объединенный
институт
ядерных
исследований
дубна

E2-88-78

V.N.Pervushin, W.Kallies, Nguyen Suan Han[1],
N.A.Sarikov[2]

PHENOMENOLOGY OF CHROMOSTATICS

Submitted to "ЯФ"

[1] Moscow State University
[2] INP Uz.SSR Academy of Sciences

1988

I. Introduction

At present a great number of phenomenological models for hadrons exists. All of them in a way are based on the QCD but each of them is not connected with the others. For example, the model most fruitful for the description of the spectroscopy of (heavy and light) quarkonia is the nonrelativistic potential one /1/ which cannot be used to describe the hadron interactions.

In this paper we propose a phenomenological model based on QCD containing the nonrelativistic hadron spectroscopy, dual-resonance amplitudes, chiral Lagrangians and in the limit of high energies turning into the quark-parton phenomenology. A model of that type is useful not only for the investigation of different transitional regions of energy but can be used as a prototype of the future theory in which an arbitrary hadron process may be calculated within a given accuracy (in total analogy with QED).

The main idea of the present paper consists in the application within the QCD of the experience of QED to the bound state problem. Recall that in QED the bound states are constructed by the expansion of the theory on the spatial components of the gauge field over the exact solution defined by the temporal one, i.e. by the Coulomb field. Let us call this perturbation theory "physical" (PPT). The PPT supposes the quantization only of the physical degrees of freedom (in contrast to the Dirac method where all components of the field are quantized).

Usually, one thinks that for the restoration of the relativistic-covariance of the bound states it is necessary to bring all components (A_0, A_i) of the field in equal conditions. There are two ways to do this: using the relativistic gauge or summing up the whole perturbation theory. But the latter cannot be accomplished and each transition to another relativistic gauge mixes nonperturbative bound effects with those coming from the perturbative radiative corrections and this is noncalculable, too /2/ (instead of a finite number of diagrams one has an infinite number of diagrams even in the lowest orders). To describe the interactions practically, it is desirable to understand how one can get relativistic-covariant bound-state wave functions in each order of the perturbation theory. At present there is no clear understanding of this problem even in QED /3/.

Concrete calculations have shown /4/ that the covariant and self-consistent description of bound states in the PPT are achieved by an appropriate choice of the time-like quantization vector. The vector is fixed by the following physical reason: it must be such a vector that the Coulomb field $(A_0 = n_\mu A_\mu)$ moves together with the bound state forming by it $(n_\mu \parallel \mathcal{P}_\mu)$. (In all other cases the relativistic dispersion law $\mathcal{P}^2 = \mu^2$, where μ is the mass of the bound state, is breaking /5/). In this way the relativistic-covariance is formally re-established. On the other hand, the covariance as a transformation property is achieved by taking account of additional diagrams induced by nonlocal Lorentz transformations of the fields in the "minimal" quantization scheme that is equivalent to the PPT /4/.

Another constructive idea to confine the phenomenology is to use nonnormalized solutions for the time component A_0 which in the given quantization scheme is not quantum and is not limited by the normalization condition. Such solutions produce changes in the Coulomb potential in the infinitesimal vicinity of the zero transfer momentum and the phenomenological parameters of that modification are defined by the quarkonia spectroscopy which plays the role of the Coulomb experiment for this case. It is natural to name such an infrared modification of the Coulomb law the phenomenological chromostatics.

To show the self-consistency of that approach, it is necessary to find out (by direct and exact calculations for the propagators of quarks and gluons which interact by the infrared potential) that the phenomenological chromostatics removes all infrared divergences of the perturbation theory written in terms of quasiparticles and that the coupling constant is small enough in the whole transfer-momentum region. Additionally, it is useful to verify that the lowest order of PPT contains different phenomenological models of hadron interactions at low-energies and there is a continuous transition to the parton model and the QCD phenomenology of small distances with the formula of asymptotic freedom.

The aim of the present paper is to give a symmary of this program. In section 2 the PPT and the basic hypothesis of the chromostatics are described. In section 3 calculated are the Green functions

of the quasiparticles and the effective coupling constant. In section 4 the spectrum of mesons and their interactions are considered.

