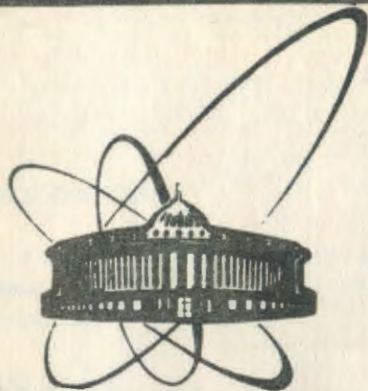


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ОБЪЕДИНЕННЫЙ  
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ON A PHASE STRUCTURE  
OF A TWO-DIMENSIONAL  
 $(\phi^2)^2$  FIELD THEORY

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## INTRODUCTION

In this paper we will investigate the problem of spontaneous symmetry breaking (SSB) or, in other words, the phase structure of quantum field models

$$\mathcal{L}(x) = \frac{1}{2} \phi(x) (\square - m^2) \phi(x) - \frac{g}{4} \phi^4(x), \quad (1)$$

$$\mathcal{L}(x) = \frac{1}{2} \sum_{i=1}^n \phi_i(x) (\square - m^2) \phi_i(x) - \frac{g}{4} \left( \sum_{i=1}^n \phi_i^2(x) \right)^2 \quad (2)$$

in a space-time of two dimensions  $\mathbb{R}^2$ . The Lagrangian (1) describes a one-component scalar field  $\phi$  and this Lagrangian is invariant under the transformation  $\phi \rightarrow -\phi$ . The Lagrangian (2) describes an isotopic multiplet of  $n$  scalar fields and it is invariant under continuous isotopic transformations of the group  $O(n)$ . The mass  $m^2$  in (1) and (2) is positive, and  $g$  is coupling constant.

There are many papers devoted to the investigation of SSB in quantum field theory (see, for example,<sup>1,2/</sup> containing many earlier references). The Goldstone phenomenon underlies our understanding of SSB. Namely, SSB of a continuous symmetry is always accompanied by the appearance of massless scalar bosons (see, for example, <sup>3/</sup>).

However, a specific situation takes place in  $\mathbb{R}^2$ . The theorem was proved<sup>4/</sup> that in two dimensions the Goldstone phenomenon cannot occur for any continuous symmetry. This proof is based on that the Goldstone particles as particles with zero masses do not exist in  $\mathbb{R}^2$ .

In addition to the above-mentioned there exist theorems proving that the second order phase transition takes place in the model (1) for a coupling constant  $g = g_c$  (see, for example, <sup>5,6/</sup>). In the paper <sup>7/</sup>, an attempt was made to construct an explicit form of these two phases. However, this construction shows the first order phase transition although one can always say that this result is connected with an approximation method.

We will investigate the models (1) and (2) by the method of canonical transformations. Namely, we construct the Hamil-

tonians for Lagrangians (1) and (2) and formulate our problem as follows:

What representation of canonical commutative relations is suitable for different values of the dimensionless coupling constant

$$G = \frac{\xi}{2\pi m^2} \quad (3)$$

and what physical picture does correspond to this representation?

It turned out that two phases exist in these models for  $G < G_c$  and  $G > G_c$ . Our result shows the first-order phase transition in accordance with<sup>7/</sup>. The critical coupling constant  $G_c$  can be found in the lowest approximation. SSB takes place for  $G > G_c$ . In the phase  $G > G_c$  these models describe particles with large masses and the "Goldstone" particles possess nonzero masses.

Now we do not have reasonable arguments for explaining the contradiction between our result and the afore-said theorems.

## 1. LAGRANGIAN $\phi^4$ IN $R^2$

Let us consider the theory of a scalar field  $\phi$  in the space-time  $R^2$  which is described by the Lagrangian density (1). We denote coordinates by  $x = (x_0, x_1)$  and momenta by  $k = (k_0, k_1)$ . Let us consider this theory in the Hamiltonian representation. The second-quantized Hamiltonian for Lagrangian (1) is written as

$$H = H_0 + H_I, \quad (1.1)$$

$$H_0 = \frac{1}{2} \int_V dx_1: [\pi^2(x_1) + (\nabla \phi(x_1))^2 + m^2 \phi^2(x_1)]:, \quad (1.2)$$

$$H_I = \frac{\xi}{4} \int_V dx_1: \phi^4(x_1):,$$

where

$$\phi(x_1) = \int \frac{dk_1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\omega(k_1)}} ( a_{k_1} e^{ik_1 x_1} + a_{k_1}^+ e^{-ik_1 x_1} ), \quad (1.3)$$

$$\pi(x_1) = \int \frac{dk_1}{\sqrt{2\pi}} i \sqrt{\frac{\omega(k_1)}{2}} (a_{k_1} e^{ik_1 x_1} - a_{k_1}^+ e^{-ik_1 x_1}),$$

$$\omega(k_1) = \sqrt{m^2 + k_1^2}, \quad [a_{k_1}, a_{k_1'}^+] = \delta(k_1 - k_1'). \quad (1.3)$$

