объединенный
институт
ядерных
исследований
дубна



E2-88-783

A.A.Bogoljubskaja, Y.L.Kalinovsky*, W.Kallies,
V.N.Pervushin

# GLUON DYNAMICS IN QCD WITH INCREASING POTENTIAL

Submitted to "Physics Letters B"

* Gomel Politechnical Institute, USSR

1988

1. The present potential phenomenology for light and heavy quarkonia definitely says that in the limit of infinitely heavy quarks only the Coulomb potential gives a contribution [1]. On the other hand, the increasing potential is very useful to describe the spectrum of massless (light) quarks and the spontaneous breaking of chiral symmetry (i.e. the origin of the structure masses of the quarks) [2,3,4].

At the same time, it is well known that the lattice QCD gives the increasing potential in the Wilson approach, based on the assumption of infinite quark masses [5]. So, in this sense one can say that the lattice and potential ideology are contradictory. The situation is redoubled by the vagueness in the description of gluons in both cases.

The authors think that the success of the potential phenomenology for light quarks justifies the attempt to use it for gluons, too, because from the phenomenological point of view the massless gluons do not differ from the light quarks.

The Hamiltonian for the potential interaction of transversal gluons is easy to write down if one replaces in the potential quark model the quark current by the conserving total color current

$$q^{+}\frac{\lambda^{a}}{2}q \longrightarrow J^{a}_{tot} = \left( q^{+}\frac{\lambda^{a}}{2}q + f^{abc}E^{b}_{i}A^{c}_{i} \right). \quad (1)$$

In the purely gluonic sector we obtain the following Hamiltonian

$$H = \int d^{3}x \left[ \frac{1}{2}\left(E^{a}_{i}(x)\right)^{2} + \frac{1}{2}\left(\partial_{i}A^{a}_{j}(x)\right)^{2} \right] + \quad (2)$$

$$+ \frac{1}{2} f^{abe} f^{ecd} \int d^{3}x\, d^{3}y\, E^{a}_{i}(x)\, A^{b}_{i}(x)\, V_{R}(x-y)\, E^{c}_{j}(y)\, A^{d}_{j}(y).$$

For simplicity we consider the oscillator potential [3] in (2)

$$V_{R}(r) = V_{0}\, r^{2}, \qquad V_{0} = (234\ \mathrm{MeV})^{3}.$$

The fields $E^{a}_{i}$ and $A^{a}_{i}$ have the following decomposition over creation and annihilation operators $a^{b(\pm)}_{r}(k)$

$$E^{b}_{j} = i\int \frac{d^{3}k}{(2\pi)^{3/2}} \sqrt{\frac{\omega(k)}{2}} \left\{ \exp\left[i(\omega(k)t - kx)\right] e^{r}_{j} a^{b(+)}_{r}(k) - \exp\left[-i(\omega(k)t + kx)\right] e^{r}_{j} a^{b(-)}_{r}(k) \right\}, \quad (3)$$

$$A^{b}_{j} = \int \frac{d^{3}k}{(2\pi)^{3/2}} \sqrt{\frac{1}{2\omega(k)}} \left\{ \exp\left[i(\omega(k)t - kx)\right] e^{r}_{j} a^{b(+)}_{r}(k) + \exp\left[-i(\omega(k)t + kx)\right] e^{r}_{j} a^{b(-)}_{r}(k) \right\}.$$



Here $k_{j}e^{r}_{j} = 0$, $e^{r}_{i}e^{r}_{j} = \delta_{ij} - \frac{k_{i}k_{j}}{k^{2}}$, $k = |k|$; the operators satisfy the commutator relations

$$\left[a^{b(-)}_{r}(k), a^{c(+)}_{r'}(q)\right] = \delta^{bc}\delta_{rr'}\delta(k-q),$$

$$\left[a^{b(\pm)}_{r}(k), a^{c(\pm)}_{r'}(q)\right] = 0,$$

and the single particle-energy $\omega(k)$ is defined as the expectation value of the Hamiltonian (2) between the one-gluon states $|b,r,k>_{g}$ with the quantum numbers b,r and momentum k

$$_{g}\langle b',r',k'|H|b,r,k\rangle_{g} = \omega(k)\,\delta(k-k')\,\delta^{bb'} \quad (4)$$

