ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

18/VIII-75

M-41

E2 - 8896

2972/2-75

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RELATIVISTIC FOURIER ANALYSIS, QUASIPOTENTIAL APPROACH AND PROTON-PROTON ELASTIC SCATTERING

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Submitted to *Physics Letters*

CONTENTS

1. Introduction

In paper [1] the power behaviour of the proton-proton elastic scattering amplitude in the energy interval from $s = 8\ (\mathrm{GeV}^2)$ to ISR energies and for four-momentum transfer squared in the interval $.2 \leq -t \leq 8.\ (\mathrm{GeV}^2)$, was tested by numerical analysis. The amplitude was obtained in Born approximation like a relativistic Fourier image of phenomenologically chosen in the relativistic relative coordinate space[2] quasipotential[3] with a simple pole structure

$$V(r,s) = \frac{\lambda(s)}{R^2(s) + r^2} .$$

To explain the detailed structure of the differential cross section a quasipotential in the form [1]

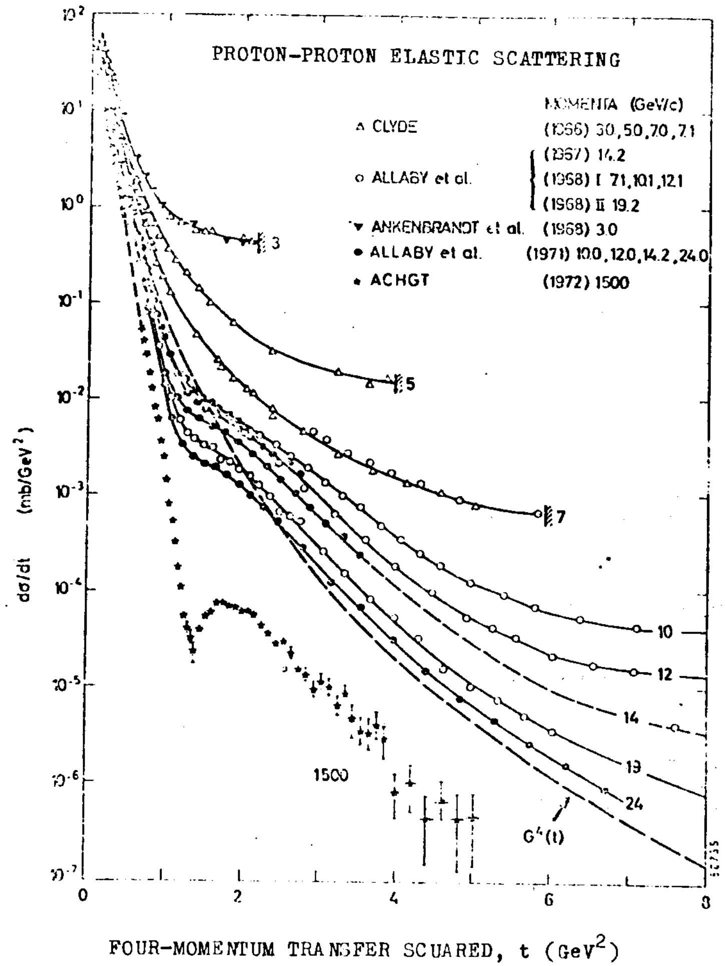
$$V(r,s) = \frac{\lambda_1(s)}{R_1^2(s) + r^2} + \frac{\lambda_2(s)}{R_2^2(s) - r^2} \qquad (1)$$

was proposed. The unknown functions $\lambda_i(s)$ and $R(i\ )$ are correspondingly coupling "constants" and interaction radii. Assuming that the spin effects are small, the amplitude in the Born approximation was obtained in the following form

$$T(s,t) = \frac{1}{F_1(t)}\left[\frac{\lambda_1(s)}{F_2(t)^{R_2(s)}} + \lambda_2(s)\cos\left(\ln F_2(t)^{R_2(s)}\right)\right], \qquad (2)$$

where

$$F_1(t) = \sqrt{-t(1-t/4)}, \quad F_2(t) = 1 - t/2 + \sqrt{-t(1-t/4)} .$$



The large angle pp elastic scattering at various laboratory momenta.

