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HADRONIC PART OF THE MUON ANOMALOUS MAGNETIC MOMENT: AN IMPROVED EVALUATION

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I. Introduction

Anomalous magnetic moments of leptons are traditional quantities for extremely detailed confrontation of QED predictions with experimental results. Unlike the electron g-2 factor, which to precision achieved is a pure leptonic effect, the consequence of a relatively large muon mass is that the interaction of non-lepton-photon origin contributes to the total muon anomaly a_μ at the level of $6.10^{-3}\%$. Due to the precise QED calculations up to four loops[1,2], yielding

$$a_\mu(\mathrm{QED}) = (116\,584\,800 \pm 30)\mathrm{x}10^{-11} \qquad (1)$$

as well as highly accurate measurements[3]

$$\begin{aligned} a_{\mu^+} &= (116\,591\,000 \pm 1200)\mathrm{x}10^{-11} \\ a_{\mu^-} &= (116\,593\,600 \pm 1200)\mathrm{x}10^{-11}\ , \end{aligned} \qquad (2)$$

this number is by far not negligible. In fact it is about six times larger than the experimental uncertainty in a_μ value. The non - QED part of the muon anomaly is dominated by the lowest-order hadronic vacuum-polarization contribution a_μ^{vac} (Fig. 1). In spite of the gradual diminishing of the error of this component in recent years[2,4], it remains to be known with the error four times larger than the pure QED part. As it has been stressed in Ref.2, to make the theoretical value of the hadronic part of a_μ more precise is crucial for the possibility to detect in measured anomaly the one-loop weak-interaction contribution evaluated as[2]

$$a_\mu(\mathrm{weak}) = (195 \pm 1)\mathrm{x}10^{-11}. \qquad (3)$$

Since a new generation of g-2 experiments with considerably improved precision is under consideration[5], it is desirable to come up with the accuracy of a_μ^{vac} as close as possible to the accuracy level of the QED contribution, thus enabling to perform an important independent test of the GWS electroweak gauge theory.

In the present work we describe an attempt to diminish the error of the lowest-order vacuum-polarization contribution to a_μ, induced by hadrons.

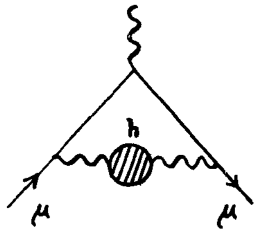

Figure.

Lowest-order hadronic vacuum polarization contribution to a_μ.

There are a few reasons one could hope to achieve this goal. First, we have developed global analytic models for pion and kaon form factors in recent years. The models formulated in terms of physical parameters reproduce the data simultaneously in the space-like and time-like regions. We use these parametrizations for the evaluation of the two-pion and two-kaon contributions to a_μ^{vac} including in this way the experimental information from the space-like region. Another reason for a possible accuracy improvement of the theoretical value of a_μ^{vac} is that besides new data on pion and kaon form factors significantly better data on the three-pion e^+e^- annihilation became available recently due to new measurements in Novosibirsk [13]. Last but not least we believe that there are possibilities to perform the error analysis in the individual channels contributing to a_μ^{vac} in a more quantitative and systematic way than it has been done in the previous works[2,4].

We describe our treatment of a_μ^{vac} and the corresponding error analysis in Sec. III, while the final results with their discussion are given in Sec. IV. In the following section the results of the last calculations[2,4] are briefly summarized.

II. Present knowledge of hadronic contributions to a_μ

The hadronic part of a_μ has been known with gradually higher

precision in connection with the improvement of information on the cross section $\sigma(e^+e^-\to$ hadrons) from the experiments on e^+e^- colliders in Novosibirsk, Frascati and Orsay. Relevance of e^+e^- annihilation measurements to a_μ^{vac} is based on the fact that $\sigma(e^+e^-\to$ hadrons) enters into the integral representation which serves as a basis for all calculations of a_μ^{vac} (see Eq. (5) below). The last two evaluations of a_μ^{vac} have been done in the year 1985 and read

$$a_\mu^{vac} = (7070 \pm 60 \pm 170)\times 10^{-11} \qquad \text{(KNO)} \qquad (4a)$$
$$a_\mu^{vac} = (7100 \pm 105 \pm 49)\times 10^{-11} \qquad \text{(CLY)} \qquad (4b)$$

where the first error is statistical, the second is systematic and the abbreviations refer to Kinoshita et al.[2] and Casas et al.[4]. In what follows we characterize the main features of both the analyses.

