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LOW-ENERGY PHYSICS OF MESONS IN THE QUANTUM CHIRAL THEORY

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Introduction.

It is well known that theoretical predictions made on the basis of current algebra are in excellent agreement with experimental data on hadron interactions at low energies. Nowadays there exists a great deal of good review articles on this subject[1]. Similar results may be obtained in a field-theoretical approach by using a chiral-invariant interaction Lagrangian. The field theory with a chiral-invariant Lagrangian has been proposed by Gürsey [2] and Gell-Mann and Levy[3]. Connection of current algebra with a chiral Lagrangian theory has been found by Weinberg [4] on the basis of consideration of "tree" diagrams only.

Note that the chiral-invariant Lagrangian has nonpolynomial form and leads to nonrenormalizable field theories. This circumstance favoured that for a long time many authors try to consider it only as a phenomenological Lagrangian on which basis one should not construct quantum field theory in the usual sense. It was used only for description of low-energy processes in a "tree" approximation.

However, when there appeared the methods allowing the description of quantum field theories with nonpolynomial Lagrangians (see, for instance, ref.[5]), more and more attempts were made to construct a quantum chiral field theory, where, in addition to "tree" diagrams, the "one-loop" approximation is considered[6-14]. We immediately notice that in this approximation we obtain not only corrections to the Born terms but also an essentially new information which is not contained in the "tree" approximation. This concerns such physical quantities as, for example, wave lengths and effective-range parameters of $\pi\pi$ -system for higher partial waves, starting from D -wave, meson form factors, its polarizabilities, and decay structure constants.

There immediately arises the question: to what extent is the perturbation theory correct for description of such interactions of particles where, apart from the weak and electromagnetic interactions, there are also the strong ones? But so far one cannot give a comprehensive answer to this question. However, there are some grounds to believe that in describing the low-energy interactions in a chiral quantum field theory at energies $\sqrt{q^2} < 4\pi F_\pi \approx$ 1,2GeV ($F_\pi \approx 92\, MeV$ is the pion decay constant) one may obtain quite reasonable results in the one-loop approximation.

The basic idea of our approach is tightly related to the following supposition firstly made by Lehnmann[7]. Since, by the Adler-Weinberg theory, in the low-energy limit the first perturbation expansion order gives the exact result then, consequantly, higher-order corrections should also be small at small energy--momentum. There, of course, remains the question concerning relative importance of subsequent orders of perturbation theory. As to the direct $\pi\pi$ - interaction the use of perturbation theory here is completely justified because there we have a small expansion parameter of the type $\left(\frac{m_\pi}{4\pi F_\pi}\right)^2 \approx 0.015$. Unfortunately this is not the case for πN -interactions, where there appears larger parameter $\left(\frac{M_N}{4\pi F_\pi}\right)^2 \approx 0.66$ (however, it is smaller than unity). Nevertheless, one may hope to consider the higher-order strong effects by performing the finite renormalization of the "strong" vertex: appearance of a factor g_A for $\frac{M_N}{F_\pi}$ (see ref.7,15).

One of interesting features of the chiral quantum theory, in particular, is that in the one-loop approximation for πN interaction there occurs the cancellation of all divergences when all diagrams of a given perturbation expansion order in $1/F_\pi^2$ are considered together.

Therefore, we shall need to use special methods (e.g., the superpropagator ones[5]), necessary for description of non-polynomial theories, only when considering direct $\pi\pi$- interactions the contribution of which to physical quantities is, as a rule, considerably smaller than that from πN - interactions. The latter are described by standard methods of the renormalizable quantum field theory.[16]

To complete this section we would like to note the following. Despite the absence of rigorous proof of validity of the employed perturbation theory, one can say with certainty that the information contained in the one-loop approximation of the chiral quantum theory gives a correct physical picture of hadron interactions at low energies. That the considered approximation works well is confirmed indirectly by the following: i) The calculated physical quantities are in good agreement with the available experimental data [17-20] as well as with predictions of other known models (e.g., the ρ -dominance model [21]); ii) the found expressions obey the conditions following from the most general requirements of field theory (say, the Martin inequalities for $\pi^0\pi^0$--scattering [22]).

In the next section we present the basic principles necessary for construction of a chiral-invariant Langrangian. In the second section we shall describe, in more detail, the perturbation theory used here. In the third section pion strong interactions are considered and $\pi\pi$ -scattering amplitudes, scattering lengths, and effective-range parameters are calculated. In the fourth section the pion electromagnetic interactions are studied and the pion electromagnetic radius and polarizability are found. The fifth section deals with the main modes of pion decays and with calculations of the decay structure constants. In the sixth section the electromagnetic interactions are investigated for kaons and their electromagnetic radius and polarizability are calculated. And finally, in the seventh section the mass difference of neutral K_L and K_S mesons is computed.

1. Chiral-invariant Lagrangian.

Nowadays there exist various methods for construction of Lagrangians invariant with respect to the chiral group $SU(2)\times SU(2)$ (see, e.g.refs.[2,3]).We attempt to describe this procedure in the most simple way.

First of all we call attention to that one should distinguish between two different symmetries; kinematical and dynamical when constructing a Lagrangian of interacting fields. The kinematical (algebraic) symmetry requires the invariance of a Lagrangian under space-time translations and rotations of coordinates.It is the condition of relativistic (Lorentz) invariance which must hold for any Lagrangian. The dynamical symmetry requires the invariance of a Lagrangian with respect to certain transformations of fields entering into the total Lagrangian and, as a rule, it corresponds to some internal symmetries of the system of fields. The most known example of this symmetry is the symmetry of the electrodynamical Lagrangian under gradient transformations of fields A_μ and under gauge transformations of fields ψ . Recall briefly a method for construction of the interaction Lagrangian in electrodynamics with the use of this symmetry.

The Lagrangian of free nucleon fields ψ

$$\mathcal{L}_0 = i\bar{\psi}\overleftrightarrow{\hat{\partial}}\psi - M_N\bar{\psi}\psi, \tag{1}$$

where $\bar{\psi}\overleftrightarrow{\hat{\partial}}\psi = \frac{1}{2}[\bar{\psi}\gamma_\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma_\mu\psi]$, M_N is the nucleon mass, is not invariant with respect to gauge transformations of the field ψ_p:

$$\psi_p' = \psi_p e^{-ie\Lambda(x)}, \tag{2}$$

where e is the proton charge, $\Lambda(x)$ is an arbitrary smooth function of x . If, however, one introduces an electromagnetic field A_μ interacting with ψ_p by the law

$$\mathcal{L}_{\psi A} = -e\bar{\psi}_p\gamma_\mu\psi_p A_\mu , \tag{3}$$

then the total Lagrangian will simultaneously be invariant with respect to transformations (2) and gradient transformations of the field A_μ

$$A_\mu' = A_\mu + \partial_\mu\Lambda(x) . \tag{4}$$

This symmetry corresponds to the well known requirement that the observables do not change under the gradient transformations of the fields A_μ .

Now we proceed to construct a chiral-invariant Lagrangian. Consider again the Lagrangian (1). It is invariant under isotopic transformations $\psi' = e^{i\vec{\tau}\vec{\omega}}\psi$, where $\vec{\tau}$ is the isotopic matrix, $\vec{\omega}$ is the constant vector in the isotopic space. Let us examine whether this Lagrangian is invariant under chiral transformations mixing up states with different parity:

$$\psi' = \mathcal{U}^{-1/2}(a)\psi, \qquad \mathcal{U}(a)\,\mathcal{U}^{*}(a) = 1. \tag{5}$$

The matrix $\mathcal{U}$ may be taken, for instance, in the form

$$\mathcal{U}_{exp}(\alpha) = \exp\{-\gamma_5 \vec{\tau}\vec{a}\}, \quad \gamma_5 = -i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{6}$$

After transformation (5) Lagrangian (1) acquires the form

$$\mathcal{L}_0(\psi') = i\bar{\psi}\hat{\partial}\psi - M_N\bar{\psi}\,\mathcal{U}^{-1/2}(a)\,\mathcal{U}^{-1/2}(a)\psi. \tag{7}$$

It is easily seen that if $M = 0$ then Lagrangian (1) is invariant under the chiral transformations. To this symmetry there corresponds the well known law of helicity conservation for neutrino. When $M_N \neq 0$ one needs to make use of the dynamical method for restoring the invariance of the total Lagrangian under the chiral transformations, by analogy with electrodynamics. There we introduce the electromagnetic field A_μ. Here, since we deal with transformations changing parity of the system, we must introduce an interaction with pseudo-scalar massless particles, "pions", which change parity. Usually, these are called the Goldstone particles

$$M_N\bar{\psi}\psi \to M_N\bar{\psi}\,\mathcal{U}\left(\frac{\vec{\pi}}{F_\pi}\right)\psi. \tag{8}$$

To the chiral transformation (5) of the field ψ there corresponds the nonlinear transformation of the field $\vec{\pi}$

$$\vec{\pi}' = \vec{\pi}'(\vec{\pi}, a), \tag{9}$$

which form is defined from the condition

$$\mathcal{U}\left(\frac{\vec{\pi}'}{F_\pi}\right) = \mathcal{U}^{1/2}(a)\,\mathcal{U}\left(\frac{\vec{\pi}}{F_\pi}\right)\mathcal{U}^{1/2}(a) \tag{10}$$

The dimensional constant F_π is introduced to make the scalar field $\vec{\pi}$ dimensionless. It is not difficult to verify that the new Lagrangian

$$\mathcal{L}(\psi,\vec{\pi}) = i\bar{\psi}\hat{\partial}\psi - M_N\bar{\psi}\,\mathcal{U}\left(\frac{\vec{\pi}}{F_\pi}\right)\psi + \mathcal{L}_{\pi\pi}, \tag{11}$$

$$\mathcal{L}_{\pi\pi} = \frac{F_\pi^2}{4}\,Sp\left\{\partial_\mu\,\mathcal{U}\left(\frac{\vec{\pi}}{F_\pi}\right)\partial_\mu\,\mathcal{U}^{+}\left(\frac{\vec{\pi}}{F_\pi}\right)\right\} \tag{12}$$

is invariant under the chiral transformations (5) and (10)*).