Fig.1

It is interesting to mention the extreme simplicity of formula (2) in terms of transferred rapidity

$$x_t = \ln\left(1 - t/2 + \sqrt{-t(1-t/4)}\right),$$

$$T(s, x_t) = \frac{1}{\mathrm{sh}\, x_t}\left[\lambda_1(s)\exp(-R_1(s)x_t) + \lambda_2(s)\cos(R_2(s)x_t)\right].$$

The aim of this paper is to apply formula (2) to describe the available experimental data [4,5,6].

2. Numerical analysis

Beoause table form of the experimental data is not available, we used Fig.11 from paper [4] and Fig.4 from paper [6] to extract the values of $\frac{d\sigma}{dt}^{expt}(s,t)$ – Fig.1.

The data thus obtained were used to solve the overdetermined nonlinear system of equations

$$\frac{d\sigma}{dt}(s,t) - \frac{d\sigma}{dt}^{expt}(s,t) = 0,$$

where

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{16\pi s(s-4)}\left|T(s,t)\right|^2$$

and $T(s,t)$ is the amplitude (2).

The numerical analysis, based on the least squares method was carried out by the regularized iterational processes of Gauss-Newton type [6] realized in the program COMPIL [7] (JINR standart program library).

The simultaneous exploitation of the above mentioned numerical analysis and some physical considerations and conditions lead to the following parametrizations of the functions $\lambda_i(s)$ and $R_i(s)$

$$\lambda_i(s) = \left(A_i + i\sqrt{s(s-4)}\,B_i\right) R^2(s),$$
$$R_1(s) = \frac{1}{g} R_2(s) = R(s) \qquad (3)$$
$$R(s) = R_0 + \frac{\sqrt{\sigma_0}}{s^{\alpha_0}} + R\left(\ln\frac{s}{4\mu^2}\right)^{\alpha}.$$

The solutions for the parameters are given in Table I.

Table I

A_1	−64.4874	R_0	1.1072
B_1	3.6622	$\sqrt{\sigma_0}$	7.0999
A_2	.5327	α_0	.5669
B_2	.0046	R	.6911
g	.4171	μ	.1377
		α	.8061

The value of

$$\chi^2 = \sum_{i=1}^{N}\left(\frac{d\sigma}{dt}(s,t) - \frac{d\sigma}{dt}(s,t)\right)^2$$

(where N is the number of the points) for the solutions in Table I is given in Table 2.

Table 2

S (GeV^2)	All curves	8.	12.	16.1	26.1	38.4	5o.	3000
N	210	12	15	30	31	29	27	66
χ^2	97.8	13.3	11.5	31.6	17.1	5.8	4.1	14.4

In Fig.2 it is given a comparison of the "experimental" points and theoretical curves obtained with the values of the parameters from Table 1.

As it can be seen from Fig.2 the amplitude (2) provides a good enough description of the experimental picture. There is a better agreement with the increasing incident energy (see Table 2) because of the relativistic analog of the Born approximation validity condition [1]

