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Summary

Theses are presented on the basis of which one should conclude that the gravitational radius of the test body can put principal restrictions on the measurability of coordinates and time in quantum theory, there appear the bounds of the type $\Delta x \Delta T \geq \frac{\hbar \kappa}{c^4}$, or rather $\Delta r_{gr} \Delta T \geq \frac{\hbar \kappa}{c}$, as a consequence of the relation $\Delta E \Delta T \geq \hbar$. The corresponding bounds arise for the measurability of the mean electrostatic field $\Delta \bar{\Sigma} \Delta T \geq \frac{\hbar \sqrt{\kappa}}{r_{gr}^2 c}$ and of the gravitational field (the Christoffel symbols $\{ {\alpha \atop \mu\gamma} \}$): $\Delta \{ {1 \atop 44} \} \cdot \Delta T \geq \frac{\hbar \kappa}{r_{gr}^2 c}$.

Thus, the conceptual contradiction arises between the modern concept of space-time continuum, which serves as the basis of the modern field theories, and the real physical properties of the space-time continuum in small regions.

1. Introduction

Quantum mechanics of particle has preserved classical concepts of space and time because within the framework of concepts of this theory the four-dimensional coordinates of particle can be measured as precisely as one wants, if the problem is not posed to determine simultaneously the momentum and energy of the particle.

The principal possibility of measuring measurable physical quantities, which are the basis of the physical theory, is one of the principal conditions for the self-consistency of the theory. As is known, Bohr and Rosenfeld, in their famous paper /1/, put the task to prove that in this respect quantum electrodynamics is not contradictory, because the mean value of the electromagnetic field can be measured with the help of the macroscopic test body as precise as one wishes, in an arbitrarily small space-time volume. In other words, the concept of the electromagnetic field, according to Bohr and Rosenfeld, does not encounter conceptual difficulties in the whole space-time continuum.

In this note the attention is called to the fact that the contradiction with such a concept of the fields and of the space-time continuum can emerge if one includes into consideration the gravitational field of the test body.

In the past one of such contradictions of the conceptual type has been indicated by Wigner /2/ and by Anderson /3/.

As Wigner noted the Heisenberg relation

$$\Delta E \Delta T \sim \hbar \qquad (1)$$

leads to the conclusion that the precision of the functioning of a clock depends on the uncertainty of its mass

$$\Delta E = c^2 \Delta m \; . \qquad (2)$$

According to this remark the precise measurement of time corresponds to the infinite large fluctuation of its mass, and, consequently, of the gravitational field. However, if one proceeds with this analysis and pays attention to the fact that if the fluctuation of the mass Δm should correspond to the fluctuation of the gravitational radius

$$\Delta r_{gr} \sim \frac{\Delta m}{c^2} \kappa \, , \qquad (3)$$

where κ is the gravitational constant, then the Heisenberg relation can be rewritten in the following form

$$\Delta r_{gr} \, \Delta T \geq \frac{h \kappa}{c^4} \, . \qquad (4)$$

If one assumes that the spatial dimensions ΔR of the clock cannot be smaller than its gravitational radius $\Delta R > \Delta r_{gr}$, then ΔR is the uncertainty of the position of the clock, showing the time with uncertainty ΔT, so that

$$\Delta R \Delta T \geq \frac{\hbar \kappa}{c^4} \, . \qquad (5)$$

If one assumes that the minimal uncertainty in measurement of time* $\Delta T_{min} \geq \frac{\Delta R}{c}$, then

$$(\Delta R_{min})^2 \geq \frac{\hbar \kappa}{c^3} \, . \qquad (6)$$

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*ΔT_{min} is the propagation time of the signal of the clock localized somewhere in ΔR.

So we arrive at the conclusion about the possible existence of individual errors of measurement of coordinates in an empty "Euclidean" space

$$\Delta R_{min} \geq \sqrt{\frac{\hbar \kappa}{c^3}} \sim 10^{-33} \text{ cm.} \qquad (7)$$

The corresponding relation could be obtained for the uncertainty of measuring of time.

One cal recall that in one of the papers of B.De Witt, where the interaction of the gravitational field with the scalar particles has been taken into account, the propagation function

$$Z(x) = \frac{i}{(2\pi)^2} \frac{1}{x^2 - \lambda^2 + i0} \, , \qquad (8)$$

where

$$\lambda = \frac{2}{\sqrt{\pi}} \sqrt{\frac{\hbar \kappa}{c^3}} \sim 10^{-33} \text{ cm} \, , \qquad (9)$$

has been obtained.

Expressions (9) for λ coincides with expression (7). In fact, this result has been obtained by De Witt in the approximation where only the so-called ladder type diagrams (the Bethe-Salpeter equation) in the special gauge of the gravitational field (in the gauge of de Donder, i.e., harmonical gauge

$$\frac{\partial G^{ik}}{\partial x^k} = 0 \,) \qquad (10)$$

have been summed. However, the previous general consideration indicates that the emerging of λ in the propagation function obtained by De Witt, apparently, is not accidental.