The fields  $\phi(x_1)$  and  $\pi(x_1)$  are canonical variables and obey the canonical commutation relations

$$[\phi(x_1), \phi(x_1')] = [\pi(x_1), \pi(x_1')] = 0, \quad (1.4)$$

$$[\phi(x_1), \pi(x_1')] = \frac{1}{i} \delta(x_1 - x_1').$$

The integration in (1.2) is performed over a large "volume"  $V$  (here it is an interval  $-L < x_1 < L$  and  $V=2L$ ). All operators in (1.2) are taken in the form of normal products.

Hamiltonian (1.1) is defined on a Fock space. This theory contains no ultraviolet divergences because the only divergences in (1.2) are removed by definition of the normal product in (1.2).

Hamiltonian (1.2) describes scalar particles with mass  $m$  interacting with each other by the interaction Hamiltonian (1.2) which is invariant under the transformation

$$\phi \rightarrow -\phi. \quad (1.5)$$

This physical picture is valid only for the small enough coupling constant  $g$  ( $G \ll 1$ ).

It is known<sup>5,6</sup> that the phase transition exists in this model for  $g = g_c$  which breaks the symmetry (1.5). Our aim is to investigate the mechanism of symmetry breaking in the model (1.2).

We want to describe the system (1.1) for  $G \gg 1$ . We will use the method of transition to another vacuum, which means the consideration of another representation of the canonical commutative relations (1.4).

Let us perform the canonical transformation of our fields

$$\pi(x_1) \rightarrow \pi_f(x_1), \quad \phi(x_1) \rightarrow \phi_f(x_1) + B, \quad (1.6)$$

where the fields  $\pi_f(x_1)$  and  $\phi_f(x_1)$  have the form (1.3) but their mass is equal to

$$M^2 = m^2(1 + f); \quad (1.7)$$

$B$  is a constant field.

The explicit form of this transformation (1.6) can be written but we will not do it here.

Now let us perform this canonical transformation (1.6) for the Hamiltonians (1.2) and go over to the normal ordering in the new fields  $\phi_f(x_1)$  and  $\pi_f(x_1)$ . After some calculations one obtains

$$H' = VE_{\text{vac}} + H_0 + H_{\text{int}} + H_1. \quad (1.8)$$

Here

$$E_{\text{vac}} = \frac{m^2 B^2}{2} + L(f) + \frac{g}{4} [B^4 - 6B^2 D(f) + 3D^2(f)],$$

$$H_0 = \frac{1}{2} \int dx_1 : [ \pi_f^2(x_1) + (\nabla \phi_f(x_1))^2 + M^2 \phi_f^2(x_1) ] :$$

$$H_{\text{int}} = \frac{g}{4} \int dx_1 : [ \phi_f^4(x_1) + 4B \phi_f^3(x_1) ] : \quad (1.9)$$

$$H_1 = \int dx_1 : \{ \phi_f^2(x_1) [ -\frac{m^2}{2} f + \frac{g}{4} (6B^2 - 6D(f)) ] + \phi_f(x_1) [ m^2 B + \frac{g}{4} (4B^3 - 12BD(f)) ] \} :$$

where

$$L(f) = \frac{1}{2} \int \frac{dk_1}{2\pi} [ \omega_f(k_1) - \omega(k_1) - \frac{m^2 f}{\omega_f(k_1)} ] = \frac{m^2}{8\pi} [ f - \ln(1+f) ], \quad (1.10)$$

$$D(f) = \int \frac{dk_1}{2\pi} [ \frac{1}{2\omega(k_1)} - \frac{1}{2\omega_f(k_1)} ] = \frac{1}{4\pi} \ln(1+f).$$

The operators in  $H_0$ ,  $H_{\text{int}}$  and  $H_1$  are normally ordered with respect to the operators  $\phi_f$  and  $\pi_f$ .

$E_{\text{vac}}$  represents the vacuum energy density which consists of the energy of the external field  $\frac{m^2}{2} B^2$  and the interaction energy containing "kinetic" and "potential" parts.