$$t_{max}/s \ll 1 .$$

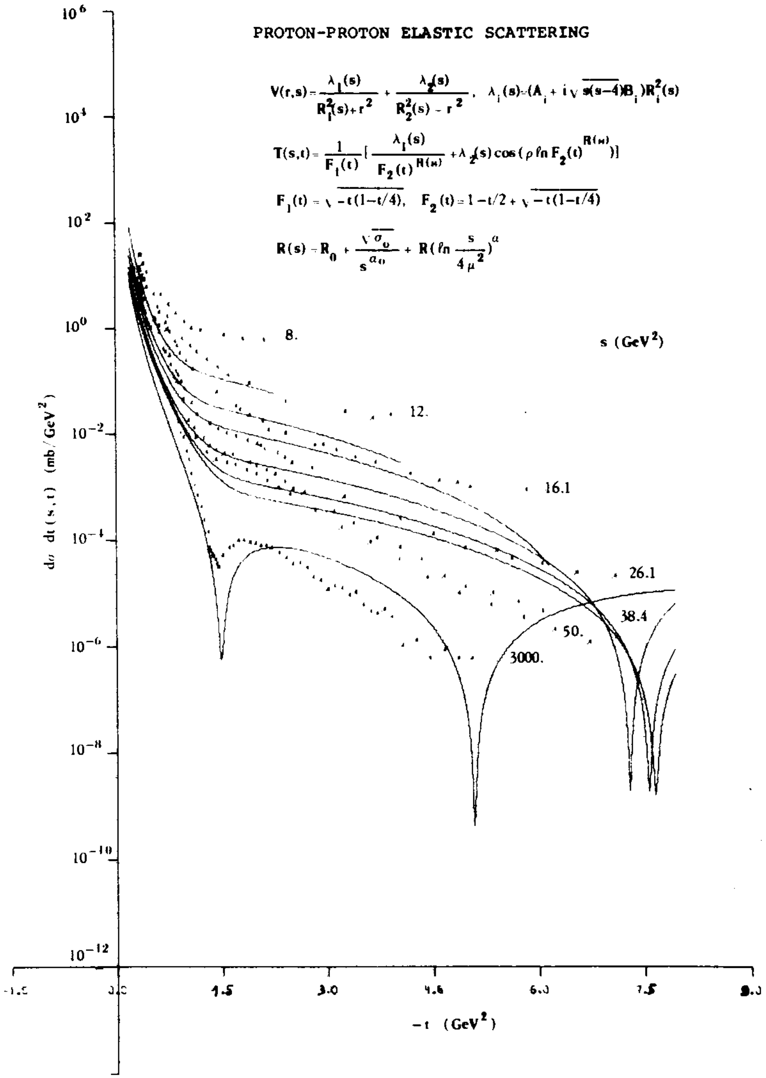


Fig. 2

In conclusion of this part of the paper we should stress that only a reliable and complete (in S and t) information about $\frac{d\sigma}{dt}(s,t)$ should allow us to investigate the uniqueness of the solution and, also, to obtain the statistic errors of the parameters.

3. Some consequences and interpretation of the results

We would like to mention that at $t \ll 1$ formula (2) becomes likewise the model-independent Barger-Phillips formula [9]

$$T = \sqrt{A}\,\exp\left(\tfrac{1}{2}Bt\right) + \sqrt{C}\,\exp\left(\tfrac{1}{2}Dt + i\phi\right).$$

Our description of the proton-proton elastic scattering differential cross section is given in Fig.3.

The first minimum of the differential cross section decreases continuously with increasing of the incident energy. The position of the first dip and the value of the second maximum agrees with the ISR experiments [10]. The so obtained solution gives a usual quantum mechanical difraction picture with increasing of the four momentum transfer.