2. Physical perturbation theory

The formulation of the perturbation theory in QCD (as the expansion on the spatial components of the gauge (gluon) field over the exact solution of the equation for the temporal component) is connected with the quantization method in which only physical degrees of freedom are quantized. A straightforward account of this quantization method, namely of the "minimal" one was done in ref. /4/. This method was formulated starting with the requirement that the relativistic transformation properties of classical and quantum fields coincide.

As the Hamiltonian of the theory, the gauge-invariant Belinfante tensor is taken

$$H_{QCD} = \int d^3x \left[\frac{1}{2} F_{0i}^{a\,2} + \frac{1}{4} F_{ij}^{a\,2} + \bar{q}\left(i\gamma_i \partial_i + m_0 + \gamma_i \hat{A}_i\right) q \right], \quad \hat{A}_i = e\,\frac{\lambda^a A_i^a}{2i},$$

which on the solutions of the Gauss equation may be expressed only in terms of physical variables as gauge-invariant functionals (A^T, q^T) of the initial fields (A, q). In the lowest order of the perturbation theory H_{QCD} has the form

$$H_{QCD} = \int d^3x \left\{ \frac{1}{2} (\dot{A}_i^{Ta})^2 + \frac{1}{4} F_{ij}^a (A_i)^2 + \bar{q}^T (i\gamma_i \partial_i + m_0) q^T + \right.$$
$$\left. + j_i^{Ta} A_i^{Ta} - \frac{e^2}{2} J_0^{Ta} \left(\frac{1}{\partial_k^2} J_0^{Ta} \right) + \cdots \right\}, \tag{1}$$

where

$$J_0^{Ta}(x) = j_{0(q)}^{Ta}(x) + j_{0(A)}^{Ta}(x),$$
$$j_{0(q)}^{Ta}(x) = j_{0(q)}^{a}(x) = \bar{q}^T \gamma_0 \frac{\lambda^a}{2} q^T,$$
$$j_{0(A)}^{Ta}(x) = f^{abc} \dot{A}_i^{Tb} A_i^{Tc} \quad ; \quad E_i^T = \dot{A}_i^T \; ; \quad A^T \text{ and } q^T$$

are functionals of the initial fields:

$$\left.\begin{array}{l}\hat{A}_i^T[A] = U[A](\hat{A}_i + \partial_i) U[A]^{-1}, \\ q^T[A,q] = U[A]\, q,\end{array}\right\} \quad U[A] = \exp\left(\frac{1}{\partial_k^2}\partial_i \hat{A}_i\right). \tag{2}$$

Exactly these functionals due to their Lorentz-transformation properties are compared to quantum fields $\hat{A}^T$ and q^T. Note, that the functionals (2) differ from the transversal variables $(\partial_i A_i = 0)$ in the Dirac method in their transformation properties under Lorentz transformations and in containing an extra physical information /4/. For example, in QCD, where the stationary gauge transformations (as the mapping of a 3-dimensional space R_3 into SU(3)-group) have non-trivial topological properties, the functionals in (1) are defined up to a phase factor degenerated in the topological index of the mapping. As a result of the degeneracy the physical fields as factors of the sources in the generating functional differ from the "undressed" field used in the diagrams of the perturbation theory. The removal of the degeneration for the physical fields makes all colour Green functions and the creation amplitudes vanish for colour particles due to the destructive interference of the phase factors of that degeneration /6,7/. Therefore we shall only consider "bare" propagators of the perturbation theory and colourless bound states.

Taking into account the fact that the Hamiltonian (2) is defined by the exact solution of the equation for the time component and the solution does not obey the normalization condition, we can redefine the expressions $(\frac{1}{\partial_k^2} J_0^T)(x)$ in (2) within the solutions of the equations $\partial_k^2 (\frac{1}{\partial_k^2} J_0^T)(x) = J_0^T(x)$ so that the effective Hamiltonian should remain translation-invariant.