The appearance, in this way, of pions in Lagrangian (1) may be interpreted as a reaction of the system aimed at restoring of the chiral symmetry broken due to the nucleon mass. The algebra of the chiral group $SU(2)\times SU(2)$ coincides with that of the group of rotation of 4-dimensional space $O(4)$ and Lagrangian (12) represented in the form

$$\mathcal{L}_{\pi\pi} = \tfrac{1}{2}\,g_{ij}(\vec{\pi})\,\partial_\mu\pi^i\,\partial_\mu\pi^j$$

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*) For $\mathcal{U}$ taken in the form (6) we have

$$\mathcal{L}_{\pi\pi} = \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{1}{2}\left[(\partial_\mu\vec{\pi})^2 - \frac{(\vec{\pi}\partial_\mu\vec{\pi})^2}{\vec{\pi}^2}\right]\left[\frac{\sin\sqrt{\vec{\pi}^2/F_\pi^2}}{\vec{\pi}^2/F_\pi^2} - 1\right] \tag{12'}$$

$$\mathcal{L}_{\pi\pi} = \left[+\frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{(\vec{\pi}\partial_\mu\vec{\pi})^2}{2\vec{\pi}^2}\right]\left[1 - \frac{\sin^2\sqrt{\vec{\pi}^2/F_\pi^2}}{\vec{\pi}^2/F_\pi^2}\right]$$

has a beautiful geometrical interpretation. g_{ij} is the metric tensor of a three-dimensional isospace of constant curvature (of a sphere in the four-dimensional Euclidean space).
Transformation (9) is a displacement of the coordinate origin on the sphere by vector $\vec{a}$, therefore (9) has the meaning of summation of vectors in a curved isospace which will be denoted as

$$\vec{\pi}' = \vec{\pi}(+)\vec{\tilde{a}}, \quad \vec{\tilde{a}} = F_{\pi}\vec{a}. \qquad (13)$$

The constant F_{π} is a parameter characterizing a curvature of the isospace. For $F_{\pi} \to \infty$ there arises the usual Euclidean isospace $\vec{\pi}(+)\vec{\tilde{a}} \to \vec{\pi} + \vec{\tilde{a}}$.

If one identifies the introduced Goldstane field with a real pion and the axial current $\vec{J}_{5\mu} = F_{\pi}\partial_{\mu}\vec{\pi} + O(\pi^3)$ with a current appearing in weak interactions, then the constant F_{π} in the Born approximation coincides with the constant of pion weak decay. Hence F_{π} = 92MeV.

The chiral invariance does not fix completely the Lagrangian form leaving an arbitrariness in choice of the matrix U which obeys only the condition $UU^{+}=1$. This arbitrariness corresponds to the arbitrariness in choice of different coordinates on a sphere. For instance, to the representation of the matrix U by exponential form (6) there correspond the normal coordinates along geodesics. The change of coordinates is made by the transformation

$$\pi = \pi' f(\pi'), \quad f(0) = 1. \qquad (14)$$

At present, the well-developed methods exist for description of theories with chiral Lagrangians. These are independent of a concrete choice of Lagrangian, i.e.,chiral invariant methods.

In the "tree" approximation, the independence of physical observables of a choice of the chiral Lagrangian form has first been proved by D.V.Volkov[23]. In the chiral quantum theory such an equivalence has been shown in papers[24,25]. It will be convenient for us to use the most simple and natural, from the geometrical point of view, exponential form of the chiral Lagrangian.

2. Perturbation Theory for Chiral Lagrangian

Let us now discuss, in more detail, the perturbation theory used here. To begin with, it is useful to see what changes appear, in the chiral theory, in the Lagrangian describing the strong interactions.

In a standard renormalizable theory, where the chiral symmetry is not taken into account when constructing the Lagrangian, the interaction Lagrangian looks as follows

$$\mathcal{L}_{int} = g :\bar{\psi}\gamma_5\vec{\tau}\vec{\pi}\psi: + h :(\vec{\pi}^2)^2: . \qquad (15)$$

Here g is the strong coupling constant ($g^2/4\pi = 14.7$), h is the second constant of the direct $\pi\pi$-interaction. Since the constant g is large in magnitude, perturbation theory expansion cannot give correct results for this theory. On the other hand, one might hope to obtain reasonable results in the low-energy limit. However, it has appeared that here also the theory with Lagrangian (15) does not describe the right behaviour of, e.g., the $\pi\pi$ scattering amplitude or $\pi\pi$ scattering lengths.

Now let us write the chiral Lagrangian (11) in the lowest orders in the constant $(1/F_\pi)$;

$$\mathcal{L}^{ch}_{int} = \frac{M_N}{F_\pi} :\bar\psi \gamma_5 \vec\tau \vec\pi \psi: + \frac{M_N}{2F_\pi^2} :\bar\psi\psi \vec\pi^2: - \frac{:\vec\pi^2(\partial_\mu \vec\pi)^2:}{4F_\pi^2} \qquad (16)$$

Comparing it with (15) we see that, first, in (16) only one constant, F_π, is present. This fact will help us to extract the complete set of diagrams of given order in the constant $(1/F_\pi)$. Second, there appears the connection with a derivative in the direct $\pi\pi$ -interaction. Third, there emerges one more term describing πN -interaction. The latter is extremely important, because, as we shall see, its consideration helps to cancel completely all divergences in loop diagrams in a given order of $(1/F_\pi)$ with the πN -vertices.

Unfortunately, in this theory also there is no small expansion parameter necessary for a successful use of perturbation theory. Nevertheless, in the low- energy limit we obtain the correct description of particle interactions corresponding to the low-energy theorems of current algebra. For instance, for the $\pi\pi$ -scattering amplitude in the Born approximation we have

$$A(s,t,u) = \frac{s}{F_\pi^2} + \dots , \qquad (17)$$

that is in good agreement with experiment.

However, if in the limit of small q^2 ($q^2 \to 0$) we have the true result, then one may expect that the higher order in q^2 corrections obtained from subsequent orders of perturbation expansion in $(1/F_\pi)^2$ will be reasonable too. Let us observe these higher orders of perturbation theory.

The second order in $(1/F_\pi)^2$, the one-loop approximation, as a rule, gives small quantities of the type $\left(\frac{q^2}{(4\pi F_\pi)^2}\right)^2$ or $\left(\frac{m_\pi^2}{(4\pi F_\pi)^2}\right)^2$ (see sects. 3-6). The third order is more complicated. Here, in addition to certainly small quantities of the type $\left(\frac{q^2}{(4\pi F_\pi)^2}\right)^3$ or $\left(\frac{m_\pi^2}{(4\pi F_\pi)^2}\right)^3$, larger quantities of the form $\left(\frac{q^2}{(4\pi F_\pi)^2}\right)^2\left(\frac{M_N}{4\pi F_\pi}\right)^2$ may appear. Just this is the influence of strong vertices of Lagrangian (16). However, one may hope that the consideration of these terms will reduce simply to renormalization of the strong vertices, namely, to appearance, in (16), of the factor g_A necessary, at the same time, for fulfilment of the Goldberger-Treiman relation *

$$g \approx g_A \frac{M_N}{F_\pi} . \qquad (18)$$

Then Lagrangian (16) can be rewritten as follows

$$\mathcal{L}^{ch}_{int} = g :\bar\psi \gamma_5 \vec\tau \vec\pi \psi: + \frac{g^2}{2M_N} :\bar\psi\psi \vec\pi^2: - \frac{1}{4F_\pi^2} :\vec\pi^2(\partial_\mu \vec\pi)^2: . \qquad (19)$$

To conclude the section, we stress once more that the chrial quantum field theory contains some energy scale equal to $4\pi F_\pi \approx 1.2\ GeV$, as it has already been mentioned in papers [7,26]. Therefore, till we are considering the low-energy interactions of hadrons with energies $q^2 < (4\pi F_\pi)^2$, we may hope to obtain reasonable corrections, in the one-loop approximation, to contributions from "tree" diagrams (the Born terms). In what follows we shall demonstrate this by calculations

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*) Further on, following Lehmann[7], we put $g_A = 1.25$.

of various quantities of strong, weak and electromagnetic hagron interactions.

3. $\pi\pi$ -Scattering (Strong Interactions)[7-9]

Further we shall omit most of details of calculations (these can be found in original papers[7-14]), but mainly discuss the results obtained. First of all, consider the elastic $\pi\pi$ -scattering.