with $\quad |b,r,k\rangle_{g} = a^{b(+)}_{r}(k)|0\rangle.$

After the substitution of (3) into (2) expression (4) can be rewritten as the following equation for $\omega(k)$

$$\frac{\omega(k)}{2} + \frac{k^{2}}{2\omega(k)} - V_{0}N_{c}\left\{ \left[ \frac{1}{2\omega(k)}\frac{d\omega(k)}{dk} \right]^{2} - \frac{1}{k^{2}} \right\} = \omega(k), \quad (5)$$

where the left-hand side corresponds to the three terms of the Hamiltonian (2). To obtain the solution of (5) two numerical methods are used : the "shooting" [6] and the Runge-Kutta-Gill methods [7]. Both give similar results ( the solution is shown in Fig. 1 ).

In dimensionless variables the asymptotic behaviour is the following

$$\underline{\omega}(\underline{k}) \xrightarrow[k \to 0]{} \frac{2}{\underline{k}^{2}}$$

$$\text{with} \begin{pmatrix} \underline{\omega} \\ \underline{k} \end{pmatrix} = (N_{c}V_{0})^{-1/3} \begin{pmatrix} \omega \\ k \end{pmatrix}. \quad (6)$$

$$\underline{\omega}(\underline{k}) \xrightarrow[k \to \infty]{} \underline{k}$$

Thus the gluons effectively acquire the structure mass depending on the momentum ( $m_g(k^2) = \sqrt{\omega^2(k) - k^2}$ ) and such that $m(0) = \infty$ (by analogy with the massless quarks [3] ). It seems that all increasing potentials give this effect if one defines the effective mass as $m_{eff} = \omega(k=0)$.

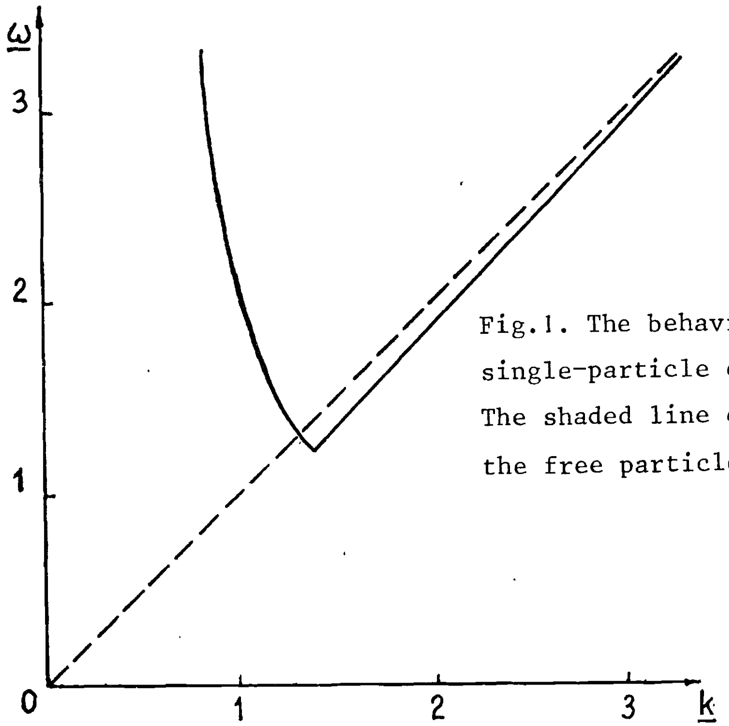


Fig.1. The behaviour of the single-particle energy $\omega(k)$. The shaded line corresponds to the free particle case.

Now let us consider the simplest bound state of two gluons : the scalar ( spinless ) glueball. It is a linear combination of the two-particle eigenstates of the Hamiltonian (2)