One can see that the above redefinition of $\left(\frac{1}{\partial_k^2} J_0^T\right)(x)$ is given by

$$e^2\left(\frac{1}{\partial^2} J_0^T\right)(x) = \int d^3y \left[-\frac{\alpha}{|\vec{x}-\vec{y}|} - 2V_0\, \vec{y}(\vec{x}-\vec{y})\right] J_0^T(y), \alpha = \frac{e^2}{4\pi},$$

the second term disappears under the action of the Laplace operator and the substitution of the expression into the Hamiltonian (1) leads to a translation-invariant potential (owing to the integration symmetry in $\vec{x}$ and $\vec{y}$):

$$H_{int} = -\frac{1}{2}\int d^3x\, d^3y\, J_0^T(x)\, J_0^T(y)\, V(|\vec{x}-\vec{y}|), \tag{4}$$

$$V(r) = -\frac{\alpha}{r} + V_0 r^2, \quad r = |\vec{x}-\vec{y}|. \tag{5}$$

The parameters of this potential will be fixed from the spectroscopy of mesons /1,5/

$$V_0^{1/3} \simeq 250 \text{ МэВ} \; ; \; \alpha \simeq 0,2 \, . \tag{6}$$

3. Green functions for quarks and gluons and the coupling constant

For light quarks ($m_L \ll V_0^{1/3}$) and massless gluons their infrared behaviour must only be defined by the chromostatic Hamiltonian with the oscillator potential:

$$H_{QCS}[q] = \int d^3x\, \bar{q}^T(i\gamma_i\partial_i + m_L) q^T - \frac{V_0}{2}\int d^3x\, d^3y\, j_{0(q)}^{Ta}(x)(\vec{x}-\vec{y})^2 j_{0(q)}^{Ta}(y) \tag{7}$$

$$H_{QCS}[A] = \int d^3x \left[\frac{1}{2}(\dot{A}_i^{Ta})^2 + \frac{1}{2}(\varepsilon_{ijk}\partial_j A_k^{Ta})^2 - \frac{1}{2}V_0 \int d^3x\, d^3y\, j_{0(A)}^{Ta}(x)(\vec{x}-\vec{y})^2 j_{0(A)}^{Ta}(y)\right]. \tag{8}$$

The Green function for the quark was found in ref. /5/

$$G_\Sigma(p) = \frac{1}{\not{p} - \Sigma(p)} \equiv \gamma_0\left[\frac{\Lambda_{(+)}(p_i)}{p_0 - E(|\vec{p}|) + i\varepsilon} + \frac{\Lambda_{-}(p_i)}{p_0 + E(|\vec{p}|) - i\varepsilon}\right], \tag{9}$$

where

$$\Lambda_\pm(p_i) = \frac{1}{2} S(p_i)\left[1 \pm \gamma_0\right] S(p_i)^{-1},$$
$$S(p_i) = \exp\left\{\frac{\gamma_i p_i}{|\vec{p}|}\left[\frac{\varphi(|\vec{p}|)}{2} + \frac{\pi}{4}\right]\right\}, \quad i = 1,2,3 \tag{10}$$

are the projective operators on states with positive and negative energies, $\Sigma(p_i)$ is the self-energy operator obeying the Schwinger--Dyson equation

$$\Sigma(p_i) = m_L + \int (dq)\, \underset{\sim}{V}(|\vec{p}-\vec{q}|)\gamma_0 G_\Sigma(q)\gamma_0, \quad (dq) = \frac{i\, d^4q}{(2\pi)^4}, \tag{11}$$

$$\underset{\sim}{V}(|\vec{q}|)=\frac{N_c^2-1}{2N_c}V_0\left[(2\pi)^3\left(\frac{\partial}{\partial\vec{p}}\right)^2\delta^3(\vec{p})\right],\quad N_c=3,$$

that after the substitution (10) reduces to a differential equation for the function $\varphi(p)$ of the sine-Gordon type

$$(\underline{p}^2\varphi')'=2\underline{p}^3\sin\varphi(p)-\sin 2\varphi(p)-2m_L\underline{p}^2\cos\varphi(p),$$

$$\underline{E}(\underline{p})=\underline{p}\sin\varphi(p)-\underline{p}^2\cos\varphi(p)-\frac{1}{2}(\varphi')^2+m_L\sin\varphi(\underline{p}),$$

$$\varphi'=\frac{d}{dp}\varphi(p),\quad p=|\vec{p}|,\quad \underline{p}=\left(\frac{4}{3}V_0\right)^{-1/3}p,\quad \underline{E}=\left(\frac{4}{3}V_0\right)^{-1/3}E,$$