Let us put equal to zero the coefficients of the operators  $\phi_f^2$  and  $\phi_f$  in  $H_1$ . The main argument is that after the canonical transformation (1.6) the total Hamiltonian cannot contain the linear term  $\phi_f$  at all and can contain only the quad-

ratic term:  $\phi^2$ : in the free Hamiltonian. In other words, the total Hamiltonian should have a "correct" form, i.e. the free Hamiltonian has a standard form and the interaction Hamiltonian can contain field operators in the degrees more than two. Thus, we have for the Hamiltonian formula (1.8) where  $H_1 = 0$ .

The above-mentioned two equations are written as follows:

$$\begin{aligned} B[m^2 + g(B^2 - 3D(f))] &= 0, \\ m^2 f - 3g(B^2 - D(f)) &= 0. \end{aligned} \tag{1.11}$$

It should be stressed that these equations define the minimum of vacuum energy  $E_{vac}$  in (1.9) with respect to the parameters  $B$  and  $f$ . Thus, the condition that the total Hamiltonian should have a "correct" form coincides with the requirement that the vacuum energy  $E_{vac}$  should have minimum.

Thus two parameters of our canonical transformation (1.6), the mass  $M^2 = m^2(1+f)$  and the constant field  $B$ , are fixed by equations (1.11).

It is convenient to rewrite the vacuum energy  $E_{vac}$  and equations (1.11) in the following form:

$$\begin{aligned} E_{vac} &= \frac{m^2}{8\pi} [A^2 + (e^s - 1 - s) + \frac{G}{4}(A^4 - 6A^2 s + 3s^2)], \\ A[1 + \frac{G}{2}(A^2 - 3s)] &= 0, \quad e^s - 1 = \frac{3}{2}G(A^2 - s), \end{aligned} \tag{1.12}$$

where

$$A^2 = 4\pi B^2, \quad G = \frac{g}{2\pi m^2}, \quad (1+f) = e^s.$$

The first equation in (1.12) has two solutions

$$A = 0, \quad A^2 = 3s - \frac{2}{G} \geq 0. \tag{1.13}$$

Consider the solution  $A = 0$ . It follows from the second equation in (1.12) that  $s = 0$  or  $f = 0$ . It means that we arrive at the initial Hamiltonian (1.2). In this case  $E_{vac} = 0$ .

Now consider the second solution in (1.13). Substituting this solution into  $E_{vac}$ ,  $A^2$ ,  $M^2$  and the second equation (1.12), and introducing the new variable  $t = s - 2/3G$  we obtain

$$E_{vac} = \frac{m^2}{8\pi} \left[ e^{(2/3G)t} - \frac{3G}{2}t^2 - 1 - \frac{1}{3G} \right],$$

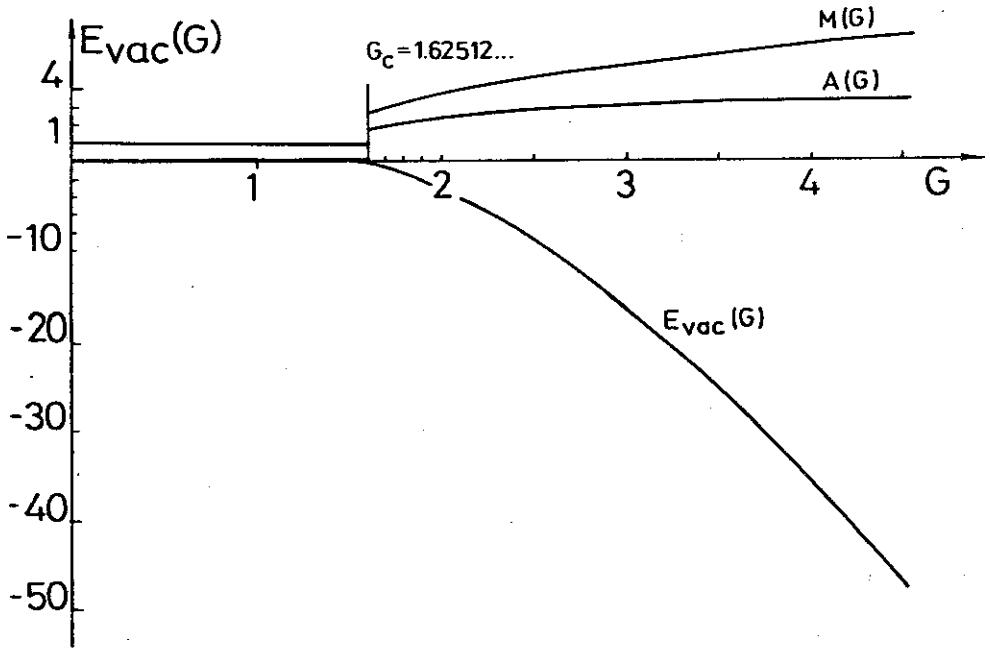


Fig.1

$$A^2(G) = 3t, \quad M^2(G) = m^2 \exp\left(t + \frac{2}{3G}\right), \quad (1.14)$$