$$H|b,r,k;c,s,-k\rangle_{gg} = M|b,r,k;c,s,-k\rangle_{gg} ,$$



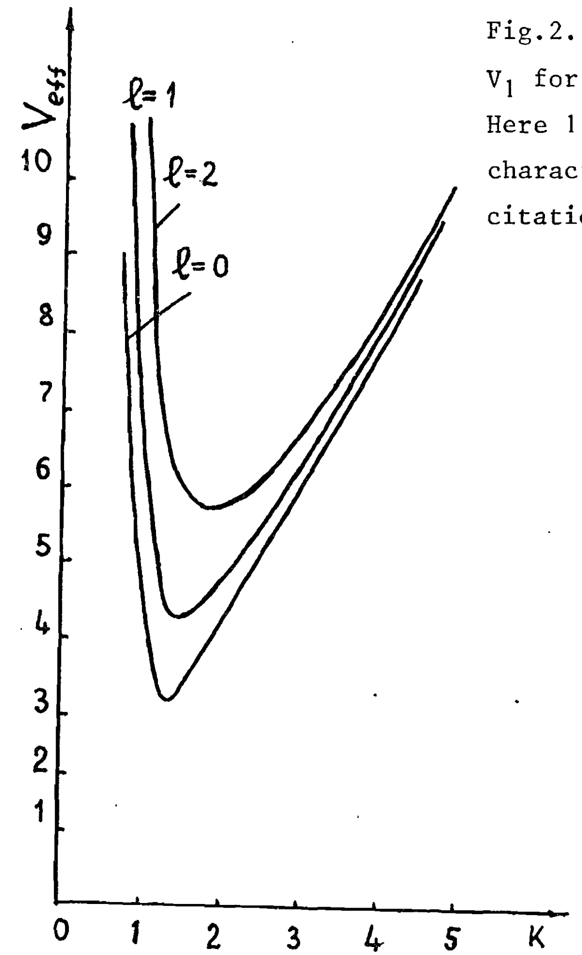


Fig.2. The effective potential $V_l$ for the spinless glueball. Here $l$ is the orbital momentum characterizing the orbital excitation of the glueball.

where M is the glueball mass and the state is defined by the action of the creation operators on the vacuum

$$|b,r,k;c,s,-k\rangle_{gg} = a_r^{b(+)}(k)\, a_s^{c(+)}(-k)\, |0\rangle .$$

For these states the spectral equation has the form

$$M\phi_G(k_o,k) = 2\omega(k)\phi_G(k_o,k) - \frac{1}{2}N_c V_o \left\{ \left[\frac{1}{\omega(k)}\frac{d\omega(k)}{dk}\right]^2 - \frac{4}{k^2} + 2\Delta_k \right\} \phi_G(k_o,k) \qquad (7)$$

with

$$\Delta_k = \frac{\delta^2}{\delta k^2}, \quad k = |k|, \quad \phi_G(k_o,k) = \sum_{b,r} |b,r,k;b,r,-k>.$$

$\phi_G$ is the wave function of the glueball G. The substituion of the standard decomposition of $\phi_G$ into (7) gives for $y_l = k\,\psi_l(k)$ the "radial" equation

$$\frac{d^2 y_l(k)}{dk^2} + \left[ M - V_l(r) \right] y_l(k) = 0.$$

Here $V_l(k)$ is the effective potential ( see Fig. 2 ) with the quantum number l of the orbital momentum. The masses M and the "radial" wave functions $\psi_l(k)$ were found numerically by the

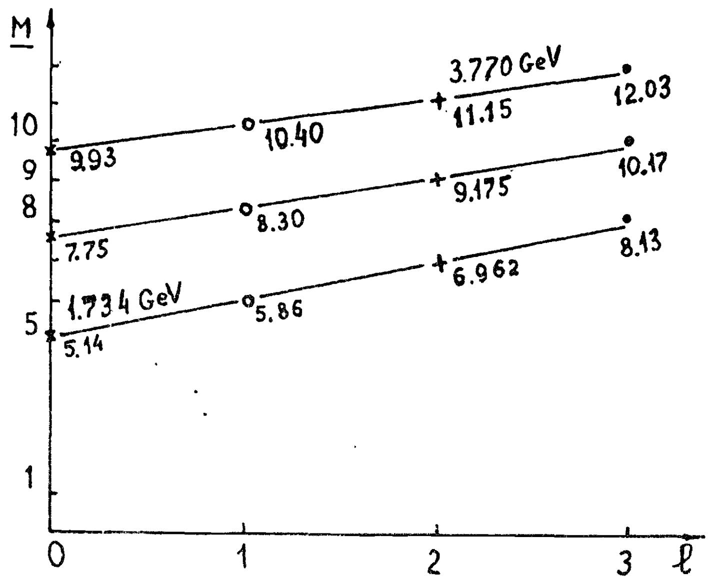


Fig. 3. The numerical values for the glueball masses. They are in the region expected up-to-date [8].