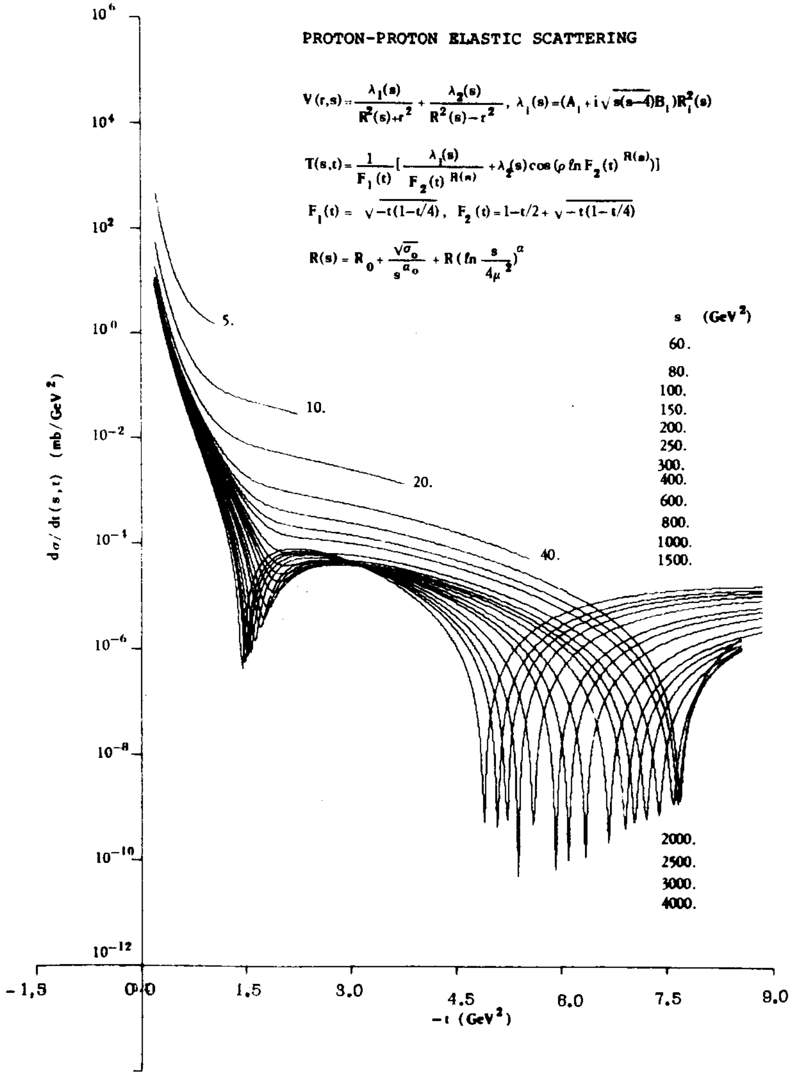


Fig. 3

An experimental test of the geometrical scaling of the proton-proton elastic scattering is given in Fig.4 [11]

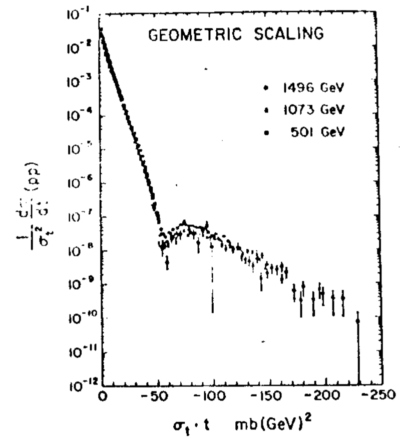


Fig.4

In Fig.5 the geometrical scaling property of our model is tested by assuming that the total cross section is proportional to the interaction radius squared.