$$\exp\left(t + \frac{2}{3G}\right) = 3Gt.$$

The behaviours of  $\epsilon_{vac}(G) = E_{vac}/(\frac{m^2}{8\pi})$ ,  $A(G)$  and  $\mu(G) = M(G)/m$  are shown in Fig.1. The critical coupling constant is defined by the condition

$$E_{vac}(G_c) = 0$$

and it equals

$$G_c = 1.62512... \quad (1.15)$$

One can see that the pictures in Fig.1 correspond to the first order phase transition.

For  $G \rightarrow \infty$  one can obtain

$$E_{vac}(G) \rightarrow -\frac{3m^2}{16\pi} G(\ln G)^2, \quad (1.16)$$

$$A^2(G) \rightarrow 3 \ln G, \quad M^2(G) \rightarrow 3m^2 G \ln G.$$

Now, let us discuss the total Hamiltonian (1.9) for  $G > G_c$

$$\begin{aligned}
 H' = & VE_{\text{vac}}(G) + \frac{1}{2} \int dx_1 : [\pi_f^2(x_1) + (\nabla \phi_f(x_1))^2 + M^2(G) \phi_f^2(x_1)] : + \\
 & + \frac{g}{4} \int dx_1 : [\phi_f^4(x_1) \pm 4B(G) \phi_f^3(x_1)] : \quad (1.17)
 \end{aligned}$$

First, this Hamiltonian is defined on the Fock space and it is written in the "correct" quantum field form. It describes particles with the mass

$$M^2(G) = m^2 \exp(t(G) + 2/3G) .$$

Second, Fig.1 shows that the first-order phase transition takes place. But this conclusion follows from the lowest approximation for the vacuum energy  $E_{\text{vac}}(G)$ . The next perturbation corrections should be taken into account. Probably, they can smooth away this sharp picture.

Third, the symmetry under the transformation  $\phi \rightarrow -\phi$  (1.15) is broken in (1.17). It means that the spontaneous symmetry breaking takes place when  $G > G_c = 1.62512\dots$

Fourth, the effective coupling constants, in which the perturbation series is constructed, are

$$G_{\text{eff}}^{(1)} = \frac{g}{2\pi M^2} = \frac{G}{\mu^2(G)}, \quad G_{\text{eff}}^{(2)} = \frac{g}{2\pi M^2} \sqrt{A(G)} = \frac{G\sqrt{A(G)}}{\mu^2(G)}. \quad (1.18)$$

Their behaviour is shown in Fig.2. The asymptotic behaviour of  $G_{\text{eff}}^{(1)}$  and  $G_{\text{eff}}^{(2)}$  as  $G \rightarrow \infty$  is the following:

$$G_{\text{eff}}^{(1)}(G) \rightarrow \frac{1}{3 \ln G}, \quad G_{\text{eff}}^{(2)}(G) \rightarrow \frac{1}{\sqrt{3 \ln G}}. \quad (1.19)$$

The matrix elements for any physical processes in the model under consideration can be written in the perturbation series of the type

$$\sum_{n_1, n_2} (G_{\text{eff}}^{(1)}(G))^{n_1} (G_{\text{eff}}^{(2)}(G))^{n_2} T_{n_1 n_2} \left( \frac{(P_i P_j)}{M^2(G)} \right), \quad (1.20)$$