"shooting" method [6] and are shown in Figs. 3 and 4 . One can see that the values for the glueball masses are in the region expected up-to-date [8].

We see that the increasing potential makes massive massless color particles, i.e. it has infrared-regularizing properties.

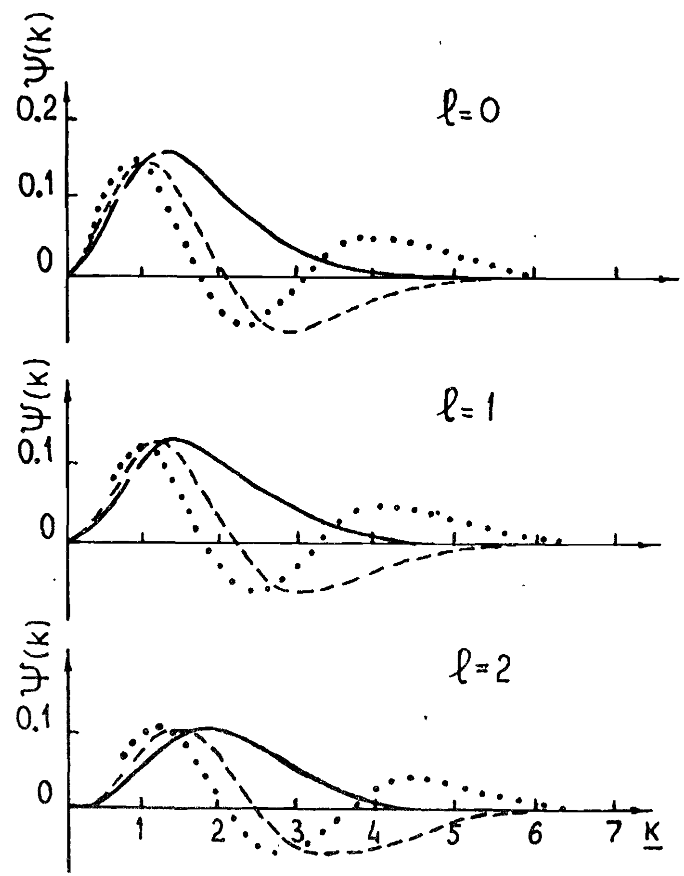


Fig.4. The "radial" wave functions of the glueball for l=0,1,2 in dimensionless variables.

2.Consequently, one can use the increasing potential as the infrared redefinition of QCD in the radiative gauge because the latter is closely connected with the potential approach. In the radiative gauge the infrared-indetermined Hamiltonian is

$$H[g^2 D] = \int d^3x \left[\frac{1}{2}(E_i^a)^2 + \frac{1}{4}(F_{ij}^a(x))^2 + \bar{q}(x)(i\gamma_k\nabla_k + \hat{m}_o)q(x) + \right.$$

$$\left. + \frac{g^2}{2} J_{tot}^a(x)\int d^3y\, D^{ab}(x-y|A)J_{tot}^b(y)\right], \qquad (8)$$

where

$$F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g\, f^{abc} A_i^b A_j^c$$

$$J_{tot}^a = q^+ \frac{\lambda}{2} q + f^{abc} E_i^b A_i^c,$$

$\nabla_k$ is a covariant derivative

$$\nabla_k = \partial_k + g \frac{A_k^a \lambda^a}{2i}$$

and the function $D^{ab}(x-y|A)$ is the solution of the following equation

$$\left[(\nabla_i \partial_i) \frac{1}{\partial^2}(\nabla_j \partial_j)\right]^{ab} D^{bc}(x-y) = \delta(x-y)\delta^{ac}$$

$$\nabla_i^{ab} = \delta^{ab}\partial_i + g\, f^{abc} A_i^c \cdot$$

There exists an opinion that for the infrared redefinition one has to use the formula of the asymptotical freedom. Inapplicability of the perturbation theory in the region of small $Q^2$ follows from the extrapolation of this formula to small $Q^2$. The procedure of this extrapolation is not theoretically proved. From the phenomenological point of view there are other ways possible to redefine the Hamiltonian. Only two requirements on them must be taken into account: they must be self-consistent and reproduce the formula of the asymptotical freedom in the region in which it is strongly proved.

