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Many-Particle Production

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БИБЛИОТЕКА

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LECTURE I.

Introduction

§ 1. Some empirical regularities in the processes of high energy multi-particle production of hadrons

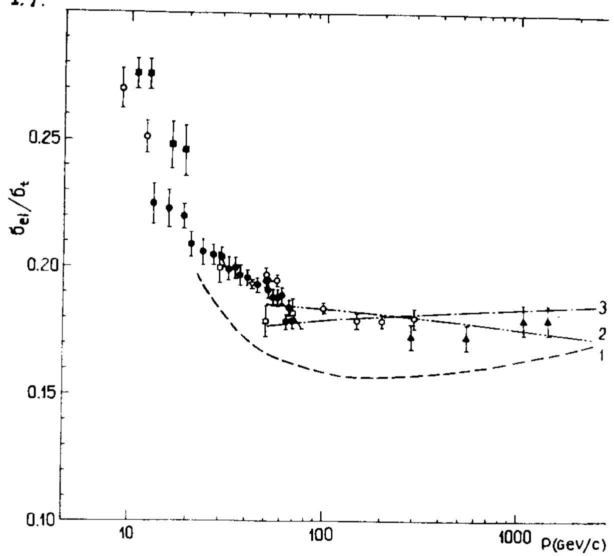
The problem of multi-particle production is one of the central problems in elementary particle physics. For a long time its study was possible only in cosmic rays. In spite of large experimental difficulties, connected with considerable errors, cosmic ray physics laid down the foundations of our notions about multiparticle production.

Modern accelerators have made it possible the intensive and detailed investigations of multi-particle production in a large energy interval (10-10^3 GeV). But no reasons so far exist to consider that we have a complete and clear description of phenomena.

At the same time a number of fundamental regularities and specific properties has been established for such processes.

We should note that the prediction by Vatagin (1934) concerning the increase of the relative number of inelastic channels at high energies is confirmed. The data from ISR (E $\sim$ 10^3 GeV) : $\frac{\sigma_{el}}{\sigma_{tot}} \sim 0.175$.

It means that under the hadron-hadron collision additional particles are not produoed only in 17 cases of a hundred. Thus the hadron-hadron collisions are mainly inelastic. The elastic ones obviously show themselves as a shadow of inelastic channels. This fact received an obvious interpretation in the Logunov-Tavkhelidze quasipotential approach. Qualitative changes of the characteristic $\frac{\sigma_{el}}{\sigma_{tot}}$ with energy is shown in fig. I.1.



Ratio of the elastic to the total cross section above 8 GeV/c.

/ V.Bartenev et al. JINR E1-8456, Dubna (1974) /

Fig.I.1

It is interesting to note that the hypothesis by G.Vatagin anticipated the prediction of π - meson by Yukawa (1935).

It was proved later that most of the secondaries are pions

$$\frac{\langle n_\pi \rangle}{\langle n_{sec} \rangle} \sim 80\%$$

(at ISR- energies).

Their relative number in inelastic processes somewhat decreases with energies. For example, with E $\sim$ 20 GeV

$$\frac{\langle n_\pi \rangle}{\langle n_{sec} \rangle} \sim 90\% .$$

Another important property of inelastic collisions at high energies is the smallness of the momentum transfer or the transverse momentum secondaries. (See fig. I.2)

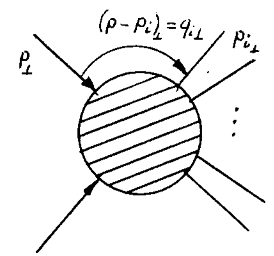


Fig. I.2

At the available energies the average value of the transverse momentum of secondaries does not depend on energy or depends on s rather weakly. It is limited by the interval:

$$\langle p_{i\perp} \rangle \sim 0.2 \div 0.4 \; GeV/c \; .$$

This empirical fact is closely related to the existence of the leading particle effect. This notion appeared and was effectively used in oosmio rays. By a leading partiole we conditionaly mean one of the colliding hadrons which looses a negligible part of its momentum under the interaction. Thus, the particles produced by oollision have mainly small momentum, compared with that of incoming hadron.

The total oross sections have been aotively investigated with putting into operation the aocelerators in Serpukhov, Batavia and CERN. Measurements of this quantity is the simpliest multi-particle experiment, while the latter is extremely critical to the theoretical models.

First unexpected results concerning the dependenoe of the total cross seotions on energy were obtained in 1971 at the Serpukhov accelerator in the energy interval from 30 to 70 GeV. The decrease of cross sections, determined at lower energies, became slower and went to oonstant in most of the hadron--hadron processes. In the case K^+p-coll. the increase of the total cross sections was found. This phenomenon comprising the change of cross section behaviour with increasing energy was

called as the Serpukhov effeot. Later the total cross sections were as well studied at ISR (1973) for the proton-proton collisions in 300 GeV to 2000 GeV range and at the accelerator in Batavia (1974) for all the hadron reactions at the energies up to 200 GeV.

The new data confirmed the Serpukhov effect and also showed that it may start a new phenomenon in high energy physics: rapid and, may be, unlimited increase of this quantity. The total cross section behaviour in the $\bar{p}p$-, pp interactions is seen from fig. I.3. It is so-far difficult to determine an analytical funotion which would describe the increase of σ_{tot}. We can make use of all the increasing functions up to the upper bound of possible increase of the total cross sections, determined by Froissart (1961) from the general principles of quantum field theory ($\sigma_{tot} \leq A \ln^2 s$).

Another rather general feature of inelastic processes is the average multiplicity. Most of the theoretical models predict its increase with energy. The models of a statistical type give us the power dependence

$$\langle n \rangle = a s^{b}$$

Multiperipheral, parton and a number of another models predict the logarithmic increase

$$\langle n \rangle = a \ln s + b \; .$$

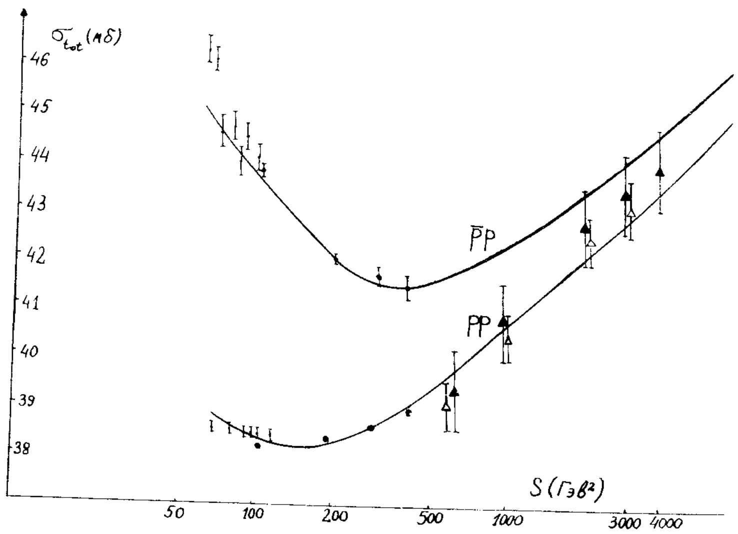


Fig.I.3

/Semenov S.V., Troshin S.M., Tyurin N.E., Khrustalev O.A.
Description of Growing Cross Sections by Means of Reaction Generalized Matrix.
Serpukhov, 1975.
p. 12. (IHEP 75-24)./

It should be noticed that maximum number of particles (pions), permitted by the energy-momentum conservation law, is written in the form:

$$n_{max} = \frac{\sqrt{s} - 2m_p}{m_\pi} \quad .$$

The observing multiplicity increases slower compared with the former equation, i.e. ,it is extremely small in comparison with that of kinematically allowed (see fig.I.4).

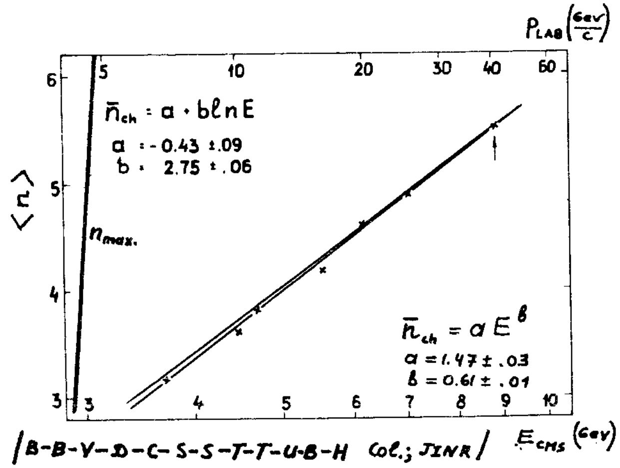


Fig.I.4

Unfortunately the comparison of the models with the experiment does not allow one to give preference to the logarithmic or power dependence of the average multiplicity on energies. One may only state the increase to be moderate.

Another important feature of the processes of high energy multi-particle production is deviation of the multiplicity distributions (or the topological cross sections) from the simple Poisson law determined at the energies higher than 250 GeV . (See fig.I.5).

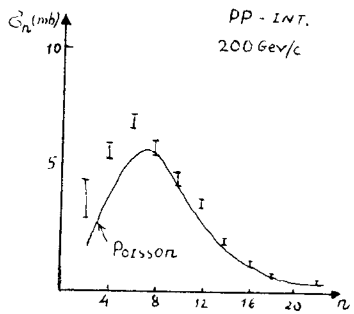


Fig.I.5

The topological cross section is that with the given number of charged particles and arbitrary number of neutral particles in the final state. If the production of particles in the given collision is considered to be of random nature, the distribution naturally assumes the Poisson form:

$$\sigma_n = \sigma_{inel}\, e^{-\nu} \frac{\nu^n}{n!} .$$

This distribution has the following properties:

$$\langle n \rangle \equiv \sum_{n=0}^{\infty} n\, P_n(\nu) = \nu ,$$

$$\langle n^2 \rangle \equiv \sum_{n=0}^{\infty} n^2 P_n(\nu) = \nu^2 + \nu .$$

Thus, for the Poisson distribution the correlation function f_2 is equal to zero:

$$f_2 \equiv \langle n(n-1) \rangle - \langle n \rangle^2 ,$$

$$\therefore f_2^{Poisson} = 0$$

However, the experimental data obtained at the accelerators in Serpukhov and Batavia show that the multiplicity distributions are broader than the Poisson distributions. And the quantity f_2 differs considerably from zero:

$$f_2 = 7.44 \pm 0.72 \quad (at\ P_{LAB} \sim 200\ GeV/c)$$

This fact shows that the production of secondaries at high energies cannot be considered a statistically independent process. Satisfactory distributions are obtained by the approaches based on consideration of productions of the whole hadron associations (or clusters). The models based on the account of two (or more) mechanisms of the hadron production, leading to the multicomponent description of distributions, are more successful for the description of experimental data. The possibility of extraction of the contributions of various mechanisms (the ranges of the n-particle volume phase) into the cross sections of multi-particle processes was first pointed out by Logunov and collaborators. The idea of two production mechanisms gives wide possibilities for the theoretical description of the correlation phenomena.

Already for the simplest distribution which is the topological cross section (depending on $n_{ch.}$) one could see that the secondaries are not independent but correlate with each other. Then the question arises about the sensitivity of neutral particles to the charged hadron production (i.e., charge-neutral correlations). Whether the particles "feel" what momentum has a produced "near" or "distant" (in the momentum scale) neighbour (short-range and long range momentum correlations).

As we have possibility to touch upon this question later, we should only note that the latest experiments at high energies gave a number of qualitatively new results. Here may be refered the detection of linear dependence of an

average number of neutral particle on a number of prongs. (See fig.I.6). Such correlation has not been observed at the energies up to 20 GeV.

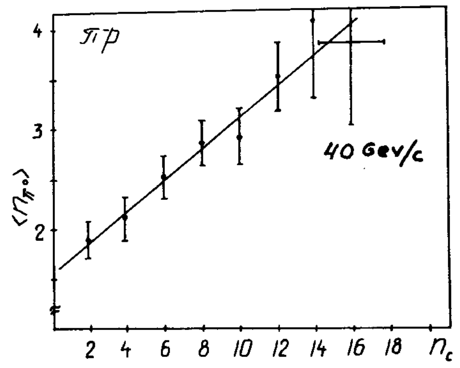


Fig.I.6

A large number of empirical facts on the dynamics of multiparticle processes makes it possible to interprete experimentally observed scaling regularities of strong interaction characteristics. These regularities are the display of a rather general principle of automodelity characteristics of a number of physical problems.

Here we mean, roughly speaking, the decrease of a number of independent variables of the studied physical quantity connected with definite similarity properties and symmetry of the problem (in the space of the given independent variables). The principle of automodelity was first suggested for the lepton-hadron and hadron-hadron processes by Matveev Muradyan and Tavkhelidze. They point out the analogy of these processes with an explosion in the gas dynamics.

Among the scaling regularities, studied in strong interactions, the hypothesis by Feynman on the decrease of a number of variables of the invariant differential cross sections when $s \to \infty$ is widely used:

$$E \frac{d\sigma}{d\vec{p}} = f(s, p_z, p_\perp) \xrightarrow[\substack{s\to\infty \\ x\text{-fixed}}]{} f\left(x = \frac{2p_z}{\sqrt{s}}, p_\perp\right)$$

Scaling regularities suggested by Koba, Nielsen, Olesen are of great use in the multiplicity distributions:

$$\langle n \rangle \frac{\sigma_n}{\sigma_{inel}} = \Psi \left(\frac{n}{\langle n \rangle}\right),$$

where σ_n — the topological cross section,

σ_{inel} — the total inelastic cross section,

$\langle n \rangle$ — the average multiplicity.

Those and a whole number of scaling laws, approximately satisfying at the available energies make it possible to assume that the strong interactions have some definite symmetry (may be not one).

The facts known at present about the hadron-hadron processes are not limited to the above-listed properties. However, these properties reflect the basic characteristics of multiple production in strong interactions. These properties are constantly exploited by theoreticians, no matter, which way they go. Those advocating the phenomenological schemes and empirical formulae seek to use these properties in constructing the models. Others who keep to field-theoretic approaches verify consistency of these properties with the basic axioms of quantum field theory and develop approximations adequate for the general properties found.

It may be hoped that these two approached, studying the same phenomena from various directions, after being united, will provide a closer description of high energy muti-particle production.

§2. Basic definition

The analysis of multiparticle production processes is very difficult both from the technical aspect and from the view-point of kinematical description. Therefore, it is especially important to obtain an information in the language of inclusive reactions. We mean here the processes where only a part of secondaries is detected. First the consideration of such reactions has been proposed by A.Logunov et al. in 1967.

It is customary to write the inclusive n-particle reaction in the form

$$a + b \to p_1 + p_2 + \dots + p_n + X , \qquad (1.1)$$

where X stands for "anything", i.e., all possible particles which are not subjected to observation in a given experiment.

Unlike the inclusive consideration, the reaction

$$a + b \to p_1 + p_2 + \dots + p_{n'} , \qquad (1.2)$$

when all the particles in a final state are detected, is characterized by the differential (exclusive) production cross section x)

$$\frac{d\sigma_{n'}}{d\vec{p}_1 \dots d\vec{p}_{n'}} = c \, | T(ab \to p_1 \dots p_{n'}) |^2 \, \delta(P - \Sigma p_{n'}) \qquad (1.3)$$

---

x) Later it will be used different realizations of the phase volume

$$d\vec{p} = \frac{d^3 p}{E} = \frac{2\pi \, dp_\perp^2 \, dp_\parallel}{E} \simeq 2\pi \, d^2 p_\perp \frac{dx}{x} = 2\pi d^2 p_\perp dy = s\,dt\,dM^2,$$

where variables are:

$$\vec{p} = (p_\parallel , \vec{p}_\perp) , \quad E = \sqrt{\vec{p}^{\,2} + m^2} , \quad x = \frac{2p_\parallel}{\sqrt{s}} , \quad y = \frac{1}{2} \ln \frac{E + p_\parallel}{E - p_\parallel} ,$$

where $T(ab \to p_1 \dots p_{n'})$ is the amplitude of transition of two particles a, b into n' particles with momenta $p_1 , \dots , p_{n'}$.

The transition from (1.3) to the inclusive distribution of the process, where only n of n' particles are identified, is achieved by integrating over momenta of the nondetected particles

$$\frac{d\sigma_{n'}}{d\vec{p}_1 \dots d\vec{p}_n} = c \int | T(ab \to p_1 \dots p_{n'}) |^2 \delta(P - \Sigma p_{n'}) \cdot d\vec{p}_{n+1} \dots d\vec{p}_{n'} \qquad (1.4)$$

If we make summation over all the channels with n particles of reaction (1.1) we arrive at the so-called n-particle inclusive distribution

$$\frac{d\sigma}{d\vec{p}_1 \dots d\vec{p}_n} = c \left\{ |T_n|^2 \delta(P - \Sigma p_n) + \sum_{n_c} \int |T_{n'}|^2 \delta(P - \Sigma p_{n'}) \, d\vec{p}_{n+1} \dots d\vec{p}_{n'} \right\} \qquad (1.5)$$

If differential cross sections were known for all the exclusive channels we could construct , using this formula, all the inclusive distributions.

And, vice versa, knowing all the inclusive distributions one could reproduce the cross sections of the channels. Thus, in principle, both the descriptions contain complete information on the two-hadron collision process.

Until we dealt with a small amount of secondaries it was more convenient to employ the exclusive description. At higher energies when ten and more particles are produced it is better to keep the inclusive consideration.

1. The one-particle distribution. Consider an example of the inclusive reaction with only one detected particle

$$a + b \to p_1 + \underbrace{X}_{p_2 + \dots + p_{n'}} \tag{1.6}$$

The one-particle cross section with a fixed multiplicity is defined as follows

$$\frac{d\sigma_{n'}}{d\vec{p}_1} = \frac{1}{(n-1)!}\int \frac{d\sigma_{n'}}{d\vec{p}_1 \dots d\vec{p}_{n'}} \prod_{i=2}^{n'} d\vec{p}_i \,. \tag{1.7}$$

Knowing this cross section one can easily go over to the topological cross section of n' particle production

$$\sigma_{n'} = \frac{1}{n'}\int \frac{d\sigma_{n'}}{d\vec{p}_1}\, d\vec{p}_1 \,. \tag{1.8}$$

Here $\sum_{n'} \sigma_{n'} = \sigma_{inel}$ is the total inelastic cross section.

As is seen from the previous definitions, summing up (1.7) over n' we get the one-particle inclusive distribution

$$\frac{d\sigma}{d\vec{p}_1} = \sum_{n'} \frac{d\sigma_{n'}}{d\vec{p}_1} \tag{I.9}$$

with the normalization

$$\int \frac{d\sigma}{d\vec{p}_1}\, d\vec{p}_1 = \sum_{n'} n'\sigma_{n'} \equiv \langle n' \rangle \sigma_{inel} \tag{1.10}$$

From the sum rule

$$\int \frac{d\sigma}{d\vec{p}}\, p_\mu \, d\vec{p} = \sigma \langle p_\mu \rangle$$

it is easy to get the definitions of average momenta $\langle p_\perp \rangle$ at $\mu = 1, 2$ and $\langle p_\parallel \rangle$ at $\mu = 3$.

Notice that relation (1.10) is the definition of mean multiplicity $\langle n_c \rangle$ of secondaries.

2. The two-particle distribution.

Analogy, one can consider the inclusive reaction with identification of two particles

$$a + b \to p_1 + p_2 + \underbrace{X}_{p_3 + \ldots + p_n} \tag{1.11}$$

Arising here two-particle inclusive distributions define a series of the widely used average quantities:

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2}\, d\vec{p}_2 = \langle n_c(p_1) \rangle \frac{d\sigma}{d\vec{p}_1},$$

$$\int \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2}\, d\vec{p}_1 d\vec{p}_2 = \sum n(n-1)\sigma_n = \langle n(n-1) \rangle \sigma_{inel} \tag{1.12}$$

The quantity $\langle n_c(p_1) \rangle$, in particular, is called the associated multiplicity.