The first group of authors[2] has calculated the contributions from individual channels of the reaction $e^+e^-\to$ hadrons separately. The four-parameter modified Gounaris-Sakurai parametrization of a pion form factor has been used for the dominant two-pion part. While the statistical error has been evaluated by the covariance matrix of the fit (χ^2/D.F.= 1.85), the systematic error has been assessed from the deviation of the mean values of a_μ in the two methods (the second one being the trapezoidal integration over the experimental points). The result 150.10^{-11} is the main contributor to the total error in (4a). The low-energy three-pion and two-kaon parts of a_μ^{vac} were treated by the Breit-Wigner formula for the ω and ϕ resonances. The statistical error was estimated from the statistical errors of the measured total and electronic widths. The systematic error has been taken equal to the systematic error of both the widths. The same error estimates were done also for the contributions of the J/Ψ and Υ resonances, treated in the narrow width approximation. The contributions of other channels have been obtained by the trapezoidal-rule integration over the experimental data for $R = \sigma(e^+e^-\to \text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ whose errors were taken as an error estimate for this part of a_μ^{vac}.

The second group of authors[4] has reduced the essential part of the errors coming from the region $s>2$ GeV^2 by employing the

$O(\alpha_s^2)$ QCD expression for the quantity R. The error in this treatment comes from the uncertainty in the value of the QCD scale parameter Λ and from the neglected higher-order terms in R. The J/ψ and Υ resonances were evaluated in the narrow-width approximation and regions of $c\bar{c}$ and $b\bar{b}$ thresholds by the experimental data on R.

The integration over experimental points has been used also in the region $0.8\ \mathrm{GeV}^2 \leq s \leq 2\ \mathrm{GeV}^2$ for the 2π, 3π, 4π, 5π, 6π, K^+K^- and $K_S^0K_L^0$ channels. The largest statistical error (~17 %) was found for the three-pion contribution and attributed to experimental uncertainties in the ϕ region. The total systematic error from this region was given implicitly in the overall systematic error of the a_μ^{vac} value (4b).

A great deal of Ref. 4 is devoted to the thorough numerical study of the dominant low-energy two-pion contribution to a_μ. It is performed in terms of a 15-parameter pion form factor representation written as a product of the Omnés function and the inelastic part with correct analytic properties, normalization and the asymptotic behaviour. The inelastic part is parametrized in terms of higher vector meson contributions, a three-parameter background function and a function providing the asymptotic behaviour of $F_\pi(t)$. The equality between the form factor phase and the phase $\delta_1^1(s)$ of $I = J = 1$ partial $\pi\pi$ scattering wave for $s \leq 0.8\ \mathrm{GeV}^2$ is used in the integrand of the Omnés function. Two methods for the evaluation of the two-pion part $a_\mu^{2\pi}$ based on different parametrizations of $\delta_1^1(s)$ were applied to assess the systematic error of $a_\mu^{2\pi}$. The value 27.10^{-11} (compared to 150.10^{-11} of KNO) is an essential source of diminishing the total error of a_μ in Ref. 4. The mean value of $a_\mu^{2\pi}$ and its statistical error were obtained by the variational analysis of the experimental data on the form factor inelastic part.

Closing this section we note that though KNO have found smaller statistical errors than CLY in all channels, the latter authors were able to diminish the total error of a_μ^{vac} for essentially two reasons: the use of QCD in the high-energy region and due to taking the deviation of mean values of $a_\mu^{2\pi}$ in two methods as a measure of the systematic uncertainty of the dominant two-pion part of a_μ.

III. Calculation of the lowest-order hadronic vacuum polarization contribution to a_μ

All calculations of a_μ^{vac} are based on the integral representation [6]

$$a_\mu^{vac} = \frac{1}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \sigma^h(s)\, K_\mu(s)\, ds, \qquad (5)$$

where α is the fine-structure constant, $\sigma^h(s)$ stands for the cross section $\sigma(e^+e^- \to \text{hadrons})$ and $K_\mu(s)$ is the function coming from the triangle Feynman diagram for a_μ corresponding to the exchange of a "particle" with the propagator $-i(q^2 - s)^{-1}$:

$$K_\mu(s) = \frac{\alpha}{\pi} \int_0^1 \frac{x^2(1-x)\, dx}{x^2 + (1-x)s/m_\mu^2}. \qquad (6)$$

A decomposition of the integrand to partial fractions leads to the explicit form

$$K_\mu(s) = \frac{\alpha}{\pi}\frac{1}{Sy}\Big\{\Big[\frac{y^2}{2}\cdot(1+S)-y\Big]\Big[1-\frac{y}{2}\cdot(1+S)\Big]\cdot\ln\Big(1-\frac{2}{y(1+S)}\Big) - \Big[\frac{y^2}{2}\cdot(1-S)-y\Big]\Big[1-\frac{y}{2}\cdot(1-S)\Big]\cdot\ln\Big(1-\frac{2}{y(1-S)}\Big) + \frac{Sy}{2} - y^2S\Big\}, \qquad (7)$$

where

$$S = \sqrt{1-4/y}\,, \quad y = s/m_\mu^2.$$

As can be seen from Eq. 6, $K_\mu(s)$ behaves as $(\alpha/\pi)m_\mu^2/3s$ for $s \gg m_\mu^2$, suppressing in this way the contributions from higher-energy region.

