qquad (B.7)$$

with

$$L_e = \ln\frac{Q^2}{m_e^2}\ . \tag{B.8}$$

The lepton type functions are the only place where in the resulting QED-corrections the logarithms L_w and L_e occur. The L_w is defined in (2.16). It becomes singular when Q^2 approaches zero. The L_e , instead, is singular for large Q^2 . In view of the large negative corrections due to single soft photon radiation at small y, large x, one should apply a soft photon exponentiation procedure as proposed for the neutral current case /5,2/. This has not been done so far.

$$\overset{(-)}{T}_e = \overset{(-)}{a}{}^2 \left[L_0 - \frac{7}{4} \right] \ , \text{ where } \quad L_0 = L_e + \ln\frac{y_1^2}{y^2(z-1)}\ , \tag{B.9}$$

$$\begin{aligned}
U_e = &- zL_0 + 3 - \frac{7}{2} y + \frac{5}{4} y^2 + \left(\frac{1}{2} y_1 + y^2 \right) R + \left[y - \frac{1}{4} - \frac{y}{2} (2-y) R \right] z \\
&- \frac{1}{4} \frac{y^2}{z} - \frac{3}{2} \frac{y_1}{z} - \frac{1}{2} \frac{y_1 (1+R^2)}{R+z} + y^3 \frac{(1+R)}{z-y} + \frac{1}{2} \frac{yz^2}{z_2} \left[(1+R)^2 + y_1(-1+4R+3R^2) \right] \\
&- \frac{yy_1 z^3}{z_2^2} (1+R^3) - \frac{1}{2} \frac{y^2(1+R)}{z_1} (z + yR) \cdot L_1 + 2\frac{z}{z-1} \ln\frac{z_2}{y_1(R+z)} \\
&+ \frac{1}{2}\left\{ z\left[2-y(1+R)^2\right] - y^2 R(1+R) - \frac{z^2 yR}{z_2}\left[2-(1+R)^2\right] + \frac{z^3 yy_1}{z_2^2} (1+R)(1+R^2) \right\} \cdot L_2 \\
&+ \frac{R+1}{z-1} \left[y^2 R + y(y-2)z \right] \ln\frac{z_2}{z_1(R+z)} \\
&+ \left\{ -y_1 + \frac{1}{2} \frac{y_1}{z} + R\left[-\frac{1}{2}(1+y) - y^2 + \frac{1}{2} \frac{y(3y-4)}{z} - \frac{1}{2} \frac{y^2+6y_1}{z^2} \right] \right\} \cdot L_z \\
&+ R\left[\frac{1}{2} - \frac{3}{2} y + \frac{5}{2} y^2 + \frac{1}{z}\left(-6 + 11y - \frac{7}{2} y^2\right) + \frac{3}{2} \frac{y^2+6y_1}{z^2} \right] \cdot \left[\left(1 + \frac{R}{z}\right) \cdot L_z - 1 \right],
\end{aligned} \tag{B.10}$$

where

$$L_1 = L_e + \ln\frac{z_1^2}{y^2(z-1)} \quad \text{and} \quad L_2 = L_e + \ln\frac{z_2^2}{y^2(z-1)R(R+z)}\ , \tag{B.11}$$

$$\begin{aligned}
\overline{U}_e = &- y_1^2 z L_0 + 3 - \frac{1}{2} y + \frac{3}{4} y^2 + \frac{1}{2} \left(1-3y-y^2\right) \cdot R + \left(-\frac{1}{4} - \frac{5}{2} y + \frac{7}{4} y^2 \right) \cdot z \\
&+ y\left(1 + \frac{1}{2} y \right) R \cdot z + 2yz_1 R^2 - \left(\frac{3}{2} + \frac{5}{2} y - \frac{7}{4} y^2 \right) \frac{1}{z} - \frac{1}{2} y_1^3 (1+R^2) \frac{1}{z+R} \\
&+ \frac{yz_1^2}{z_2} \cdot \left[R - 2R^2(1+R) + \frac{1}{2} y(1+R^2) \right] - \frac{yy_1 zz_1^2}{z_2^2} \cdot (1+R^3)
\end{aligned}$$