Making use of the two- and one-particle inclusive cross sections one can construct the two-particle correlation function

$$C_2 = \frac{1}{\sigma_{inel}} \frac{d^2\sigma}{d\vec{p}_1 d\vec{p}_2} - \frac{1}{\sigma^2_{inel}} \frac{d\sigma}{d\vec{p}_1} \frac{d\sigma}{d\vec{p}_2} \tag{I.13}$$

and the corresponding moment of distribution

$$f_2(s) = \int C_2(p_1, p_2)\, d\vec{p}_1 d\vec{p}_2 = \langle n(n-1) \rangle - \langle n \rangle^2 = D^2 - \langle n \rangle, \tag{I.14}$$

where $D \equiv \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$ is the dspersion.

Higher correlation functions and moments $C_3, f_3, \ldots, C_n, f_n$ are defined analogously. It is natural in the oase of independent production of particles all the C_n and f_n are zero. This has been demonstrated above when considering the Poisson law.

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Problems for Discussion to Lec.I

1. Find relations between the variables in various systems. ($p_\perp$, $p_\parallel$, x, y in (CMS, LAB, PROJ))

2. Let the form of distribution be given find $\langle p_\perp \rangle$.

$$\frac{d\sigma}{d\vec{p}} \sim \begin{cases} e^{-A(s)p_\perp^2 f(x,s)} \\ \frac{1}{p_\perp^N} \end{cases}$$

3. Find the relation between $\langle n \rangle$ and $p_\perp$ in the assumption given in (2).

LECTURE II

MULTIPLICITY DISTRIBUTIONS

§1. Multicomponent descriptions of multi-particle production

The multiplicity distributions or the topological cross sections are referred to a number of the most simple characteristics of the processes of multi-particle production. They are determined by a number of events with a given number of secondaries. As a rule the charged secondary particles are taken into consideration. In the high energy range up to 20 GeV the experimental topological cross sections have been very well described by a series of theoretical models and phenomenological formulae.

Firstly, one may successfully use a usual Poisson formula

$$P(n_\pm) = \frac{\langle n_\pm \rangle^{n_\pm}}{n_\pm!} \cdot exp\left[-\langle n_\pm \rangle\right]$$

describing an independent production of particles. Two models have been applied for the description of charged distributions, were suggested by Wang.

The first one started from the assumption of uncorrelated production of the hadron pairs $\pi^+\pi^-$. In this case the multiplicity distribution has a simple quasi-Poisson formula:

$$P(n_\pm) = \frac{\left(\frac{1}{2}\langle n_\pm - \alpha \rangle\right)^{\frac{1}{2}(n_\pm - \alpha)}}{\left[\frac{1}{2}(n_\pm - \alpha)\right]!} \; exp\left[-\frac{1}{2}(n_\pm - \alpha)\right]$$

where α is the number of charged particles in the initial state.

The second one, suggested by Wang, led to the Poisson distribution for the charged secondary particles subtracting the leading parti cles:

$$P(n_{\pm}) = \frac{(\langle n_{\pm} - \alpha \rangle)^{n_{\pm}-\alpha}}{(n_{\pm} - \alpha)!} \cdot exp\left(-\langle n_{\pm} - \alpha \rangle\right)$$

It was assumed in the Chou and Pignotty multiperipheral model that the Poisson dependence describes the distribution over a number of secondary pions with excluding events of the pure neutral particle production (0- prong events). Considerable deviations of the topological cross sections from the Poisson law are observed from energies ~ 25 GeV. (see fig. II.1). This testifies to failure of the models based

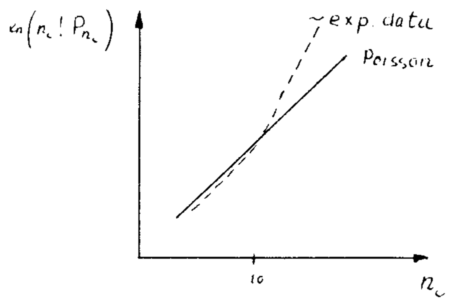

Fig.II.1

on the assumption of unocorrelated production of single particles. The experiments performed in Serpukhov on the 2-meter propan chamber, irradiated by π^-- mesons, when $pc = 40$ GeV proved to be especially critical to the multiplicity distributions.

If at the energy 25 GeV Wang and Chow-Pignotty models were in satisfactory agreement with experiment, then the data from the 2-meter propan chamber in combination with the recent experiments in Batavia and at ISR give evidence in favour of the multicomponent description of the multi-partic le produc tion.

The attempts to combine the two extreme approaches to multiparticle production at high energies became a starting point to the origin of the multi-component description.One of them, the diffraction dissociation, proceeds from the assumption that the secondaries are produoed due to the leading particle fragmentation (target partiole and incoming particle). We may say that the secondaries have information about the colliding hadrons, they may be combined with one of the initial particles. Figuratively speaking, they remember their "parents". The diffraction dissociation approach leads to the topological cross sections of the type $\sigma_n' \sim n^{-2}$ which disagree with the recent experimental data as well as the Poisson distributions.

Another approach deals with the secondaries which do not "remember" their origin from one or another initial particle. To this category we may refer the models of independent emission some of which have been discussed above. It is convenient to classify these approaches, having determined the correlations of the produced particles. The difference between the correlations depends on whether the secondaries are in the same (short-range) or different (long-range) ranges of phase space volume of n-particles.

If $y_1, y_2, \left(y = \frac{1}{2} \ln \frac{E+p_{\parallel}}{E-p_{\parallel}} \right)$ are the rapidities of the secondaries, then

1) Short-range (SR) correlations exist between the particles, produced with approximately equal rapidities and with increasing $|y_1 - y_2|$ tends to zero, as

$$C_2(y_1; y_2) \sim e^{-\gamma \cdot |y_1 - y_2|}$$

when

$$|y_1 - y_2| \gg \frac{1}{\gamma}, \quad \gamma \neq 0$$

(LR)

2) Long-range correlations exist between the particles, produced in distant ranges of the y - space, i.e., for $|y_1 - y_2| \gg \frac{1}{\gamma}$ and the two-particle function of the distribution increases rapidly when both particles come from one "cluster".

In other words when observing the particle with y_1 the information about possible presence of another secondary with any admissible rapidity is the LR effects. And vice versa, the information about probability of presence of another particle with similar rapidity is the SR-effect.

In the diffraction dissociation approach there are strong LR-correlations. Concrete realizations of the second approach, i.e.,the models of independent emission, are characteristic of either absence of correlations (Poisson, $C_2 = 0$), or presence of small SR-correlations.

We should note that the possibility of extraction of contributions of various mechanisms (ranges of phase volume of n-particles) into multiple cross sections was first pointed out by Logunov and collaborators.

In this connection in recent years there has been changes in the philosophy of approach to the mechanism of high energy multi-particle production. Wilson and Feynman proposed the two- component model. The simplest version of this model is based on the multiplicity distribution, written in the form of the sum:

$$\sigma_n = \alpha \cdot n^{-2} + \beta\, P(n)$$

with the chosen contributions of each component. In particular, the parameters may be chosen so that there is left only a term, corresponding to one of the approaches.

Supposing, that both components are present at all the energies, the first moments $\langle n^k \rangle$ of the distribution σ_n are of the form

$$\langle n \rangle = a + (\alpha_1 + \beta_1) \ln s$$
$$\langle n^2 \rangle = b + \alpha_2 \sqrt{s} + \beta_2 (\ln^2 s + 2 \ln s - 4);$$
$$\langle n^3 \rangle = c + \alpha_3 s + \beta_3 (\ln^3 s + \ldots);$$

where the contributions with the coefficient α_i are consistent with the first component, and those with the coefficient β_i are consistent with the second one. It follows, that the first component (the component of the diffraction dissociation) dominates beginning from the second order moment at high energies.

It should be noted that in the given approach the "play" of these two components leads to the existence of a weak deep in the multiplicity distribution. The deep becomes more noticeable with increasing energy (see fig.II.2).

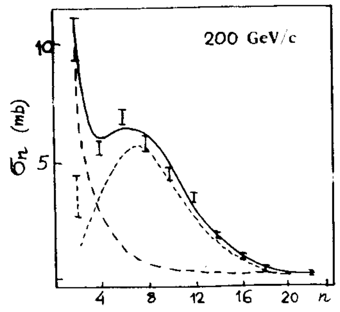

Fig II.2

In spite of a number of virtue (for example, the increase of the second correlation parameter f_2 is in an excellent agreement with experiment) the two-component model results in discrepancy of higher correlation moments with experimental data. Note, that such discrepancy cannot be eliminated in the models with a large $(n>2)$ number of components.

§2. Model of "two mechanisms"

Let us consider the multi-component description of multi-particle production, realized due to phenomenological model of "two mechanisms", suggested by the Dubna group (Matveev, Kuleshov, Sissakian, Grishin, Jancso) in 1972.

The IMP-model appeared as a concrete phenomenological scheme on the basis of the study of the processes of multi-particle production in the framework of the straight-line path approximation in quantum field theory (SLPA) . The physical essence of SLPA is the following: At high energies the main contribution to the process amplitude in the form of the Bogolubov-Feynman functional integral, over the particle paths give the trajectories which are nearly straight-lines having the same direction as the momentum vectors of the leading particles before and after the correlation. In field theoretic language SLPA rests on the assumption of a leading particle. To the most important results of SLPA we should refer the generalized Poisson law for the topological cross sections, automodel or point-like behaviour of the cross sections and the prediction about the dependence of the average multiplicities on a transverse momentum of an extracted particle. We shall appeal to some of these results, when considering the picture of multi-particle production.

The main point of the TMP-model is the hypothesis on the existence of two mechanisms of production of secondaries.

i) There exist the leading particles, dissociating with the local conservation of isospin

ii) in the process of interaction in a statistically independent way there appear as well the hadron associations of clusters which then decay into mesons.

It is natural to suppose that the average numbers of these associations at high energies are independent of a type of the colliding particles.

According to these assumptions, one can see that in the TMP-model the probability of production of clusters at the given dissociation channels of the leading particles (i,j) takes the form:

$$W^{i,j}_{n_1,n_2,\dots} = \alpha_i \, \beta_j \cdot P_{n_1}(\langle n_1\rangle)\, P_{n_2}(\langle n_2\rangle)\dots \; ; \qquad \text{(II.1)}$$

where α_i, β_j is the probability of the dissociation channels. $n_1, n_2, \dots$ is a number of clusters, produced according to the Poisson law. Thus, the distribution over a number of secondaries in the given model has the form of superposition of the Poisson factors. Multi-components character appears as a result of summation over a number of channels of the leading particle dissociation. Now, consider a concrete example of description by the TMP- model of the charged distributions in the π^-p and π^-n interactions. Comparing with experiment we use the data received on a two-meter propane chamber of JINR at the Serpukhov accelerator $(E_{\pi^-} = 40\ GeV)$

In this case it is sufficient to consider only the simplest channels of dissociation of the colliding particles and the hadron clusters with the isospin I=0. Thus, we consider dissociation of the leading nucleon in the following scheme:

1. $N \rightarrow N$ $\quad - \alpha_1$

2. $N \rightarrow N\pi^0$ $\quad - \alpha_2$

3. $N' \rightarrow N''\pi^{\pm}$ $\quad - \alpha_3$

where $\sum\limits_{i=1}^{3} \alpha_i = 1$, and $\alpha_3 = 2\alpha_2$, by the assumption on local isospin conservation. As another source for secondary particle production we introduce the σ- and ω- associations, produced by the Poisson law, with isospin $I = 0$ and G-parity $G = \pm 1$

We confine ourselves to the main schemes for the decay of the σ- and ω- associations:

1. $\sigma \longrightarrow \pi^+\pi^-,\ \pi^0\pi^0$

2. $\omega \rightarrow \pi^+\pi^-\pi^0$

In accordance with the assumptions of the TMP-model, one can easily see from eq. (II.1) that the production probability for the pion pairs $(n_{\pm}, n_0)$ and the triplets of pions n_3 at the given channel of the nucleon dissociation is defined by the expression:

$$W^i_{n_{\pm}, n_0, n_3} = \alpha_i \cdot P_{n_{\pm}}(a_{\pm}) \cdot P_{n_0}(a_0) \cdot P_{n_3}(b) ; \qquad \text{(II.2)}$$

where $P_n(\langle n \rangle)$ the Poisson factor,

$a_{\pm}, a_0, b$ are the average numbers of pion pairs and of pion triplets, correspondingly.

From the condition that the pairs are produced with the isospin I=0, it follows that

$$a_{\pm} = 2a_0 \equiv a$$

It is evident that the number of charged particles n_c and neutral pions n_{π^0} can be written as follows:

$$n^i_c = 2n_{\pm} + 2n_3 + \ell^i_c$$

$$n^i_{\pi^0} = 2n_0 + n_3 + \ell^i_{\pi^0} \qquad \text{(II.3)}$$

where ℓ_c^i, $\ell_{\pi^c}^i$ are, respectively, the numbers of charged particles and π^c- mesons among the dissociation products of the leading particles in the i-th dissociation channel (See Table II.1).

Table II.1

	$i=1$		$i=2$		$i=3$	
	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$	$\pi^- p$	$\pi^- n$
ℓ_c	2	1	2	1	2	3
ℓ_{π^c}	0	0	1	1	0	0

From eqs. (II.2) and (II.3) for distributions over the number of charged particles, it follows:

for the $\pi^- p$ - interaction

$$W_{n_c}^{\pi^- p} = P_{\frac{n_c-2}{2}}(a') ; \qquad \text{(II.4)}$$

for the $\pi^- n$ - interaction

$$W_{n_c}^{\pi^- n} = (1-2\alpha_2)\cdot P_{\frac{n_c-1}{2}}(a') + 2\alpha_2 \cdot P_{\frac{n_c-3}{2}}(a') ; \qquad \text{(II.5)}$$

where $a' = a + \ell$ has essence of the average number of pairs $\pi^+\pi^-$, including the contribution from the similar combinations among the pion triplets $\pi^+\pi^-\pi^0$.

A comparison of eqs. (II.4) and (II.5) with experimental data at $E_{\pi^-} = 40$ GeV shows a good agreement (see fig. II.3, II.4 and table II.2)

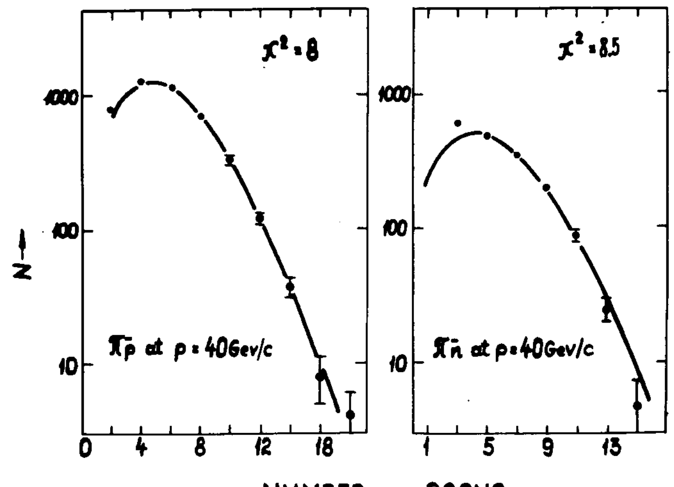

Fig. II.3 Fig. II.4

Table II.2

Type of interaction	Number of events	$\bar{n}$	$\sqrt{D}$	χ^2 fit by Wang I model	χ^2 fit by suggested model	Degrees of freedom
$\pi^- p$	4400	5.62 ± 0.4	2.75	8	8	8
$\pi^- n$	1860	5.32 ± 0.7	2.82	13	8.5	7

Note, in this simple case of distribution over the number of charged particles, only two components are important, each of which corresponding to the pair independent emission. However, it appears to be sufficient to describe broadenning of distributions, which is characteristic of high energies.

Note as well, that unlike the Wang-I model, the case of distributions of the type – superposition of the Poisson factors with the same number of parameters gives a good joint description for the $\pi^- p$ and $\pi^- n$ collisions with the same average value of the $\pi^+\pi^-$ combinations. It is consistent with a natural physical hypothesis on independence of particle production in non-diffraction region of a type of colliding hadrons. Multi-component structure of distirbutions arises also in the case if under the same assumptions. heavy strange particles are included into consideration.

The following additional channels of dissociation of nucleons are possible:

$$p \to \Lambda^0 K^+ , \qquad n \to \Lambda^0 K^0$$

$$p \to \Sigma^0 K^+ , \qquad n \to \Sigma^0 K^0$$

$$p \to \Sigma^+ K^0 , \qquad n \to \Sigma^- K^+$$

Under the given assumptions the pion dissociates, with the most probability, according to the schemes:

$$\pi^- \to \pi^-$$

$$\pi \to 2\pi^-\pi^+$$

$$\pi \to 2\pi^0\pi^- .$$

It is necessary to include independent production of heavy Λ– associations besides the pion pairs and pion triplets

$$\Lambda \to K^+K^-, \; K^0\bar{K}^0 .$$

The scheme leads to the following distribution over the charged particles (π^+, π^-, K^+, K^-)

$$W_{n_c}^{\pi^- p} = \mu \cdot P_{\frac{n_c-2}{2}}(a'') + \nu \cdot P_{\frac{n_c-4}{2}}(a'') , \qquad \text{(II.6)}$$

$$W_{n_c}^{\pi^- n} = f_1 \cdot P_{\frac{n_c-1}{2}}(a'') + f_2 \cdot P_{\frac{n_c-3}{2}}(a'') + f_3 \, P_{\frac{n_c-5}{2}}(a'') ;$$

where the parameters μ, ν, f_i are connected with probabilities of the channels of dissociation , and $\bar{a}''$ is the average number of combinations, including charged pairs.

It is seen from the above consideration that the idea of joining two-opposite viewpoints on the mechanism at secondary particle production, namely:

i) independent emission;

ii) dissociation (or fragmentation) of leading particles, may turn out to be rather fruitful.

Simplicity of such a synthetic approach is very attractive. The assumption on uncorrelated production of associations (or olusters) makes it possible to combine advantages of the models of independent emission with possibility (it will be shown in the following section) to study the correlation dependencies. Apart from the suggested approach still more models are available based on the idea of joining two mechanisms. Note, that the old schemes are reconstructed in accordance with the new ideology. To explain, in multi-Regge scheme, experimental data on charged distributions and correlation dependencies the assumption is used on the necessity of consideration of the diagrams with a large number of showers (or clusters) at high energies. The latter is also equivalent to the multi-component structure of distributions over multiplicity.

§ 3. Scaling properties of topological cross sections

As mentioned before, one of the characteristic features of topological cross sections is "broadening" of distribution with increasing energy. Consideration of normalized topological cross sections

$$P(n,s) = \frac{\sigma_n}{\sum_n \sigma_n} , \tag{II.7}$$

as a function of the number of particles and energy s shows that curves strongly change their form with increasings s (See fig. II.5)

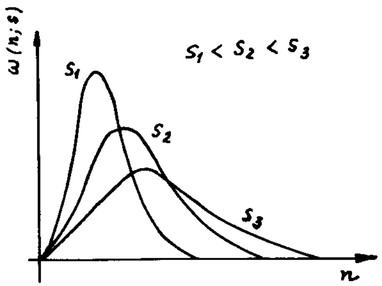

Fig.II.5.

If one plots a function $\langle n \rangle \frac{\sigma_n}{\sigma}$ in the scale $n/\langle n \rangle$ it appears that at high energies the family of distributions over multiplicity for various energies S will be on the same universal curve (See fig.II.6)

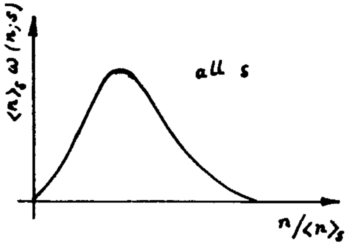

Fig.II.6

In fact, it means that the function $\langle n \rangle \cdot \sigma_n / \sigma$ depends only on the ratio $n/\langle n \rangle$

$$\langle n \rangle \cdot \frac{\sigma_n}{\sum\limits_n \sigma_n} \xrightarrow[S \to \infty]{} \Psi\left(\frac{n}{\langle n \rangle}\right) \qquad \text{(II.8)}$$

The existence of such a regularity was first pointed out by Koba, Nielsen and Olesen. Thus, it is called KNO-scalling. KNO-scaling was obtained under the assumption of the Feynman scaling, i.e., of scale properties with respect to $X = \frac{2P_{\parallel}}{\sqrt{S}}$. At present, this universal property is thoroughly verified by the experiments for various types of particle interactions at the accelerators in Serpukhov and Batavia. This favours the statement that at high energies the hadron-hadron collisions tend to be similar.