Formula (5) can be derived by replacing the free photon propagator in the $O(\alpha)$ amplitude for a_μ by the exact photon propagator, defined in terms of the (hadronic) polarization operator $\Pi^h(s)$. Writing a dispersion integral for the latter and isolating the invariant function at the tensor structure $\sigma_{\mu\nu}k^\nu$, which at $k^2 = 0$ defines the anomalous magnetic moment (k is the four-momentum of the external photon), one obtains a_μ^{vac} as a superposition of the amplitudes $K_\mu(s)$ with the weight function $\mathrm{Im}\Pi(s)/\pi s$. The usefulness of this representation for a_μ^{vac} follows from the well-known relation

$$\mathrm{Im}\Pi^h(s) = \frac{s\sigma^h(s)}{4\pi^2\alpha} = \frac{1}{12\pi} R(s) \tag{8}$$

providing the possibility to employ rich experimental information from the reaction $e^+e^- \to$ hadrons for the calculation of a_μ^{vac} via the relation (5). As a consequence, the accuracy of the result depends primarily on the precision of the measured cross section for individual annihilation channels. However, as we shall try to demonstrate, one can non-negligibly reduce the errors of a_μ^{vac} by choosing more realistic and adequate models for the cross sections $\sigma^h(s)$.

In our calculation of a_μ^{vac} we have devided the integral in (5) into the low-energy ($s<s_0 = 2$ GeV^2) and high-energy ($s>s_0$) parts. Following CLY we have used QCD in the latter, including, however, the $O(\alpha_s^3)$ term to the perturbative expansion of the ratio $R = \sigma(e^+e^- \to$ hadrons$)/\sigma(e^+e^- \to \mu^+\mu^-)$ and confronting this calculation with the result obtained by integration over experimental data on R (subsection B). As to the chosen position of the point s_0, it is dictated from one side by the validity of perturbative QCD and from the other side by the fact that we are able to estimate 2π and 2K contributions by means of the reliable form factor models in the whole region $4m_\pi^2 \le s \le s_0$, in which the corresponding integrals are saturated almost completely.

A. The low-energy region

We treat each channel in this region separately. In order to achieve realistic and quantitative error estimates, we adhere to the following scheme: the statistic errors will be computed as a rule from the covariance matrices of the corresponding fits while for systematic errors we will take the errors calculated from experimental systematic errors by the CERN program TRAPER which uses the trapezoidal-rule integration over experimental points. To be on safe grounds we add also the second sort of systematic errors , namely those induced by the models used for the cross section $\sigma^h(s)$. They will be called hereafter the model errors. Their actual value in each channel will be determined from the deviation of TRAPER integration in (5) and the integration using model parametrizations for the corresponding cross sections.

According to the remark after Eq.(7) it will be the contribution of the process $e^+e^- \to \pi^+\pi^-$ which will dominate in a_μ^{vac}. Its cross section is given by

$$\sigma^{2\pi}(s) = \frac{\pi\alpha^2\beta^3}{3s} \left| F_\pi(s) + \xi e^{i\phi} \frac{m_\omega^2}{m_\omega^2 - s - im_\omega\Gamma_\omega} \right|^2, \tag{9}$$

where $\beta = (1-4m_\pi^2/s)^{1/2}$ is the velocity of an outgoing pion in CMS and the second term in (9) describes the part of the $\pi^+\pi^-$ final state due to the ω meson. Parameters ξ and ϕ are the ρ-ω interference amplitude and phase, respectively /7/

It turns out that it is of crucial importance to find suitable and adequate parametrization of the complex function $F_\pi(s)$. For example, the modified Gounaris-Sakurai formula used by KNO which takes into account the inelastic ρ-ω channel by the effective factor with three parameters fixed by hand does not give a fully satisfactory description of the data. It manifests itself in a rather large deviation of the final result for the two-pion part $a_\mu^{2\pi}$ from the value obtained by direct integration over the data points of $\sigma^{2\pi}$. Problems with a simultaneous description of the spacelike and timelike pion-form-factor data (and data on δ_1^1) /8/ indicate a possible inconsistency also in the model of CLY caused probably by the choice of the parametrization of the inelastic part of $F_\pi(s)$. The nonadequate description of the data above 1 GeV^2 is likely the reason why the authors compute the contribution from the region $0.8 \le s \le 2$ GeV^2 directly by means of the data instead of the model.

For our calculation of $a_\mu^{2\pi}$ we choose the analytic pion form factor model /9/ formulated in the conformally mapped cut-free variable W:

$$W = i\frac{(q_1+q)^{1/2} - (q_1-q)^{1/2}}{(q_1+q)^{1/2} + (q_1-q)^{1/2}}, \quad q = \tfrac{1}{2}(s-4m_\pi^2), \quad q_1 = q(s_1), \tag{10}$$

where s_1 is the position of the square-root branch point which together with the elastic branch point at $s=4m_\pi^2$, corresponding cuts and complex-conjugated pairs of resonance poles define the pion form factor analytic structure in the complex s-plane (for a more complete treatment of the model see Refs.9). The formula for