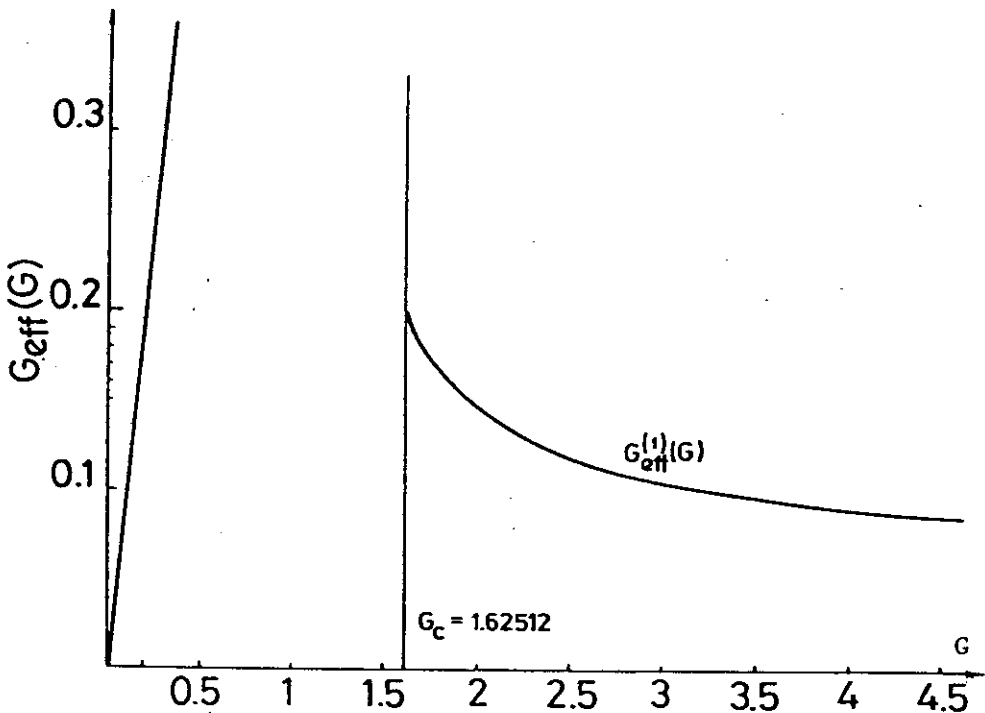


Fig. 2

where  $p_1$  and  $p_2$  are external momenta. The decreasing of the effective coupling constants as  $G \rightarrow \infty$  means that the Hamiltonian (1.17) describes the system with weak coupling. In other words, the strong coupling regime in the sense of the standard quantum field does not exist in the system described by Lagrangian (1.1).

Thus, for  $G < G_c$  Lagrangian (1.1) describes particles with the mass  $m^2$  and coupling constant  $G < G_c = 1.62512\dots$ , the symmetry  $\phi \rightarrow -\phi$  being unbroken. For  $G > G_c$  it describes particles with the mass  $M^2(G)$  and weak coupling  $G_{\text{eff}}^{(1,2)} < 1/2$ , the symmetry  $\phi \rightarrow -\phi$  being broken.

## 2. LAGRANGIAN $(\sum_i \phi_i^2)^2$ IN $R^2$

Let us consider a system of  $n$  scalar fields  $\phi_i$  ( $i = 1, \dots, n$ ) described by Lagrangian (2). As above this Lagrangian is invariant under isotopic transformations of the group  $O(n)$ .

The Hamiltonian is as follows:

$$H = H_0 + H_1, \quad (2.1)$$

$$H_0 = \frac{1}{2} \int_V dx_1 \sum_{i=1}^n : [\pi_i^2(x_1) + (\nabla \phi_i(x_1))^2 + m^2 \phi_i^2(x_1)] : , \quad (2.2)$$

$$H_1 = \frac{g}{4} \int_V dx_1 : \left( \sum_{i=1}^n \phi_i^2(x_1) \right)^2 : .$$

The notation here is the same as in (1.2). The normal ordering in the interaction Hamiltonian is defined in a standard way:

$$: \left( \sum_{i=1}^n \phi_i^2 \right)^2 : = \left( \sum_{i=1}^n \phi_i^2 \right) - 2(n+2)\Delta \sum_{i=1}^n \phi_i^2 + n(n+2)\Delta^2 ,$$

$$\Delta = \int \frac{dk_1}{2\pi} \cdot \frac{1}{2\omega(k_1)} .$$

The Hamiltonian (2.2) taken in the normal form does not contain any ultraviolet divergences. It is invariant under the rotations  $O(n)$  in the isotopic space.

Let us proceed to investigate the problem of spontaneous symmetry breaking for (2.2). We perform the following canonical transformation of our fields:

$$\begin{aligned} \pi_i(x_1) &\rightarrow \Pi_i(x_1) \\ &\quad (i = 1, 2, \dots, n-1), \\ \phi_i(x_1) &\rightarrow \Phi_i(x_1) \\ \pi_n(x_1) &\rightarrow \Pi(x_1), \\ \phi_n(x_1) &\rightarrow \Phi(x_1) + B, \end{aligned} \quad (2.3)$$

where the fields  $\Phi_i$  ( $i = 1, \dots, n-1$ ) have the mass  $M^2 = m^2(1+f_0)$  and the field  $\Phi$  has the mass  $M^2 = m^2(1+f)^0$ .

The explicit form of this transformation (2.3) can be written by analogy with (1.6).