The scattering amplitude has the form

$$\langle i_1 i_2 | S | i_3 i_4 \rangle = \left[(2\pi)^6 4\sqrt{p_1^0 p_2^0 p_3^0 p_4^0}\right]^{-1} \Big\{ I + i(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \times$$
$$\times \left[\delta_{i_1 i_2}\delta_{i_3 i_4} A(s,t,u) + \delta_{i_1 i_3}\delta_{i_2 i_4} A(t,s,u) + \delta_{i_1 i_4}\delta_{i_2 i_3} A(u,t,s)\right]\Big\}, \qquad (20)$$

where I is the unit matrix, i_K are the pion isotopic indices, δ_{ij} are the Kronecker symbols, $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, $u=(p_1-p_4)^2$. In Fig. 1 the diagrams are drawn which correspond to the one-loop approximation (the order is not higher than $1/F_\pi^4$). Diagram 1a

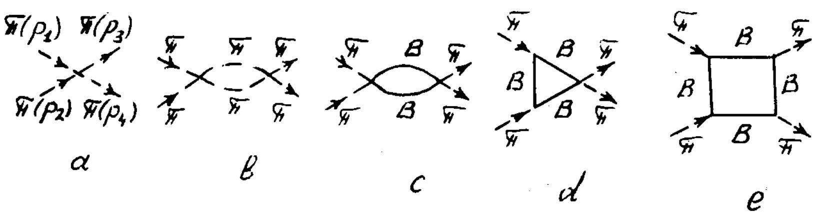

Fig. 1

corresponds to the "tree" approximation (the Born term (17)). The contribution to the amplitude from diagram 1b is calculated by using the superpropagator method[5]. The contributions from all the other diagrams are calculated by standard methods of renormalizable field theories and here only the terms quadratic in variables s, t, u are kept, as the terms of higher powers will be small, of the type $O\left(\frac{s}{M_N^2}\right)$. Taking into account of the contributions from all members of the baryon octet is performed by the use of the $SU(3)$ theory [8, 27]. As a result, in the $1/F_\pi^4$ approximation we obtain the following expression for $A(s,t,u)$ [9]:

$$A(s,t,u)/(4\pi)^2 = \alpha_0(3\bar{s}-1) + \alpha_0^2 \Pi(\bar{s},\bar{t},\bar{u}),$$
$$\Pi(\bar{s},\bar{t},\bar{u}) = A + B\bar{s} + C\bar{s}^2 + D(\bar{t}^2+\bar{u}^2) - (3\bar{s}-1)^2 J(\bar{s}) -$$
$$-[3(\bar{u}-1)(\bar{u}-\bar{t}) + 3\bar{u}-1]J(\bar{u}) - [3(\bar{t}-1)(\bar{t}-\bar{u}) + 3\bar{t}-1]J(\bar{t}), \qquad (21)$$

where $\bar{\xi} = \xi/4m_\pi^2$ $(\xi = s,t,u)$, $A=-\frac{3}{2}$, $B=3$, $C=0.63$, $D=20.5$, $\alpha_0 = \frac{1}{3}\left(\frac{m_\pi}{2\pi F_\pi}\right)^2$,

$$J(\bar{\xi}) = 1 - \frac{1}{2}\sum_1^\infty (4\bar{\xi})^n \frac{n!(n-1)!}{(2n+1)!} =$$
$$= \begin{cases} x \,\mathrm{arctg}\, x^{-1} & x = \left(\frac{1}{\bar{\xi}}-1\right)^{1/2}, \quad 0<\bar{\xi}<1 \\ \frac{y}{2}\left[-i\pi + \ln\frac{1+y}{1-y}\right] & \\ \frac{y}{2}\ln\frac{y+1}{y-1} & y = \left(1-\frac{1}{\bar{\xi}}\right)^{1/2}, \quad \bar{\xi}>1, \ \bar{\xi}<0 \end{cases} \qquad (22)$$

At energies considerably smaller than $4\pi F_\pi$ formula (21) is good expansion of the $\pi\pi$ -scattering amplitude in powers of small parameter $\alpha_0 \approx 0.02$. The form of the amplitude in channels with isospins 0, 1, and 2 is defined by the formulae

$$A^0 = 3A(s,t,u) + A(t,s,u) + A(u,t,s),$$
$$A^1 = A(t,s,u) - A(u,t,s), \quad A^2 = A(t,s,u) + A(u,t,s). \qquad (23)$$

Following paper [28] we introduce the notations

$$\alpha_\ell^{I(n)} = \lim_{\bar{s}\to 1} 4^{-n} \frac{\partial^n}{\partial \bar{s}^n} A_\ell^I(\bar{s}),$$

$$\alpha_\ell^{I(0)} = a_\ell^I, \quad \alpha_\ell^{I(1)} = b_\ell^I, \quad \alpha_\ell^{I(2)} = c_\ell^I, \quad \left(\begin{array}{l}\bar{t} = \frac{1}{2}(1-\bar{s})(1-x), \\ \bar{u} = \frac{1}{2}(1-\bar{s})(1+x)\end{array}\right)$$

$$A_\ell^I(\bar{s}) = \frac{1}{2(\bar{s}-1)^\ell} \int_{-1}^{1} dx \, P_\ell(x) A^I(\bar{s}, x). \tag{24}$$

Here a_ℓ^I are the scattering lengths, b_ℓ^I and c_ℓ^I are the effective range parameters, $P_\ell(x)$ is the Legendre polynomial. Then, for the $\pi\pi$ -scattering lengths and effective range parameters we obtain the values presented in the Table.

Table

$\alpha_\ell^{I(n)}$	Experiment	Our values	Values from ref [28]
a_0^0	0,10; 0,60	0,15	0,15 ± 0,02
a_0^2	-0,10; -0,03	-0,042	-0,065 ± 0,025
a_1^1	0.032; 0,040	0,031	0,0341 ± 0,0036
b_1^1		$1,14.10^{-3}$	$(1,07 \pm 0,27).10^{-3}$
a_2^0	$1,4.10^{-3}$; $1,8.10^{-3}$	$1,85.10^{-3}$	$(1,48 \pm 0,08).10^{-3}$
a_2^2	$-2,10^{-4}$; 3.10^{-4}	$2,6.10^{-4}$	$(-3 \pm 8).10^{-5}$
b_2^0		$-1,02.10^{-4}$	$(-3,8 \pm 1,1).10^{-5}$
b_2^2		$-5,1.10^{-5}$	$(-4,4 \pm 1,1).10^{-5}$
c_2^0		2.10^{-5}	$(1,13 \pm 0,36).10^{-5}$
c_2^2		$1,06.10^{-5}$	$(1,27 \pm 0,36).10^{-5}$
a_3^1		$1,33.10^{-5}$	$(3,8 \pm 0,5).10^{-5}$
a_4^0		5.10^{-6}	$(4.8 \pm 0,8).10^{-6}$
a_4^2		2.10^{-6}	$(1,7 \pm 0,8).10^{-6}$

For $\ell \geq 3$, the above formula allows us to obtain the following simple expressions for the scattering lengths:

$$a_\ell^0 = (2\ell+1)(4\ell+7) Z_\ell,$$

$$a_\ell^1 = 1/3(4\ell^2-2\ell-1) Z_\ell, \qquad Z_\ell = 3\pi \alpha_0^2 4^{\ell-1} \frac{(\ell!)^3(\ell-3)!}{[(2\ell+1)!]^2}.$$

$$a_\ell^2 = (4\ell^2+3\ell+8) Z_\ell. \tag{25}$$

The results given in the Table are in good agreement with the known experimental data [17a,d; 28], as well as with the results of the Paloн-Yndurain phenomenological approach [28], where the Gribov-Froissart representation is used. The results we have found correspond to choice of parameters in the mentioned model which gives the value $a_0^0 = 0.15$ (see Table II in ref.[28]).

All the scattering lengths for $\ell \geq 3$ obey the inequalities

$$a_{\ell+2}^I \leq a_\ell^I \frac{(\ell+1)(\ell+2)}{4(2\ell+3)(2\ell+5)}, \tag{26}$$

derived in papers[22] from the requirements of unitarity and analyticity of scattering amplitude.

Notice, that though the values of scattering lengths of the S and P waves are mainly defined by the Born term(17)(Fig.1a), the Born term contribution to the scattering lengths of higher partial waves, starting from D-wave, is entirely absent, and their values are determined by the contribution of pion loop diagram 1b.

Expanding the amplitude A^I in partial waves and making use of the formula $(ctg\,\delta_\ell^I - i)^{-1} = (1-\frac{1}{\bar{s}})^{1/2} A_\ell^I$, one may gain information on a behaviour of the $\pi\pi$ -phases. In Figs. 2,3 the corresponding graphs are drawn. The dashed line shows the behaviour of phases in the limit $m_\pi = 0$ (the case considered in refs.[7,8]). In the P-wave one clearly sees the ρ-meson

resonance at energy $\sim 800\,MeV$ with width $\sim 150\,MeV$.
The points ⧫ and + are taken from ref.[17a], ⧫ and · from ref.[17b], and ⧫ from ref.[17c] (for the other notation see ref.[8]).