Similar propagation functions, shifted from cone to hyperboloid, have been previously considered ad hoc without connection with gravitational field. They were introduced with the aim to overcome the known difficulties with ultraviolet divergences in the field theory /5/. In the Lehmann representation the corresponding propagation function $D(s')$ can be written as follows *

$$D(s') = \frac{1}{\pi} \iint \Delta(s\kappa^2) \cos(\kappa^2\beta - \frac{\lambda^2}{4\beta}) d\beta \, d\kappa^2 , \qquad (11)$$

where $s' = s - \lambda^2$, $s = x^2$, $\Delta(s\kappa^2)$ satisfies the Klein-Gordon equation with mass κ. Therefore the Lehmann function $\rho(\kappa^2)$ has the following form

$$\rho(\kappa^2) = \int_0^\infty \cos(\kappa^2\beta - \frac{\lambda^2}{4\beta}) d\beta . \qquad (12)$$

In this theory the Lehmann function $\rho(\kappa^2)$ is sign-alternating. In the contemporary field theory $\rho(\kappa^2) > 0$. In other words, one considers a regularization function $\rho(\kappa^2)$ of the Pauli-Villars type /6/, i.e., the so-called realistic regularization. However, one should note that in this theory with such a propagation function introduced in the theory ad hoc, as in very nonlocal theory, the violation of the principle of causality occurs: the signal spreads through the domain λ with velocity larger than the velocity of light: the corresponding commutators of the physical quantities on the spacelike surface do not become equal to zero. De Witt interpretes his result as arising of the rigid sphere of diameter λ around the scalar particle. Such an interpretation corresponds to the obtained by De Witt mathematical form of the propagation function. However, one could present arguments in favour of the fact that in the final form of the propagation function obtained by De Witt the physical meaning of the quantity λ has been lost. As follows from the previous analysis of the quantity λ this possibly is not a diameter of the rigid sphere, but rather a diameter of the corresponding Schwarzschild sphere. The gravitational radius of this Schwarzschild sphere corresponds to the mass

$$m_p \sim \sqrt{\frac{\hbar c}{\kappa}} . \qquad (13)$$

It is essential that the same mass can be obtained from the considerations based on the Heisenberg uncertainty relation. Let us assume that for the mass m the nonrelativistic movement is considered inside the domain $\lambda = \Delta R$.

For the kinetic energy of the mass m inside the domain λ on the basis of the relation $\Delta p \Delta R \sim h$ one obtains the expression

$$\frac{\Delta p^2}{2m} = \frac{\hbar^2}{\lambda 2m} \sim \frac{\hbar c}{\kappa} \frac{c^2}{2m} = Mc^2 .$$

In this case for the external observer the total mass of the object is equal to

$$M \sim \frac{\hbar c}{\kappa m} = m_p \cdot \frac{m_p}{m} ,$$

where $m_p = \sqrt{\frac{\hbar c}{\kappa}}$ is the Planck mass.

If $m < m_p$, then $M > \sqrt{\frac{\hbar c}{\kappa}}$.

In this case we arrive at the contradiction; the radius of the Schwarzschild sphere is larger than λ. The agreement with the uncertainty relation arises only in the case when $m = m_p = \sqrt{\frac{\hbar c}{\kappa}}$. Moreover, the case $M > m_p$ testifies that the assumption $m < \sqrt{\frac{\hbar c}{\kappa}}$ contradicts the nonrelativistic expression for the kinetical

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* In this paper the signature is 1-1-1-1,i.e., in contrast to the result of De Witt, the displacement from the cone to the hyperboloid takes place in the timelike region.

energy of the particle. The case $m \sim m_p$ does not lead to contradictions of any kind. In the relativistic case for the mass m one has $pc\Delta T \geq \hbar$. If $c\Delta T \sim \Delta R \sim \lambda$, then

$$p \sim \frac{\hbar}{\lambda} = mc . \qquad \text{From here also}$$

$$m = \frac{\hbar}{\lambda c} = \sqrt{\frac{\hbar c}{\kappa}} .$$

In other words, there is no contradiction between the Heisenberg uncertainty relation and the interpretation of λ as a dimension of a sphere in which the particle of the mass m is contained, in that case if this mass equals the Planck mass $m_p = \sqrt{\frac{\hbar c}{\kappa}}$. On the other hand, just to this mass corresponds the Schwarzschild sphere with dimensions λ.

