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К решению проблемы квантования заряда

Рассмотрена квантовая теория поля, в которой пространство виртуальных 4-импульсов обладает геометрией Де Ситтера с кривизной, определяемой фундаментальной длиной ℓ_0. Обычной локальной теории с p-пространством Минковского отвечает предельный переход $\ell_0 \to 0$.

На основе требования инвариантности относительно локальных калибровочных преобразований введено взаимодействие со скалярной компонентой ϕ электромагнитного поля. Установлено, что величина ϕ в новой схеме с необходимостью является угловой переменной: $|\phi| \le \frac{\hbar c}{\ell_0 e}\pi$. Этот факт может объяснить наблюдаемую на опыте целочисленность электрических зарядов частиц.

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On a Solution of the Charge Quantization Problem

Quantum field theory is consideres, in which the virtual 4-momentum space has De Sitter geometry with curvature defined by the fundamental length ℓ_0. The limit $\ell_0 \to 0$ corresponds to the usual local theory with Minkowsky p-space.

Basing on the invariance under local gauge transformation the interaction with the scalar component ϕ of the electromagnetic field is introduced. It is shown that the quantity ϕ in the new scheme is necessarily an angular variable: $|\phi| \le \frac{\hbar c}{\ell_0 e}\pi$. This fact can explain the observed integrality of the particle electric charges.

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The electric charge Q of all known elementary particles is integer multiple of the electron charge e. This fact, however, has no convincing theoretical interpretation. It has been noticed /1-3/ that Q is integer multiple of e in theories where compactness of the group of gauge transformations is postulated. Examples of such theories are the gauge field theories on lattice space-time /4/.

We shall demonstrate that electric charge quantization appears necessarily in the framework of quantum field theory (QFT) grounded on new physical ideas about momentum space /5-7/. This approach is based on the following reasoning:

1) In the ordinary local QFT the pseudoeuclidean character of the four-dimensional momentum space in which fields, currents, Green's functions, etc., are defined, does not follow logically from the basic axioms of the theory and is, essentially, an independent postulate. Therefore one can try to change the structure of the p-space at high energies and large momenta where the theory confronts difficulties (divergencies) in order to avoid them.

2) The axiomatic field theory in the form of Bogolubov /8,9/ can be reformulated assuming that the 4-momentum space possesses

De Sitter geometry. Concrete realization of such a curved p-space is the surface:

$$p_0^2 - \vec{p}^{\,2} + M^2 p_4^2 = M^2 = \frac{1}{\ell_0^2} \, . \tag{1}$$

The constant ℓ_0 is the "fundamental length" and M is the "fundamental mass". The formal limit $\ell_0 \to 0$ corresponds to usual "flat" theory. In the system of units $\hbar = c = M = \ell_0 = 1$, which we shall use, the region

$$p_0 , \; |\vec{p}| \ll 1 \; , \quad p_4 \simeq 1 \tag{2}$$

corresponds to the flat theory.

The Klein-Gordon equation has the following form in the new scheme:

$$2\,(p_4 - m_4)\,\psi(p) = 0 \, , \tag{3}$$

where $m_4 = \sqrt{1-m^2}$, $|p_4| = \sqrt{1-p^2}$. Introducing on the surface (1) coordinates $(\omega, \vec{p}\,)$:

$$\begin{aligned} p_0 &= \sqrt{1+\vec{p}^{\,2}} \, \sin\omega \\ p_4 &= \sqrt{1+\vec{p}^{\,2}} \, \cos\omega \end{aligned} \tag{4}$$

we have instead of (3):

$$2\left(\sqrt{1+\vec{p}^{\,2}} \, \cos\omega - m_4\right)\psi(\omega, \vec{p}\,) = 0 \, . \tag{5}$$

Let us consider the 4-vector:

$$\hat{M}_{4\mu} = -i p_4 \frac{\partial}{\partial p_\mu} \qquad , \qquad \mu = 0,1,2,3 \tag{6}$$

which is the generator of the 5-rotations in the (4μ)-plane. The time-component of the operator (6) in coordinates (5) has the form:

$$\hat{M}_{40} = -i \frac{\partial}{\partial \omega} \, . \tag{7}$$

The discrete eigenvalues n of this operator play the role of time intervals*). The correspondent eigenfunctions can be easily written:

$$\langle n | \omega \rangle = e^{in\omega} \, . \tag{8}$$

Of principal importance is the fact that in this scheme invariant T-product can be introduced in terms of the discrete time n. Here a specific step function of integer argument arises naturally:

$$\vartheta(n-n') = \frac{1}{4\pi i} \int_{-\pi}^{\pi} \frac{e^{i(n-n')\omega}}{tg\,\frac{\omega}{2} - i\varepsilon} \, d\omega = \begin{cases} 1 & n > n' \\ 0 & n < n' \end{cases} \tag{9}$$

It may be shown that the causal Green's function of the equation (3) is vacuum expectation value of the T_n-product of ψ-fields /6/. This circumstance allowed to construct a generalization of the Bogolubov's causality condition.