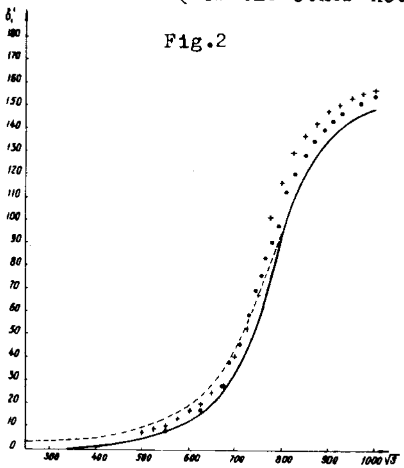

Fig.2

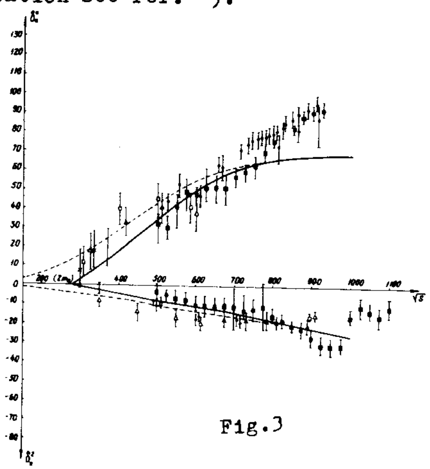

Fig.3

To complete this section we also mention some inequalities found by Martin [22] in the subthreshold region from the conditions of unitarity and crossing-symmetry for the S-wave of the process $\pi^0\pi^0 \to \pi^0\pi^0$

$$f_0^{00}(s) = \frac{1}{64\pi}\int_{-1}^{1} dx\,[A(s,t,u) + A(t,s,u) + A(u,t,s)] \qquad (27)$$

Let us write down these inequalities

$$1)\ f_0^{00}(s) < f_0^{00}(1), \quad 0 \le s \le 1$$

$$2)\ \frac{d f_0^{00}(s)}{ds} > 0, \quad 0.5 \le s \le 1$$

$$3)\ f_0^{00}(s) \ge 2\int_{0.5}^{1} ds\, f_0^{00}(s),$$

$$4)\ f_0^{00}(s) > f_0^{00}\left(\tfrac{1}{2}\left(1+\tfrac{1}{\sqrt{3}}\right)\right),$$

$$5)\ \frac{d f_0^{00}(s)}{ds} < 0, \quad 0 \le s \le \frac{1{,}24}{4}$$

$$6)\ \frac{d f_0^{00}(s)}{ds} > 0, \quad 1{,}7 \le 4s \le 1{,}76$$

$$7)\ f_0^{00}(0{,}8) > f_0^{00}\left(\frac{0{,}21}{4}\right) > f_0^{00}\left(\frac{2{,}98}{4}\right). \qquad (28)$$

Direct calculations show that amplitude (21) obeys completely all these inequalities.

4. Pion Electromagnetic Form Factor and Polarizability (Electromagnetic Interactions)[10,11]

The interaction with electromagnetic field A_μ is introduced into Lagrangian (11) by the standard gauge-invariant way

$$\partial_\mu \psi_p \to (\partial_\mu + ieA_\mu)\psi_p,$$
$$\partial_\mu \pi^\pm \to (\partial_\mu \pm ieA_\mu)\pi^\pm. \qquad (29)$$

Then, in addition to Lagrangian (3) for the pA -interaction, we obtain the following Lagrangian of the πA -interactions

$$\mathcal{L}_{\pi A} = ie\,[A_\mu(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+) + eA_\mu^2\pi^+\pi^-]\,\frac{\sin^2\sqrt{\vec{\pi}^2/F_\pi^2}}{\vec{\pi}^2/F_\pi^2}; \quad (30)$$

a) The matrix element for the pion in an external electromagnetic field A_μ equals

$$\langle \pi^+ | S(A) | \pi^+ \rangle = ie \frac{p_\mu A_\mu(q)}{(2\pi)^3 2\sqrt{p_1^0 p_2^0}} \Phi_\pi(q), \tag{31}$$

where p_1 and p_2 are the pion momenta, $p = p_1 + p_2$, $q = p_1 - p_2$ and

$$\Phi_\pi(q) = 1 + \Phi_\pi^{(\pi)}(q) + \Phi_\pi^{(B)}(q) + \dots \tag{32}$$

is the pion form factor. $\Phi_\pi^{(\pi)}(q)$ is the contribution to the form factor from pion diagram 4b and $\Phi_\pi^{(B)}(q)$ – that from baryon diagrams 4c,d,e in the e/F_π^2 -approximation.

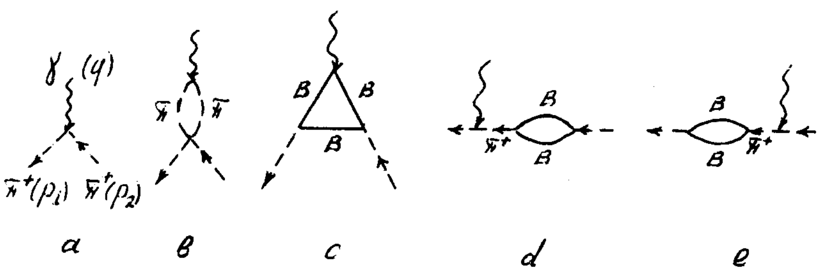

Fig.4

To calculate the function $\Phi_\pi^{(\pi)}(q)$ we again employ the superpropagator method. As a result, we have

$$\Phi_\pi^{(\pi)}(q) = \alpha_0 \left\{ \bar q^2 \left[\frac{13}{24} - \frac{3}{2}C + \ln \frac{2\pi F_\pi}{m_\pi} \right] - 1 + \frac{4}{3}\bar q^2 + (1-\bar q^2) J(\bar q^2) \right\}, \tag{33}$$

where $C = 0.577\ldots$, $\bar q^2 = \frac{q^2}{4m_\pi^2}$; α_0 and $J(\bar q^2)$ are the same as in formula (21),From (33) it is seen that the contribution from the pion loop to the pion radius equals

$$\langle r^2 \rangle_\pi^{(\pi)} = \frac{3}{2} \frac{\alpha_0}{m_\pi^2} \left[\frac{13}{24} - \frac{3}{2}C + \ln \frac{2\pi F_\pi}{m_\pi} \right] \simeq 0.065 (fm)^2 . \tag{34}$$

The contribution from baryon diagrams is again calculated up to q^2 -term due to smallness of the other terms. All the divergences in diagrams 4c, d,e cancel and for $\Phi_\pi^{(B)}(q)$ we obtain the expression*)

$$\Phi_\pi^{(B)}(q) \simeq \frac{1.7}{6(2\pi)^2} g^2 \frac{q^2}{M_N^2} . \tag{35}$$

Hence, for the pion mean square radius we find the contribution

$$\langle r^2 \rangle_\pi^{(B)} \simeq 0.36 (fm)^2 \tag{36}$$

From (34) and (36) we have finally:

$$\sqrt{\langle r^2 \rangle_\pi} \simeq \sqrt{\langle r^2 \rangle_\pi^{(\pi)} + \langle r^2 \rangle_\pi^{(B)}} \simeq 0.65 (fm), \tag{37}$$

that is in a satisfactory agreement with the recent experimental data[18].

Inserting the functions (33), (35) into (32) we arrive at the following expression of the pion form factor

$$\Phi_\pi = 1 + \alpha_0 \{ -1 + 8.6\bar q^2 + (1-\bar q^2) J(\bar q^2) \} . \tag{38}$$

This formula describes the behaviour of pion form factor at energies $\sqrt{|q^2|} < 1\, GeV$ in good agreement with the experimental data recently obtained at Dubna and Serpukhov[18] (see Fig 5,6: Points ⊥ are from ref[18a], ⊥ from ref.[18b]).

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*) Factor 1.7 arises as a result of consideration of all members of the baryon octet (see [10]). The π-K -interactions make a very small contribution to the pion form factor and these will not be discussed here.

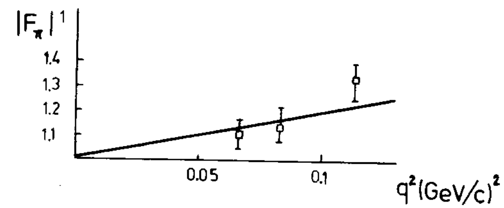

Fig.5

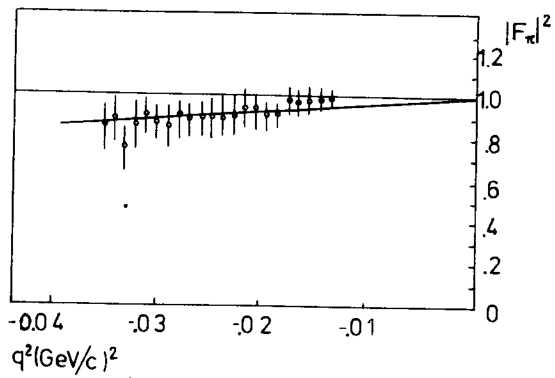

Fig. 6

It is interesting to notice that the pion radius is defined almost completely by the contribution of the baryon loop diagrams. The value of the radius we have found is close to predictions on the basis of the ρ -dominance model $\left(\sqrt{\langle r^2\rangle_\pi} \sim \sqrt{\frac{6}{m_\rho^2}} \sim 0.64 (fm)\right)$.

b) Now let us write the matrix element corresponding to the Compton effect by pion

$$\langle \pi^a(p_1)\pi^b(p_2)|S|\gamma_{\lambda_1}(q_1)\gamma_{\lambda_2}(q_2)\rangle = \frac{i\delta^{(4)}(p_1+p_2-q_1-q_2)\varepsilon^\mu_{\lambda_1}\varepsilon^\nu_{\lambda_2}}{(2\pi)^2 4\sqrt{p_1^0 p_2^0 q_1^0 q_2^0}} T^{\mu\nu}_{ab}(p_1p_2|q_1q_2), \quad (39)$$

where q_1, q_2 are the photon momenta, $\varepsilon^\mu_{\lambda_1}, \varepsilon^\nu_{\lambda_2}$ are polarizabilities, p_1, p_2 are the pion momenta, a,b isotopic indices. Note immediately that for this process in the one-loop approximation divergences cancel not only in the baryon loop diagrams but

also in the pion ones. Therefore we shall not need to employ the superpropagator method and shall be able to confine our consideration to the lowest orders of the nonpolynomial chiral Lagrangian. On introducing the mass term $-\frac{m_\pi^2}{2}\vec{\pi}^2$, the Lagrangian (12') in the F_π^{-2} order takes the form*)

$$\mathcal{L}^{(2)}_{\pi\pi} = -(2F_\pi)^{-2} : \vec{\pi}^2 [(\partial_\mu \vec{\pi})^2 - \frac{m_\pi^2}{3}\vec{\pi}^2] : \quad (12'')$$