Now, let us perform this canonical transformation in the total Hamiltonian (2.2) and go over to the normal ordering in the new fields  $\Phi_i$  ( $i = 1, \dots, n-1$ ) and  $\Phi$ . The transformed Hamiltonian in the normal form with respect to the field with the new masses is written after some calculations

$$H' = VE_{\text{vac}} + H_0 + H_{\text{int}} + H_1. \quad (2.4)$$

Here

$$E_{\text{vac}} = \frac{m^2}{2} B^2 + L(f) + (n-1)L(f_0) + \frac{g}{4} [B^4 - 2(3D(f) + (n-1)D(f_0))B^2 + (D(f) + (n-1)D(f_0))^2 + 2(D^2(f) + (n-1)D^2(f_0))], \quad (2.5)$$

$$H_0 = \frac{1}{2} \int_V dx_1 : \left\{ \sum_{i=1}^{n-1} [\Pi_i^2 + (\nabla \Phi_i)^2 + M_0^2 \Phi_i^2] + [\Pi^2 + (\nabla \Phi)^2 + M^2 \Phi^2] \right\}, \quad (2.6)$$

$$H_{\text{int}} = \frac{g}{4} \int_V dx_1 : \left\{ \left[ \sum_{i=1}^{n-1} \Phi_i^2 + \Phi^2 \right]^2 + 4B\Phi \left[ \sum_{i=1}^{n-1} \Phi_i^2 + \Phi^2 \right] \right\}, \quad (2.7)$$

$$H_1 = \int dx_1 : \left\{ \Phi(x_1) \left[ m^2 B + \frac{g}{4} (4B^3 - 4(3D(f) + (n-1)D(f_0))B) \right] + \Phi^2(x_1) \left[ -\frac{m^2 f}{2} + \frac{g}{4} (8B^2 - 2(3D(f) + (n-1)D(f_0))) \right] + \sum_{i=1}^{n-1} \Phi_i^2(x_1) \left[ -\frac{m^2 f_0}{2} + \frac{g}{4} (2B^2 - 2(D(f) + (n+1)D(f_0))) \right] \right\}; \quad (2.8)$$

where the notation is the same as in (1.10).

In the interaction Hamiltonian  $H_1$  (2.8) we put equal to zero the coefficients of the operators  $\Phi(x_1)$ ,  $:\Phi^2(x_1):$  and  $:\sum_{i=1}^{n-1} \Phi_i^2(x_1):$  and obtain the following equations:

$$B \left[ 1 + \frac{g}{m^2} (B^2 - 3D(f) - (n-1)D(f_0)) \right] = 0, \quad (2.9)$$

$$f = \frac{g}{m^2} [3B^2 - 3D(f) - (n-1)D(f_0)],$$

$$f_0 = \frac{g}{m^2} [B^2 - D(f) - (n+1)D(f_0)].$$

We want to stress again that these equations define the minimum of the vacuum energy  $E_{\text{vac}}$  (2.5).

The solution  $B = 0$  of the first equation in (2.9) leads to the initial Hamiltonian (2.2). The second solution is

$$B^2 = 3D(f) + (n-1)D(f_0) - \frac{m^2}{g}. \quad (2.10)$$

Substituting this solution into  $E_{vac}$  and two equations (2.9), one can get

$$E_{vac} = \frac{m^2}{8\pi} \left[ f + 2\ln(1+f) + (n-1)f_0 - \frac{1}{G} - \frac{G}{4} (3(\ln(1+f))^2 + 2(n-1)\ln(1+f)\ln(1+f_0) - (n-1)(\ln(1+f_0))^2) \right] \quad (2.11)$$

$$f + 3 = G[3\ln(1+f) + (n-1)\ln(1+f_0)], \quad (2.12)$$

$$f_0 + 1 = G[\ln(1+f) - \ln(1+f_0)],$$

where  $G = \frac{g}{2\pi m^2}$ .