F_π reflecting this analytic structure in the W-plane is

$$F_\pi(W) = \frac{(W^2-1)^2(W-W_z)\sum_0^4 A_n W^n}{(W-W_p)\,D_\rho(W)\,D_{\rho'}(W)\,D_{\rho''}(W)} \qquad (11)$$

The factor $(W^2-1)^2$ ensures the asymptotic behaviour $\sim 1/s$, $W_z = 0.21$ and $W_p = 0.23$ simulate the left-hand cut from the second Riemann sheet [10] and

$$D_\rho(W) = (W-W_\rho)(W-W_\rho^*)(W-W_\rho^{-1})(W-W_\rho^{*-1})$$

$$D_v(W) = (W-W_v)(W+W_v)(W-W_v^*)(W+W_v^*), \quad v = \rho', \rho''$$

with W_v ($v=\rho,\rho';\rho''$) being the positions of resonance poles. Five real coefficients A_n can be expressed in terms of resonance masses m_v , widths Γ_v and coupling-constant ratios $g_v = f_{v\pi\pi}/f_v$ ($f_{v\pi\pi}$, f_v correspond to the transitions $v \to \pi^+\pi^-$, $v \to \gamma$, respectively) by requiring correct normalization $F_\pi(0) = 1$ and threshold behaviour $\delta_1^1 \sim q^3$ for $q \to 0$ together with taking into account a connection of VMD pion form factor representation with formula (11) in the limit $\Gamma_v \to 0$ separately for ρ, ρ' and ρ'' resonances [9].

Formula (11) has been compared with 288 data on F_π (see [11], references therein and [12,13]) from the spacelike and timelike regions. The fitted parameters A_i ($i=1,..9$) were Re W_ρ , Re $W_{\rho''}$, Im W_ρ , Im $W_{\rho''}$,coupling constant ratios g_ρ, $g_{\rho'}$, $g_{\rho''}$, position of the effective inelastic threshold s_1 and the modulus of the ρ-ω interference amplitude ξ. The interference phase ϕ can be expressed by m_ρ, Γ_ρ and m_ω [7]. The parameters Re $W_{\rho'}$ and Im $W_{\rho'}$ of the resonance $\rho'(1250)$ have been fixed at the values corresponding to $m_{\rho'} \sim 1310$ MeV, $\Gamma_{\rho'} \sim 400$ MeV which are typical for a few fits with small modifications of formula (11). Presence of the resonance ρ' is important for the quality of the fit, however fixing its parameters is necessary due to the fact that data points are rather scattered in this region and making $m_{\rho'}$ and $\Gamma_{\rho'}$ free would introduce rather strong correlations to the covariance matrix.

The results of the best fit (transformed to the s-plane) are:

$$\begin{array}{lll} m_\rho = 760 \pm 4 \text{ MeV} & \Gamma_\rho = 143 \pm 3 \text{ MeV} & g_\rho = 1.19 \pm 0.03 \\ m_{\rho''} = 1743 \pm 110 \text{MeV} & \Gamma_{\rho''} = 280 \pm 96 \text{MeV} & g_{\rho''} = -0.06 \pm 0.02 \\ s_1 = 1.42 \pm 0.06 \text{ GeV}^2 & \xi = 0.0146 \pm 0.0006 & g_{\rho'} = -0.40 \pm 0.06 \, . \end{array} \qquad (12)$$

A good simultaneous description of the spacelike and timelike data has been achieved with $\chi^2/\text{D.F.} = 1.45$. Numerical evaluation of the integral (5) with K_μ, $\sigma^{2\pi}$ and F_π given in (7), (9) and (11) yields

$$a_\mu^{2\pi} = (4985 \pm 28).10^{-11}, \qquad (13)$$

where the statistical error 28.10^{-11} (to be compared with the values 22.10^{-11} of KNO and 43.10^{-11} of CLY) has been obtained by the formula

$$\sigma^2 = \sum_{ij} C_{ij} D_i D_j \quad , \; D_i = \frac{\partial a_\mu}{\partial A_i} \; . \qquad (14)$$

C_{ij} is the nine by nine external covariance matrix of the fit as given by the Hesse subroutine of the MINUIT program (with the parameter UP adjusted to 9 parameters). The values of diagonal matrix elements have been checked by MINOS subroutine. Evaluating the same integral by the trapezoidal rule we find

$$a_\mu^{2\pi} = (4906 \pm 24)\times 10^{-11}, \qquad (15)$$

where the systematic uncertainty 24×10^{-11} is the error as given by TRAPER integration over the experimental cross sections supplemented by systematic errors of measurements. Its magnitude is close to the value 27×10^{-11} found by CLY who however use in fact a model error as the systematic uncertainty of $a_\mu^{2\pi}$. Our model error is 40×10^{-11} to be compared with the value 150×10^{-11} of KNO obtained by the same method but being used again as the systematic error. Adding our three errors quadratically gives the total error of the dominant two-pion part of a_μ^{vac} to be 54×10^{-11}.