The results obtained above show the following possibility of redefining the infrared behaviour of the Hamiltonian (8)

$$H\,[g^2 D(r|A)] \longrightarrow H\,[V_R(r) + g^2 D(r|A)]\,.$$

In this case the perturbation theory in $g^2$ has to be formulated in terms of quasiparticles ( quarks and gluons with nonzero structure masses ). It is easy to see that the Green function of the transversal gluon

$$D_{ij}^{mod}(k_o,\underline{k}) = (\delta_{ij} - \underline{k}_i \underline{k}_j/\underline{k}^2)\frac{1}{k_o^2 - \omega^2(\underline{k}) - i\varepsilon} \qquad (9)$$

vanishes in the region of small $\underline{k}^2$ and changes into the standard parton Green function for large momenta $\underline{k}$( $k \geq 300$ MeV ).

The asymptotical behaviour of (6) in the propagator of (9) eliminates all infrared divergences and modifies the formula of asymptotic freedom in the region of small momenta. To compare the modified formula with the experimental data, it

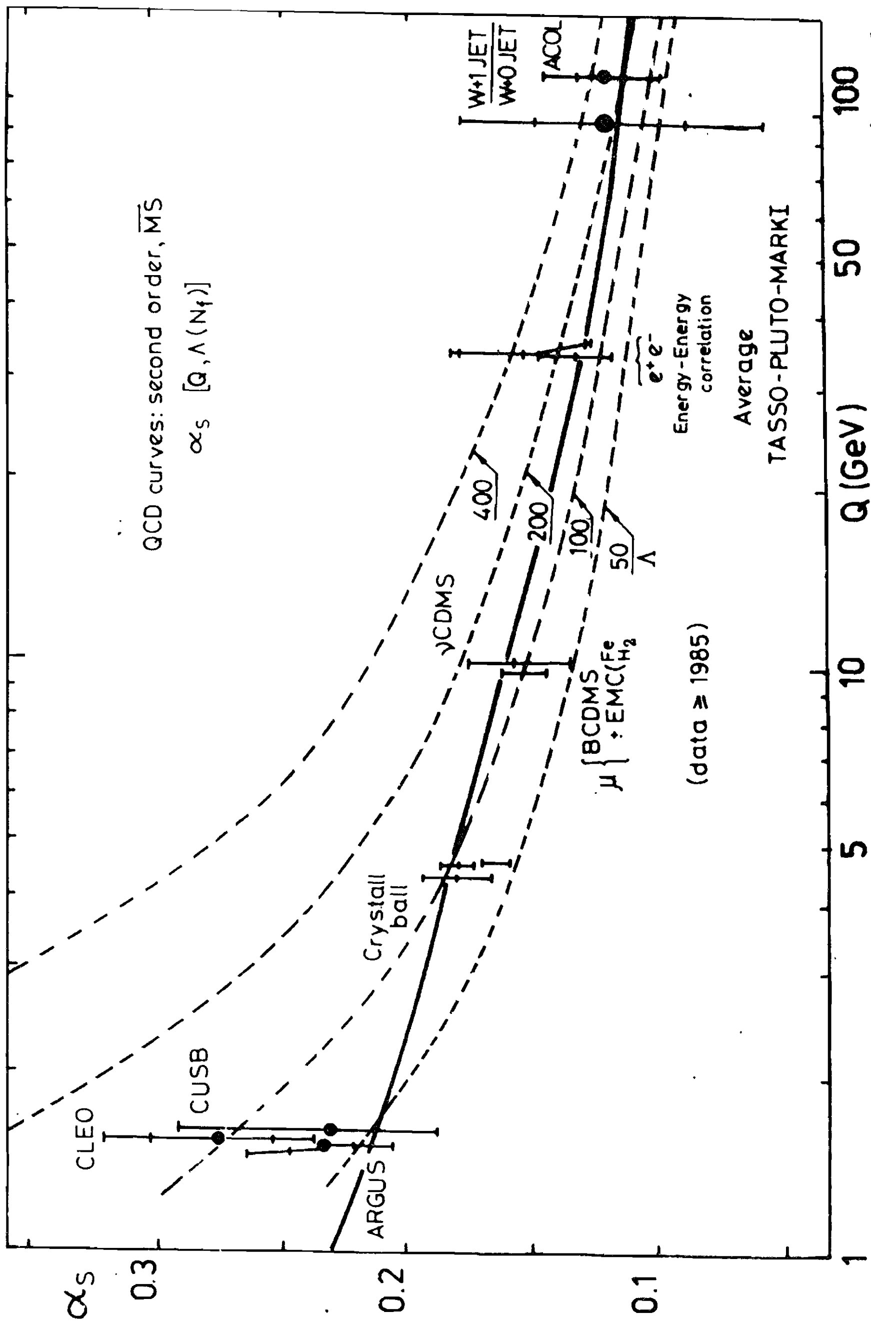


Fig.5. The dependence of the strong coupling constant α on Q. Given are the experimental and theoretical values (shaded lines) obtained by using the standard asymptotic freedom formula. The thick line corresponds to the modified formula for $\alpha_{eff}$ .



is enough to estimate it with a one-loop diagram and propagator (9). The contribution of that one-loop diagram is defined by the modified polarization operator

$$\Pi^{mod}(Q^2) = \frac{i}{\pi^2} \int d^4q\, D^{mod}(q+Q)\, D^{mod}(q), \qquad D^{mod} = \frac{1}{q_0^2 - \omega^2(q)}$$

which, with taking into account the asymptotic behaviour for $\omega(k)$ (6), gives the infrared modified running coupling constant having a finite limit

$$\alpha_s^{mod} = \frac{1}{\beta\,[1 + \ln(\frac{4N_c\, V_0^{1/3}}{\Lambda})^2]} \approx 0.2, \qquad \beta = \frac{11}{4\pi} .$$

This modified running constant $\alpha_s^{mod}$ $(Q^2)$ is in the whole region smaller than $\alpha_s^{mod}$ (0) $\lesssim$ 0.2 and therefore one can use the perturbation theory for all transfer momenta $Q^2$ ( see Fig.5 ).