In ref./5/ for massless quarks it was shown that solutions with spontaneous chiral symmetry breaking

$$|\Sigma(p)|=\begin{cases}2\left(\frac{4}{3}V_0\right)^{1/3}, & p\to 0,\\ 0, & p\to\infty\end{cases}$$

are energetically preferable as compared with the trivial solution $\Sigma=0$. In the same way we consider the gluon propagator

$$D_{ij}(q_0,\vec{q})=\frac{1}{q_0^2-\omega(|\vec{q}|)^2-i\varepsilon}\left(\delta_{ij}-q_i\frac{1}{\vec{q}^2}q_j\right) \qquad (12)$$

with the one-particle energy $\omega(p)$ defined by the equation

$$\langle p_1|\,H_{QCS}[A]\,|p_2\rangle=\omega(p_1)\langle p_1|p_2\rangle,$$

or

$$\frac{\omega(p)^2-p^2}{2\omega(p)}+N_c\int\frac{d^3q}{(2\pi)^3}(2\pi)^3V_0\left(\frac{\partial}{\partial\vec{q}}\right)^2\delta^3(\vec{q})\,W(p+q|p)^2=\omega(p),$$

$$W(p|q)=\frac{1}{2}\left(\sqrt{\frac{\omega(p)}{\omega(q)}}+\sqrt{\frac{\omega(q)}{\omega(p)}}\right).$$

In dimensionalless variables

$$\underline{\omega}=\omega/\varkappa,\quad \underline{P}=P/\varkappa,\quad \varkappa=(N_cV_0)^{1/3},$$

we get the equation

$$\frac{\underline{\omega}^2-\underline{p}^2}{\underline{\omega}}=\left(\frac{\underline{\omega}'}{\underline{\omega}}\right)^2,\quad \left(\underline{\omega}'=\frac{d}{d\underline{p}}\underline{\omega}(\underline{p})\right). \qquad (13)$$

In the limit of large momenta ($\underline{p}\to\infty$, $\underline{\omega}'\to 1$) we have

$$\underline{\omega}(\underline{p})^2-\underline{p}^2=\underline{\mu}^2(\underline{p})=\frac{1}{\underline{p}}\to 0,$$

and we can see that the gluon mass vanishes as the quark mass, too. In the limit of small momenta equation (13) becomes a nonlinear first-order equation which can be solved

$$\underline{\mu}'(\underline{p})=\pm\,\underline{\mu}(\underline{p})^{3/2},\quad \underline{\mu}(\underline{p}=0)=\underline{\mu}_0\neq 0,$$

and which needs nontrivial boundary conditions while zero boundary conditions lead only to an energy $\omega(p)$ identically equal to zero.

As a result, the gluon and quark become massive and their structure masses vanish at large momenta.

On the other hand, the existence of the structure mass for the gluon leads to the infrared modification of the asymptotic freedom formula

$$\alpha(Q^2)=\frac{1}{\beta\ln\frac{Q^2}{\Lambda^2}},\quad \beta=\frac{1}{4\pi}\left(11-\frac{2}{3}N_f\right). \qquad (14)$$

To find a rough estimate of this modification at small enough Q^2, one can change $\ln Q^2/\Lambda^2$ by the expression for a massive loop

$$\ln\frac{Q^2}{\Lambda^2}\Rightarrow\int_0^1 dx\ln\frac{\mu_0^2+x(1-x)Q^2}{x(1-x)\Lambda^2}\Bigg|_{\mu_0^2=0}=\ln\frac{Q^2}{\Lambda^2}.$$

The asymptotic freedom formula takes the form

$$\alpha^{inf}(Q^2)=\frac{1}{\beta\int_0^1 dx\ln\frac{\mu_0^2+x(1-x)Q^2}{x(1-x)\Lambda^2}}\Bigg|_{Q^2\simeq 0}=\frac{1}{\beta\left(\ln\frac{\mu_0}{\Lambda^2}+2\right)} \qquad (15)$$

and, unlike (14), has no singularities in Q^2 in the whole Euclidean region, but at $Q^2\simeq 0$ defines the effective coupling constant of "quasiquarks" (9) and "quasigluons" (12).