Without giving the general expression for the covariant amplitude $T^{\mu\nu}_{ab}$, we write here only the two first orders of perturbation expansion of this amplitude

$$T^{\mu\nu}_{ab} = 2e^2(\delta_{ab} - \delta_{3a}\delta_{3b})\Big\{g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 q_1} - \frac{p_1^\nu p_2^\mu}{p_1 q_2} + (g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu) \times$$
$$\times \left[\beta_\pi^{(\pi)}(q_1q_2) + \beta_\pi^{(B)}(q_1q_2)\right]\Big\} + 4e^2\delta_{3a}\delta_{3b}(g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu)\beta_\pi^{(\pi)}(q_1q_2) \quad (40)$$

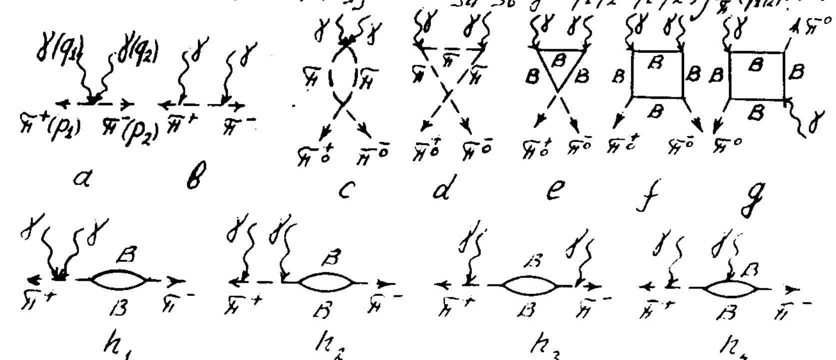

Fig. 7

The first three terms in the braces are the Born terms (diagrams 7a and 7b), $\beta_\pi^{(\pi)}(q_1q_2)$ is the contribution from the pion loops (diagrams 7c, 7d), $\beta_\pi^{(B)}(q_1q_2)$ that from

----

*) The mass term can be introduced in a different way. Nevertheless, it changes slightly the final results (see ref11)).

the baryon loops (7e,f,g,h). In $\beta_\pi^{(B)}$ only the constant terms are kept because of smallness of the other terms of expansion in powers of ($q_1 q_2$). Besides, in deriving (40) the equalities $(q_1\varepsilon_1)=(q_2\varepsilon_2)=0,\ q_1^2=q_2^2=0,\ p_1^2=p_2^2=m_\pi^2$ have been used.

In combined calculation of the contribution to the amplitude from diagrams 7c and 7**d** one arrives at the finite expression:

$$\beta_\pi^{(\pi)}(q_1q_2)=(4\pi F_\pi)^{-2}\left(1-\frac{2}{3}\frac{m_\pi^2}{q_1q_2}\right)\left\{\frac{2m_\pi^2}{q_1q_2}\left[\operatorname{arctg}\left(\frac{2m_\pi^2}{q_1q_2}-1\right)^{-1/2}\right]^2-1\right\} \qquad (41)$$

When considering the Compton effect by a neutral pion all the contributions from diagrams 7e,f,g cancel . For charged pions, the contributions from nucleon diagrams 7e,f, h_i equal

$$\beta_\pi^{(N)}=\frac{2}{3}\frac{g_A^2}{(4\pi F_\pi)^2}\ , \qquad (42)$$

If one takes into account the contribution from the other m-embers of the baryon octet the factor 1.7 again appears in (42).

Defining the pion polarizability as a coefficient for the effective interaction of a pion with an external electromagnetic field A_μ *)

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*)The factor $(g^{\mu\nu}q_1q_2-q_1^\mu q_2^\nu)$, always entering into the one-loop approximation of the amplitude $T_{\alpha\beta}^{\mu\nu}$ (see formula (40)), in a language of quantum mechanics corresponds to the combination $(\vec{E}^2-\vec{H}^2)$. Hence it follows that the pion electric and magnetic polarizabilities are equal in magnitude and opposite in sign.

$$V_{int.}=-\frac{\alpha_\pi}{2}(\vec{E}^2-\vec{H}^2), \qquad (43)$$

we get

$$\alpha_{\pi^\pm}=\alpha_{\pi^\pm}(q_1q_2)\Big|_{q_1q_2=0}=\frac{e^2}{m_\pi}\left(\beta_\pi^{(\pi)}(0)+\beta_\pi^{(B)}(0)\right)=0.33\frac{\alpha}{m_\pi^3}\approx 7\cdot10^{-3}(fm)^3,\quad (\alpha=e^2/4\pi)$$

$$\alpha_{\pi^0}=\alpha_{\pi^0}(q_1q_2)\Big|_{q_1q_2=0}=2\frac{e^2}{m_\pi}\beta_\pi^{(\pi)}(0)=-0.04\frac{\alpha}{m_\pi^3}=-8\cdot10^{-4}(fm)^3. \qquad (44)$$

It is interesting to note that the function $\beta_\pi^{(\pi)}(q_1q_2)$ is rapidly varying in the threshold region. As a result, at the threshold of two-pion production we obtain

$$\alpha_{\pi^\pm}(2m_\pi^2)=0.51\frac{\alpha}{m_\pi^3}\quad ;\quad \alpha_{\pi^0}(2m_\pi^2)=0.36\frac{\alpha}{m_\pi^3} \qquad (45)$$

The found values of $\alpha_{\pi^\pm}$ coincide in the order of magnitude, with the estimates made on the basis of current algebra [29] and quark models [30], but differ by a factor of 2 from predictions of ref.[29]. The value of α_{π^0} differs essentially from the result of ref [29]: $\alpha_{\pi^0}=0$.

5. Decays of Charged Pions (Weak Interactions)[12].

Consider now main decays of charged mesons and calculate the structure constant of these decays. To this end we shall need to complement the chiral Lagrangian with the part responsible for the weak interactions. It is as follows

$$\mathcal{L}_{int.}^{(G)}=:L_\mu^{(+)}\{-\sqrt{2}F_\pi\partial_\mu\pi^- - i\sqrt{2}(\pi^-\partial_\mu\pi^0-\pi^0\partial_\mu\pi^-)+ \qquad (46)$$
$$+\bar\psi_p\gamma_\mu(1-ig_A\gamma_5)\psi_n+ie\sqrt{2}F_\pi\pi^-A_\mu\}:,$$

where $L_\mu^{(+)} = \frac{G}{\sqrt{2}}\cos\theta\bar{\mu}(\bar{e})\gamma_\mu(1-i\gamma_5)\nu$, G is the weak coupling constant, θ the Cabibbo angle, μ, e and ν the muon, electron, and neutrino fields.

The process amplitudes T will be defined as usual. For instance, for the process $\pi^\pm \to \mu^\pm \nu\gamma$ this definition has the form

$$\langle \mu\nu(\ell)\gamma_\lambda(q)|_f S|\pi(p)\rangle = i\pi\frac{\delta^{(4)}(p-q-\ell)}{\sqrt{p^0 q^0}}\varepsilon_\lambda^\mu T_\mu , \qquad (47)$$

where ε_λ^μ is the photon polarization, p, q and ℓ are the momenta of pion, photon, and lepton pair, respectively. Since the baryon loop contributions are much larger than those from pion loops, as can be easily seen from the example of earlier calculations, we here shall consider only the baryon loop contributions.

a) We start with study of the main pion decay $\pi^\pm \to \mu^\pm\nu(e^\pm\nu)$. On the basis of this process the only parameter of the chiral theory - F_π is fixed. It appears that the perturbation expansion order next to the Born one gives only a small correction to F_π and in loop diagrams 8b and 8c again all the divergences cancel.

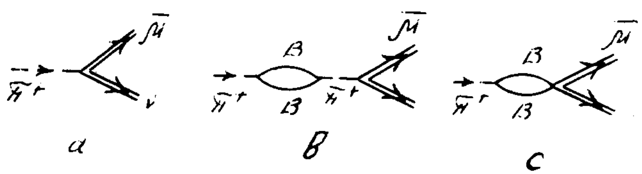

Fig. 8

As a result, in the one-loop approximation we obtain

$$T_{(\pi\to\mu\nu)} = i\sqrt{2}F_\pi\left[1-\frac{1}{6}\left(\frac{g_A m_\pi}{2\pi F_\pi}\right)^2\right]p_\mu \ell_\mu^{(+)} , \qquad (48)$$

where p_μ is the pion momentum, $\ell_\mu^{(+)} = \frac{G}{\sqrt{2}}\cos\theta\bar{u}_\mu\gamma_\mu(1-i\gamma_5)u_\nu$ is the lepton current. The second term in brackets is essentially smaller than unity. Comparing (48) with experiment gives $F_\pi \approx$ $\approx 93 MeV$.

b) Now consider the process $\pi^\pm \to \mu^\pm\nu\gamma$. A detailed discussion on this process can be found in papers [31,29].