Note, that at asymptotically high energies

$$\sum_n n^q \cdot \frac{\sigma_n(s)}{\sigma} \sim \int dn \cdot n^q \, \frac{\sigma_n(s)}{\sigma} \underset{S \to \infty}{\approx}$$

$$\approx \int dn \cdot n^q \frac{1}{\langle n \rangle} \cdot \Psi\left(\frac{n}{\langle n \rangle}\right) =$$

$$= \langle n \rangle^q \cdot \int dz \cdot z^q \, \Psi(z),$$

$$q \ll \langle n \rangle ,$$

i.e., dependence (2.8) may be given in the form:

$$\langle n^q \rangle \xrightarrow[s \to \infty]{} C_q \cdot \langle n \rangle^q .$$

Thus, universality of eq. (2.8) is equivalent to the statement that the ratio of moments $C_q = \frac{\langle n^q \rangle}{\langle n \rangle^q}, q=1,...$ does not depend on energy. Such dependence, as it was mentioned above, yields in the models of independent emission (i.e., at SR-correlations), where

$$\langle n \rangle \sim \ln s$$
$$\langle n^2 \rangle \sim \ln^2 s, .$$

However, the KNO-scaling describes such processes in which one cannot do without taking into consideration the LR-correlations. Most probably, a mechanism, leading to the KNO-behaviour of distributions, unites many components which lead to a non-trivial disappearance of dependence $C_q = \frac{\langle n^q \rangle}{\langle n \rangle^q}$ at sufficiently high energies. Various modifications of the KNO-scaling,

derivation of this regularity from different approaches, and fit by empirical functions are intensively discussed at present in many theoretical and experimental works. To some of these questions we shall return when considering inclusive or semi-inclusive reactions.

Note, that the KNO-scaling is one of the most interesting displays of a general principle of automodelity in the hadron--hadron interactions at high energies. Further investigation of this regularity and of divergence from it, makes it possible to understand the dynamics of multiparticle processes more profoundly.

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Lecture III

CORRELATION DEPENDENCIES

§1. The problem of correlations

Correlation dependencies in the multi-particle production prooesses can be conditionally separated into two groups. The first group is the correlations between different parameters of a single particle. For instance, the dependence $P_\perp$ on $P_{\parallel}(x)$

The second group of correlation effects arises in studying the two-particle distributions in the inclusive experiments. To this group one can relate the dependenoies between different particles. For instance, the correlations between neutral and charged particles, only at $E \approx 25 GeV$, the problem of factorization in the distributions of different particle contributions (it is usually conneoted with the problem of deviation from independent emission) and a number of other effects. Two-particle, three-particle, . . . correlations are considered. There exist many reasons as to the appearance of correlations. Among them, the production of associations, clusters and other dynamic phenomena are important . There may exist other less evident reasons.

In the present paper we mainly consider the problem of two-particle correlations and, espeoially, the dependence between

charged and neutral particles, since these effeots owe to the latest investigations at modern acoelerators. We try to interpret these phenomena from the point of view of multi-component desoription of mutli-particle processes, since it makes possible to understand the nature of such correlations from the point of view of an important hypothesis on clusterization in multi-particle production.

§2. Two-particle correlations

If one oonsiders an arbitrary multi-component reaction

$$a + b \longrightarrow P_1 + P_2 + \dots + P_n \qquad (3.1)$$

then the invariant n-particle cross section can be written in the form:

$$f_n(s, \vec{P}_1, \dots, \vec{P}_n) = \left(\prod_{i=1}^{n} E_i\right) \frac{d^{3n}\sigma_n}{\prod_{i=1}^{n} d^3\vec{P}_i} , \qquad (3.2)$$

where E_i, $\vec{P}_i$ is the energy and three-dimensional momentum of i-th secondary particle, correspondingly, $s = (P_a + P_b)^2$ is the known Mandelstam variable. The corresponding distribution density may be reoeived by separating f_n into total inelastic cross seotions σ_{inel}

$$\rho_n(s,\vec{P}_1,\ldots,\vec{P}_n) = \frac{1}{\sigma_{inel}} f_n(s,\vec{P}_1,\ldots,\vec{P}_n). \quad (3.3)$$

In the given lecture we consider only two-particle $\rho_2(s,\vec{P}_1,\vec{P}_2)$ distributions.

If all the particles are independent, then the ρ_2 distribution is simply connected with the one-particle distribution

$$\rho_2(\vec{P}_1,\vec{P}_2) = \rho_1(\vec{P}_1)\rho_1(\vec{P}_2). \quad (3.4)$$

However, if there are correlations between the particles 1 and 2, then simple factorization is not present here, i.e. , it is necessary to introduce the correlation term:

$$\rho_2(\vec{P}_1,\vec{P}_2) = \rho_1(\vec{P}_1)\rho_1(\vec{P}_2) + C_2(\vec{P}_1,\vec{P}_2), \quad (3.5)$$

where C_2 is the two particle correlation function. The meaning of C_2 is that it is a measure of influence of particle 1 (with momentum $\vec{P}_1$) on the probability that another particle 2 has a momentum P_2 for any distribution over momenta of remaining particles.

The correlation function, determined in the rapidity space

$$C_2(y_1,y_2) = \frac{1}{\sigma}\frac{d\sigma}{dy_1 dy_2} - \frac{1}{\sigma^2}\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2} \quad (3.6)$$

is widely used,

where σ is the cross section for the given class of events. Sometimes it is convenient to consider the given correlation function

$$R_2(y_1,y_2) = C_2(y_1,y_2)\,\sigma^2 \Big/ \frac{d\sigma}{dy_1}\cdot\frac{d\sigma}{dy_2}. \quad (3.7)$$

The two-particle correlation function C_2 is simply connected with f_2

$$f_2 = \int C_2 \frac{d^3\vec{P}_1}{E_1}\frac{d^3\vec{P}_2}{E_2} = \begin{cases} \langle n(n-1)\rangle - \langle n\rangle^2 \\ \langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle \end{cases} \quad (3.8)$$

to which we have appealed considering the distributions over multiplicity, i.e., f_2 is the completely integrable correlation function C_2 .

Note that the models of a multiperipheral type (i.e., SR the models with the SR-correlations) predict a logarithmic dependence of the function s on f_2

$$f_2 \sim a \ln s$$

and the diffraction dissociation approach (where the LR-correlations are taken into account) gives the power dependence

$$f_2 \sim A s^{1/2}.$$

In the multi-component description, we separate the contributions of different mechanisms into multi-particle cross section. In this case the behaviour of correlation functions is determined by superposition of the correlators, corresponding to each of mechanisms. Their concrete form depends on the way of realization of multi-component approach. In particular, one may consider decomposition of such a type

$$\sigma_n = \sum_a \sigma_n^{(a)} \tag{3.9}$$

where

$$\sum \sigma_n^{(a)} = C_a \sigma , \qquad a = 1, 2, \dots$$

Contributions into the average multiplicity and higher distribution moments are received for different mechanisms separately:

$$\langle n \rangle_a = \frac{\sum n \sigma_n^{(a)}}{\sum \sigma_n^{(a)}} , \quad f_2^{(a)} = \langle n(n-1) \rangle_a - \langle n \rangle_a^2 ,$$

where

$$a = 1, 2, \dots$$

the total (observed) quantites are correspondingly equal

$$\langle n \rangle = \sum C_a \langle n \rangle_a$$

$$f_2 = C_1 f_2^{(1)} + C_2 f_2^{(2)} + \dots + C_1 C_2 (\langle n \rangle_1 - \langle n \rangle_2)^2 + \dots \tag{3.10}$$

where $\sum_a C_a = 1$

The following formula

$$C_2 = \alpha_d C_2^d + \alpha_\pi C_2^\pi + \frac{\alpha_d}{\alpha_\pi}\left[\frac{1}{\sigma}\frac{d\sigma}{dy_1} - \frac{1}{\sigma_d}\frac{d\sigma_d}{dy_1}\right]\left[\frac{1}{\sigma}\frac{d\sigma}{dy_2} - \frac{1}{\sigma_d}\frac{d\sigma_d}{dy_2}\right] \tag{3.11}$$

is widely used in practice for the two-particle correlation function. ($\alpha_\pi + \alpha_d = 1$).

This formula shows the character of correlation function in the case of two-component description, i.e., when the inelastic collisions may be described by the fraction α_d of the diffraction dissociation processes and the fraction α_π of the pionization process (the processes with the SR-correlations are often called so).

Note, that in this case as seen from (3.11), the resulting two-particle correlation function C_2 is not an average quantity of C_2^{π}, C_2^{d} calculated for each of the components. It may be sufficiently large even if the two-particle correlation functions are very small for each of the components taken separately. It is sufficient to assume the one-particle distributions different for both components in order that the last term in (3.11) would be large.

Note, that a large number of correlation functions and parameters has been proposed for consideration. We shall determine only the widely accepted ones.

Point some experimental data. The experimental values for the function f_2 obtained in the pp interaction, are shown in fig. III.1.

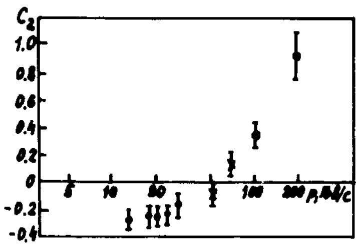

Fig.III.1

This curve has a rather characteristic trend showing that independent emission (the Poisson distribution over multiplicity) occurs only at $E_0 \sim 50$ GeV.

Here $f_2(s)=0$. The integrated correlation function has small but negative values, corresponding to approximately independent emission (for instance, the model of uncorrelated beams) at $E_0 < 50$ GeV.

At energies larger than 50 GeV, the two-particle correlation parameter f_2 increases rather rapidly. Note that multicomponent description will be most useful for describing this energy region. (see for instance /3.11/) .

In 1972 Gaugili and Malhotra considered the dependence of the two-particle structural function $\langle n(n-1)\rangle$ on s. They compared experimental data in a large energy interval with the predictions of the maximum fragmentation model (Benecke, Chou, Yang, Yen) and the multi-peripheral model (Horn, 1972). The first model predicts the dependence $\langle n(n-1)\rangle \sim \sqrt{s}$ the second one predicts $\sim \ln s$ or $\sim \ln^2 s$. If one assumes $\langle n\rangle \sim \ln s$, then in the asymptotic region the quantity

$$\frac{\langle n\rangle}{D} = \frac{\langle n\rangle}{\langle n^2\rangle - \langle n\rangle^2}$$

is expected to be proportional to $\ln s / s^{1/4}$ in the maximum fragmentation model and to $(\ln s)^{-1}$ in the multi-peripheral model. The authors made a conclusion that the multi-peripheral model fits better the dependence of $\langle n(n-1)\rangle$ and $\frac{\langle n\rangle}{D}$ on s.

In the range of energies with data available from the bubble chamber both models have small differences, and the conclusion is based on the data from cosmic ray research at $E \sim 2\cdot 10^4\ GeV$.

However, this argument for the model with SR-correlations is not essential due to a weak sensitivity of the studied dependences of the function $\langle n(n-1)\rangle$ to experiment. The dependence of the dispersion D on $\langle n\rangle$ is more .

Consider it by the example.

If the SR- correlations dominate and do not depend on energy then f_2 determined according to (3.8) by the integral

$$f_2 = \int C_2\, dy_1\, dy_2 \qquad (3.12)$$

receives the main contribution from the integration region, from diagonal $y_1 \approx y_2$ and has an order $f_2 \sim \ln s$ due to the fact that the surface of kinematic region in the y_1, y_2 plane is broadening with energy like $(\ln s)^2$. As has already been mentioned, in the models with SR-correlations an average multiplicity increases with energy like $\ln s$. Thus, for the integrated two-particle function we have

$$f_2 \sim \langle n\rangle .$$

Correlation (3.12) leads to the following dependence of D on $\langle n\rangle$

$$D^2 = f_2 + \langle n\rangle \sim \langle n\rangle . \qquad (3.13)$$

However, this dependence is not confirmed experimentally. It is seen from fig. III.2

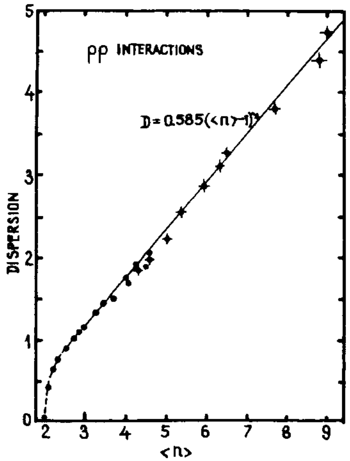

fig.III.2

Linear dependence between D and $\langle n \rangle$ holds for all available now pp data. It is confirmed, though with a different slope, for the data on the meson-proton collisions. (See Fig.III.3)

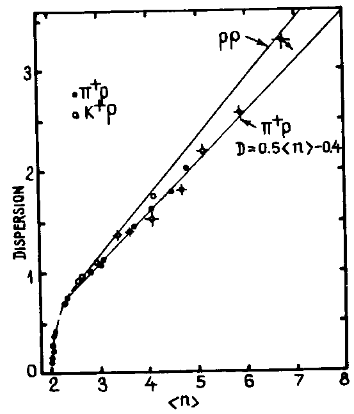

Fig.III.3

Note, however, that the assumption on energetic dependence of the SR-correlations allowed Bialas to construct a model where a linear dependence between D and $\langle n \rangle$ holds approximately in some region, including almost ISR-energies (see fig.III.4).

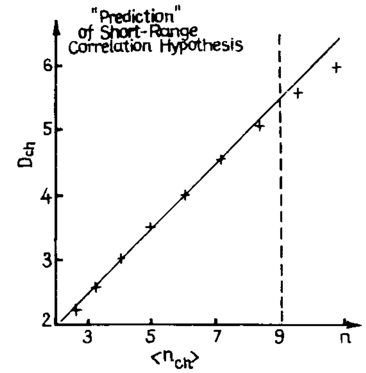

Fig.III.4

When considering the SR-correlations it is convenient to use the concept of clusters. It appeared in the gas theory.
If there is SR-interaction between the particles, then it is natural to consider grouping of particles into clusters, i.e., into the particle associations which are so close to each other that they may interact. Explain this by the picture

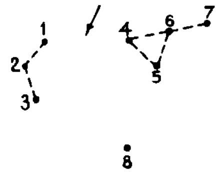

Fig.III.5

Here, the particles 1,2,3 make one cluster, 4,5,6,7 make another cluster, and the particle 8 alone makes a cluster. If the particle 8 is moved into the position indicated by the arrow, it will interact with the particles 1 and 4 and we shall receive one large cluster. As a matter of fact we have introduced the SR-interactions between clusters. Thus, if the clusters interact in the SR-way, they merge into one large cluster. Clusters in the above determined sense may be introduced at the SR-interactions. For elementary particles there are SR-interactions in the rapidity space thus consideration of cluesters is justified.

To realize in terms of associations or clusters, concrete dependencies in multi-particle production it is necessary to make some additional assumptions on the character of their production and on their quantum numbers. Concrete of the notion of hadron associations has been considered above in the multi-component model of two mechanisms.

§3. Charged-neutral correlations and the model of two mechanisms

Consider the charged-neutral correlations between secondaries making use of the model of two mechanisms.

As is seen from experimental data (see fig. III.4).

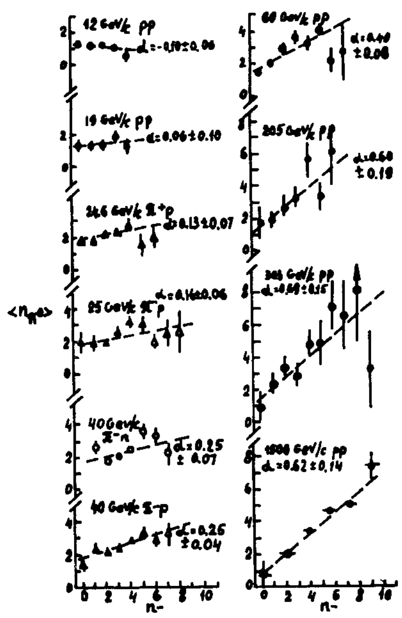

Fig. III.6

The presence of linear dependence of an average number of neutral pions on number of charged traojs is only indicated at $\sim$ 25 GeV, and becomes obvious at $\sim$ 400 GeV. The experiments performed on ISR at CERN and, also in NAL at Batavia verified this regularity by the example of pp collisions at energies 1500 and 200 GeV. Note here that at lower energies the effect of any correlation between neutral and charged pions is considerably weaker.

Now we turn to the model of two mechanisms at first For the simplicity , we do not take into account strange partiole. As has already been mentioned, the initial points of the model:

i) Dissociation of the leading particles with local conservation of isospin and

ii) Independent production of associations (see fig. III.7) leads in the given case, to the distribution

$$W^{i}_{n_\pm, n_0, n_3} = \alpha_i P_{n_\pm}(a_\pm) P_{n_0}(a_0) P_{n_3}(l) \qquad (3.14)$$

$$a_\pm = 2a_0 \equiv a$$

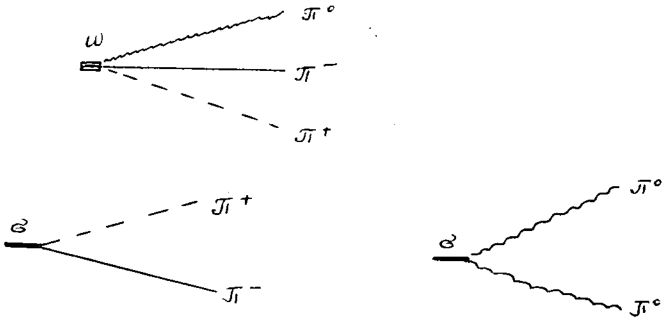

Fig. III.7

Taking into account that a number of neutral pions may be presented in the following way (see formula 2.6 and table II 1).

$$n^{i}_{\pi^0} = 2n_0 + n_3 + l^{i}_{\pi_0}$$

one can easily obtain the average number of neutral pions.

$$\langle n_{\pi^0}\rangle_{n_c} = \frac{2\langle n_0\rangle_{n_c} + \langle n_3\rangle_{n_c} + \langle l_{\pi^0}\rangle_{n_c}}{W_{n_c}} . \qquad (3.15)$$

Formulae (3.15) and (3.14) lead to a linear correlation between an average number of neutral particles and a number of charged particles.

$$\langle n_{\pi^0}\rangle_{n_c} = K_1 + K_2(n_c - \bar{n}_c) \qquad (3.16)$$

where

$$K_1 = a + l + \lambda_2 ,$$

$$K_2 = \frac{l}{2(a+l)} .$$

and an average number of charged particles

$$\bar{n}_c = \begin{cases} 2(a+b)+2 & \text{(for } \pi^- p \text{ collisions)} \\ 2(a+b+d_2)+1 & \text{(for } \pi^- n \text{ collisions)} \end{cases}$$

The case of $\pi^- N$ interactions is specially given here, in order to illustrate quantitative comparison of the model with experimental data, obtained at Serpukhov with a two-meter propane chamber irradiated with 40 GeV π^- mesons. The results are given in fig. III.8

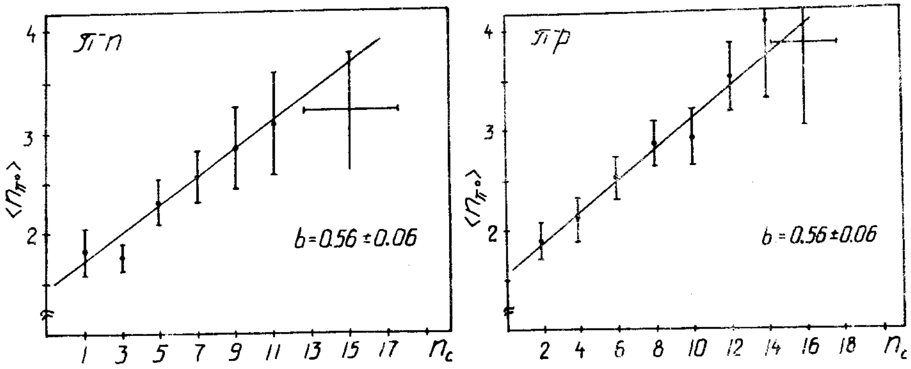

Fig.III.8

Good agreement with experiment ($\chi^2 \sim$ 0.5 on one degree of freedom) confirms the prediction of the model about the

linear form of correlation

$$\langle n_0\rangle_{n_c} = A + B n_c . \qquad (3.17)$$

One of the conclusions of the model is that the slope B does not depend on the type of colliding particles ($B_{\pi^- p} = 0.16 \pm 0.02$, $B_{\pi^- n} = 0.15 \pm 0.02$).