In this approach it seems better to work with the parameters $\alpha_s^{mod}(0)$ and $V_0$ and not with the parameter Λ ( the parameter Λ is a function of $\alpha_s^{mod}(0)$ and $V_0$).

The authors thank A.V.Efremov, V.G.Kadyshevsky and D.V.Volkov for helpful discussions.

REFERENCES

1.A.A. Bykov, I.M. Dremin, A.V. Leonidov, Usp. Fiz. Nauk, 143 (1984) 3.

2.S.L.Adler and A.C. Davis, Nucl. Phys.,B224 (1984) 469.

3.A. Le Yaouanc, L. Oliver, O. Pene, J.-C. Raunal, Phys. Rev., D29 (1984) 1233; ibid. D31 (1985) 137.

4.I.V.Amirchanov et al., JINR Preprint P11-88-506, Dubna (1988).

5.R. Petronzio, in: Proceedings of XXIV. Int. Conf. on High Energy Physics, Münich, 1988.

6.N. S. Bachvalov, Numerical Methods, vol.1 (Nauka, Moscow, 1975) 548.

7.M. Abramowitz and A. Stegun, Handbook of Mathematical Functions (National Bureau of Standards,1964) 896.

8. A. Le Yaouanc, L. Oliver, O. Pene and J.-C. Raynal,Preprint LPTHE Orsay 88/21 (1988).

Received by Publishing Department
on November 1, 1988.



Боголюбская А.А. и др. E2-88-783
Динамика глюонов в КХД с растущим потенциалом

Делается обобщение потенциальной феноменологии легких кварков на глюоны. Выводятся и решаются численно уравнения на спектр глюбола и глюонов. Предлагается инфракрасное доопределение гамильтониана КХД с помощью растущего потенциала, которое ведет к теории возмущений с малой константой связи в области низких энергий и делает согласованной потенциальную модель с КХД.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

**Препринт Объединенного института ядерных исследований. Дубна 1988**

Bogoljubskaja A.A. et al. E2-88-783
Gluon Dynamics in QCD with Increasing Potential

The potential phenomenological experience of light quarks is generalized to gluons. Equations describing the glueball and gluon spectra are obtained and numerically solved. The proposed infrared modification of the QCD Hamiltonian by using an increasing potential leads to the perturbation theory with a small coupling constant in the low energy region and makes the potential model self-consistent with QCD.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

**Preprint of the Joint Institute for Nuclear Research. Dubna 1988**