One can see that equations (2.12) define the minimum of the vacuum energy  $E_{vac}$  in (2.11). It is convenient to introduce new variables

$$1+f = Gt, \quad 1+f_0 = Gt_0. \quad (2.13)$$

Equations (2.12) become

$$t = t_0 e^{t_0}, \quad (2.14)$$

$$t_0 e^{t_0} - 3t_0 - (n+2)\ln t_0 = -\frac{2}{G} + (n+2)\ln G.$$

In this paper we consider the case  $n = 4$ . The behaviour of  $\epsilon_{vac}(G) = E_{vac}(G) / (\frac{m^2}{8\pi})$  is shown in Fig.3. The critical coupling constant is defined by the condition

$$E_{vac}(G_c) = 0$$

and it equals

$$G_c = 1.317 \dots \quad (2.15)$$

The behaviour of  $A(G) = \sqrt{4\pi B(G)}$  and the masses

$$\mu(G) = \frac{M(G)}{m} = \sqrt{1+f(G)}, \quad \mu_0(G) = \frac{M_0(G)}{m} = \sqrt{1+f_0(G)}$$

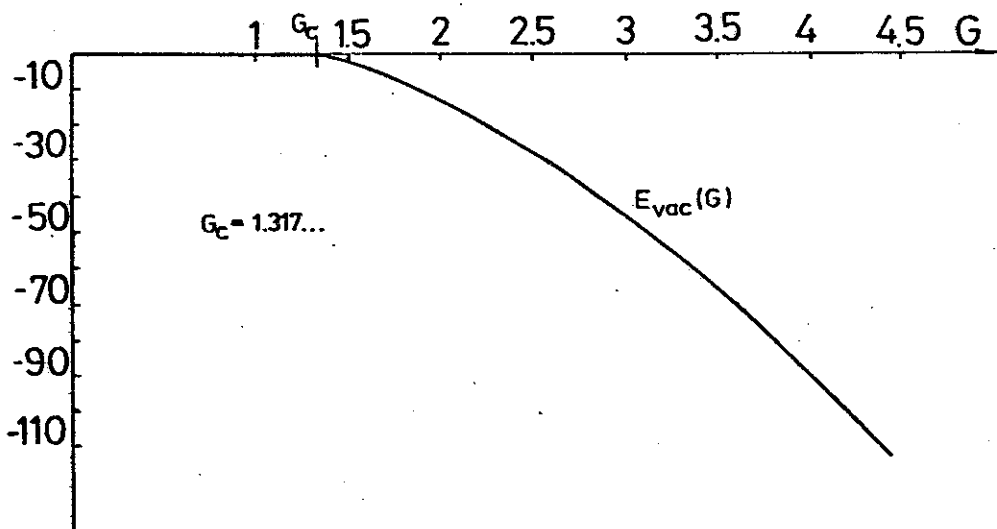


Fig. 3

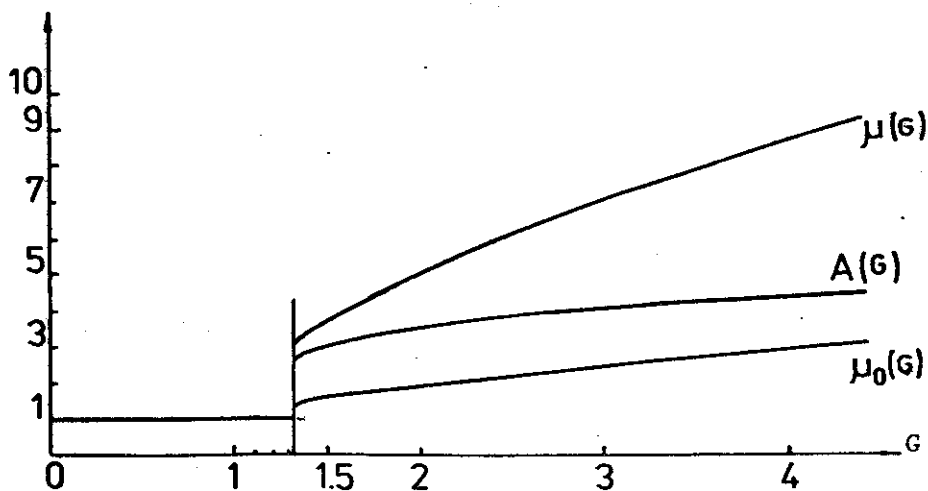


Fig. 4

are shown in Fig.4. One can see that these pictures correspond to the first order phase transition.

As  $G \rightarrow \infty$  one can obtain

$$f(G) \rightarrow (n+2)G \ln G + O(G \ln \ln G),$$

$$f_0(G) \rightarrow G \ln \ln G + O(G \ln \ln \ln G)$$

and

$$E_{\text{vac}}(G) \rightarrow -\frac{m^2}{8\pi} \cdot \frac{3}{4} G (\ln G)^2,$$

$$M^2(G) \rightarrow m^2(n+2) G \ln G, \quad (2.16)$$

$$M_0^2(G) \rightarrow m^2 G \ln \ln G,$$

$$A^2(G) \rightarrow 3 \ln G.$$

Now let us examine the Hamiltonian (2.6) and (2.7)

$$H = H_0 + H_{\text{int}}, \quad (2.17)$$

where all parameters are defined by equations (2.12).