It is seen from (3.16) that the slope is expressed in terms of the parameters of independently produced clusters. If one assumes that probability of production of multi-particle clusters increases with increasing energy, then as a result one obtains the increase of the slope with energy.

Indeed, the slope (see formula (3.16)) is connected with the relation of average numbers of the hadron associations (clusters).

$$B = \frac{1}{2}\,\frac{\bar{N}(\omega \to \pi^+\pi^-\pi^0)}{\bar{N}(\rho \to \pi^+\pi^-) + \bar{N}(\omega \to \pi^+\pi^-\pi^0)} \qquad (3.18)$$

As extreme cases (in the given assumption) from (3.18) it follows:

1) at $\bar{N}(\omega) \gg \bar{N}(\rho)$, $s \gg s_{\text{threshold}}$

$B \to \frac{1}{2}$;

2) at $\bar{N}(\omega) \ll \bar{N}(\rho)$, $s \ll s_{\text{threshold}}$

$B \to 0$

These conclusions are consistent with experimental data. The experimental data at ISR in the pp collisions ($E \sim$ 2000 GeV) also demonstrate a dependence of the type (317) with the slope $\beta \sim$ 1/2. An absence of such a correlation at low energies means that $\beta \sim 0$. Making use of the multi-component distribution (26) the scheme presented above can easily be extended to the case of multi-particle production involving strange particles.

The model of two mechanisms in this case gives a distribution over the number of charged particles in the form of the superposition of Poisson functions, and predicts correlations between multiplicities K^+ and K^- as well as between K^o and $\bar{K}^o$ mesons. The average number of K^o, Λ^o, and Σ^o in the region under consideration (when the production of clusters off three heavy strange particles is hardly probable) is independent of the number of charged particles in πp collisions and reaches its constant value at sufficiently large number of charged particles in $\pi^- n$ collisions. The processing of results from the two-meter propane chamber has produced good agreement of the model with experiment. (See Fig. III 9, III 10)

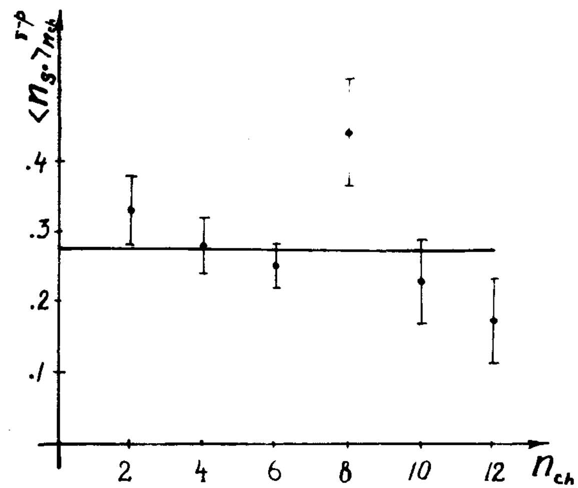

Fig. III.9

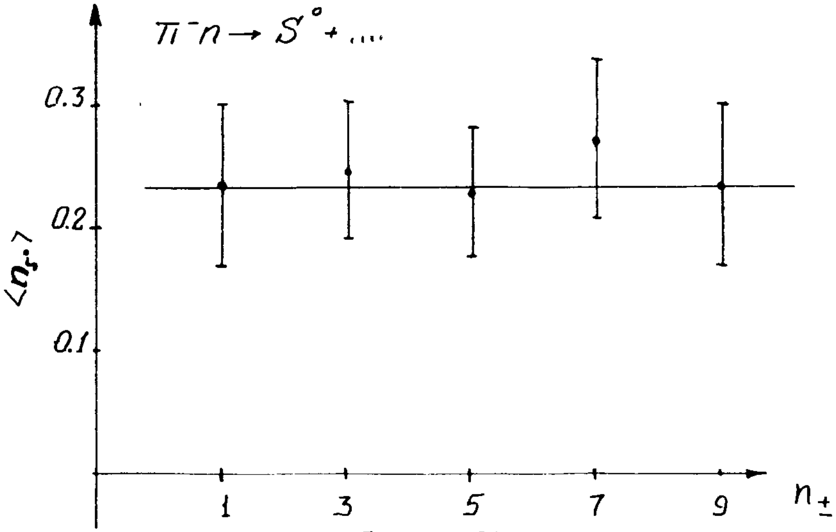

Fig. III.10

The model of two mechanisms, realizing the idea of multi-component description, and its comparison with the results of experiments for the $\pi^- n$ and $\pi^- p$ interactions show that at high energies many characteristics of multi-particle processes for various collisions have a tendency to be similar. Such a tendency is observed experimentally. In the spirit of TMP-model this looks quite natural: dissociation gives relatively small contribution into multiplicity, at increasing energies multiple characteristics are determined by increasing number of clusters (with the tendency to increase weight) which are produced independently of eaoh other and of the leading particles.

Indeed everything may be more complicated. Most probable little distinctions in the characteristics of different types of interactions

$$(pp, \bar{p}p, \pi p, Kp, \cdots)$$

will provide a closer desoription of multi-particle production. However, one may hope that a rough scheme being observed at modern energies, as well as its simple and obvious realizations will be a convenient frame for future theories.

Note for conclusion that now there are many new approaches to the oorrelation problem.

In the same oonneotion I should like to mention that it was proposed at the same school in 1970 by El.Mikhul to use a new variable which is available exclusively for multi-particle production. It is the determinant of the matrix formed with the components of four 4-momenta. Consider the process

$$a + b \longrightarrow a_1 + a_2 + a_3 + a_4$$

and

$$\Delta = \det \{ P_i^k \}_{i=1,2,3,4}, \quad k = 0,1,2,3$$

as only one variable built with the particles of final state. In the center of mass system of colliding particles

$$\Delta = \sqrt{s}\,(\vec{P}_1 \times \vec{P}_2)\cdot\vec{P}_3 = \sqrt{s}\,(\vec{P}_1 \times \vec{P}_3)\cdot\vec{P}_4 = \sqrt{s}\,(\vec{P}_2 \times \vec{P}_3)\cdot\vec{P}_4$$

where $\sqrt{s}$ is c.m.s. energy. For a fixed s the variable we consider is the measure of the volume of the parallelepiped on any three 3-vectors of four.

The experimental distributions on Δ for $p+p \to p+p+\pi^+\pi^-$ have been performed for ten values of energy between 4.0 GeV and w4.8 GeV. A strong shrinkage (Fig.III.11) with respect to the energy is obtained when they are compared with the phase space distributions. Δ equals to zero define the singular domain of the physical region. So for increasing energy this region becomes prevalent.

In connection with this approach it is interesting to find from experimental data the answer to the following questions:

a. Do Δ distributions as functions of energy ,i.e., depend on the nature of colliding particle or final particles (neutrino-production, foto-production,etc.)?

b. For four inclusive reactions (four prong events) one can divide the interval of the energy of the four particles in their center of mass system $(P_1+P_2+P_3+P_4)^2$ into intervals of the "fixed" energy. For the events corresponding to every certain energy to get the Δ histogram. Will be the shrinkage the same with respect to energy.

c. Important is to know from the reactions with more then four particles in final state for which a few independent determinants exist if they are simultaneously going to zero for a given event. (There are $n(n-1)(n-2)(n-3)/24$ determinants but not all independent 3 since

$$\Delta_{i_1 i_2 i_3 i_4}\Delta_{k_1 k_2 k_3 k_4} = \det\{P_i P_k\}_{i=i_1,i_2,i_3,i_4,\ k=k_1,k_2,k_3,k_4}$$

d. Finally Δ being a pseudoscalar it is interesting to investigate if there exists any asymmetry in the Δ distribution with respeot to $\Delta=0$.
It is possible to perform it for the channels where the four particles in final state are different, thus, they can be uniquely labelled, and the ordering of them permits to introduce the orientation of the spaoe.

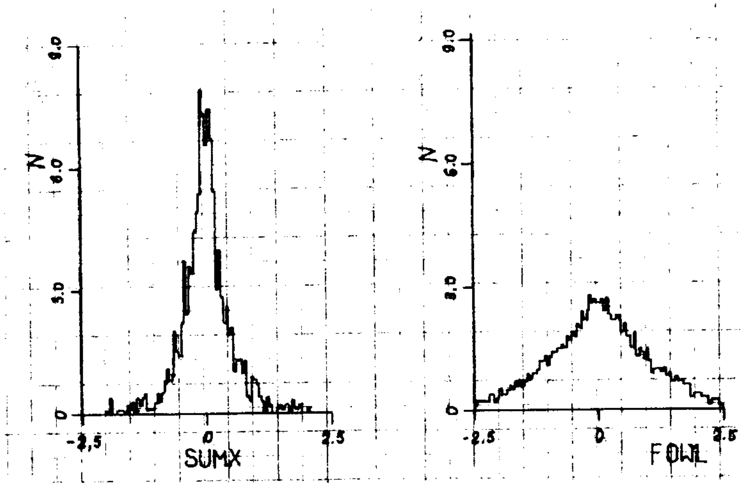

Fig. III.11. Comparison of experimental Δ distribution for $pp\to pp\pi^+\pi^-$ at 10 GeV/c with events generated by FOWL.

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Problems for Discussion to Lec.II and III

1. Find $\langle n \rangle, \langle n^2 \rangle, D, f_2, \ldots, \langle n^k \rangle$ in the oase of two-component model, when

$$\sigma_n = \alpha \sigma_n^{(1)} + \beta \sigma_n^{(2)}$$

2. Show, that in the assumption on independent production of particle pairs, the distribution over multiplicity has the form of Wang I - distribution.

3. Obtain (2.4) from (2.2) and (2.3) .

4. Write out the correlation parameters f_3 and f_4 .

Lecture IV

Inclusive and semi-inclusive processes

§ 1. The problems of desoription of multi-particle processes

The analysis of the processes of multi-particle production is important for inderstanding the nature of hadron interactions at high energies. It has considerable difficulties both from a technical point of view and from a point of view of kinematical description.

It is necessary to find such summation oharacteristics of inelastic processes which give a sufficiently complete information on the hadron interactions at high energies and at the same time are rather simple both for theoretical and for experimental analysis.

Characteristics of such a type were, first introduced in 1967 (Logunov, Mestvirishvili, Nguyen Van Hieu) . Later a set of processes, giving the oontribution into these characteristics was called "the inclusive processes". Thus, the first stage of experiments is mainly concentrated on the measurements of the most direct quantities: the inclusive and topological characteristics of the particle production spectra.

It was first experimentally determined at the Serpukhov-aocelerator (Yu.Bushmin et al.) that the ratio of the production probabilities of K-mesons and anti-protons to the production probabilities of π - mesons depends only on the ratio of

momenta. $\frac{p}{p_{max}}$ The experimental consideration of the scaling invariance at high energies, as well as the difficulties of microscopic description of multi-particle processes (first of all, absence of strict mathematical apparatus) lead to the appearance of phenomenological approaches and models (the parton mode, "droplet" model) and on their basis, to the appearance of hypothesis of limiting fragmentation and scaling (Feynman, Yang).

The later determine a number of limiting relations and restriotions to the cross seotions of inclusive processes, correlations and other characteristics. The principle of automodelity (Matveev, Muradyan, Tavkhelidze) on the basis of a generalized dimensional analysis makes it possible to classify the scaling relations at high energies.

In the present lectures we shall not dwell on the problem of strong interaotions, but only remind the listeners about the review by M.Jacob, A.Logunov and Mestvirishvili, Muradyan . (See References).

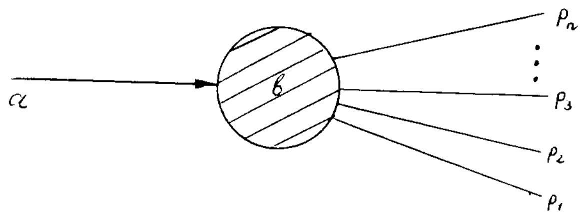
§ 2. Semi-inclusive prooesses and their charaoteristios

The one-partiole inclusive reactions have a number of practical advantages: they are easily obtained experimentally, the study of average values by the particles non-fixed in the reaction clears up the collective properties of the system of secondaries. On the other hand they represent a limited part of dynamics as the one-particle characteristics have been integrated over particles and summed over all the inclusive channels. In fact, in the inclusive approach are mised various mechanisms of particle production, responsible for different phenomena. There arises a question to explain the dependence of these effects on multiplicity. To solve such problems we can make use of the so-called semi-inclusive processes of multi-particle production, i.e., of the characteristics of reactions with the fixed multiplicity (topology) without averaging of the inclusive approach and, thus, evidently giving the contributions of different multiplicities to physical effects.

Basic definitions.

Consider the process of particle production as a result of collision at high energies

$$a + b \to p_1 + p_2 + \dots + p_n + \dots$$

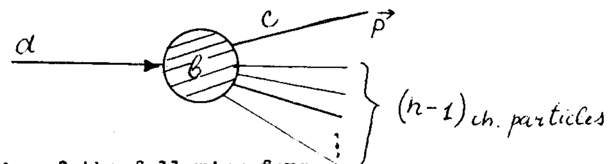


Denote the differential cross section of production of n-charged particles (with a different number of neutral ones) through

$$\frac{d\sigma}{d\vec p_1 d\vec p_2 \dots d\vec p_n} = \sum_{k=n+1} \frac{1}{(k-n)!} \int \frac{d\sigma}{d\vec p_1 d\vec p_2 \dots d\vec p_n \dots d\vec p_k} \prod_{j=n+1}^{k} d\vec p_j \quad (4.1)$$

Then the semi-inclusive cross section of particle $c(\vec p)$ production with a given $(n-1)$ number of charged particles

$$a + b \to c(\vec p) + \underbrace{\dots}_{(n-1)_{ch.\ particles}} + \dots$$



will be of the following form

$$\frac{d\sigma_n^c}{d\vec p} = \frac{1}{(n-1)!} \int \frac{d\sigma}{d\vec p \dots d\vec p_n} \prod_{i=2}^{n} d\vec p_i \quad (4.2)$$

with the normalizations (see. Lec.I):

$$\frac{1}{n} \int \frac{d\sigma_n^c}{d\vec p} d\vec p = \sigma_n^c, \quad \sum_{n=2} \frac{d\sigma_n^c}{d\vec p} = \frac{d\sigma^c}{d\vec p}, \quad \int \frac{d\sigma^c}{d\vec p} d\vec p = \langle n \rangle \sigma; \quad (4.3)$$

$$\langle n \rangle \sigma \equiv \sum n \sigma_n,$$

where σ_n , σ is the partial (topological) and total (inelastic) cross sections of interaction $(ab \to \dots)$ correspondingly;

$\langle n \rangle$ – is the average multiplicity of final particles.

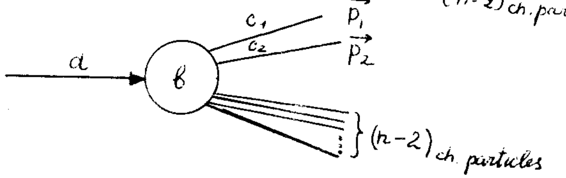
Define now the first moment of the semi-inclusive distribution (4.2)

$$\langle n(\vec{p})\rangle = \frac{\sum_{n=2}^{N(s)} (n-1)\, d\sigma_n^c/d\vec{p}}{\sum_n d\sigma_n^c/d\vec{p}} \; ; \tag{4.4}$$

eq.(4.4) defines the average multiplicity of charged particles, produced together (in association) with an extracted fixed particle „C" from momentum $\vec{p}$ and is called the associative multiplicity of charged particles.

Pay attention to the correlation character of the introduced value (4.4). In this connection consider the reaction with two (inclusively) extracted particles

$$a + b \to c_1(\vec{p}_1) + c_2(\vec{p}_2) + \underbrace{\;\cdots\;}_{(n-2)\,\text{ch. particles}} + \ldots$$



and determine the corresponding two-particle spectra: the semi-inclusive distributions with fixed multiplicity

$$\frac{d\sigma_n^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2} = \frac{1}{(n-2)!} \int \prod_{i=3}^{n} d\vec{p}_i \, \frac{d\sigma}{d\vec{p}_1 \ldots d\vec{p}_n} \tag{4.5}$$

and the corresponding two-particle inclusive spectrum

$$\frac{d\sigma^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2} = \sum_{n=3} \frac{d\sigma_n^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2} \tag{4.6}$$

with the normalizations:

$$\begin{aligned} &\frac{1}{n(n-1)} \int \frac{d\sigma_n^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2}\, d\vec{p}_1\, d\vec{p}_2 = \sigma_n , \\ &\frac{1}{n-1} \int \frac{d\sigma_n^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2}\, d\vec{p}_2 = \frac{d\sigma_n^{c_1}}{d\vec{p}_1} \\ &\int \frac{d\sigma^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2}\, d\vec{p}_1\, d\vec{p}_2 = \sum n(n-1)\sigma_n = \langle n(n-1)\rangle \sigma . \end{aligned} \tag{4.7}$$

Having partially integrated (4.5) over the phase volume of the particle $c_2(\vec{p}_2)$ taking into account (4.7) and (I)

$$\sum_n \int \frac{d\sigma_n^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2}\, d\vec{p}_2 = \sum \frac{d\sigma_n^{c_1}}{d\vec{p}_1}(n-1) =$$

$$= \int \frac{d\sigma^{c_1 c_2}}{d\vec{p}_1\, d\vec{p}_2}\, d\vec{p}_2 = \langle n(p_1)\rangle \frac{d\sigma^{c_1}}{d\vec{p}_1} ,$$

and using the definition of the two-particle correlation function $C_2(\vec{p}_1, \vec{p}_2)$ (see I), we receive necessary relation

$$\langle n(\vec{p}_1)\rangle = \frac{1}{\frac{1}{\sigma}\cdot\frac{d\sigma}{d\vec{p}_1}} \int C_2(\vec{p}_1, \vec{p}_2) + \langle n\rangle , \tag{4.8}$$

i.e., in absence of correlations between the particles c_1 and c_2 the associative average multiplicity does not depend on the momentum $\vec{p}_1$ and

$$\langle n(p_1)\rangle = \langle n\rangle - 1 .$$

Reduce as well the formula determining the semi-inclusive two-particle correlations. Defining

$$\rho_n(\vec{p}) = \frac{1\cdot d\sigma_n}{\sigma_n \cdot d\vec{p}} \tag{4.9}$$

by (4.9) put by analogy with I:

$$C_n^{(2)}(\vec{p}_1,\vec{p}_2) = \frac{1}{\sigma_n}\,\frac{d^2\sigma_n}{d\vec{p}_1\, d\vec{p}_2} - \rho_n(\vec{p}_1)\rho_n(\vec{p}_2), \tag{4.10}$$

$$R_n^{(2)}(\vec{p}_1,\vec{p}_2) = \frac{C_n^{(2)}(\vec{p}_1,\vec{p}_2)}{\rho_n(\vec{p}_1)\,\rho_n(\vec{p}_2)} =$$

$$= \frac{\sigma_n\cdot \dfrac{d^2\sigma_n}{d\vec{p}_1\, d\vec{p}_2}}{\dfrac{d\sigma_n}{d\vec{p}_1}\cdot\dfrac{d\sigma_n}{d p_2}} - 1 .$$

§3. The experimental situation

Now we shall make use of the experimental data on semi--inclusive distributions and give a brief classification of the basic facts.