Let the field $\psi(\omega, \vec{p}\,)$ describes scalar particles with electric charge $Q = qe$. How can one introduce interaction with electromagnetic field in equation (5)? Let us require simply that the equation for the ψ-field remains invariant under local gauge transformations. In the usual theory the corresponding analysis is carried out in configuration space. For simplicity let us perform one dimensional Fourier transform in the variable ω and let us work in "mixed" $(n, \vec{p}\,)$-representation:

*) Because of the translation invariance of the theory only <u>relative</u> times are quantized /5-7/.

$$\psi(n,\vec{p}) = \frac{1}{2\pi}\int e^{in\omega}\,\psi(\omega,\vec{p})\,d\omega\ . \tag{10}$$

Equation (5) in this representation takes the form:

$$\left[\sqrt{1+\vec{p}^{\,2}}\left(e^{\frac{\partial}{\partial n}} + e^{-\frac{\partial}{\partial n}}\right) - 2m_4\right]\psi(n,\vec{p}) = 0\ . \tag{11}$$

Let us now transform the function $\psi(n,\vec{p})$:

$$\psi(n,\vec{p}) \rightarrow e^{iQ\lambda(n)}\,\psi(n,\vec{p})\ ,\quad Q = qe\ . \tag{12}$$

The equation of motion for the ψ -field will be invariant under (12) if one introduces gauge field $\varphi(n)$ with the following transformation law*):

$$\varphi(n) \rightarrow \varphi(n) + \Delta\lambda(n)\ , \tag{13}$$

where

$$\Delta\lambda(n) \equiv \lambda(n+1) - \lambda(n) \tag{14}$$

is the finite-difference derivative of the gauge function $\lambda(n)$. The equation itself gets the following form:

$$\sqrt{1+\vec{p}^{\,2}}\left(e^{-iQ\varphi}e^{\frac{\partial}{\partial n}} + e^{-\frac{\partial}{\partial n}}e^{iQ\varphi}\right)\psi(n,\vec{p}) = m^2\psi(n,\vec{p}) \tag{15}$$

$$Q = qe$$

In the flat limit (3) equation (15) becomes:

$$\left[\left(i\frac{\partial}{\partial t} - Q\varphi\right)^2 - \vec{p}^{\,2}\right]\psi(t,\vec{p}) = m^2\,\psi(t,\vec{p})\ . \tag{16}$$

*) Here it is sufficient to consider φ -field as a function of one argument n .

In such a way φ in (15) is a precise analog of the scalar potential of the electromagnetic field. Analyzing the equation (15) it is easy to see that the variable $Q\varphi$ has an angle variable nature like the parameter ω. But the region of definition of the gauge field cannot depend on the value of the electric charge Q of the field ψ . For this reason one has to consider the quantity $e\varphi$ as the analog of ω . Hence, the limits of variation of $e\varphi$ are the same as those for ω : $(-\pi,\pi)$. Putting (compare with (4)):

$$\begin{aligned} eA_0 &= \sqrt{1+e^2\vec{A}^2}\ \sin e\varphi \\ A_4 &= \sqrt{1+e^2\vec{A}^2}\ \cos e\varphi \qquad |e\varphi| \le \pi \end{aligned} \tag{17}$$

one can introduce Cartesian coordinates of the electromagnetic field $(eA_0, e\vec{A}, A_4)$ under the condition (compare with (1)):

$$e^2A_0^2 - e^2\vec{A}^2 + A_4^2 = 1\ . \tag{18}$$

So in our approach the electromagnetic field is momentum-like unit 5-vector. Taking into account dimension reasoning one can consider the quantity eA as a more "fundamental" object than the field A itself.

Since addition to the potential φ quantities multiple of $2\pi/e$ must not influence the physical results, then in the equation (15) only electric charges Q of the type

$$Q = qe = Ne \qquad \text{(N is integer number)} \tag{19}$$

are allowed.

In conclusion let us notice the following.

1) The conclusion that the charge is integer multiple of e is obtained in the framework of QFT rigorously satisfying the

requirements of the Lorentz and translation invariance. The time quantization is consistent with these requirements. Therefore the field equations of motion are relativistic and translation invariant (in this point a principle difference from lattice gauge field theories appears).

2) The charge quantization may be considered as implicit confirmation of existence of fundamental length in Nature.

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