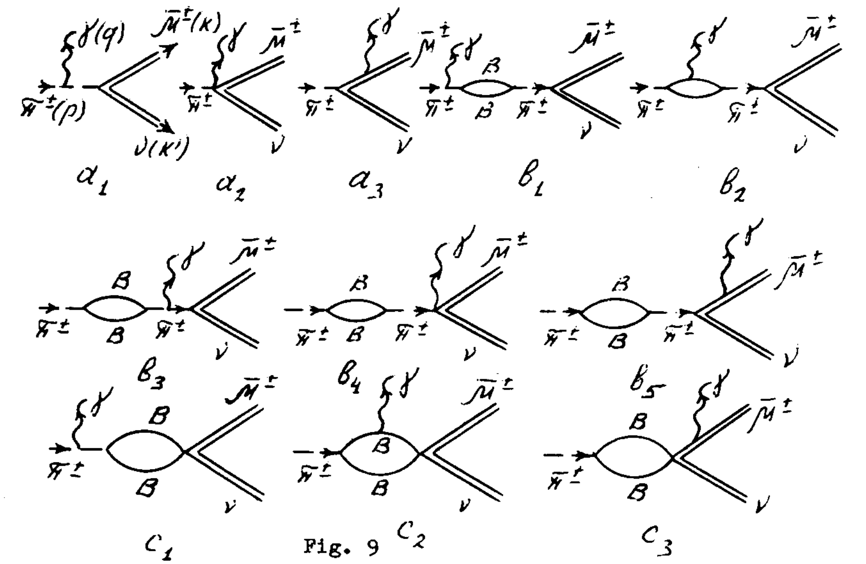

Fig. 9

The Born approximation is defined by diagrams $9a_i$

$$T_\mu^{Born} = ieF_\pi\left\{\sqrt{2}\left(g_{\mu\nu}+\frac{p_\mu(p\cdot q)_\nu}{pq}\right)\ell_\nu^{(+)} - G\cos\theta\bar{u}_\mu\gamma_\mu(\hat{k}+\hat{q}-m_\mu)^{-1}\hat{p}(1-\gamma_5)u_\nu\right\}. \qquad (49)$$

The one-loop approximation is mainly determined by diagrams $9b_i$ and $9c_i$. Their contributions are of the form

$$T_\mu^{0} = -\frac{1}{6}\left(\frac{g_A m_\pi}{2\pi F_\pi}\right)^2 T_\mu^{Born} - ie\sqrt{2}\left[ih_V \varepsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta - h_A\left(g_{\mu\nu} pq - p_\mu q_\nu\right)\right]\ell_\nu^{(+)}, \qquad (50)$$

where

$$h_V = \frac{g_A}{8\pi^2 F_\pi}, \qquad h_A = \frac{g_A^2}{6(2\pi)^2 F_\pi^2} \qquad (51)$$

and $\varepsilon_{\mu\nu\alpha\beta}$ is the fully antisymmetric tensor. Thus, taking account of the nucleon loops is reduced: 1) to renormalization of the constant F_π (see (48)) and 2) to appearance of the terms describing the structure constants of emission.

For the ratio $h_A/h_V = \gamma$ we have

$$\gamma = g_A/3 \approx 0.41, \qquad (52)$$

whereas experiment gives two possible values $\gamma = \{0.4; -2\}$ [32].

Note, that in our approach there hold the following from current algebra [29] relations between the constant h_V and constant f of the decay $\pi^0 \to \gamma\gamma$, as well as between the constant h_A and pion polarizability β_π.

The amplitude for the process $\pi^0 \to \gamma\gamma$ has first been calculated in the paper by Steinberger [33] in the one-loop approximation (see Fig.10):

$$T_{\mu\nu} = f\varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta; \quad f = -\frac{e^2 g_A}{(2\pi)^2 F_\pi} \approx 0.59\frac{\alpha}{m_\pi}; \qquad (53)$$

where q_i are the photon momenta. The experimental values of f are as follows

$$|f| = \{0.45^{[34]}; 0.57^{[35]}\}\frac{\alpha}{m_\pi}. \qquad (54)$$

The relation following from current algebra is

$$h_V = -f/2e^2 \qquad (55)$$

Comparing (53) with (51) one can easily see that this equality is fulfilled.

The pion polarizability, at energies $(q_1 q_2) = 0$, is basically determined by the baryon contributions (see 42)). Comparison (42) with (51) gives

$$h_A = F_\pi \beta^{(N)} \qquad (56)$$

It is just the relation following from current algebra [29].

The consideration of contributions from the other members of the baryon octet results in the appearance of factor 1.7 in coefficients with g_A^2 and of factor 1.2 in those with g_A. Therefore equalities (55) and (56) are not violated.

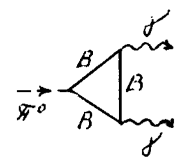

Fig. 10

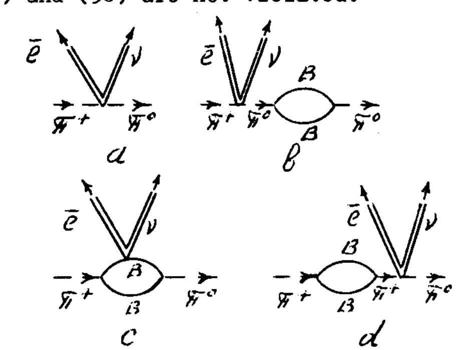

Fig.11

c) And finally, consider the process $\pi^+ \to \pi^0 e^+ \nu$ (Fig.11). The calculation of its amplitude is analogous to that of the pion form factor, and results in the following expression

$$T_{(\pi^+ \to \pi^0 e^+ \nu)} = T^{Born}\left[1 + \frac{1}{6}\left(\frac{g_A}{2\pi F_\pi}\right)^2 q^2\right] + \frac{\sqrt{2}}{6}\left(\frac{g_A}{2\pi F_\pi}\right)^2 (m^2_{\pi^0} - m^2_{\pi^+}) q^\nu \ell^{(+)}_\nu , \tag{57}$$

where

$$T^{Born} = \sqrt{2}(p_{\pi^+} + p_{\pi^0})^\nu \ell^{(+)}_\nu , \quad q = p_{\pi^+} - p_{\pi^0} .$$

This result completes our study of the pion interactions. Further, we shall demonstrate in what way it is possible to employ such an approach for description of kaon physics.

6. Kaon Electromagnetic Interactions [13)]

To describe the kaon interactions in the framework of the chiral theory one should use a Lagrangian invariant under the group $SU(3) \times SU(3)$. The mass terms violating both the $SU(3)\times SU(3)$ and $SU(2)\times SU(2)$ symmetry of the Lagrangian are introduced according to papers [36,37)]. The interaction with an electromagnetic field is again introduced by a gauge-invariant way

$$\partial_\mu \chi^\pm \to (\partial_\mu \pm ie A_\mu)\chi^\pm , \tag{58}$$

where

$$\chi^\pm = (\pi^\pm, K^\pm, p^\pm, \Sigma^\pm, \Xi^-)$$

Then in addition to Lagrangians (3), (12') and (30) we obtain

the following parts:

$$\mathcal{L}_{\pi K} = -i\bar{\psi}_K \overset{\leftrightarrow}{\partial}_\mu \tau_\alpha \psi_K \varepsilon_{\alpha\beta\gamma} \frac{\pi^\beta \partial_\mu \pi^\gamma}{4F_\pi^2} \frac{\sin^2\sqrt{\vec{\pi}^2/F_\pi^2}}{\vec{\pi}^2/F_\pi^2} : , \tag{59}$$

where $\psi_K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$, $\bar{\psi}_K = (K^-, \bar{K}^0)$, $\varepsilon_{\alpha\beta\gamma}$ is the antisymmetric tensor. We need this nonpolynomial Lagrangian, together with Lagrangian (30), to calculate the contribution from diagrams of type 4b to the kaon form factor by the superpropagator method. Since in other calculations we shall not use superpropagators, the remaining parts of the Lagrangian are written here only in the lowest orders of $(1/F_\pi)$ with taking account of the mass terms*)

$$\mathcal{L}_{\pi K} = -2(4F_\pi)^{-2} : \vec{\pi}^2 [(\partial_\mu \vec{K})^2 - m_K^2 \vec{K}^2] : ; \tag{59'}$$

$$\mathcal{L}_{KK} = -(4F_\pi)^{-2} : \vec{K}^2 [(\partial_\mu \vec{K})^2 - m_K^2 \vec{K}^2] : , \tag{60}$$

$$\mathcal{L}_{KA} = :ie A_\mu (K^+ \partial_\mu K^- - K^- \partial_\mu K^+) : + e^2 : A_\mu^2 K^+ K^- : ; \tag{61}$$

$$\mathcal{L}_{BA} = \mp e : A_\mu \bar{B}^\pm \gamma_\mu B^\pm : , \tag{62}$$

$$\mathcal{L}^B_{(K^-)} = \sqrt{2} g_A \frac{M_N}{F_\pi} :K^- \{\bar{\Xi}^0 \Sigma^+ + (2\alpha-1)\bar{\Sigma}^- n + \frac{(2\alpha-1)}{\sqrt{2}}\bar{\Sigma}^0 p - \frac{\bar{\Xi}^- \Sigma^0}{\sqrt{2}} - \frac{(3-2\alpha)}{\sqrt{6}}\bar{\Lambda} p - \frac{(4\alpha-3)}{\sqrt{6}}\bar{\Xi}^- \Lambda\}: , \tag{63}$$

$$\mathcal{L}^B_{(K^-K^+)} = g_A^2 \frac{M_N}{F_\pi^2} :K^- K^+ \{\bar{\Sigma}\Sigma^- + (1-2\alpha)^2 \bar{\Sigma}^+ \Sigma^+ + 2\left[\frac{\alpha^2}{3} + (1-\alpha)^2 - \frac{\alpha(1-\alpha)}{4}\right]\bar{p}p + 2\left[\frac{\alpha^2}{3} + (1-\alpha)^2 + \frac{\alpha(1-\alpha)}{4}\right]\bar{\Xi}^- \Xi^- + (\text{neutral Baryons})\}: . \tag{64}$$

Here $\vec{K}^2 = 2K^+K^- + 2\bar{K}^0 K^0$; $\vec{\pi}^2 = 2\pi^+\pi^- + (\pi^0)^2$, $\bar{B}\cdot B = \bar{B}\gamma_5 B$, B are the baryon fields, α is the parameter of mixing of f and d coupling ($\alpha \approx 2/3$),

---

*) We put $F_\pi \approx F_K$. As an example, for baryons we write only the part of the Lagrangian which contains K^-. The other parts can be easily obtained with the use, e.g., of monograph [27)].