For $\alpha^{inf}(0)=0.2$ and $\Lambda=100$ MeV we get the following estimate for the gluon mass: $\mu_0\simeq 700$ MeV. Formula (15) does not contradict the available experimental data and explains the tendency of the experimental change of the parameter Λ (the larger Q^2 the larger Λ) (see fig. 1).

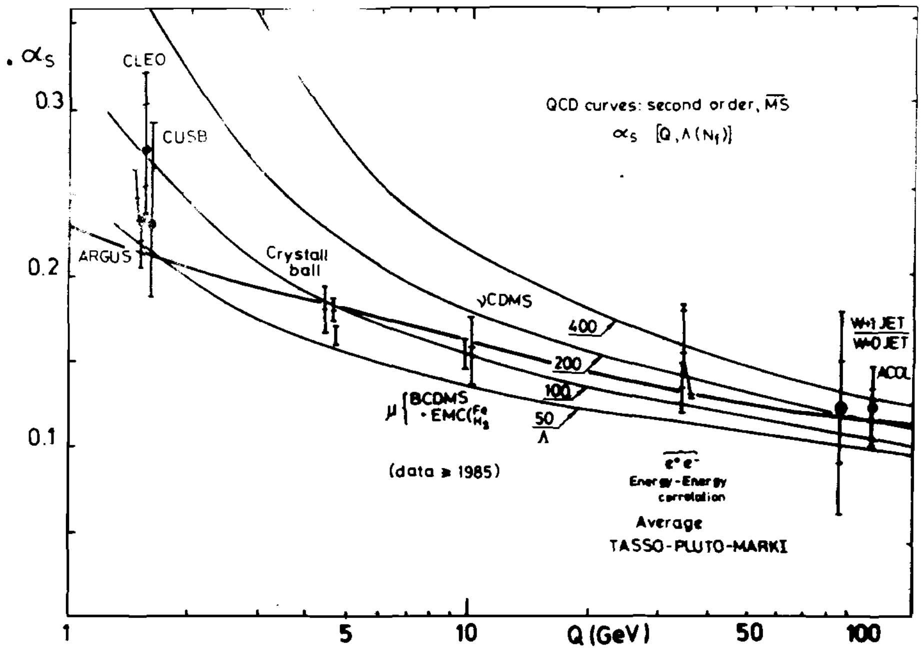

Fig. 1. The dependence of the strong coupling constant on α . Given are experimental and theoretical values (thin lines) obtained by using the standard asymptotic freedom formula. The thick line corresponds to the modified formula values with $\alpha(Q^2=0)=0.24$ when $N_f=3$, $\Lambda=110$ MeV and $\mu_0=700$ MeV. (The authors would like to thank A.Roussarie (Saclay) for supplying with the data collected by him).

A more precise parametrization in the region of low energies seems to be done not with the parameter Λ , but with the coupling constant, as in QED, and the structure quark and gluon masses which eliminate, as shown above, the infrared catastrophe of the PPT.

Therefore one can expect that the radiative corrections to the spectrum of the bound states for the theory (2)-(15) will be small enough $\left(\sim O\left(\alpha(0)/4\pi\right)\right)$.