In principle the same procedure can be applied to the two-kaon contributions. A suitable generalized VMD model for the charged and neutral kaon form factors with correct analytic properties has been derived in Ref. 14. The final formulae for the isoscalar (s) and isovector (v) parts read

$$F_K^s(V) = \left[\frac{1-V^2}{1-V_N^2}\right]^2 \sum_{s=\omega,\phi,\phi'} \frac{f_{sK\bar K}}{f_s} \frac{(V_N-V_s)(V_N-V_s^*)(V_N-\bar V_s)(V_N-\bar V_s^*)}{(V-V_s)(V-V_s^*)(V-\bar V_s)(V-\bar V_s^*)} \qquad (16a)$$

$$F_K^v(W) = \left[\frac{1-W^2}{1-W_N^2}\right]^2 \sum_{v=\rho,\rho',\rho''} \frac{f_{vK\bar K}}{f_v} \frac{(W_N-W_v)(W_N-W_v^*)(W_N-\bar W_v)(W_N-\bar W_v^*)}{(W-W_v)(W-W_v^*)(W-\bar W_v)(W-\bar W_v^*)}. \qquad (16b)$$

The variable W is the same as in (10), the variable V is defined in a similar way by means of the three-momentum $r=\frac{1}{3}(s-9m_\pi^2)^{1/2}$. An effective inelastic threshold in the r-plane is assumed analogously to q_1 in the q-plane. The points V_N and W_N correspond to the normalization point $s=0$. The factors in front of the sums in (16a,b) give the asymptotic behaviour $\sim s^{-1}$ to the form factors. The ratios of the VMD coupling constants are restricted by the conditions

$$\sum_{s=\omega,\phi,\phi'} \frac{f_{sK\bar K}}{f_s} = \sum_{v=\rho,\rho',\rho''} \frac{f_{vK\bar K}}{f_v} = \frac{1}{2} \qquad (17)$$

which are the consequence of the normalization of F_K^s, F_K^v. As can be seen from (16a,b), each resonance is represented by four poles lying in the complex V, W planes (i.e. $\Gamma_v \neq 0$, $\Gamma_s \neq 0$) with

$$\bar V_s = V_s^{-1}\ ,\ \bar W_v = W_v^{-1} \quad \text{or} \quad \bar V_s = -V_s\ ,\ \bar W_v = -W_v \qquad (18)$$

depending on the relative position of the resonance and the effective threshold. Finally, the form factors of the charged and neutral kaons are given by linear combinations of F_K^s and F_K^v

$$F_{K^\pm} = F_K^s + F_K^v \quad , \quad F_{K^0} = F_K^s - F_K^v \ . \qquad (19)$$

The number of free parameters of the model can be reduced to 14 by Eq. (17) and by fixing m_ρ, Γ_ρ, m_ω, Γ_ω at their table values as $\rho(770)$ and $\omega(783)$ lie in the unphysical region and one could hardly expect to be able to determine them with a sufficient accuracy from the fit.

The optimal values of the fitted parameters (two inelastic thresholds, four ratios of coupling constants and positions of four resonances in complex V and W-planes) from the analysis of all 138 available data (see Ref. 14) of the charged and neutral kaon form factor are

$$\begin{array}{lll}
m_\phi = 1019.4 \pm 0.7\ \text{MeV} & \Gamma_\phi = 4.3 \pm 0.8\ \text{MeV} & f_{\phi K\bar K}/f_\phi = 0.33 \pm 0.01 \\
m_{\phi'} = 1660 \pm 21\ \text{MeV} & \Gamma_{\phi'} = 158 \pm 37\ \text{MeV} & f_{\omega K\bar K}/f_\omega = 0.20 \pm 0.01 \\
s^s_{inl} = 1.68 \pm 0.03\ \text{GeV}^2 & s^v_{inl} = 1.72 \pm 0.04\ \text{GeV}^2 & \qquad (20) \\
m_{\rho'} = 1315 \pm 183\text{MeV} & \Gamma_{\rho'} = 245 \pm 167\text{MeV} & f_{\rho K\bar K}/f_\rho = 0.57 \pm 0.01 \\
m_{\rho''} = 2114 \pm 140\text{MeV} & \Gamma_{\rho''} = 150 \pm 104\text{MeV} & f_{\rho' K\bar K}/f_{\rho'} = -0.04 \pm 0.01 .
\end{array}$$

The data are reproduced well (χ^2/D.F.= 1.44) for s>0 as well as for s<0. Since kaons are pseudoscalars, the cross section of the reactions $e^+e^- \to K^+K^-$, $K_S^0K_L^0$ is completely analogous to (9) (without the second term, of course) and the rest of the analysis goes as for the pion contribution. The results are

$$\begin{array}{l}
a_\mu^{K^+K^-} = \left[\ 235 \pm 16(\text{stat}) \pm 9(\text{syst}) \pm 3(\text{model})\right] \times 10^{-11} \\
a_\mu^{2K^0} = \left[\ 183 \pm 12(\text{stat}) \pm 9(\text{syst}) \pm 3(\text{model})\ \right] \times 10^{-11}.
\end{array} \qquad (21)$$

The statistical errors are smaller in our approach due to the influence of the space-like data in the fits. Since the data on F_{K^0} do not cover the whole ϕ meson region, the systematic and model errors of the $K_S^0K_L^0$ contribution have been estimated conservatively by the corresponding errors of the K^+K^- one. The model errors are very small reflecting the reliable description of the data by the parametrization (16a,b).