Now proceeding in the same way as in sect.4, we calculate the kaon electromagnetic form factor and polarizability.

a) Defining the kaon form factor by analogy with (31) we write it in the form

$$\Phi_K(q) = 1 + \Phi_K^{(\pi)}(q) + \Phi_K^{(K)}(q) + \Phi_K^{(B)}(q) + \dots \quad . \tag{65}$$

Here $\Phi_K^{(\pi)}(q)$ is the contribution to the form factor from the pion loop diagram of type 4b, $\Phi_K^{(K)}(q)$ that from the kaon loop diagram, and $\Phi_K^{(B)}(q)$ from the baryon loops of type 4c and 4d but with kaon external lines. These contributions again correspond to the e/f_π^2 approximation. As for the pion form factor, the contribution from the kaon loop can be neglected, and we write here only the expression for $\Phi_K^{(\pi)}(q)$ and $\Phi_K^{(B)}(q)$.

$$\Phi_K^{(\pi)}(q) = \alpha_0 \left\{ \frac{q^2}{8m_\pi^2}\left[\ln\left(2\left(\frac{2\pi f_\pi}{m_\pi}\right)^2\right) - 3C + 1\right] - 1 + \frac{1}{3}\frac{q^2}{m_\pi^2} + \left(1 - \frac{q^2}{4m_\pi^2}\right) J\left(\frac{q^2}{4m_\pi^2}\right)\right\}. \tag{66}$$

Here the constants α_0, C and function $J\left(\frac{q^2}{4m_\pi^2}\right)$ are the same as in formulae (21) and (33). The term in brackets contributes to the kaon mean square radius. It equals

$$\langle r^2 \rangle_{K^\pm}^{(\pi)} = 0.03 (fm)^2 . \tag{67}$$

The contribution from baryon diagrams again turns out to be

essentially larger and equals*)

$$\Phi_K^{(B)}(q) = \frac{1.4}{6(2\pi)^2} g^2 \frac{q^2}{M_N^2} . \tag{68}$$

Hence it follows

$$\langle r^2 \rangle_{K^\pm}^{(B)} \approx 0.3 (fm)^2 . \tag{69}$$

From (67) and (69) the charged kaon radius is obtained to equal

$$\sqrt{\langle r^2 \rangle_{K^+}} \approx 0.61 (fm) . \tag{70}$$

For the neutral kaon the baryon loop contribution is zero and that from the pion loop is the same as for the charged kaons. Thus, we have

$$\sqrt{\langle r^2 \rangle_{K^0}} \approx 0.28 (fm) . \tag{71}$$

This result is in good agreement with the predictions following from the vector dominance model (a variant of the model with current mixing [21]).

b) Now we present the calculation results for the amplitude on the Compton effect by kaon.

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*) Factor 1.4 arises due to account of all contributions from the whole baryon octet. It is interesting to note that in case of the exact $SU(3)$ symmetry all the one-loop diagrams for the meson self-energy, form factor and Compton amplitude appear to be proportional to the function $f(\alpha) = 3(1-\alpha)^2 + \frac{5}{3}\alpha^2$. Its minimum corresponds to α =0.65 , that agrees well with experiment.

In addition to diagrams in Fig.7, but with kaon external lines instead of pion ones, we will consider also two diagrams of type 7c and 7d, with the kaon internal lines. The latter diagrams give the negligible contribution to the Compton amplitude by pion but in the case of kaon these diagrams should be taken into account. Then, without the Born terms, for the amplitudes with charged and neutral external kaons in the e^2/F_π^2 approximation we obtain:

$$T_+^{\mu\nu} = 2e^2(g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu)[\beta_K^{(\pi)}(q_1q_2) + \beta_K^{(K^+)}(q_1q_2) + \beta_K^{(B)}], \quad (72)$$

$$T_0^{\mu\nu} = 2e^2(g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu)[\beta_K^{(\pi)}(q_1q_2) + \beta_K^{(K^0)}(q_1q_2)]. \quad (73)$$

The function $\beta_K^{(\pi)}(q_1q_2)$ corresponds to the contribution from two diagrams with the pion internal lines (of type 7c and 7d). Being calculated together these diagrams give the following finite contribution

$$\beta_K^{(\pi)}(q_1q_2) = (4\pi F)^{-2}\frac{q_1q_2}{4m_\pi^2}\,\mathcal{J}\left(\frac{q_1q_2}{2m_\pi^2}\right), \quad (74)$$

where

$$\mathcal{J}(\xi) = \frac{1}{\xi}\left\{\frac{1}{\xi}\left[\mathrm{arctg}\left(\frac{1}{\xi}-1\right)^{-1/2}\right]^2 - 1\right\}. \quad (75)$$

The function $\mathcal{J}$ rapidly changes with increasing ξ. Therefore the contribution from $\beta_K^{(\pi)}(q_1q_2)$ to the amplitude $T^{\mu\nu}$ equal to zero at $q_1q_2 = 0$ may become considerable at sufficiently large q_1q_2.

. The function $\beta_K^{(K^+)}(q_1q_2)$ corresponds to the contribution from two diagrams with the internal kaon lines and external charged kaons:

$$\beta_K^{(K^+)}(q_1q_2) = (8\pi F_\pi)^{-2}\left(1 + \frac{q_1q_2}{2m_K^2}\right)\mathcal{J}\left(\frac{q_1q_2}{2m_K^2}\right). \quad (76)$$

For the external neutral kaons such a function has the form

$$\beta_K^{(K^0)}(q_1q_2) = (4\pi F_\pi)^{-2}\frac{q_1q_2}{4m_K^2}\,\mathcal{J}\left(\frac{q_1q_2}{2m_K^2}\right). \quad (77)$$

Hence it is seen that at $(q_1q_2) = 0$ only the function $\beta_K^{(K^+)}(q_1q_2)$ gives a nonzero contribution to $T_+^{\mu\nu}$ $(\beta_K^{(K^+)}(0) \approx 0.08/(4\pi F_\pi)^2)$.

In the case of the neutral external kaons (like in the case of the neutral external pion lines) the contribution to $T_0^{\mu\nu}$ from the baryon loop diagrams is zero. For the charged external kaons the total contribution from diagrams of type 7e, 7f and 7h equals

$$\beta_K^{(B)} \approx 1.4(4\pi F_\pi)^{-2}. \quad (78)$$

Here we again have kept only the constant terms because of smallness of subsequent terms of expansion in powers of (q_1q_2) (of the type $o\left(\frac{q_1q_2}{M_N^2}\right)$) Hence it is seen that at $(q_1q_2) = 0$ the baryon loops give the main contribution to the amplitude of the Compton effect $T_+^{\mu\nu}(0)$. The amplitude $T_0^{\mu\nu}(0) = 0$.

Making use of formulae analogous to (43), we find the following values of the kaon polarizability

$$\alpha_{K^\pm} = \frac{1.5}{m_K}\left(\frac{e}{4\pi F_\pi}\right)^2 \approx 1.6\ 10^{-3}(fm)^3, \quad (79)$$

$$\alpha_{K^0} = 0. \qquad (80)$$

These values are consistent both with theoretical estimates found recently on the basis of current algebra and PCAC [38]: $\alpha_{K^+} \sim 10^{-3}(fm)^3$, and with the experimental data [19]: $\alpha_{K^+}^{exp} \sim -(4 \pm 11)\cdot 10^{-3}(fm)^3$.

7. $K_L - K_S$ mass difference

We complete the study of low-energy meson interactions calculating the neutral-kaon mass difference.

To this end, we introduce one more Lagrangian describing πK -interactions and corresponding to the rule $\Delta T = 1/2$. The simplest chiral Lagrangian of such a type, having no derivative coupling is of the form [14]:

$$\mathcal{L}^{(1/2)}_{\pi K} = a : \mathcal{U}\left(\frac{\pi}{F_\pi}\right)\varphi_K : + c.c., \qquad (81)$$

where $\varphi_K = \binom{K^+}{K^0}$, and $\mathcal{U}\left(\frac{\pi}{F_\pi}\right)$ is the same chiral matrix as in Lagrangian (11). Taking again the exponential form for the matrix $\mathcal{U}\left(\frac{\pi}{F_\pi}\right)$ one may write the part of the Lagrangian responsible for the neutral kaon interaction in the following form

$$\mathcal{L}^{(1/2)}_{\pi K^0} = a\cdot\left\{K_S\left[\cos\sqrt{\frac{\vec{\pi}^2}{F_\pi^2}} - 1\right] + K_L\frac{\pi^0}{F_\pi}\frac{\sin\sqrt{\vec{\pi}^2/F_\pi^2}}{\sqrt{\vec{\pi}^2/F_\pi^2}}\right\}:, \qquad (82)$$

where $K_S = \frac{\bar{K}_0 + K_0}{\sqrt{2}}$, $K_L = i\,\frac{\bar{K}_0 - K_0}{\sqrt{2}}$. The Born approximation of this Lagrangian reproduces correctly the low-energy theorems of current algebra concerning the nonlepton decays of neutral kaons into two and three pions.