The one-particle spectra with fixed multiplicity

1. The linear growth of semi-inclusive one-particle densities $\rho_n(\vec{p})$ for the fixed y, $(p_\perp)$

$$\rho_n(y) = \left.\frac{1.d\sigma_n/dy}{\sigma_n}\right|_{y\text{-central}} = A + Bn: \quad \begin{matrix}(FNAL, IS.R, IHEP)\\ pp\to\pi + X_N\\ \pi p \to \pi + X_N\end{matrix}$$

see, e.g., Fig. IV.1

$$\rho_n(p_\perp) = \frac{1}{\sigma_n}\,\frac{d\sigma_n}{d p_\perp} = a + bn:$$

(IHEP)

$\pi p \to \pi + X_N$

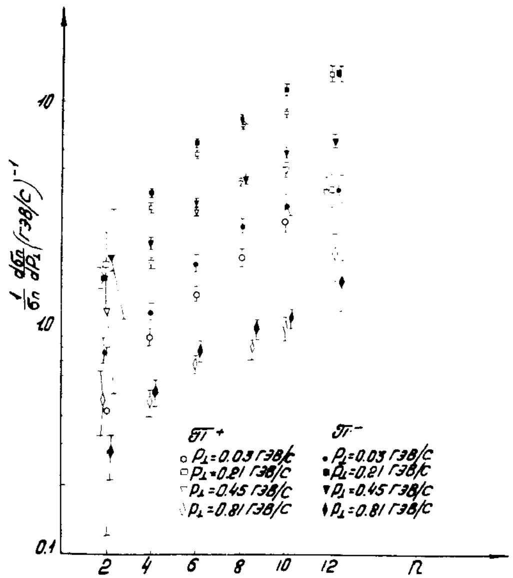


Fig. IV.1.

$\left(\frac{1}{\sigma_n}\,\frac{d\sigma_n}{dp_\perp}\right)_{max}$ - distribution over n (for π^+ and π^-) in $\pi^- p$ -coll. at P_L = 40 GeV/c (2-meter propane chamber, JINR, IHEP - accelerator).

2. The shrinkage of semi-inclusive spectra with increasing multiplicity

$$\rho_n(y) = \frac{1}{\sigma_n}\,\frac{d\sigma_n}{dy} = \frac{n}{S_n\sqrt{2\pi}}\,e^{-y^2/2S_n^2} \quad ; \quad \begin{matrix}(FNAL,\ ISR)\\ pp \to \pi^- + X_N\end{matrix}$$

where $S_n \sim 2/n$

$$\begin{cases} \dfrac{dN}{dp_\parallel} = B_\parallel\, e^{-a p_\parallel}, & a = (0.39n - 0.23)\ \text{Gev/c} \\[2mm] \dfrac{dN}{dp_\perp} = B_T\, p_\perp^{3/2} e^{-b p_\perp}, & b = (0.31n + 5.36)\ \text{Gev/c} \end{cases} \quad : \quad \begin{matrix}(BNL)\\ \pi^- p \to \pi + X_N\end{matrix}$$

$$\begin{cases} \dfrac{dN}{dp_\parallel} = N_\parallel\, a_\parallel\, e^{-a_\parallel p_\parallel}, & a_\parallel \simeq \dfrac{10 + 5n}{W_{cm}} \\[2mm] \dfrac{dN}{dp_\perp} = N_\perp\, a_\perp^{5/2} p_\perp^{3/2} e^{-a_\perp p_\perp}, & a_\perp = (b + 0.3n) + \dfrac{6(1+n)}{W_{cm}} \end{cases} \quad : \quad \begin{matrix}(BNL)\\ pp \to \pi + X_N\end{matrix}$$

$$\rho_n(y, p_\perp) = N\, e^{-a n m_\perp \operatorname{ch}(y-y')} \quad ; \quad \begin{matrix}(IHEP)\\ \pi^- p \to \pi^\pm + X_N\end{matrix}$$

The properties of the experimental data 1 and 2 are clearly seen, for example, in the experiment Pisa-Stony Brook coll. at ISR (Fig. IV.2).

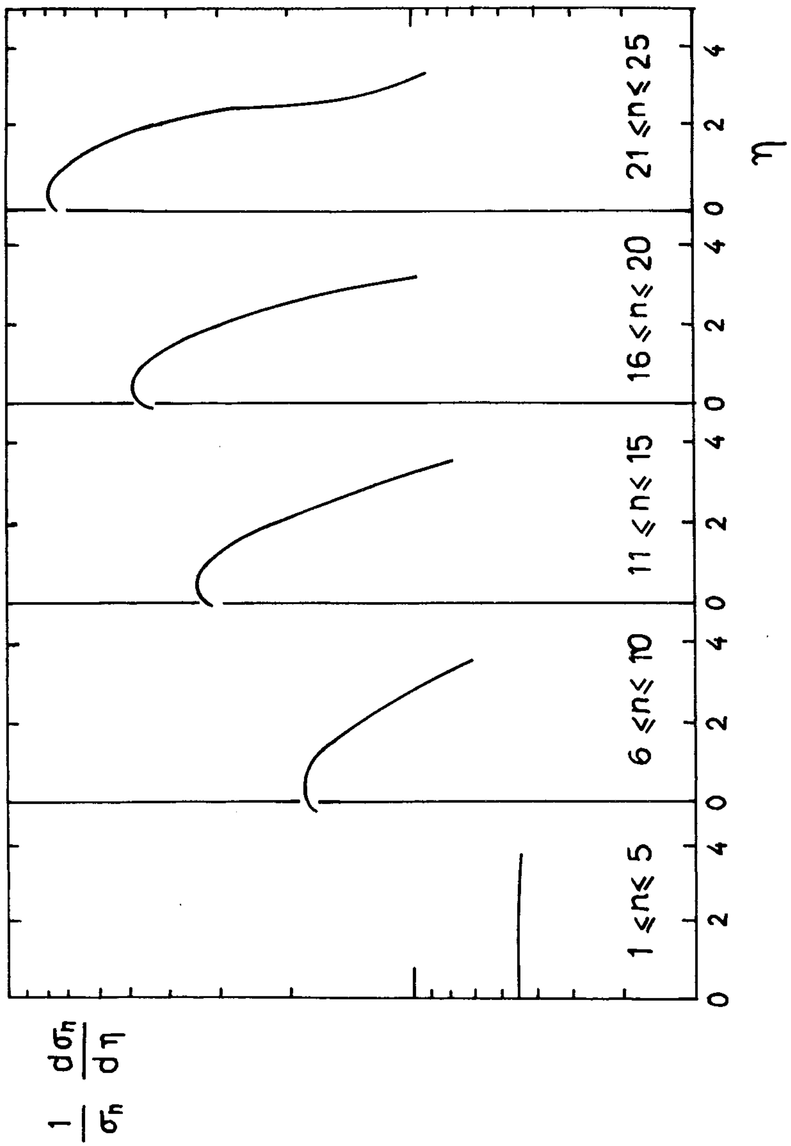
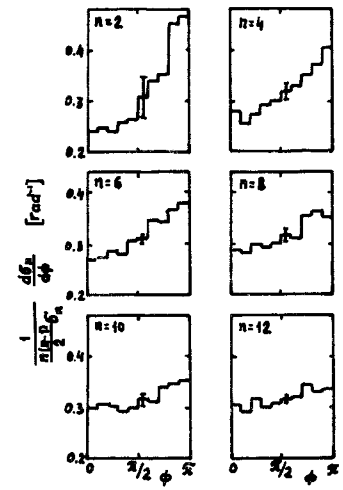


Fig.IV.2. The semi-inclusive spectrum over the rapidities of charged particles for different multiplicities.

3. The asymmetry in azimuthal semi-inclusive distributions, which is not explained by energy-momentum conservation low and has the character of LR-correlations

Fig.IV.3

Azimuthal semi-inclusive distributions for different combinations of charged particles



The semi-inclusive correlations

1. The central region $y_1 \sim y_2 \sim 0$

FNAL, ISR

$R_n^{(2)}(0,0) \sim \frac{1}{n}$

$pp \to \pi\pi$

$C_n^{(2)}(0,0) \sim n$

The correlations have a typical short range character

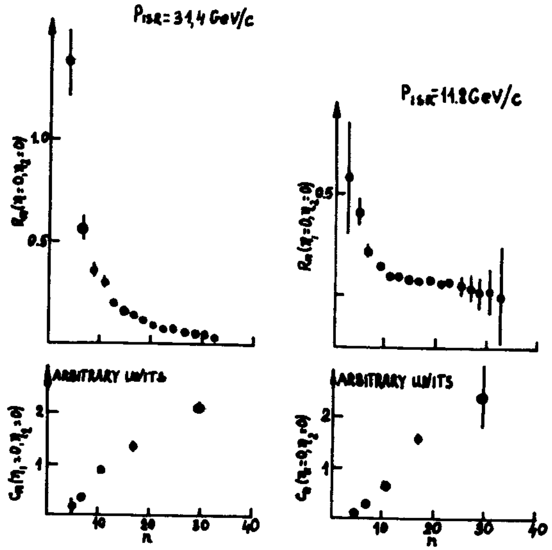


Fig.IV.4

2.In the range of large $\Delta y = y_1 - y_2$

The correlations are maximum for small n.

This effect increases with energy (long range) and $C_n^2(y_1 - y_2) \sim n$, $R_n^2(y_1 - y_2) \sim 1/n$.

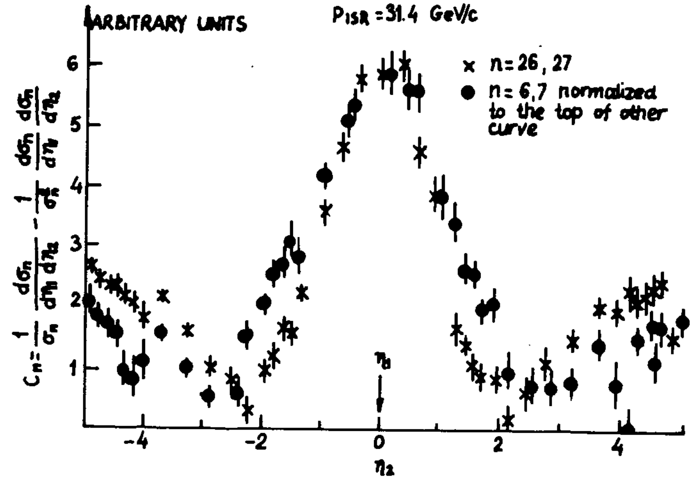


Fig.IV.5

3. The associative multiplicities. The correlations between n and $(y, p_\perp)$

The associative multiplicity as a function of various variables has been calculated in many experiments up to the ISR energies.

Consider the typical data.

a) There has not been found an essential dependence of $\langle n(p_\perp)\rangle$ on the transverse momentum of secondaries (p, π, K, n) for $p_\perp \lesssim 1$ GeV/c when $p_L = 19$ GeV/c in the pp - interaction (The Scandinavian group) as well as in the p interaction at p_L =40 GeV/c when though the same data in the same-opposite selection show an essential dependence on $p_\perp$. The value $\langle n(x)\rangle (\langle n(y)\rangle)$ is gradually decreasing with the growth of transverse momenta $x(y)$.

b) The BNL-collaboration points out the increasing dependence of $\langle n(p_\perp)\rangle$ on the transverse momentum of a leading proton in the range $p_{\perp(p)} \leq 2$ GeV/c and for different missing masses MM x). The data from FNAL-ISR confirm this effect in a wide energy range and $p_\perp$ (a detailed discussion of the range of large $p_\perp$ see in V).

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x) Note that experimental observation of the linear relation of the average multiplicity with the transverse momentum of one of the final protons was first presented in the paper by Anderson, Collins, 1967.

c. The associative multiplicity as a function of missing masses, produced with a leading particle, increases according to the same low as the average multiplicity as a function of $\sqrt{s}$.

§4. Theoretical approaches. Cluster models

Various experimental informations on correlations, e.g., data on $f_2(s)$, $R^{(2)}(y_1 - y_2)$, an approximate validity of the KNO-scaling for mutli-particle distributions, etc., point out the fact that the multi-particle production (in any case, most of it) proceeds through multi- cluster intermediate states.

In particular, the assumption of the independent emission of isotropic clusters makes it possible to understand a positive short-range character of the completely inclusive two-particle correlation functions with respect to rapidities in the central region.

The central idea of this approach is that the hadron associations (clusters) are produced according to a definite dynamics and that the secondaries observed are products of decay of these clusters.

Nowadays it is not yet clear whether clusters have intrinsic dynamical meaning or represent simply a phenomenological method, i.e., suitable initiation of more complicated dynamics.

The cluster models have been extensively studied recently (see review articles of Berger, J.Ranft) in connection with the experimental information of FNAL-ISR on correlations with respect to rapidieis at multiplicity fixed (on semi-inclusive correlations).

We list here some model consequences:

1. It is convenient to split σ_n and $d\sigma_n/dy$ into "diffractive" and "nondiffractive" parts;

2. The correlation length 2δ $\left(C_n^{(2)} \sim e^{-(y_1-y_2)^2/4\delta^2} \right)$ does not depend on n and s.

3. In the model of independent clusters: $\frac{1}{\sigma_n}\frac{d\sigma_n}{dy} \approx \frac{n}{Y}$.

4. The behaviour of semi-inclusive correlations is consistent with experimental data of FNAL–ISR:

$$C_n^{(2)}(0,0) \sim \frac{n}{\log s} f(\langle k \rangle),$$

$$R_n^{(2)}(0,0) \sim \frac{\log s}{n} f(\langle k \rangle),$$

where $\langle k \rangle$ stands for the average number of hadrons in a cluster.

5. The n-dependence of the semi-inclusive correlation functions reflects the structure of multi-particle distribution inside a cluster.

In the concrete cluster model with diffractive excitation the one-particle distribution at a fixed multiplicity reduces to the following form:

$$\frac{d\sigma_n}{dy} = A_n \int^{N(s)} dM \, \rho(M)\, \delta(M - n/\gamma) \frac{e^{-\lambda^2 m^2 sh^2(y-\varphi)}}{ch^2(y-\varphi)}, \tag{4.11}$$

where M and φ are resp. the cluster mass and rapidity.

In present variants of the cluster diffractive models, agreement of the slow decrease of topological cross sections σ_n and nondecreasing character of spectra relative to rapidities in the central region is achieved if one gives up the assumption of the isotropy of the cluster decay.

Note that if one takes as the cluster decay amplitude a modified distribution of the Bose gas it is possible to avoid an artificial introduction of non-isotropy. In this case, in particular, observable properties of "shrinkage" of distributions are obtained and, unlike the standard models (DEM) resulting in a fall of the spectra in the central region, there is obtained the increase of maximal values of distributions

$$\rho_n(y) \sim e^{-an\, sh^2 y/2}, \quad \rho_n(y=0) \sim c\sqrt{n}\,(1+nm) \tag{4.12}$$

$$\rho_n(p_\perp) \sim \frac{a + b\, p_\perp}{m_\perp^{1/2}} e^{-nb(m_\perp - m)}, \quad \rho_n(p_\perp = max) \sim cn$$

Scaling in Semi-Inclusive Distributions

a. Uncorrelated Production. KNO II.

Keeping to the same ideas which have resulted in the similarity law for multi-particle distributions (See Lec. II) Koba, Nielsen and Olesen have obtained the law of automodel behaviour for semi-inclusive cross sections $\rho_n(\vec{p})$. Assuming the noncorrelated particle production (or weak short--range correlations) and the Feynman scaling for the one--particle spectral densities at a fixed multiplicity they have found the asymptotical formula:

$$\frac{1}{\sigma_n}\,\frac{d\sigma_n(\vec{p},s)}{d\vec{p}} \xrightarrow[s\to\infty]{} h\left(\frac{n}{\langle n\rangle_s}, x, p_\perp\right)\left[1+O\left(\frac{1}{\langle n\rangle}\right)\right] \qquad (4.13)$$

for the reaction $a + b \to c(\vec{p}) + (n-1)$ charged + anything neutral; where $x = 2p_{\parallel}/\sqrt{s}$, $\langle n\rangle_s$ is the mean multiplicity at energy s. The relation (4.13) predicts that if one compares two or more semi-inclusive experiments at different (high enough) energies taking the same values of $n/\langle n\rangle_s$, then the distributions over momenta (in the variables x and $p_\perp$) normalized to σ_n will be almost equal to each other, i.e., the cross sections $\frac{d\sigma}{dx\,dp_\perp}$ for different s and

different topologies but with the same ratio $n/\langle n\rangle$ should be the same. For instance, for the experimental distributions shown in Fig. IV.2 the relation (4.13) means that, if one performs a selection over $n/\langle n\rangle$ and rejects $\frac{1}{\sigma_n}\frac{d\sigma_n}{dy}$ with respect to y, the curves corresponding to the same $\frac{n}{\langle n\rangle}$ will coincide approximately for different energies.

Due to non rigorous character of the arguments resulting in (4.13) it is interesting to check, through models, this relation. This has been made in the two cases:

1. The Feynman gas model (Olesen)
2. The uncorrelated beam model.

In the first case, by using the Mueller method of the generating functionals the proper relations (4.13) are found for semi-inclusive cross sections and it is shown, on the example of the reaction $K^+ + p \to K^0 + n_{ch} +$ (anything neutral) at $p_L = (5, 8.2, 16)$ GeV/c. That the spectra are in qualitative agreement with the Feynman gas model within a good accuracy (except for boundary regions of phase space where effects of the energy-momentum conservation laws are important).

As n is the discrete variable a convenient way to check the prediction (4.13) is to obtain an analytical expression which then can be fitted to experimental data. Such an expression :

$$\frac{1}{\sigma_n}\frac{d\sigma_n}{d\vec{p}} = C\,\frac{n}{\langle n\rangle}\,(1-x)^{\lambda_1\frac{n}{\langle n\rangle}+\lambda_2}\left[1+O\!\left(\frac{1}{\langle n\rangle}\right)\right] \quad (4.14)$$

has been found in the model of noncorrelated beams. Here λ_1, λ_2 are constants. Formula (4.14) has been fitted to experimental data at p = 19 GeV/c in the reaction $pp \to \pi^+ + (n_{\pi^-}-1)_c$ + anything neutral.

Applicability of the semi-inclusive scaling (4.13) (KNO II) to the one particle spectra has been verified experimentally for the corresponding cross sections with pion production in pp-collisions at 2o5 GeV/c (FNAL). A comparison has been made at fixed $n/\langle n\rangle$ with data at low energies from 13 to 28.4 GeV3c. By relation (4.13), it follows that if one takes two energies s_1 and s_2 and two multiplicities n_1 and n_2 then $\frac{1}{\sigma_n}\frac{d\sigma_n}{d\vec{p}}$ quantities should be equal if $\frac{n_1}{\langle n_1(s_1)\rangle} = \frac{n_2}{\langle n_2(s_2)\rangle}$ (up to corrections $O\left(\frac{1}{\langle n\rangle}\right)$). Though a qualitative agreement holds for such a behaviour (except for the region $x \simeq 0$) essential deviations are observed in data on the semi-inclusive scaling. These are considerably larger than for corresponding inclusive cross sections. Analogous results have been obtained for semi-inclusive distributions of pions in $\pi^- p$-collisions at p = 40 GeV/c (IHEP - collaboration).

Since $\langle n(s)\rangle$ is a slowly varying function of energy and $n/\langle n\rangle$ is roughly constant for a constant multiplicity n from (4.13) the usual unclusive scaling should be valid in a wide energy interval. This prediction was compared with experiment (Shliapnikov et al.). In the K^+p-interaction at p = 5.82, 16 GeV/c the quantity $\frac{1}{\sigma_n}\frac{d\sigma_n}{d\vec{p}}$ has been found to be independent of energy, for n = 2,4,6 though the corresponding change considerably in this energy range.

b) Strongly Correlated Production

Experimental data on $C_n^{(2)}$, $R_n^{(2)}$ and essential dependence of the associative momenta $\langle n(\vec{p})\rangle$ on $\vec{p}$ point out noticeable correlations in processes of the multiparticle production. Studies of the correlation dependences of average characteristics of hadron production processes can give evidences only to existence of certain relation between secondaries. In studying the semi-inelastic characteristics there arises the question: what restrictions on the shape and character of dependence of the one-particle distributions on n and $\vec{p}$ do correlations between the average multiplicity and magnitude of the momentum or transfer momentum result?