First, this Hamiltonian is defined on the Fock space. The free Hamiltonian  $H_0$  (2.6) contains two kinds of scalar fields: the fields  $\Phi_i$  ( $i = 1, \dots, n-1$ ) which describe scalar particles with the mass  $M_0^2 = m^2(1+f_0)$  and belong to the isotopic multiplet  $O(n-1)$  and the field  $\Phi$  which describes particles with the mass  $M^2 = m^2(1+f)$ . Moreover, we have

$$1 < \frac{M^2(G)}{M_0^2(G)} \xrightarrow{G \rightarrow \infty} (n+2) \frac{\ln G}{\ln \ln G}.$$

Thus, the mass of the field  $\Phi$  is larger than the mass of the multiplet particles and this difference increases as  $G \rightarrow \infty$ .

Second, the symmetry  $O(n)$  is broken in Hamiltonian (2.7), i.e. the spontaneous symmetry breaking takes place for  $G > G_c$ . The Hamiltonian is invariant under the transformations of the group  $O(n-1)$  and is not invariant under the transformation  $\Phi \rightarrow -\Phi$ . Moreover, the fields  $\Phi_i$  ( $i = 1, \dots, n-1$ ) have non-zero masses, although from the point of view of the standard SSB picture these particles are "Goldstones" and should have masses to be equal to zero.

Third, the effective coupling constants in which the perturbation series is constructed are

$$G_{\text{eff}}^0 = \frac{g}{2\pi M_0^2} = \frac{G}{1+f_0(G)}, \quad G_{\text{eff}} = \frac{g}{2\pi M^2} = \frac{G}{1+f(G)}. \quad (2.18)$$

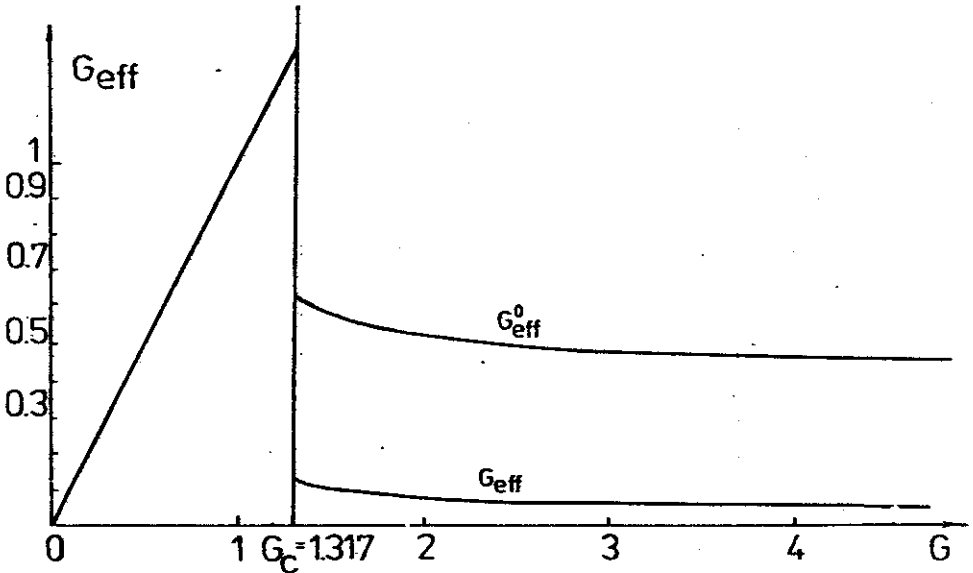


Fig. 5

Their behaviour is shown in Fig.5. For  $G \rightarrow \infty$  these constants decrease

$$G_{\text{eff}}^0 = O\left(\frac{1}{\ln \ln G}\right), \quad G_{\text{eff}} = O\left(\frac{1}{\ln G}\right).$$

Thus, one can say that the strong coupling regime does not exist in the theory with Lagrangian (2).

Fourth, the theorems proved in <sup>5,6/</sup> indicate that the second order phase transition takes place in the models (1.1) and (2.1). The method of canonical transformations used above gives the first order phase transition. Probably, perturbation corrections should be taken into account and their contribution reduces the first order phase transition to the second one.

Nevertheless, for  $G \gg 1$  we have the phase containing the  $O(n-1)$  multiplet of the "Goldstone" particles with the mass  $M_0^2 > 0$  and one particle with the mass  $M^2 > M_0^2$ . The interaction of these particles is described by the Hamiltonian (2.7) with the broken symmetry and the effective coupling is weak.

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