2. *Electrodynamics*

The error in measuring the mean electric field is expressed, according to Bohr and Rosenfeld, in the following form

$$\Delta\bar{\Sigma} \sim \frac{\hbar}{\epsilon \Delta x \Delta T} , \quad \bar{\Sigma}(R) = \int_R \Sigma(x) d^4x , \tag{14}$$

where $\hbar$ is the Planck constant, ϵ is electric charge of the test body, Δx are dimensions of the test body, ΔT is duration of experiment. This inaccuracy of measuring of field can be arbitrarily small for small ΔT and Δx, because the electric charge in the classical test body with dimension Δx can be arbitrarily large: in classical physics, as the authors stress, the atomism of charge does not exist.

However, in this classical consideration the classical gravitational field, its mass or rather its gravitational radius has not been taken into account. As is known, Bohr considered it necessary to take into account the gravitational field in situations discussed by him when interpreting other Gedanken experiments.

We call attention to the fact that the gravitational radius of the charged test body grows with growing charge ϵ.

Really, mass of the electrostatic energy of the test charge ϵ equals

$$m \sim \frac{\epsilon^2}{\Delta x \, c^2} . \tag{15}$$

The gravitational radius of this mass is

$$r_{gr}^{\kappa} \sim \frac{\kappa m}{c^2} \sim \kappa \frac{\kappa^2}{\Delta x \, c^4} , \tag{16}$$

where κ is the gravitational constant. If one assumes that the dimensions of the test body cannot be smaller than its gravitation radius then one has

$$\Delta x \sim r_{gr}^{\epsilon} \sim \frac{\epsilon \sqrt{\kappa}}{c^2} . \tag{17}$$

Hence, though the error of measurement of the field really decreases with growing charge of the test body but the region over which one averages the field unlimitedly grows with increasing charge of the test body

$$\Delta\bar{\Sigma} \sim \frac{\hbar\sqrt{\kappa}}{(r_{gr}^{\epsilon})^2 \Delta T \, c^2} .$$

In other words, if there exists the gravitational radius of the test body, then the minimal dimensions of the test body are bounded due to the value of its charge ϵ. If ϵ is equal to electron charge e then

$$\Delta x_{min} \sim r^{e}_{gr} \sim \frac{e\sqrt{\kappa}}{c^2} \sim 10^{-33}\,\mathrm{cm} \qquad (18)$$

and from here

$$\Delta \bar{\Sigma} \sim \frac{\hbar c}{e} \frac{1}{r^{e}_{gr}\, c\, \Delta T}\, . \qquad (19)$$

This relation shows that for example the electrostatic field of electron can be measured as precisely as one wants during the time $\Delta T \to \infty$. However, this measurement, in principle, cannot be carried out in regions $\Delta x < \frac{e\sqrt{\kappa}}{c^2} \sim 10^{-33}$ cm. Large charge is useless for measuring the mean field in a small region. The above discussion is given within the framework of classical theory of the gravitational field. However, quantum considerations of the possible fluctuations of the metrics itself, according to the above discussion, can enlarge the errors of the minimal dimensions of the test body $(\Delta x \sim \sqrt{\hbar c}\frac{\sqrt{\kappa}}{c^2})$ by an order of magnitude.

3. *Gravitational Field*

As is known, according to refs.[2,3] the measurability of the gravitational field (we have in mind the Christoffel expressions $\{ {\alpha \atop \mu\gamma} \}$) is given by the expression

$$\Delta \{ {1 \atop 44} \} \geq \frac{\hbar}{m \Delta x \Delta T}\, , \qquad (20)$$

where m is the mass of the test body. Inaccuracy in the measurements of the gravitational field is the smaller the larger the mass m of the test body.

However, the region where the gravitational field is measured cannot be smaller than the dimensions of the test body and the dimensions of the test body for the external observer cannot be smaller than its gravitational radius

$$r_{gr} \sim \frac{m\kappa}{c^2} \sim \Delta x\, . \qquad (21)$$

Therefore

$$\Delta \{ {1 \atop 44} \} \geq \frac{\hbar\kappa}{r^2_{gr}\, \Delta T\, c^2}\, . \qquad (22)$$

Wigner and Anderson [2,3] have stressed from a different point of view that the large mass of the clock does not provide high accuracy of the measurement of the field in the small space-time region. Namely, the large mass of the clock causes the large gravitational field, which, due to the nonlinearity of the equations, cannot be separated from the measuring field.

In our consideration, if the initial statement about the role of the gravitational radius of the test body is valid, then the quantitative estimates of the principal inaccuracies arise.

Let us consider critically our initial theses.

1. Our analysis of the problem assumes that measurements are carried out in the system of coordinates in which one has singularity on the Schwarzschild sphere. However, one can conceive the falling reference frames, in which there is no singularity on the Schwarzschild sphere. One could answer this objection in the following way. A real observer is an external observer with respect to the apparatus. The observer cannot make use of the falling (into the clock!) reference frame to interpret the functioning of clock because when such a reference frame

intersects the Schwarzschild sphere the connection between the external observer and the reading of the clock ceases.