Further important contribution to a_μ^{vac} is the three-pion one. It is this portion which is determined in our work with substantially improved precision and contributes significantly to the reduction of the total error of a_μ^{vac}. The improvement comes from two sources: new precise measurements /15/ of the $e^+e^- \to \pi^+\pi^-\pi^0$

cross section in the ω region performed recently in Novosibirsk and, to some extent, from the use of Breit-Wigner formulae to fit the data on $\sigma^{3\pi}$. Though KNO have also used Breit-Wigner parametrizations for the ω and ϕ resonances, they have performed error estimates by means of the statistical errors of the measured total and leptonic widths of ω and ϕ and not by fitting the experimental cross sections. On the other hand, CLY integrate over experimental data with large resultant statistical error of ~ 17%.

For our calculation of the three-pion contribution we employ the Breit-Wigner parametrization of the form

$$\sigma^{3\pi}(s) = \left| \sqrt{\sigma(\omega)}\, \frac{m_\omega \Gamma_\omega}{m_\omega^2 - s - i s^{1/2}\Gamma_\omega(s)} - \sqrt{\sigma(\phi)}\, \frac{m_\phi \Gamma_\phi}{m_\phi^2 - s - i s^{1/2}\Gamma_\phi(s)} \right|^2, \quad (22)$$

where $\sigma(\omega)$ and $\sigma(\phi)$ are the cross section values in the ω and ϕ peaks and $\Gamma_i(s) = \Gamma_i s^{3/2} m_i^{-3}$, $i=\omega,\phi$. To take into account the ω-ϕ interference with negative relative sign is important for the correct description of the data in the region between the two resonances and above ϕ[17]. We have used essentially the same data above 0.66 GeV^2 as KNO and CLY with the addition of the 17 data points from Ref.18. On the other hand, for s < 0.66 GeV^2 new high-quality data from the experiment with cryogenic magnetic detector in Novosibirsk have become available recently[15]. In the experiment a new method of resonance depolarization for the beam energy calibration has been applied for the first time. This procedure led to significant suppression of the systematic errors. Since the statistical errors of the measurement have also been reduced in comparison with earlier experiments and the results[15] are fully compatible with the world averages[16], we take only these data for s < 0.66GeV^2. The optimal values of the fitted parameters obtained by comparing formula (22) with 76 data points from the interval $9m_\pi^2 \le s \le 2$ GeV^2 are

$$\begin{aligned} m_\omega &= 781.8 \pm 0.3 \text{ MeV} \quad \Gamma_\omega = 9.5 \pm 0.8 \text{ MeV} \quad \sigma(\omega) = 1519 \pm 120 \text{ nb} \\ m_\phi &= 1019.6 \pm 0.3 \text{ MeV} \quad \Gamma_\phi = 4.3 \pm 0.7 \text{ MeV} \quad \sigma(\phi) = 623 \pm 92 \text{ nb}. \end{aligned} \quad (23)$$

Evaluation of the three-pion portion of the integral (5) by means of CERN program RIWIAD using the Breit-Wigner cross section (22) with resonance parameters (23) yields

$$a_\mu^{3\pi} = \left[569 \pm 17(\text{stat}) \pm 9(\text{syst}) \pm 18(\text{model}) \right] \times 10^{-11}. \quad (24)$$

The statistical error obtained from the covariance matrix is 17×10^{-11}. The model error is rather large because we have included in it the contribution coming from our ignorance of the experimental behaviour of $\sigma^{3\pi}$ below 0.5 GeV^2. The value 16×10^{-11} was estimated from the difference of $a_\mu^{3\pi}$ values obtained by extrapolating the model curve (22) to the three-pion threshold and by the TRAPER integration over the experimental cross section starting at the point 0.7502 GeV.

The last contributions to a_μ^{vac} from the region below 2 GeV^2 come from processes $e^+e^- \to 4\pi, 5\pi, 6\pi$. We perform TRAPER integration and error analysis for these components of a_μ^{vac}. One could in principle try to fit the data by the Breit-Wigner functions in $2\pi^+2\pi^-$ and $\pi^0\pi^0\pi^+\pi^-$ channels , but the intermediate resonance states are not completely clear for these processes [18,19] and, moreover, for our purposes we need only a part of the corresponding cross sections below the peak. We use the same data as in Refs. 2 and 4 supplemented however by important new measurements[18] for both the four-pion channels. The results are displayed together with all other low-energy contributions in Table I.