Consider a semi-inclusive reaction of the type

$a + b \longrightarrow$ particle of large $p_\perp$ $+ n_{ch} +$
+ anything neutral,
where one of secondaries which receives after interaction a large transverse momentum is produced inclusively.

When choosing a special form of the dependence of the average number on the transverse momentum one should allow for considerations on a mechanism of multi-particle production.

Proceeding from the assumption on the coherent excitation of particles colliding at high energies (Matveev, Tavkhelidze) one may find that the average number of secondaries grows linearly with the squared transverse momentum transferred:

$$\langle n(p_\perp) \rangle = \alpha + \beta p_\perp^2 .$$

This result for the diffractive production of particles has been obtained in the framework of the straight-line path method. Such a behaviour is in a qualitative agreement with experimental data obtained in pp-collisions at the lab. momentum of incident proton $p_{LAB} \simeq 30$ GeV/c. An analogous phenomenon follows also from the fragmentation principle (Yang) where the growth of $\langle n \rangle$ with $p_\perp$ arises due to that it is impossible to give a large transverse momentum to a proton without its disintegration. Note that in the multiperipheral model the average multiplicity decreases

logarithmically with increasing $p_\perp$ x). As we think, this decrease follows from that the multiperipheral model corresponds mainly to the secondary production mechanism due to appearance of the hadron clusters in the central region, while the results of the coherent state model, of the straight-line path method and fragmentation principle correspond to the mechanisms of diffractive dissociation of colliding particles.

Besides, keeping ideas on the physical similarity, seen in a number of observed properties of particle interactions at high energies, we may assume that the shape of dependence $\langle n(\vec{p}_\perp) \rangle = f(\vec{p}_\perp)$ will affect the character of asymptotic behaviour of cross sections of the semi-inclusive processes.

Let us assume, for instance, that the semi-inclusive cross sections obey the similarity relations:

$$\frac{d\sigma_n}{d\vec{p}_\perp} = A(p_\perp^2)\, \Psi(n / f(\vec{p}_\perp)) . \qquad (4.15)$$

Substituting this relation into formula (44) for the associative multiplicity and changing the summation by integration, we find

x) At the same time, in the framework of the multiperipheral model it is possible to reproduce the growth of specra with energy and their power decrease $p_\perp^{-8}$ at large transverse momenta (Dremin, Amati , Caneschi, Testa).

$$\langle n(\vec{p}_\perp)\rangle = \frac{\sum^{N_s} n\, F_n(\vec{p}_\perp, s)}{\sum F_n(\vec{p}_\perp, s)} = \frac{\int^{N_s} n\, dn\, \Psi(n/f(\vec{p}_\perp))}{\int^{N_s} dn\, \Psi(n/f(\vec{p}_\perp))} = \tag{4.16}$$

$$= f(\vec{p}_\perp)\, g(N_s/f(p_\perp)),$$

where $N_s \sim \sqrt{s}$.

Thus, the function $f(\vec{p}_\perp)$ really represents the dependence of the associative multiplicity $\langle n(\vec{p}_\perp)\rangle$ on momentum if $g(N_s/f(\vec{p}_\perp)) \to 1$ for $s \to \infty$ and $p_\perp$—fixed. The deviation from this asymptotic limit may appear only in the region, where $f p_\perp/\sqrt{s} \sim 1$. If the function $f p_\perp \sim p_\perp^{\alpha}$ has the power asymptotic behaviour, this condition corresponds to relatively small momentum transfers $p_\perp \sim s^{1/2\alpha}$, i.e., to values of the parameter $X_\perp = 2p_\perp/\sqrt{s}$ tending to zero with increasing "s".

Note further that the function $A(p_\perp^2)$ defined by (4.15) can be related to the inclusive cross section

$$\frac{d\sigma}{d\vec{p}_\perp} = \sum_n \frac{d\sigma_n}{d\vec{p}_\perp} \sim A(p_\perp^2)\, f(\vec{p}_\perp). \tag{4.17}$$

Making use of formula (4.15), (4.16), (4.17) one can easily establish the validity of the following relation (Matveev, Sissakian, Slepchenko)

$$\langle n(\vec{p}_\perp)\rangle \frac{d\sigma_n}{d\vec{p}_\perp} \Big/ \frac{d\sigma^{incl}}{d\vec{p}_\perp} = \Psi(n/\langle n(p_\perp)\rangle. \tag{4.18}$$

We stress here, that the similarity relation (4.18) analogous to the KNO-scaling is based only on general ideas of the physical similarity and, in particular, not on the assumption on the Feynman scaling.

As is known (see the review of experiment) to the decreasing character of the associative multiplicity there corresponds a "shrinkage" of the semi-inclusive distributions, i.e., at small $p_\perp$ the probabilities of production of a large number of particles drop much faster than those for small multiplicities. On the other hand, the growth of $\langle n(p_\perp)\rangle \sim p_\perp$ corresponds to the transition to a new regime: at increasing $p_\perp$ the cross sections with large n become more smooth than for small multiplicities - the so-called "broadening" of distributions. Thus, the region of small and large $p_\perp$ are clearly separated by essentially different regimes of behaviours both for the inclusive and semi-inclusive cross sections and for the moments of these distributions.

A relation between the semi-inclusive distributions and associative multiplicities in definite combination (4.18) with an essentially different behaviour at small and large transverse momenta indicate, as we think, a certain universality of the similarity law obtained (scaling law) for diffractive semi-inclusive spectra (4.18)

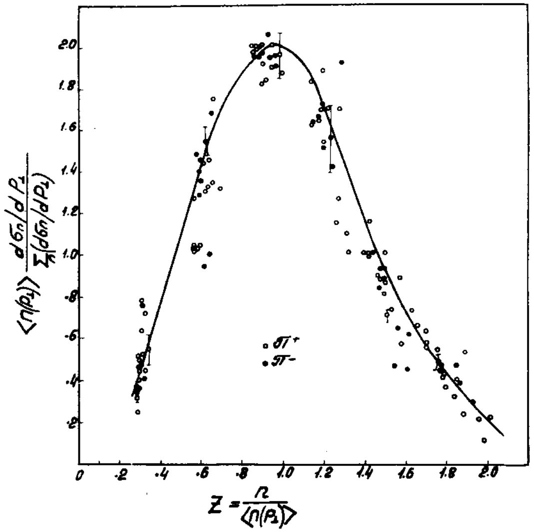

Fig. IV.6

Figure IV.6 shows a distribution of the experimental quantity corresponding to the l.h.s. of (4.18), obtained for $\pi^\pm$ mesons from processing of about 6000 inelastic $\pi^- p$ – events in the $\pi^- p$ – interaction at p = 40 GeV/c (IHEP- in collaboration). We emphasize here that experimental points, corresponding to the two-dimensional distributions $\frac{d\sigma_n}{dp_\perp}(n, p_\perp)$ with different magnitudes of the charged-particle multiplicity $n = 2 \div 12$ and to the whole measured range of $p_\perp$ (in the scale $z = n/\langle n\rangle$ according to formula (4.18)), are on the same universal curve.

Thus, this relation can be considered as a particular manifestation of automodelity specific for a wide class of phenomena in particle interactions at high energies.

c) The models with weak correlations

We have already mentioned that within the framework of the KNO-scaling the result (4.13) is valid under the assumption of absence of correlations between the secondaries. The question arises: what will happen if we introduce the correlations? We have partly mentioned such examples when having considered the Feynman gas models and the model of non-correlative beams. Let us consider in more detail the Feynman gas model (Mueller, Olesen) in which only the two particle correlations are taken into account. Define the function $\tau_n^{(1)}(\vec p)$ which determines the divergence from a non-correlative case

$$\tau_n^{(1)}(\vec p) = \frac{1}{\sigma_n}\frac{d\sigma_n}{d\vec p}(ab \to c(\vec p) + (n-1)_{ch} + \text{anyth. neutral}) - \frac{1}{\sigma}\frac{n}{\langle n\rangle}\frac{d\sigma^{incl}}{d\vec p}(ab \to c(\vec p) + \text{anything}) \qquad (4.19)$$

It appeared that the scaling law in the form (4.13) is valid for the considered model in the case of the short-range correlations. There, the function is factorized with respect to the momentum and multiplicity

$$\tau_n^{(1)}(\vec p) \simeq H(s, x, p_\perp)\,\psi(n, s)$$

In agreement with the scaling (4.13) at high energies

$$\lim_{s \to \infty} H(s, x, p_\perp) = H(x, p_\perp), \tag{4.20}$$

$$\lim_{s \to \infty} \psi(n, s) = \psi'\left(\frac{n}{\langle n \rangle}\right), \quad \frac{n}{\langle n \rangle} - \text{fixed}$$

This means the factorization of the semi-inclusive distribution. Note, that these results can be obtained when considering the sum rules for the semi-inclusive cross sections and correlations. The factorization of semi-inclusive spectra in a general case can be written in the form:

$$\frac{1}{\sigma_n} \frac{d\sigma_n}{d\vec{p}} = A(n) f(\vec{p}) \left[1 + \Phi(n, \vec{p}) \right], \tag{4.21}$$

where $\Phi(n, \vec{p})$ is the divergence measure (analogous to 4.19). It can be written in the form:

$$\Phi(n, \vec{p}) = \frac{\frac{1}{\sigma_n} \frac{d\sigma_n}{d\vec{p}}}{n \frac{d\sigma^{incl}}{d\vec{p}}} - 1 \tag{4.22}$$

Thus, the function $\tau_n(\vec{p})$ and $\Phi(n, \vec{p})$ may be considered as some analogues to the correlation n functions $C_n^{(2)}$ and $R_n^{(2)}$ respectively (see 4.10).

Due to a weak decrease (constancy) of the associative multiplicity as a function of the transverse momentum of π-mesons we can make a conclusion on the smallness of the transverse correlations of charged particles. It is concerned the form $\frac{d\sigma}{d\vec{p}}$ of distributions. In particular, when analyzing the experimental data on the semi-inclusive distributions of π^+ mesons in the $\pi^- p$-interaction when p = 40 GeV/c (IHER - accelerater 2 meter propane chamber JINR).
it was found that these distributions as multiplicity functions are similar in form at different fixed values $p_\perp$ (see Fig. IV.7

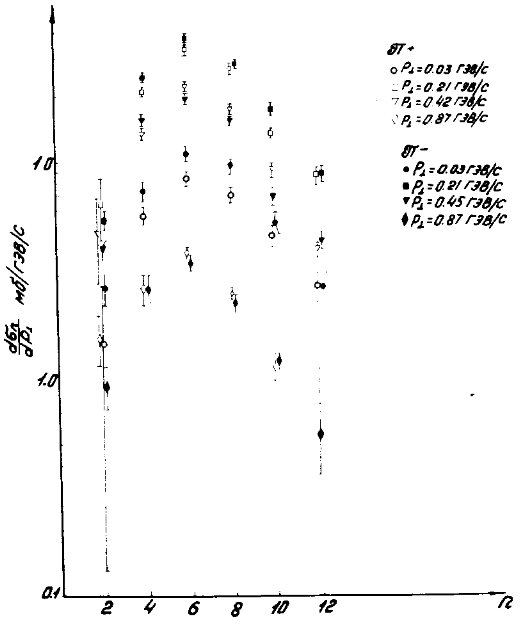

Fig. IV.7.

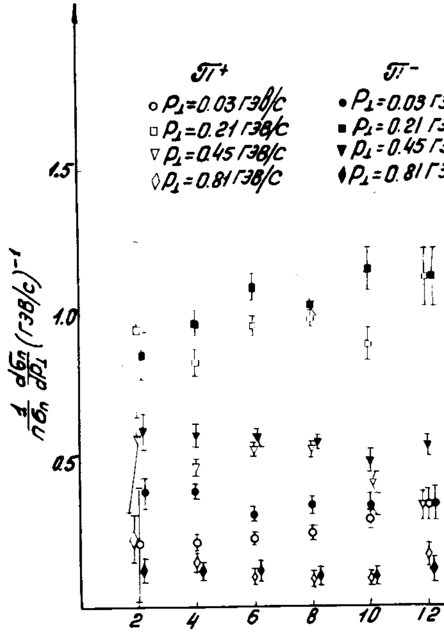

Fig. IV.8.

i.e., the parametrization (4.21) holds there. This property is more convincing in Fig. IV.8 . It follows that except for the range of small $p_\perp$ ($p_\perp \leq 0.2$) the correlation $\Phi(n, p_\perp)$ is weak and there holds, with good accuracy, the factorization of the n and $p_\perp$ variables.

$$\frac{d\sigma_n}{d\vec{p}} = F(n) f(p_\perp) \qquad (4.23)$$

$$F(n) = n\sigma_n \quad , \quad f(p_\perp) = \frac{d\sigma^{incl}}{d\vec{p}} \cdot$$

Note that for the semi-inclusive spectra with the non-correlative $n \leftrightarrow \vec{p}$ dependence (4.23) from the similarity law, it follows the relation of the KNO-scaling for the multiplicity distributions

$$\langle n \rangle \sigma_n / \sigma = \psi(n / \langle n \rangle) .$$

The relation of the moments of distribution over multiplicity with the multi-particle inclusive spectra and the correlation functions made it possible to investigate the properties of automodelity distributions over multiplicity to obtain in the case of weak (SR) correlations, a number of rather interesting results for the inolusive and semi-inclusive reactions. In particular, for the process

$$a + b \rightarrow c(p_1) + c(p_2) + \ldots + c(p_{k+1}) + \\ + \text{anything}$$

assuming the existence of scaling for the inclusive multiparticle distributions (Chliapnikov, Gerdyukov, Manyukov,

Minakata) there was obtained the asymptotic scaling behaviour of the associative moments like

$$\langle n^k(s, \vec{p}) \rangle \underset{s \to \infty}{=} a_k(x, \vec{p}_\perp^{\,2}) \ln^k(s) + O(\ln^{k-1}(s)) . \quad (4.24)$$

Thus for example, the average multiplicity of charged particles $\langle n(M^2) \rangle$ in the reaction $a + b \rightarrow c + X_M$ associated with the quantity M^2 of the system X_M is:

$$\langle n(M^2) \rangle = a(M^2/s) \ln s + b(M^2/s), \qquad (4.25)$$

where a, b are the functions depending only on M^2/s .
The experimental test of these relations is of interest.

The presence of weak correlations between the dependence of semi-inclusive spectra, both on multiplicity and the secondaries momentum made it possible to assume the existence (experimentally) of so-called "scaling in the mean" (Dao et al.). The authors confirm the independence of the forms of the $p_\perp$- and $p_\parallel$- spectra of produced particles on the multiplicity or the colliding particle momentum. The data on pp for different multiplicites between 13 GeV/c and 300 GeV/c were analyzed. It was found that this hypothesis does not qualitatively contradict the production of π^- - mesons.

Thus, the cross sections, expressed in terms of normalization variables, must be of universal form:

$$\frac{\langle V\rangle_n}{\sigma_n}\frac{d\sigma_n}{dV} \sim \phi\left(\frac{V}{\langle V\rangle_n}\right), \tag{4.26}$$

where V is the transverse or longitudinal variable, and $\varphi\left(\frac{V}{\langle V\rangle_n}\right)$ is the universal function independent of S or the multiplicity. Though one has no grounds to consider that this behaviour has a quantitative support it may serve as a useful approximate parametrization.

Fig. IV.9, IV.10.

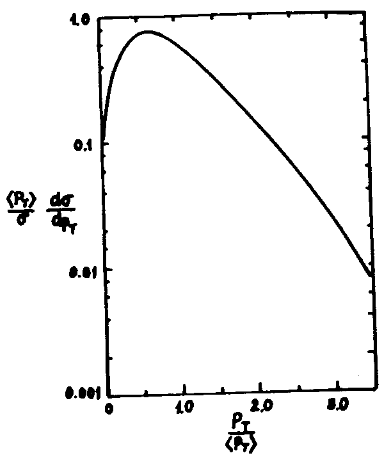

Fig. IV.9.

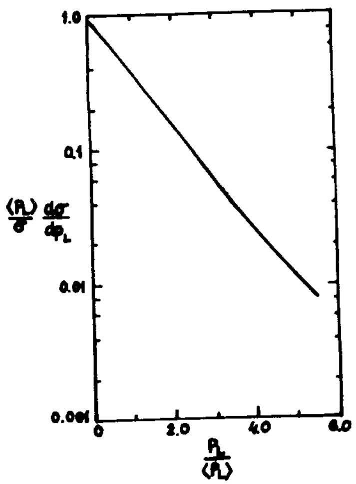

Fig.IV.10

§5. The relation of elastic and inelastic processes

It is convenient to study the elastic and inelastic processes at high energies making use of the approach based on the unitarity condition in quantum field theory. The unitarity equation of the amplitude of scattering of the 2 – spinless particles has the form

$$Im\,T(s,t) = \int d\omega \; T(s,t')\, T^{-1}(s,t'') + F(s,t), \tag{4.27}$$

where $t = -(\vec{p}-\vec{k})^2$, $t' = -(\vec{p}-\vec{q})^2$, $t'' = -(\vec{q}-\vec{k})^2$

$s = 4(m^2+\vec{p}^2)$, $|\vec{p}| = |\vec{k}| = |\vec{q}|$

and

$$d\omega = \frac{1}{8\pi^2}\frac{d\vec{q}\,d\vec{q}'}{2q_0\,2q_0'}\,\delta(p+p'-q-q')$$

are connected with the two particle phase volume.

Graphically the condition (4.27) is the following:

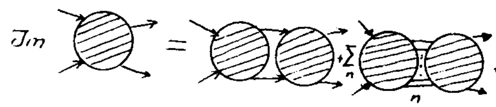

Fig.IV.11

The illustraction of the S-channel unitarity condition. The value $F_n(s,t)=\sum_n F_n(s,t)$ which is called the Van Hove overlap function is the contribution of inelastic (multi-particle) states to (4.27). According to this the elastic amplitude at high energies is nothing else than the shadow of multitudes of inelastic channels. Under the definite assumptions on the character of amplitude of scattering at high energies one can obtain in the range $s \to \infty$ and $t/s \ll 1$ the known formula

$$\mathrm{Im}\, T = \frac{1}{8\pi^2 s}\, T \cdot T^{+} + F . \tag{4.28}$$

Representing the impact parameter we rewrite (4.28) in the form

$$\mathrm{Im}\, f(b,s) = \frac{1}{2}\left| f(b,s)\right|^2 + \rho(b,s) , \tag{4.29}$$

where b is the impact parameter, $f(b,s) \equiv T(s,b)$.

Thus, an important result from the angular moment conservation law is thet the amplitude of elastic scattering with a given impact parameter is produced by the absorption into inelastic channels with the same impact parameter. According to the definition of the overlap function

$$\rho(s,b) \equiv \sum_n \left| T_n(s,b)\right|^2 \equiv \frac{d\sigma_{inel}}{db^2} , \tag{4.30}$$

where $T_n(s,b)$ is the amplitude of the inelastic state production with the n-particles with the impact parameter b. If the phase of elastic amplitude is known we can solve the equation (4.29). In particular, for the parametrization $f(b,s) = i\left(1-e^{2i\delta(b,s)}\right)$ we obtain

$$\rho(b,s) = \frac{1}{2}\left(1 - e^{4\,\mathrm{Im}\,\delta(b,s)}\right) . \tag{4.31}$$

The value $\rho(b,s) = \frac{1}{2}$ corresponds to the unitarity limit reached in the case of a full absorption.

The approach to the diffraction scattering, considered as a shadow of inelastic processes, puts a number of interesting questions. How close to the maximum absorption is $\rho(s,b)$ when $b=0$? What is its form and an average radius?

How do individual n-particle amplitudes construct $\rho(s,b)$? What processes (impact parameters) are responsible for growing cross sections with energy? and so on.

In papers of the Serpukhov group (Khrustalev, Savrin, Semenov, Troshin, Tyurin) as well as of the group of Bialas, Buras, Dias de Dens, Miettinen some interesting similarity properties for $\rho(s,b)$ are found and discussed. An analysis of the ISR experimental data on elastic pp-scattering in the diffraction region makes it possible to draws several important conclusions about properties of inelastic channels.