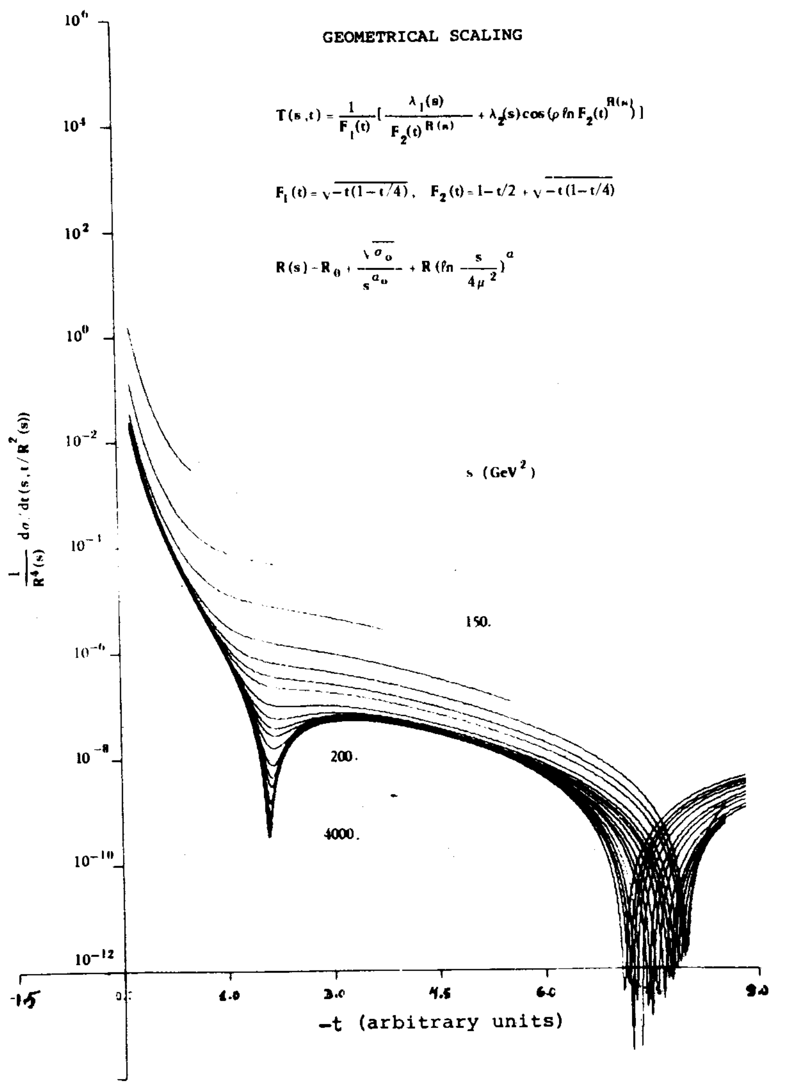


Fig. 5

It can be seen from Fig.4 that the geometrical scaling appears at incident energy $S=150\ (\mathrm{GeV})^2$.

The energy dependence of the interaction radius in the interval $4 \leq S \leq 4000\ (\mathrm{GeV})^2$ is shown in Fig.5.

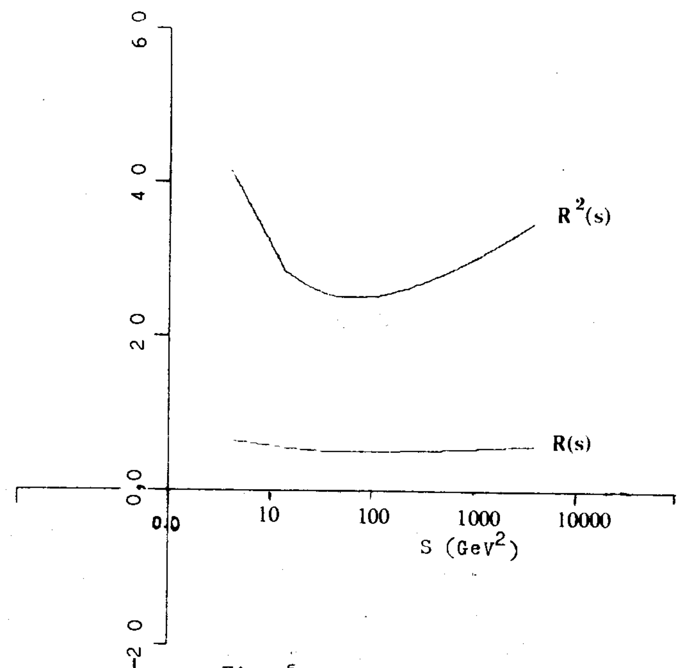


Fig. 5

Obviously, the interaction radius squared is in qualitative agreement with the present total cross section data.

The quantum mechanical language of our model allows us to treat the quasipotential of type (1) as an effective interaction potential of the nuclear matter. The picture of differential cross section of proton-nucleus elastic scattering gives a reason for such a conclusion, if supposing, as usual, the connection between number of the diffraction peaks and the number of the closed shells.

4. Conclusion

In conclusion we should list the main results:

1. The relativistic Fourier analysis and the simple phenomenological quasipotential give amplitude consistent with the experiment.

2. The position of the first minimum (in t) and the behaviour of the second maximum agree with the experimental data.

3. The geometrical scaling sets in at incident energy $s = 150\ (\mathrm{GeV}^2)$.

4. It appears that the total cross section is proportional to the interaction radius squared.

5. Our model predicts a difraction picture of the proton--proton elastic scattering likewise the experimentally observed picture of the elastic proton-nucleus scattering.

The quasipotential (1) is a phenomenological one, so in the future we should aim at investigating the type of the interaction "constants" and the radius from the view point of the unitarity and crossing symmetry conditions.

Another possible application of the relativistic Fourier analysis (the plane wave for arbitrary spin effective particles) should be done to describe the polarization phenomena in the proton-proton scattering.

Acknowledgements.

The authors express their gratitude to Drs. V.G.Kadyshevsky, M.D. Mateev, V.A.Meshcheryakov, R.M.Mir-Kasimov, V.I.Zuravlev and especially to V.A.Vinogradov for discussions.

It is a pleasant duty of the authors to thank Dr.L.Alexandrov for the decisive help in the initial stage of the computer calculations.

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Received by Publishing Department on May 19, 1975.