$$+ 2\,\frac{y_1^2 z}{z-1}\,\ln\frac{z_2}{y_1(R+z)} + y_1 z\left\{1 + \frac{yz_1}{z_2}\cdot\left[1 - R + \frac{1}{2}\frac{yz_1}{z_2}(1+R^2)\right]\right\}\cdot L_z$$

$$+ \left\{-1-y + \frac{1}{2}y^2 + \frac{1}{2z}(1+y+y^2) + R\left[-\frac{1}{2}y_1 + \frac{1}{2}\frac{y(y-4)}{z} - \frac{1}{2}\frac{6-14y+5y^2}{z^2}\right]\right\}\cdot L_z$$

$$+ R\left[\frac{1}{2}y_1 + \frac{1}{z}\left(-6+5y-\frac{1}{2}y^2\right) + \frac{3}{2}\frac{y^2+6y_1}{z^2}\right]\cdot\left[\left(1+\frac{R}{z}\right)\cdot L_z - 1\right], \qquad (B.12)$$

In the limit $Q^2 \ll M_W^2$, we reproduce here the most complicated of the QED functions. They get the following simple form:

$$\frac{1}{z}U_e = \frac{1}{4}yy_1 + \frac{2}{z-1}\ln\frac{z_1}{y_1} + y(1+y-\frac{1}{4}y^2)\frac{z}{z_1} + \frac{yy_1(3+y)z^2}{2z_1^2} +$$

$$+ \frac{3yy_1^2z^3}{2z_1^3} + \left(-y\frac{z}{z_1} - \frac{yy_1z^2}{2z_1^2} - \frac{yy_1^2z^3}{2z_1^3}\right)\cdot L_1, \qquad (B.13)$$

$$\frac{1}{z}\bar{U}_e = -\frac{3}{4}yy_1 + \frac{2y_1^2}{z-1}\ln\frac{z_1}{y_1} + \left(\frac{1}{2}y\,y_1 - \frac{yy_1^2z}{2z_1}\right)\cdot L_1, \qquad (B.14)$$

Appendix C. A bremsstrahlung integral

The bremsstrahlung integrals have been calculated using method and tables of integrals as have been quoted in /2/. There have been left to be calculated two additional integrals which read in the notations of the quoted references as follows:

$$\left[\frac{Z_-^2}{|M^2-m^2|^2}\right] = \frac{1}{2i\Gamma_V M_V}\left\{\left[\frac{Z_-^2}{M^2-m^2}\right] - \left[\frac{Z_-^2}{M^2-m^2}\right]^*\right\}, \qquad (C.1)$$

$$\left[\frac{Z_-^2}{M^2-m^2}\right] = \left[\frac{1}{M^2-m^2}\right]\frac{1}{X^2}\left\{t^2S_X^2 + (-X+M^2)\cdot\left[-XS_t^2 + 2tS_tS_X + \right.\right.$$

$$\left.\left. + M^2\left(S_t^2 - 6\frac{tX_tSS_X}{X^2}\right)\right]\right\} - \frac{S_X^2}{X}\left(3\frac{t^2}{X^2} - \frac{tS_t}{X\tau} - \frac{1}{2}\frac{S_t^2}{\tau^2}\right)$$

$$+ \frac{S_X(X-M^2)}{X^2}\left(\frac{S_t^2}{\tau} - 6\frac{tX_tS}{X^2}\right). \qquad (C.2)$$

The integral [A] is performed over the photon angles (θ_R, ϕ_R) in the system defined by the three-momentum relation $p(Q')+p(\gamma)=0$, where Q' and γ are produced quark and photon:

$$[A] = \int_{-1}^{+1} d\cos\theta_R \int_0^{2\pi} d\phi_R\,\frac{S-X}{4\pi\tau}\,A(\theta_R,\phi_R). \qquad (C.3)$$

In the scattering channel considered here, the integral (C.1) may also be got from (C.2) by differentiation with respect to $M^2 = M_V^2 - iM_V\Gamma_V \approx M_V^2$.

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