B. High-energy region

As noticed by CLY, one can considerably reduce the errors of the integral (5) coming from the region s > 2 GeV^2 by considering the QCD expression for the quantity R instead of experimental data from individual channels. Really, KNO, who have used data, quote a rather large systematic error for example for the contribution of more than two hadrons (43×10^{-11}). On the other hand, there is no systematic error if one uses QCD. In our opinion, however, it is necessary in this case to check that the results obtained with the help of QCD expression for quantity R and by integrating over experimental data on R really coincide. In this section we describe our work along this direction.

TABLE I. Contributions from the region $s<2$ GeV^2 to $10^{11} a_\mu$.

Channel	Central value	Stat. error	Syst. error	Model error
$\pi^+\pi^-$	4985	28	24	40
K^+K^-	235	16	9	3
$K^o_S K^o_L$	183	12	9	2
$\pi^o\pi^+\pi^-$	569	17	9	18
$\pi^o\pi^o\pi^+\pi^-$	140	4	6	–
$\pi^+\pi^-\pi^+\pi^-$	55	2	3	–
5π, 6π	7	2	2	–
Total	6174	39	29	44

TABLE II. Contributions from the region $s>2$ GeV^2 to $10^{11} a_\mu$.

Interval(GeV^2) and method	Central value	Stat. error	Syst. error	Model error
$2 \le s \le 9.61$ QCD	586	17	–	2
$9.61 \le s \le 20.21$ $c\bar{c}$ thres., data	98	3	5	–
$20.21 \le s \le 81.0$ QCD	90	1	–	0
$81.0 \le s \le 196.0$ $b\bar{b}$ thres., data	19	0	1	–
$s > 196$, QCD	20	0	–	0
ψ, Υ resonances	71	4	4	–
Total	884	18	7	2

The experimental information on R(s) is rather rich. In the analysis we use data from 21 different experiments published during the last ten years as collected by Marshall [20] The author has performed a simultaneous fit of these data sets to reliably determine the strong coupling constant α_s. One of his conclusions is that three data sets [21,22,23] should be renormalized modestly in order to be consistent with the remaining sets. We follow this prescription in the evaluation of the high-energy contribution to a_μ^{vac} by means of data on R. The result of TRAPER integration is

$$a_\mu^R = \left[\, 817 \pm 13(\text{stat}) \,\right] \times 10^{-11} \qquad (25)$$

and the effect of the downward renormalization is to decrease the a_μ value by 55×10^{-11}. Of course the above result concerns only the continuum. The contributions from the J/Ψ and Υ resonance families should be added. In the narrow-width approximation they are expressed as

$$a_\mu^{res} = \frac{3\Gamma_{ee}}{\pi m_{res}} K_\mu(m^2_{res}) \,, \qquad (26)$$

where Γ_{ee} is the e^+e^- width of a given resonance, whose statistical and systematic errors induce the corresponding errors of a_μ^{res}. The individual contributions from the J/Ψ and Υ resonances are listed in Table II.

In the QCD calculation of the continuum contribution we have excluded the threshold regions 9.61 - 20.20 GeV^2 ($c\bar{c}$) and 81.0 - 196.0 GeV^2 ($b\bar{b}$) where the data have to be used. The QCD expression for R(s) calculated recently to the order $O(\alpha_s^3)$ is [24]

$$R(s) = 3\sum_f Q_f^2 \left[1 + \frac{\alpha_s(s)}{\pi} + (1.986 - 0.115 n_f)\left[\frac{\alpha_s(s)}{\pi}\right]^2 + (70.985 - 1.200 n_f - 0.005 n_f^2)\left[\frac{\alpha_s(s)}{\pi}\right]^3\right] - \left[\left(\sum Q_f\right)^2 \times 1.679\left[\frac{\alpha_s(s)}{\pi}\right]^3\right], \qquad (27)$$

where Q_f is the electric charge of the quark of flavour n_f. It is interesting that the coefficient of the $O(\alpha_s^3)$ correction is unexpectedly large, affecting significantly the value of the extracted QCD scale parameter $\Lambda_{\overline{MS}}$ [24]. The effect of the next-next-to-leading term on the value of a_μ^R may therefore also be non-negligible. Indeed, we have found for example for the contribution from the region $2 \le s \le 9.61\,\mathrm{GeV}^2$

$$O(\alpha_s^2):\quad a_\mu^R = (562 \pm 8)\times10^{-11},\quad O(\alpha_s^3):\quad a_\mu^R = (586 \pm 17)\times10^{-11}. \quad (28)$$

As can be seen the inclusion of the $O(\alpha_s^3)$ correction into R increases twice the error induced by the uncertainty of the parameter Λ (for the latter we took $\Lambda = 150 \pm 50$ MeV [25]). Summing up all contributions from Table II, we find

$$a_\mu^R = \left[\,884 \pm 18(\mathrm{stat}) \pm 6(\mathrm{syst}) \pm 2(\mathrm{mod})\right]\times10^{-11}. \quad (29)$$

The systematic error comes from the $c\bar{c}$ and $b\bar{b}$ threshold regions. The model error is negligible, since after the slight renormalization of three R data sets and the inclusion of the third order term in (28) both the methods used yield the same value of a_μ^R.