In particular, the observed growth of the total inelastic cross section occurs, as the authors think, due to peripheral inelastic interactions, and in the energy region under consideration $\rho(b,s)$ appears to depend only on the ratio $\pi b^2/\sigma_{inel}(s)$. This relation is a manifestation of the geometrical similarity in inelastic processes at high energies.

$$\rho\,(b,s) \xrightarrow[s\to\infty]{} \rho\left(\frac{b^2}{R(s)}\right). \tag{4.32}$$

All the approaches and models studying the inelastic collisions in the language of impact parameter and also connections with a character of the behaviour of elastic collisions at high energies have been called geometrical approaches. These models accentuate the geometrical nature of collisions, an elementary act of collisions, productions being considered (in general of weakly correlated particles) to occur at a fixed impact parameter b . The total inelastic cross section is derived by integrating over all the impact parameters. Accordingly, inclusive (semi-inclusive) characteristics include mixtures of a large number of elementary components with given b .

Following Van Hove, one has

$$F(s,t) = \int d^2b\; e^{i\vec{\Delta}\vec{b}}\,\sigma(b)\;, \quad t=-\vec{\Delta}^2, \tag{4.33}$$

where the total inelastic cross section with given b (4.30)

$$\sigma(b) = \sum_n \sigma_n(b) = \frac{d\sigma_{inel}}{db^2} \tag{4.34}$$

and the n-particle production (topological) cross section is written as a superposition of n-particle cross sections at fixed impact parameters b :

$$\sigma_n = \int d^2b\; \sigma_n(b) = \int d^2b \cdot \frac{d\sigma_n}{db^2}\,. \tag{4.35}$$

To obtain the multiplicity distributions we need $\sigma(b)$, $\sigma_n(b)$. By using the relation (4.33) and its partial analog overlap (semi-inclusive) function $F_n(s,t)$

$$F_n(s,t) = \int d^2 b \; e^{i\vec{\Delta}\vec{b}} \sigma_n(b) \qquad (4.36)$$

and also the corresponding formula for transformation (the Fourier-Bessel transformation) one may find relations between the functions $\rho(s,b)$ and multiplicity distributions $\sigma_n(s)$. In particular, for the contribution of inelastic channels (4.36) obtained in the framework of probability approach to description of the scattering processes at high energies (Logunov, Khrustalev) it is shown, proceeding from the universality of function $\rho(b,s)$ (4.32), that $\rho(s,b)$ is connected by the Lehman transformation, with the function $\psi(z,s)$ characterising the multiplicity distribution in proton-proton collisions at high energies

$$\psi(z) = \langle n \rangle \frac{\sigma_n}{\sigma} , \qquad z = \frac{n}{\langle n \rangle} . \qquad (4.37)$$

The three-component model for $\psi(z)$ found and analysed further describes well both the multiplicity distribution and

the first ten moments of distribution (see discussion concerning difficulties of the two-component description, Lect.II). It would be of interest to study what follows in the language of geometrical models, from separation of the mechanisms of multiparticle production into a sum of contributions from different components and in this connection to study the relation between such concept as range of correlation, diffraction, fragmentation and independent emission of produced particles.

The existence of connection between elastic and inelastic processes following from the unitarity condition (4.27), (4.29) is supported also by that the result of two particle collision is defined by the internal struoture of hadrons. The structure of interacting particles, displaying in the smoothness of an effective quasipotential of interaction, defines also the multiparticle production processes. An attempt is, therefore, natural to gain information on some simple characteristics of inelastio processes by using the quantity characterising the elastio scattering.

Consider now several aspects of this problem:

a) A connection of parameters of the elastic scattering with inclusive and semi-inclusive distributions.

b) A behaviour of the associative multiplicity as a function of $t = -\Delta^2$ and elastic re-scattering on a compound system.

c) A relation between slopes of the elastic scattering amplitude and average multiplicity of seoondaries.

a) In considering the model of independent emission of soft pions as a result of collisions of two scalar nucleons, the differential cross section of production of m mesons (semi-inclusive distribution) can be written in the form (Khrustalev, Savrin, Semenov, Tyurin)

$$\frac{d\sigma_n}{d\vec{K}} = \frac{4\pi}{(n-1)!} \sum_{\ell} (2\ell+1) \int \prod_{i=2}^{n} d\vec{K}_i \left| f_\ell (\ldots K_i, q) \right|^2 \tag{4.38}$$

If one introduces the density of meson distribution $\rho(\ldots K_i)$ and the corresponding quantity in r-space, then the assumption on independent emission of mesons, together with partial unitarity, allows one to connect the quantity $\rho_\ell(0)$ with the phase of elastic scattering of two nucleons

$$\bar{\rho}_\ell(0) = 4 \operatorname{Im} \delta_\ell \tag{4.39}$$

and the cross section for n-meson production takes the form

$$\frac{d\sigma_n}{d\vec{K}} = \frac{\pi}{(2\pi)^3 q^2} \sum_{\ell} (2\ell+1)\, e^{-\bar{\rho}_\ell(0)} \frac{\bar{\rho}_\ell^{\,n-1}(0)}{(n-1)!} \rho_\ell(\vec{K}), \tag{4.40}$$

i.e., the inclusive one-particle distribution is

$$\frac{d\sigma}{d\vec{K}} = \frac{\pi}{(2\pi)^3 q^2} \sum_{\ell} (2\ell+1)\, \rho_\ell(\vec{K}).$$

In the impact parameter representation one has

$$\frac{d\sigma}{d\vec{K}} = \frac{1}{(2\pi)^3} \int d^2 b\, \rho(\vec{K}, \vec{b}). \tag{4.41}$$

In this way, we arrive at the explicit relation between the inclusive distribution over transverse momentum and the imaginary part of the phase of elastic scattering of two nucleons (simultaneously with spatial distribution of the hadron matter in nucleon):

$$\frac{d\sigma}{d\vec{K}_\perp} = \frac{1}{(2\pi)^3} g^2\left(\tfrac{1}{2}\vec{K}_\perp, 0\right) \tag{4.42}$$

$$g(\xi, 0) = 2\sqrt{\operatorname{Im} \delta(2\xi)}\,.$$

b) Consider now, in the framework of eikonal approach and impact parameter representation, the interaction of a fast particle with a compound system, the target particle in the final state dissociating into n – constituents. Consideration of the interaction with a compound system allows one (Kvinikhidze, Slepchenko) to obtain information on dependence of the one-particle distribution functions on the number of particles in the final state (on the number of constituents) and in this way to simulate the inclusive and semi-inclusive characteristics of multi-particle processes.

Consider a contribution of a multiple interaction to the one-particle distribution function of final particles. For n=2, by definition, one has

$$\frac{d\sigma_{n=2}}{dx\, d\Delta^2} \simeq F^{-1} \int \prod_i^3 \frac{d^3 p_i}{2p_{0i}} \delta^4\Big(G - \sum^3 p_i\Big)\, \delta\big(\vec{\Delta} - (\vec{p}_3 - \vec{q}_3)^2\big) \cdot$$
$$\cdot\; \delta\Big(x - \frac{p_{1z}}{p_{1z}+p_{2z}}\Big) \left| M_{n=2}(x, p_\perp, \vec{\Delta}) \right|^2, \qquad (4.43)$$

where $F = 2(2\pi)^2 \lambda^{1/2}(s, m_3^2, M^2)$ and M_n defines contributions of double interaction:

$$M_2 \sim \int d^2 b\, d^2 b_n\, e^{i\vec{b}\vec{\Delta}_\perp + i\vec{p}_{i\perp}\vec{b}_{12}}\, \chi(\vec{b}_{12}, x) \cdot$$
$$\cdot\; f_1(\vec{b} + x\vec{b}_{12})\, f_2(\vec{b} - (1-x)\vec{b}_{12}), \qquad (4.44)$$

where $\vec{b}_{12} = \vec{b}_1 - \vec{b}_2$, $\vec{b} = (1-x)\vec{b}_1 + x\vec{b}_2$ and $\vec{b}_1, \vec{b}_2$ are individual impact parameters of interaction of the fast particle with constituents 1,2. Substituting (4.44) into definition (4.43) we get

$$\frac{d\sigma_{n=2}}{dx\, d\vec{\Delta}^2} = c \int d^2 b\, d^2 b'\, e^{i\vec{\Delta}(\vec{b} - \vec{b}')} \int d^2 \vec{b}_{12} \left| \psi(b_{12}, x) \right|^2 \cdot \qquad (4.45)$$
$$\cdot\; f_1^*(\vec{b}' + x\vec{b}_{12})\, f_2(\vec{b}' - (1-x)\vec{b}_{12})\, f_1(\vec{b} + x\vec{b}_{12}) \cdot$$
$$\cdot\; f_2(\vec{b} - (1-x)\vec{b}_{12}),$$

where $f_i(\vec{b}, x)$ are the two-particle elastic amplitudes and the wave function $\psi(x, \vec{b}_{12})$ plays now the role of the probability amplitude of dissociation (fragmentation) of a compound system into constituents. From (4.45) one can easily see that the distribution over the squared momentum transfer is defined essentially by re-scatterings of an incident particle with a compound system. On the other hand, the distribution over the relative momentum of particles composing a system

$$\frac{d\sigma_{n=2}}{dx\, dp_\perp^2} = c' \int d^2 b_{12}\, d^2 b_{12}'\, \psi^*(\vec{b}_{12}', x)\, \psi(\vec{b}_{12}, x)\, e^{i\vec{p}_\perp(\vec{b}_{12} - \vec{b}_{12}')} \cdot \qquad (4.46)$$
$$\cdot \int d^2 b\, f_1^*(\vec{b} + x\vec{b}_{12}')\, f_2^*(\vec{b} - (1-x)\vec{b}_{12}') \cdot$$
$$\cdot\; f_1(\vec{b} + x\vec{b}_{12})\, f_2(\vec{b} - (1-x)\vec{b}_{12}).$$

strongly depends on a character of the wave function (i.e., on properties of fragmentation of a target into constituents).

In the general case of an arbitrary number (n) of constituents one has:

$$M_n(x_1 \dots x_n, \Delta_\perp, \vec{p}_{1\perp} \dots \vec{p}_{(n-1)\perp}; s) =$$

$$= \int d^2 b \, e^{i\vec{b}\vec{\Delta}} \prod_{i=1}^{n} \left[\int d^2 b_i \, e^{i\vec{p}_{\perp i}\vec{b}_i} f_i(\vec{b}_i + x_i \vec{b}) \right] \cdot \tag{4.47}$$

$$\cdot \Psi_{(n)}(\{x_i, b_i\}) \cdot \delta(\sum_1^n x_i b_i) \, \delta(\sum_i^n x_i - 1).$$

As has been mentioned above, the distribution $d\sigma_n / dx d\vec{\Delta}^2$ corresponding to (4.47) is sensitive to the form of the two--particle amplitude of scattering on constituents $f_i(b_i, x)$.

Making different assumptions on the structure of the local two-particle quasipotentials one may obtain a detailed informa-rion concerning the behaviour of a compound system.

In particular, if one assumes that in the region of large $\vec{\Delta}^2$ the incident particle scatters on all n constituents of a target at least once. In this case, if the scattering angle is the same for each individual amplitude, then under

rather general assumptions on the function $\Psi(\vec{b}, x)$ for X fixed one can show that

$$\frac{d\sigma^n}{d\vec{\Delta}^2} = C(n, \dots) f^n((\Delta/n)^2), \tag{4.48}$$

where

$$f_1(\Delta) = f_2(\Delta) = \dots = f(\Delta)$$

i.e., (4.48) results in the so-called "broadening" of the effective slope of the $\vec{\Delta}^2$- distribution as a function of R (becomes more smooth in the region of large $\vec{\Delta}$).

Composing the first moment (4.48), i.e., the corresponding associative multiplicity, under the assumptions made above, leading to the automodel behaviour of the dependence $d\sigma_n / d\vec{\Delta}^2 \to f(z)$, $z \to \vec{\Delta}/n$ (see (4.18)), one may obtain the growing behaviour

$$\langle n(\Delta) \rangle \sim c\vec{\Delta}^2. \tag{4.49}$$

c) On the relationship $\langle n_{diffr} \rangle$ with slope of the diffraction peak and σ_{tot}.

Let us consider some results concerning multi-particle production in the framework of the straight-line path approximation (SLPA) in quantum field theory. As is known, this approximation has been suggested and developed by the Dubna group (Tavkhelidze, Barbashov, Matveev, Kuleshov, Pervushin, Sissakian) for high energies and fixed momentum transfers. This method leads to number of interesting results for high-energy multi-particle production processes.

One of them is that the total differential cross section obtained by summing over the number of all emitted mesons is found to be independent of t

$$\frac{d\sigma^{tot}}{dt} = \left(\frac{d\sigma^{d}}{dt}\right)_0 = const \qquad (4.50)$$

in a certain range of secondary particle momenta.

This is, in a certain sense, analogous to the point-like or automodel behaviour of the cross sections for deep inelastic hadron-lepton processes.

The real content of the result (4) 50) consists in the fact that the total differential cross section can change noticeably only by changing $\Delta t \sim t_{eff}$. which greatly exceeds the sizes of the diffraction domain.

To estimate t_{eff}. , we may make use of the unitarity condition which yields

$$-t_{eff} \le \frac{8\pi}{\sigma^{tot}} . \qquad (4.51)$$

This value of t_{eff} can be employed for estimating the average number of secondary particles $\bar{n}_{diffr.}$ produced in the diffraction collisions of hadrons at high energies

$$\bar{n}_{diffr}(s) = \frac{1}{\sigma^{tot}} \int_0^{t_{eff}} \frac{d\sigma^{tot}}{dt} A(s)dt \le \frac{const\, A(s)}{\sigma^{tot}} .$$

Thus, the diffraction or peripheral part of the average multiplicity is defined by the parameters of the elastic zero-angle scattering amplitude. The conclusion about the behaviour of the total particle number $\bar{n}(s)$ can be drawn only under definite assumptions about the contribution of small distances to high-energy multiple production processes. In particular, if the assumption about the disappearance of "pionization" effects at high energies, i.e., the production of secondaries with limited momenta in the c.m.s. of the colliding hadrons, is used, then relation (4 . 51) will define the behaviour of the total average multiplicity

$$\bar{n}(s) = \frac{const\, A(s)}{\sigma^{tot}} + \tilde{\nu} , \qquad (4.52)$$

where $\tilde{\nu}$ is the number of "leading" particles.

From the viewpoint of attempts to connect the regularities observable in multiple productions with the parameters of elastic scattering, this result can be treated as a contribution to the magnitude of the slope of the elastic scattering amplitude (this contribution is due to the diffraction mechanism). It is known that within the model of uncorrelated streams very small values are obtained for the elastic slope, and a mechanism of the multi-peripheral type gives very fast narrowing of the slope with increasing energy. In this respect it would be rather interesting to estimate the value of A(s) within the models allowing for the two mechanisms.

Using the well-known restriction on the asymptotic behaviour of the diffraction peak width in quantum field theory (Logunov, et al., R. Eden) from Eqs. (4.52), we get in the general case

$$\bar{A}(s) \leq \frac{const}{\sigma^{tot}} \ln^2 s \,. \qquad (4.53)$$

This relation is an interesting interpretation of the increase of the strong interaction radius.

Indeed, A(s) is the "visible" hadron size, σ^{tot} defines the minimal distance R for which the automodel behaviour holds.

One can see from Eq. (4.52) that

$$A(s) \sim R^2 = \pi R_0^2 \,.$$

Thus, the strong interaction radius increases under the condition of the constant cross section, at the expense of the "swelling out" of hadrons associated with the "clouds" of secondary particles.

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Problems for Discussion to Lec. IV

1. **Find** $\langle n(p_\perp)\rangle$ and $\frac{d\sigma^{incl}}{dp_\perp}$ if $\psi(z)$ in (4.15) has the form $\psi(z) = z^{-2} e^{-c/z}$, $z = n/p_\perp^{2\alpha}$.

2. **Find** from (4.18) KNO (I) for the factorized distribution.

$$\frac{d\sigma_n}{dp_\perp} = F(n)\, f(p_\perp) \cdot \quad \left(KNO\ I - \langle n\rangle \frac{\sigma_n}{\sigma} = \psi(n/\langle n\rangle) \right)$$

3. **Find** $\tau_n^{(1)} = 0$ for the non-correlation case according to (4.19).

4. Solve the unitarity condition (4.29), find $f = f(\rho)$ and vice versa.

Lecture V.

PHYSICS AT HIGH $p_\perp$

§1. New regularities in high energy production

In this part we present a review both of experimental and theoretical results on large transverse momentum inclusive processes. An interest to these processes is due to the present experimental possibilities of getting large $p_\perp$ or momentum transfers on new accelerators. On the other hand, there are some theoretical arguments to expect that the interaction mechanism at large momentum transfers differs essentially from that which determines the region of small transverse momentum.

Recent experiments on production of particles with large transverse momentum in hadron-hadron collisions at high energies have revealed definite changes in the cross section behaviour compared with that in the small transfer momentum region.
Some of specific features of the processes in question are as follows. A steep decrease of the cross sections with growing $p_\perp$ at fixed S, the increase of cross sections with energy at large fixed transverse momenta $p_\perp$, the appearance of appreciable correlations between particles with large $p_\perp$ and other secondaries, etc. A general view on a behaviour of these processes as a result of analysis of experimental data, is given in Table.

Table

	Small $p_\perp$	Large $p_\perp$
S fixed $p_\perp$ increases	Rapid decrease of the cross sections with increasing $p_\perp$ $\sim exp(-a p_\perp)$	Less rapid (steeper) decrease of the cross section with increasing $p_\perp$ $\sim p_\perp^{N}$
$p_\perp$ fixed S increases	Weak dependence of the cross sections on S	Growing cross sections with increasing S
Particle Ratios	Among secondaries the pions dominate $(\pi/K) \sim 10\%$ $\pi^+/\pi^- \sim 1$	Heavy particles are produced relatively more oopiously $\pi^+/\pi^- > 1$ pp (collosions)
Associated Multi-particle	Weak dependence of the ass. multiplicity on $p_\perp$ $\langle n(p_\perp)\rangle \sim$ Const.	Growth of the associated multiplicity with increasing $p_\perp$ $\langle n(p_\perp)\rangle \sim p_\perp^{\alpha}$
Correlations	Small	Large positive correlations between two large $p_\perp$ particles $C_2(p_{\perp 1}, p_{\perp 2})$

The first indication of surprisingly high cross sections at large $p_\perp$ came from CERN ISR where the cross section was found to be several orders of magnitude higher than the extrapolation of an exponential fit to the invariant inclusive cross section found for $p_\perp <$ 1 GeV/c. Data also displayed a very strong S-dependence at large $p_\perp$

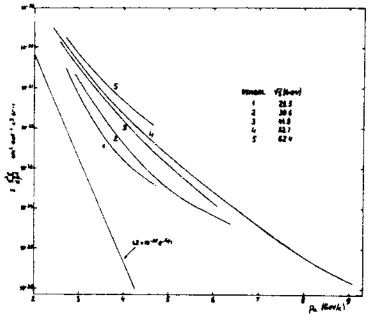

Fig. V.1.

Thus the exponential drop $e^{-a p_\perp}$ of the invariant cross section for production of charged particles established for $p_\perp < 1$ GeV/c does not last for $p_\perp >$ 2 GeV/c. In this region the $p_\perp$ dependence of the invariant cross section is much less steep than in the low $p_\perp$- region

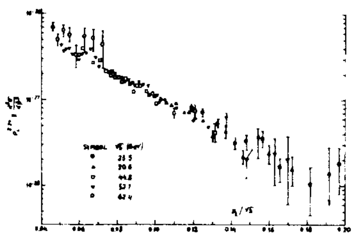

Fig. V.2.