The coupling constant a can be fixed by using the probability of decay $K_S \to \pi\pi$ ($w(2\pi)$). As a result, we have;

$$a^2 = \frac{2\pi m_K (2F_\pi)^4}{3\left[1 - \left(2\frac{m_\pi}{m_K}\right)^2\right]^{1/2}}\, w(2\pi). \qquad (83)$$

Now we proceed to calculate the $K_L - K_S$ mass difference. The mass difference of these mesons is due tc the different virtual states into which these mesons can go over, with the account of their CP parity (see Fig.12)

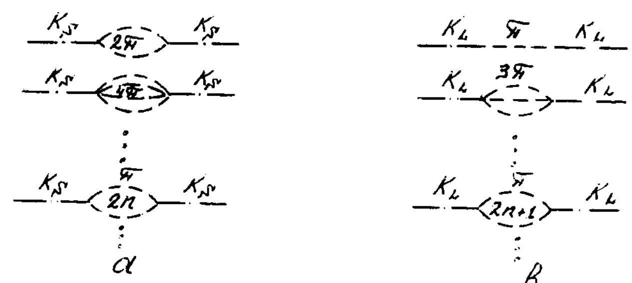

Fig.12

Therefore for the mass difference Δm_{K^0} one may write the following formula

$$\Delta m_{K^0} = m_{K_L} - m_{K_S} = 2\left[f_S - f_L\right], \qquad (84)$$

where f_S is the sum of matrix elements corresponding to the infinite set of diagrams with even number of virtual pions

(Fig.12a), and f_L is the same for diagrams with odd number of pions (Fig.12b.)

The quantities f_S and f_L can easily be calculated by using the superpropagator method[5]. In so doing, the difference Δm_{K^0} appe-ars to be almost completely determined by the two-pion diagram. Contributions to Δm_{K^0} from diagrams with three and more virtual pions are smaller than 1%. The contribution from the one-pion diagram should be calculated together with that from the diagram with one virtual η -meson. In the framework of the exact $SU(3)$ theory these contributions cancel. The consideration of η -meson for the loop diagrams is nonessential.

Now let us write the matrix element corresponding to the two-pion diagram

$$f_S^{(2\pi)} = \frac{3(2\pi a)^2}{m_K(4\pi F_\pi)^4}\left[\ln\left(\frac{4\pi F_\pi}{m_\pi}\right) - \frac{3}{2}C + \frac{13}{12} - J\left(\frac{m_K^2}{4m_\pi^2}\right)\right], \quad (85)$$

where C is the Euler constant and $J\left(\frac{m_K^2}{4m_\pi^2}\right)$ is given by (22). Inserting (85) and (83) into (84) we have

$$Re\,\Delta m_{K^0} = 0.52\, w^{(2\pi)}, \quad (86)$$

whereas the experimental value for Δm_{K^0} equals 0.48 $w^{(2\pi)}$ (see ref.[20]).

8. Conclusion.

Summarizing all the above examples of utilization of the chiral quantum field theory for description of low-energy meson interactions we can note the following:

The chiral quantum theory not only in the tree but also in the one-loop approximation is in good agreement with the consequences of current algebra and PCAC. Reproducing correctly all the relations following from current algebra the chiral theory, besides, allows one to calculate also the absolute values of various physical quantities. The results obtained reproduce, at least, the real qualitative picture of various physical processes, giving in most cases good quantitative agreement with experiment.

Due to that there is no rapidly convergent perturbation series for strong interactions, one must carefully treat some quantitative results of this theory. Nevertheless, even now it is possible to point out some very reliable results due to their weak dependence on the contribution from diagrams with strong vertices. These results concern, for instance, the values calculated here for $\pi\pi$ -scattering lengths of higher partial waves which are defined by the pion loop diagram 1b, a noticeable increase of the pion polarizability in the threshold region, in comparison with the region where $(q_1 q_2) = 0$.

Consideration performed in sect.2 allows one to hope that the strong interaction effect may be taken into account correctly by renormalizing the strong vertices (see also ref. [15]). This question, however, requires a more careful investigation.

In conclusion, I should like to express my belief that the chiral quantum field theory, giving in numerous cases, good agreement with experiment and having a number of remarkable intrinsic features (for instance, cancellation of divergences in loops with the strong vertices) undoubtedly requires the most thorough study and represents a highly perspective trend.

References

1. S.L.Adler and R.Dashen:"Current Algebras", W.A.Benjamin, Inc., New York 1968.
F.Gürsey in "Particles, Currents, Symmetries", Springer Verlag, Wien, New York 1968.
S.Weinberg in "Lecture on Element-ary Particles and Quantum Field Theory",1970 Brandeis Summer Institute, Vol.I.MIT Press. Cambridge, Mass. 1970.
2. F.Gürsey, Nuovo Cim. 16,230 (1960); Phys.Rev. 164, 1752 (1967).
3. M.Gell-Mann and M.Levy, Nuovo Cim. 16, 703 (1960)
4. S.Weinberg, Phys.Rev. 166, 1568 (1968)
5. M.K.Volkov Commun.Math.Phys. 7, 289 (1968); Ann.Phys. (N.Y) 49,202, (1968); Teor.Mat.Fizika 6, 21 (1971); Fortschr. der Phys. 22, 499 (1974).
A. Salam, J.Strathdee, Phys.Rev. D1, 3296 (1970); D3, 1805 (1971).
H.Lehmann. K.Pohlmeyer.Commun.Math.Phys. 20, 101 (1971).
6. L.D.Faddeev, A.A.Slavnov, Teor.Mat.Fizika 8, 297 (1971);
B.W.Keck and J.G.Taylor, Phys.Rev. D3, 1846 (1971);
A.P.Hunt, K.Koller and G.Shafi, Imperial College London, preprint ICTP 69/19 (1970).
7. H.Lehmann and H.Trute, Nuclear Phys. 52, 280 (1973);
H.Lehmann,Phys.Lett. 41B, 529 (1972); Preprint DESY-72/54 (1972); DESY-73/26 (1973).
8. G.Ecker and J.Honerkapm, Nuclear Phys. B52, 211 (1973); B62, 509 (1973).

9. V.N.Pervushin and M.K.Volkov Yadern.Fiz. 20, 762 (1974); Preprint JINR E2-8097, Dubna (1974)
10. V.N.Pervushin and M.K.Volkov Yadern.Fiz. 19, 652 (1974); Phys.Letters 51B, 356 (1974).
11. V.N.Pervushin and M.K.Volkov Preprint JINR E2-8098, Dubna (1974); Phys. Lett., 55B, 405 (1975).
12. V.N.Pervushin and M.K.Volkov Preprint JINR P2-8165, Dubna (1974).
13. V.N.Pervushin and M.K.Volkov Preprint JINR P2-8655; P2-8659 Dubna (1975).
14. V.N.Pervushin and M.K.Volkov Yadern.Fiz. 21, 214 (1975); Phys.Lett. 51B, 499 (1974).
15. R.Dashen and M.Weinstein, Phys.Rev.183, 1261 (1969).
16. N.N.Bogoliubov and D.V.Shirkov, "Introduction to the Theory of Quantized Fields",Interscience , New York 1959 .
17. a) J.P.Baton et al., Phys.Letters 33B, 525 and 528 (1970);
b) S.D.Protopopescu et al., LBL preprint 787 (1972);
c) P.Baillon et al., Phys.Letters 38B, 555 (1972);
d) Y.Batusov, S.Buniatov et al., Yadern.Fiz.21,308 (1975).
18. a) Berezhnev et al., Yadern.Fiz. 18, 102 (1973);
b) G.Adylob et al., Phys.Letters 51B,402 (1974).
19. G.Backenstoss et al., Phys.Letters B43, 431 (1973).
20. Rev.Mod.Phys., 45 W2 part II (1973).
21. N.M.Kroll, T.D.Lee and B.Zumino Phys.Rev., 157, 1376 (1967)
22. A.Martin Nuovo Cim. 47, 265 (1967); 58A, 303 (1968)
23. D.V.Volkov "Particles and Nucleus" 4, 3 (1973).
24. V.N.Pervushin Preprint JINR E2-7540 (1973).
25. J.Honerkamp et al. Nucl.Phys. B69, 618 (1974).

26. L.S.Brawn and R.L. Goble Phys.Rev. D4, 723 (1971).

27. S.Gasiorowitz "Elementary Particle Physics" John Willy-Sons Inc. New-York-London-Sydney.

28. F.P.Palou and F.J.Yndurain. Nuovo Cim., 19A, 245 (1974).

29. M.V.Terentiev Yadern.Fiz. 16, 162 (1972)

30. T.E.ERicson and J.Hüfner Nuclear Phys. B47, 205 (1972).

31. D.Y.Bardin and S.M.Bilenkiy Yadern.Fiz. 16, 557 (1972).

32. D.Depommier et al. Phys.Letters 7, 885 (1963).

33. I.Steinberger. Phys.Rev. 76, 1180 (1949).

34. A.Rittenberg et al. Rev.Mod.Phys. 43, 2 (1971).

35. G.Belletini, C.Bemporad, P.Braccini, Nuovo Cim. A66, 243 (1970).

36. M.Gell-Mann, R.J.Oakes and B.Renner, Phys.Rev. 175, 2195 (1968).

37. F.Gürsey and M.Serdaroglu Nuovo Cim. 7A 584(1972); 9A, 263 (1972).

38. M.V.Terentiev Yadern.Fiz. 19, 1298 (1974).

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