IV. Summary and conclusions

Our final result obtained by summing up all entries in Tables I and II is

$$a_\mu^{vac} = \left[\,7058 \pm 43(\mathrm{stat}) \pm 30(\mathrm{syst}) \pm 44(\mathrm{model})\,\right]\times10^{-11}, \quad (30)$$

where the errors have been added quadratically. Comparing (30) with the previous results (4a,b) we see that while our mean value is very close to them confirming thus the overall consistency of all three results, the real improvement over the last analysis [4] rests in diminishing the total error almost twice down to the value 68×10^{-11}. The increase in the accuracy of a_μ^{vac} stems from the low-energy region. First, the statistical errors of the two-pion and two-kaon contributions have been reduced by a factor of 1.5 and 2, respectively, due to the use of rather accurate global analytic models of the pion and kaon electromagnetic form factors. Second, new precise experimental data on the cross section $\sigma(e^+e^- \to \pi^0\pi^+\pi^-)$ analyzed by means of the interfering ω and ϕ Breit-Wigner amplitudes led to significant reduction of the errors of this channel. New data on the four-pion e^+e^- annihilation [18] contributed to partial suppression of the total statistical error, too.

We have done independent integrations directly over experimental data in the channels where models have been used. Besides obtaining in this way the estimates of the corresponding systematic errors it was possible to use the deviations of the two methods as a measure of possible model dependence of our results. That the model errors are sufficiently small gives a certain credit to the final results on a_μ^{vac}.

Taking into account the QED and weak contributions as quoted in introduction together with the new value of higher hadronic contributions [2]

$$a_\mu(\mathrm{h.h.}) = (-41 \pm 7)\times10^{-11} \quad (31)$$

new value of the total anomalous magnetic moment of the muon will be

$$a_\mu = (116\,592\,012 \pm 75)\times10^{-11}. \quad (32)$$

The error is about 38 % of the one-loop effect of the weak interactions. This creates a real chance to detect this contribution (and also the possible one of the same order of magnitude predicted by some superstring-inspired models [26]) in the experimental a_μ value after the improved g-2 measurements will be accomplished.

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References

1. Kinoshita T., Nizic B., Okamoto Y.-Phys.Rev.Lett., 1984, 52, p.717.
2. Kinoshita T., Nizic B., Okamoto Y.-Phys.Rev., 1985, 31, p.2108.
3. Bailey J. et al.-Phys.Lett., 1977, 68B, p.191.
4. Casas J. A., Lopéz C., Yndurain F. J.-Phys.Rev.,1985,32,p.736.
5. Hughes V.W., in:Proceedings of the Third LAMPF II Workshop, Los Alamos National Laboratory, 1983.
6. Lautrup B.E., Peterman A., de Rafael E.-Phys.Rep., 1972, C3, p.193.
7. Dubnička S., Martinovič L., JINR Preprint, E4-88-240, Dubna, 1988, and Phys.Rev.D, to be published.
8. Alonso J.A., Casas J.A.-Phys.Lett., 1987, 197B, p.239.
9. Dubnička S., Martinovič L.- Czech.J.Phys., 1986, B36, p.1311; Proc. of XXI Rencontre de Moriond, Les Arcs, 1986, p.297 and submitted for publication.
10. Dubnička S., Martinovič L.-Czech.J.Phys., 1979, B29, p.1382.
11. Dubničková A.Z. et al. Czech.J.Phys., 1987, B37, p.815.
12. Amendolia S.R. et al. Nucl.Phys., 1986, B277, p.168.
13. Bisello D. et al., Preprint LAL/85-15, ORSAY, 1985.
14. Dubnička S., JINR Preprint, E2-88-840, Dubna, 1988.
15. Barkov L.M. et al.-Pisma v ZHETF, 1987, 46, p.164.
16. Review of Particle Properties, Phys.Lett., 1988, 204B.
17. Kurdadze L.M. et al., Preprint INP, 84-7, Novosibirsk, 1984.
18. Aulchenko V.M. et al., Preprint INP, 86-106, Novosibirsk,1986.
19. Kurdadze L.M. et al., Preprint INP, 79-69, Novosibirsk, 1979.
20. Marshall R., Preprint RAL, 88-049, Oxon, 1988.
21. Siegrist J.L. et al.-Phys.Rev., 1982, D26, p.969.
22. Bacci C. et al.-Phys.Lett., 1979, 86B, p.234.
23. Jakubowski Z. et al.-Z.Phys., 1988, C40, p.49.
24. Gorishny S.G., Kataev A.L., Larin S.A.-Phys.Lett., 1988, 212B, p.238.
25. Narison S., Pich A.-Phys. Lett., 1988, 211B, p.183.
26. Grifols J.A., Mendez C, Solà J.-Phys.Rev.Lett., 1986 , 57, p.2348.

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