This data is consistent with a $p_\perp$ dependence given by

$$E \frac{d^3\sigma}{dp^3} = p_\perp^{-N} f(x_\perp) \quad \frac{mb}{(GeV)^2}, \tag{5.1}$$

where $x_\perp = 2p_\perp/\sqrt{s}$ and $N \simeq 8$, $f(x_\perp) \sim e^{-13x_\perp}$ for $pp \to \pi^{\pm}(90^\circ) + \dots$

The parametrization $E\frac{d\sigma}{dp^3} = p_\perp^{-N} f(x_\perp)$ with $f(x_\perp) \sim e^{-a x_\perp}$ gives a fair description of pion data at large $p_\perp$ ($x_\perp$) however, with different values of the parameters N and a depending on the region of $p_\perp$ and S over which the fits are made, typically giving $N \sim 8$ at the ISR for $x_\perp \leq 0.5$ and $N = 11$ at the larger values of $x_\perp$ at FNAL. Fig. V.3 shows the variation of N as a function of $x_\perp$ requires to bring the charged pion data of different energies of the FNAL together.

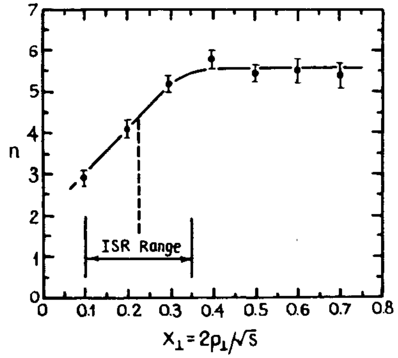

Fig. V.3

Variation of exponent in the parametrization $E\frac{d\sigma}{d^3p} \sim p_\perp^{-2n}$ as function of $x_\perp$

Most of experiments on production of particles with high transverse momenta are purely inclusive. They give only the $p_\perp$- distribution of secondaries of a given type without telling us what kind of collisions leads to the emission of high transverse momentum particles. A study of particle correlations in high energy collisions leading to high transverse momentum of secondaries can provide further insight into the dynamics of these processes. Knowledge of the correlations between the high $p_\perp$ particle and the other secondaries in an interaction is thus essential for a complete understanding of the production process at large transverse momentum. The experimental information presently available on such correlations at very high energy comes from ISR measurements involving photons (π^{0}) and $\pi^{\pm}$ mesons with large transverse momentum.

To make this problem more clear the distributions of the charged particles emitted in proton-proton collisions in association with a photon of high transverse momentum were studied at the CERN ISR . The normalized total multiplicity, associated with the photon is plotted in Fig.V.4 as a function of $p_\perp$ and for different c.m. energies. The multiplicity

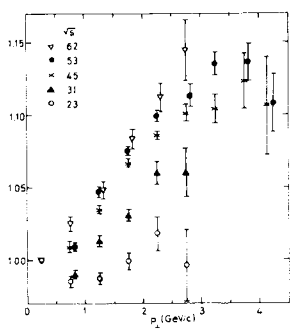

Fig.V.4.

Average total multiplicity of charged particles at $\sqrt{s}$ = 23,31, 45,53 and 62 GeV as a function of $p_\perp$ of the photons, detected at θ_{CM}= 90°.

increases moderately with $p_\perp$ the growth being more pronounced at higher energies; above $p_\perp \simeq 3$ GeV/c the distribution is flattenning. In order to understand such a behaviour, the $p_\perp$ dependence was studied for the multiplicities observed in the two hemispheres: towards the observed photon and away from it (or in the same and opposite directions) . Figures V.5a and V.5b show the normalized hemisphere multiplicities as a function of $p_\perp$ and for the same c.m. energies as in Fig.V.4. The multiplicity away from the photon increases linearly with $p_\perp$ and displays a little s-dependence. The dependence on energy seems to be entirely concentrated in the hemispheres towards the photon. Here the multiplicity decreases with $p_\perp$ at the lowest c.m. energies while only a slight increase is observed at the highest energy.

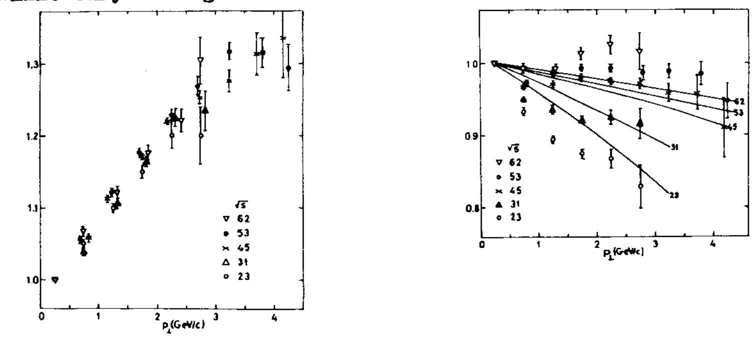

Fig.V.5a,b . Normalized partial multiplicities as a function of photon transverse momentum in the hemispheres a) towards and b) away from the detected photon

The following analysis has been repeated for photons emitted at $\theta_{CM} = 17.5°$ ($y^* \approx 2$) and for cm.m. energy $\sqrt{s}$ = 53 GeV. The data are compared with the corresponding 90° data at the same $p_\perp$ value. The normalized total multiplicity, shown in Fig.V.6, displays a rise with $p_\perp$ which is rapid than that observed at 90^0 and which begins at larger $p_\perp$.

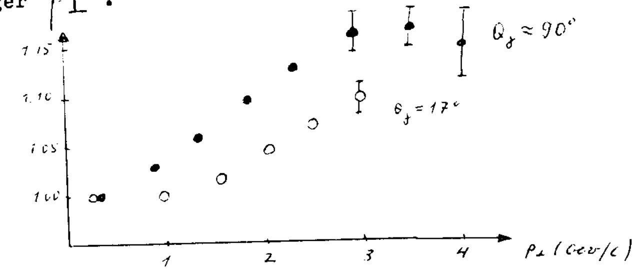

Fig.V.6

Average total multiplicities of charged particles as a function of $p_\perp$ of the detected photon for $\theta_{CM} = 90^0$ and 17.5^0. (See. T. Deb Prete, 1974).

One can summarize the relevant features of these data as follows:

1. The charged particle multiplicity increases with $p_\perp$ in a wide cone opposite to the detected photon. The growth of multiplicity is roughly linear in $p_\perp$ and energy independent.

2. The mean multiplicity of charged particles, emitted in the same direction as the photon generally decreases with increasing $p_\perp$; only at the highest ISR energy a tiny rise is observed.

3. At small angles towards the beam directions the multiplicity decreases at all energies.

4. The forward photon data show also some observable increase of multiplicity in the "towards" hemisphere.

A similar effect has been obtained in a somewhat different type of high $p_\perp$ correlation experiment which has been performed at BNL at the relatively low beam momentum of 28.5 GeV/c. In this experiment the reaction is $pp \to p(\pi) + MM$ where the charged multiplicity of the fixed missing mass (MM) is measured as a function of the transverse momentum of the fast towards proton (pion). As is seen in Fig.V.7 the multiplicity is roughly independent of $p_\perp$ below 1 GeV/c but rises moderately as $p_\perp$ increases from 1 to 2 GeV/c.

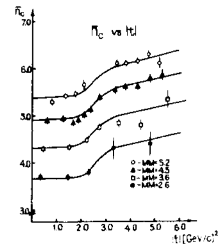

Fig.V.7

Variation of the average charged particle multiplicity, $\bar{n}_c$, with $|t| = p_\perp^2$ for four intervals of MM

At the CERN ISR the measurements were also performed of π^c correlation as a function of transverse momentum when two neutral pions are detected at large angles on opposite sides of the SIR intersection. The correlations at $\sqrt{s}$ = 5 3 GeV are shown in Fig.V.8 . One finds that when a large $p_\perp$ pion is detected on one side, the probability of having another π^c with large $p_\perp$ on the opposite side is several orders of magnitude larger than would be expected from uncorrelated pion production.

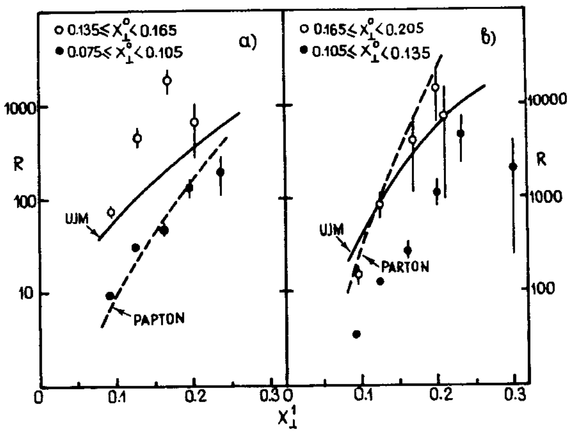

Fig.V.8 a,b

The correlation coefficient for two π^o,s near $\theta_{cM}=90^\circ$ as a function of $X_\perp$ (azimuthal separation $\sim 180^\circ$)

Here by correlation function we mean

$$R(x_{1\perp},x_{2\perp}) = \frac{\sigma_{in}\cdot\dfrac{d^6\sigma}{dp_1^3\,dp_2^3}}{\dfrac{d^3\sigma}{dp_1^3}\,\dfrac{d^3\sigma}{dp_2^3}} \quad , \qquad (5.2)$$

where $X_\perp = 2p_\perp/\sqrt{s}$.

The correlation is seen to increase with increasing $X_\perp$ of either π^o, and R is as high as $\sim 10^4$ for $X_{1\perp}=X_{2\perp}=0.2$. This behaviour might be rather a consequence of momentum conservation, however, the function R for the same-side π^o's (see Fig.V.9) is also positive and large (at $X_{1\perp}=X_{2\perp}=0.1$) an effect of which cannot be explained by kinematics

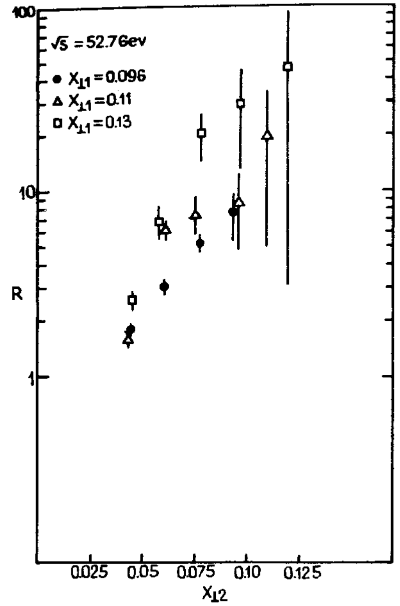

Fig.V.9

The correlation coefficient R for two π^o's with azimuthal separation $\sim 0^\circ$

2. Hadron structure and high transverse momentum

A common view is that the collisions with small $t = p_\perp^2$ are determined by a global structure of hadrons, for example, by the effective range of interactions of order 1 fermi which is related to the slope parameter of the cross section.

It is naturaly to expect that in collisions with extremely large transverse momentum (or momentum transfers) $p_\perp \sim p_\parallel \sim E$, $E \to \infty$ an inner local structure of hadrons which is presently assumed to have "hard" of "point-like" character becomes more important. In inclusive reactions at large $p_\perp$ the "hard" point-like structure of hadrons can be revealed. From automodelity point of view the processes with large have an analogy with the phenomenon of poin-like explosion and, therefore, they must be described by a usual dimensional analysis. For large s by simple dimensional considerations, it follows that instead of general form $d\sigma/d\vec{p} = f(s, p_{\parallel}, p_{\perp})$ we may have the following asymptotic formula

$$\frac{d\sigma}{d\vec{p}} = p_{\perp}^{-N} \quad f(x, x_{\perp}).$$

In the framework of the quark model using the principle of automodelity it was shown (by Matveev, Muradyan and Tavkhelidze) that the above mentioned power law at large angles depends essentially on a number of hadron constituents, i.e., on "a degree of complexity" of particles.

Attempts to derive the power character of the asymptotic behaviour of cross sections at large angles $(p_{\perp})$ have been made in a number of recent works under the various model assumptions.

Recently various composite models such as quark model, parton model and others, have extensively been used in the elementary particle theory. In this connection the problem of a self-consistent-relativistic description of interactions of composite particles is of much importance. An effective method of describing the properties of relativistic composite systems is the quasipotential approach of Logunov-Tavkhelidze in quantum field theory. This approach has turned out to be more suitable in explanation of general regularities of elastic and inelastic (inclusive) processes at high energy and transverse momentum .

Quasipotential formulation in terms of the light-front **variables gives us** in particular within rather general **assumptions about** the behaviour of wave-functions of composite **system, the intrinsic** power dependence of measured quantities, e.g. , $d\sigma/dp_{\perp} \sim s^{-N} \cdot f(x_{\perp})$, where f is a scaled function, in region of high $p_{\perp} \sim \sqrt{s}$, $s \to \infty$. Such a behaviour is obtained in the framework of various models in which a hadron is assumed to be a composite object with many point-like constituents. When these constituents are called partons (quarks) there are two possible mechanisms of the interaction of two colliding hadrons: the parton-parton scattering and the parton interchange. According to that there exist, in fact, two parton models of the high $p_{\perp}$ particle production. In the mechanisms of the parton-parton scattering discussed by Berman, Bjorken and Kogut two colliding hadrons are considered as two colliding beams of partons.
The interaction of hadrons occurs when a pair of partons interacts via a gluon exchange, scattered one against another according to the parton model the cross section for production of a high $p_{\perp}$ particle is given by $E\frac{d\sigma}{dp^3} \sim \frac{1}{p_{\perp}^4} \cdot f(p_{\perp}, s)$ where factor $p_{\perp}^{-4}$ comes from the vector gluon exchange in scattering of two partons. The function $f(p_{\perp}, s)$ is determined by the probability to have a parton with the momentum x and then to obtain from this scattered parton a particle with the transverse momentum $p_{\perp}$.

The second possible mechanisms of the interaction of two hadrons was discussed by Blanckenbecler, Brodsky and Gunion. They assumed that in a collision of two composite objects their constituents can be interchanged.

The probability a parton with a large transverse momentum can be evaluated from the form factor of a hadron and same power law for high $p_\perp$ production

$E \frac{d\sigma}{dp} \sim p_\perp^{-N} \mathcal{E}\left(\frac{p_\perp}{\sqrt{s}}\right)$, where exponent N can be calculated when we know the form factor of a pion.

The last group of models to be discussed are the cluster and multi-peripheral approaches. Berger and Branson suggested that high $p_\perp$ particles observed in high energy collisions are the decay products of two clusters which decay anisotropically and their decay products are collimated along the line-of-flight of clusters. The cluster models predict that a high transverse momentum particles are often accompanied by other particles with high transverse momenta all of them being the decay products of the same cluster.

The production of high transverse momentum particles is strongly damped by the multiperipheral mechanism of the particle production. In some recent versions of this model attempts were made for the description of high $p_\perp$ data a serious difficulty of the model is the observed increase of the multiplicity associated with high $p_\perp$ particles. The model requires that masses of many-particle systems should be small and therefore multi-particles of these systems to be low.

Instead the multiperipheral model describes correctly the increase of the heavier particle component at high $p_\perp$.

§ 3. Associated multiplicities.

As was mentioned in the lecture, a dependence of the growth of average multiplicities on the transverse momentum was considered (by Matveev, Sissakian and Slepchenko) under the assumption on the automodel character of the behaviour of semi-inclusive spectra. To demonstrate more clearly the correlation character of the associated multiplicity $\langle n(p_\perp)\rangle$ one can also introduce the equivalent to definition

$$\langle n(\vec{p})\rangle = \frac{1}{\frac{1}{\sigma}\cdot\frac{d\sigma}{d\vec{p}_1}} \cdot \int C_2\left(\vec{p}_1, \vec{p}_2\right) d\vec{p}_2 + \langle n\rangle , \qquad (5.3)$$

where $C_2\left(p_1, p_2\right)$ is defined in I.

From (5.3) it is seen, in particular, that if there are no correlations between particles with momenta $\vec{p}$ and $\vec{q}$ the associated multiplicity for the inclusive production of a particle with momentum $\vec{q}$ does not depend on $\vec{p}$

$$\langle n(\vec{p})\rangle = \langle n\rangle_{tot} - 1$$

Note that in accordance with the total momentum conservation the large transverse momentum $p_\perp$ of the detected particle is balanced by the whole transverse momentum of the group of other particles that causes a strong correlation between them.

When choosing a concrete form of dependence of the average number of particles on the transverse momentum one should take considerations of the multi-particle production mechanisms. Proceedings from the assumption on coherent excitation of the particles, colliding at high energies one can obtain that the average number of secondaries increases linearly with the squared transfer momentum

$$\langle n(p_\perp)\rangle = a + b\cdot p_\perp^2 \ . \qquad (5.4)$$

Within the framework of the straight line path method, this result has been derived for the diffractive production of secondaries. Such a behaviour is in qualitative agreement with the experimental data on pp - collisions at the laboratory momentum of the incident proton $p_{lab} \simeq 30$ GeV/c. (See fig. Y.7).

An analogous phenomenon follows also from the hypothesis of limiting fragmentation where the growth of $\langle n\rangle$ with $p_\perp$ arises due to the impossibility to give a large transverse momentum to a hadron without its break up.

Note that in the multiperipheral model the mean multiplicity decreases logarithmically with growing $p_\perp$ x). This decrease, apparently, is a consequence of that the multiperipheral model corresponds mainly to the mechanisms of secondary production connected with appearance of hadron clusters in a central region, while the results of the coherent state model (Matveev, Tavkhelidze), the straightline path method and fragmentation picture correspond to the mechanism of diffractive dissociation of colliding particles. The inclusive cross sections for a diffractive production of high $p_\perp$- particle corresponding to topological (semi-inclusive) distribution, satisfying the differential scaling law eq. are consistent with a power asymptotic behaviour on the form

$$\frac{d\sigma}{dp_\perp^2} \sim \frac{1}{(p_\perp^2)^{\varkappa+2}} \cdot F\left(\frac{p_\perp^{2\varkappa}}{\sqrt{s}}\right)$$

$$F(z) = e^{-cz} - e^{c\sqrt{s}/2} \qquad (5.5)$$

The associated multiplicity has approximately rising dependence on $p_\perp$

$$\langle n(p_\perp)\rangle \sim (a\, p_\perp)^{2\varkappa} \qquad (5.6)$$

---

x) Within the NP scheme it is possible to reproduce the growth of spectra with energy and their power decrease $p_\perp^{-\tau}$ at large transverse momenta .

In this connection note the fact that the assumptions, made in the framework of our consideration, make it possible to establish a relation between the effective degree of fall for the inclusive cross sections at large $p_\perp$ (taking in account the factor $F(x_\perp)$) and the increasing character of the associative multiplicity relative to $p_\perp$.
This correlation depends on the range of $x_\perp$.

It is of interest to turn back to (Fig.V.3) where the correlation of such type is drown, i.e., an effective dependence of degree of power decreases on the interval of the variable X_1.

In particular it may serve as some evidence of the possibility to describe the inclusive spectra at large $p_\perp$ not by a single term of the type (5.2), but by their superposition with various N. The value of N for the given region $x_\perp$ decreases with increasing energy.

Note, that from the theoretical point of view the appearance of an effective dependence of the degree of the value $x_\perp$ may be interpreted as a result of the competitions of several different dynamic mechanisms.

In the language of quarks, the process if inclusive production of meson M with large $p_\perp$ is determined by one of the exclusive interactions.

$$
\begin{array}{ll}
Mq \to Mq & N = 2n_M = 4 \\
q\bar{q} \to M\bar{M} & \text{—"—} \\
\bar{q}B \to M2q & N = n_M + n_B = 5 \\
q\,2q \to MB & \text{—"—} \\
\ldots & \ldots
\end{array}
$$

The extrapolation of the found dependence into the region gives n $\approx$ 2 that would correspond to the point-like behaviour of a cross section and could be defined by the elementary process $qq \to qq$ with the subsequent fragmentation of quarks into real particles. A direct experimental examination of a dependence of the associated multiplicity on the particle transverse momentum is thus of great interest for test of theoretical models.

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Problems for Discussion to Lec.V

1. Find the relations between different variables

$$\vec{p},\ p_\perp,\ t,\ \theta,\ \eta \equiv \ln tg\,\theta/2,$$
$$y,\ x_\perp.$$

2. How do you understand the following conditions in the terms of variables pointed out in 1): large angles (or mom. transfers);

fixed angles (or mom transfers).

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