2. It is possible that there exist objects having properties of the bare singularities which are not covered by the Schwarzschild sphere. In this case, our consideration is not applicable. However, one can make a statement that such an object, even if it exists, must disappear by means of an explosion, due to the creation of pairs of a different nature round this singularity.

The point is that with the discovery of neutral current of weak interactions the neutral matter is charged with charge-source of the neutrino-antineutrino fields. This field is of a relatively long range ($\sim \frac{1}{r^5}$) if there is no intermediate boson and is analogous to the Coulomb field ($\frac{1}{r}$) if in weak interaction there exists an intermediate boson with a mass smaller than the Planck mass $\sqrt{\frac{\hbar c}{\kappa}} \sim 10^{-5}$ gr. One can even make an assumption that weak neutral currents of the neutrino-antineutrino field are able to stop at some distances, relatively large in comparison with the quantum length ($\sim 10^{-33}$ cm), the collapse of large electrically neutral masses. Such a possibility for Coulomb forces has been indicated by Novikov /8/.

True, that the last possibility is an interesting but purely abstract example, because, in fact, one speaks about the collapse of electrically neutral nucleon matter. But a similar matter inevitably is charged with the huge charge source of the neutrino field of weak neutral currents.

One can also object in such a way that according to Bohr the test body is a body of classical physics and the creation of pairs is purely quantum effect. Such an objection is a misunderstanding. The uncertainty relation for the macroscopic test body (quantum effects) is taken into account by Bohr in all his considerations

of the problem of measurements, but they are suppressed either by a large mass or by a large charge of the classical body.

The appearance of the parameter λ in the propagation function of the type of the De Witt function attracts the attention from many points of view. First of all, this function leads to the direct conflict with the causality principle in the same way as every "rigid" universal length in nonlocal theories.

True, if one considers the Schwarzschild metrics in isotropic reference frame

$$ds^2 = \Phi^{-1} dt^2 c^2 - \Phi \, [dR^2 + R^2 (\sin^2\theta d\phi^2 + d\theta^2)] \, ,$$

where $\Phi = \left(\frac{R}{R+r_0}\right)^2$, then for the spreading of light one obtains the expression

$$\frac{dR}{dt} = \left(\frac{R}{R+r_0}\right)^2 c$$

and for $R \to 0$, i.e., when light reaches the Schwarzschild surface its velocity in the Schwarzschild reference frame tends to zero. However, such an interpretation by no means does follow from the propagation function obtained by De Witt.

Moreover, the propagation function allowing the interpretation resulting from our consideration, must be of a completely different sort, it is not simply a dipslacement of the propagation function from a cone onto a hyperboloid, but is rather something in the spirit of intuitive considerations of Pauli /7/ about the possibility of some kind of diffusion of the light cone in the case of gravitation. In the spirit of our analysis this changed propagation function should arise also in the case of the free spreading of fields with account of gravitation caused by them *.

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* De Witt function $1/s'^2$ itself is invariant with respect to the translation transformation:

$$s'^2 = (x_1 - x_1')^2 + (x_2 - x_2')^2 + (x_3 - x_3')^2 - (x_4 - x_4')^2 - \lambda^2 .$$

However, in the whole above consideration we did not stress the fundamental characteristics of the quantity λ . Namely, one connects with the quantity λ the quantum fluctuation of metrics, this follows also from our previous analysis within the framework of uncertainty principle. λ is essentially connected with the violation of the concept of the spatial distances in small. Previous considerations of the Schwarzschild metrics are too classical in this region and, strictly speaking, are not applicable in this situation.

In the end we approach the basic question which is of a conceptual nature. It is possible, without intrinsic contradictions, to combine the general theory of relativity, with its rigorous concept of continuum, with the formalism of quantum field theories, which contradicts the concept of field in the small region and even the existence of the small region itself in the space-time continuum? One should stress that the last statement is equivalent by no means to the statement that the space is quantized and discrete.

Nevertheless, the known Zenon paradox about Ahille and tortoise at small distances gains a different meaning. The assumption cannot be excluded that in future theory the propagation function at small distances, more exactly the stringent fulfilment of the special principle of relativity, will loose the meaning because there will loose the meaning the concept of distance itself. It cannot be excluded that the De Witt propagation function is just the peculiar expression of the conceptual contradiation discussed above. Perhaps, one should accept it as a fact which corresponds to the Nature. Perhaps, this fact requires only the corresponding interpretation and that more rigorous derivation of the propagation function in the unified field theory, including gravitation, is unable to lead to essentially different expression for the propagation function (we mean it conflicts with causality principle).

It is clear to the author that the last phrases contain more questions than answers.

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