4. Bilocal chiral Lagrangians

As noted above, the relativistic description of bound states is achieved in each order of PPT in the QED not by the relativization of the interaction potential but by the choice of the reference frame and boundary conditions. The choice must be made so that the "nonrelativistic" potential should move together with the particles, a bound state of which it forms. In full analogy, we get the following system of the Schwinger-Dyson and Bethe-Salpeter equations

$$\Sigma_{1,2}(p_\perp) = m_{1,2} + \int (dk)\, \underset{\sim}{V}(k_\perp - p_\perp)\, \gamma_{\parallel\mu}\, G_\Sigma(k)\, \gamma_{\parallel\mu},$$

$$G_\Sigma(p) = \frac{1}{\not p - \Sigma(p_\perp)}, \quad (dk) = \frac{i\,d^4k}{(2\pi)^4}, \quad k_\parallel = \mathcal{P}\frac{\mathcal{P}\cdot k}{\mathcal{P}^2}, \quad k_\perp = k - k_\parallel, \tag{16}$$

and

$$G^{-1}_{\Sigma_1}(k+\eta_1\mathcal{P})\, \underset{\sim}{\chi}_{\mathcal{P}}(k)\, G^{-1}_{\Sigma_2}(k+\eta_2\mathcal{P}) = \int (dq)\, \underset{\sim}{V}(k_\perp - q_\perp)\, \gamma_{\parallel\mu}\, \underset{\sim}{\chi}_{\mathcal{P}}(q)\, \gamma_{\parallel\mu}, \tag{17}$$

where $\mathcal{P}$ denotes the total momentum of the two-particle system, η_1 and η_2 are factors depending on the masses m_1 and m_2 of the particles $\eta_1+\eta_2=1$ /10/, $\underset{\sim}{V}(p)$ is the potential defined by the temporal component of the field

$$\underset{\sim}{V}(|\vec{p}|) = \frac{N_c^2-1}{2N_c}\left[-\frac{4\pi\alpha}{|\vec{p}|^2} + (2\pi)^3 V_0 \left(\frac{\partial}{\partial\vec{p}}\right)^2 \delta^3(\vec{p})\right]. \tag{18}$$

For the system (16), (17) the dispersion law $\mathcal{P}^2=\mu_H^2$ is fulfilled and the system is free from the defects of the relativistic equations used in ref./5/, where the "Coulomb" field and the bound state move in different directions. In the rest frame $\mathcal{P}_\mu=(\mu_H,0,0,0)$ the equations (16) and (17) coincide with the equations in ref./5/, and for massless quarks (for which the oscillator potential dominates) all results of these works (where the mass spectrum for quarkonia with the pion as a Goldstone particle was found) remain correct.

The four-quark chromostatics (7) with the operator

$$K(\mathcal{P},q) = \underset{\sim}{V}(q_\perp)\, \gamma_{\parallel\mu} \otimes \gamma_{\parallel\mu} \tag{19}$$

leads not only to the bound state spectrum but to its coherent interaction, too. It is useful to formulate this interaction in terms of bilocal meson fields (see, for example,/8/) going from quark fields to bilocal meson fields with the action

$$S[M] = \mathrm{Tr}\left[-\tfrac{1}{2}(M-m_0)\,K^{-1}(M-m_0) - i\ln(i\not\partial - M)\right], \tag{20}$$

where Tr means the trace over all discrete and continuous variables $Tr\, M^2 = \int d^4x\, d^4y\; tr\; M(x,y)\, M(y,x)$; $M(x,y)$ describes the bilocal meson field and $M = M^a \lambda^a/2$ is the corresponding matrix with flavour and Lorentz indices.

In terms of bilocal variables the Schwinger-Dyson equation (16) comes out from (20) as a "stationary" point

$$\frac{\delta S[M]}{\delta M}\Big|_{M=\Sigma} = 0 \Rightarrow \Sigma = m_0 + iKG_\Sigma ,$$

the expansion around which $(M = \Sigma + m')$ defines the free action in bilocal fields and their interactions.

$$S_{free} = Tr\left[-\frac{1}{2}(m' K^{-1} m') + \frac{i}{2}(G_\Sigma m')^2\right], \tag{21}$$

$$S_{int} = i\, Tr\left[\sum_{n=3}^{\infty} \frac{(-1)^n}{n}(G_\Sigma m')^n\right]. \tag{22}$$

The free action leads to the Bethe-Salpeter equation for the vertex function

$$m' = i K G_\Sigma m' G_\Sigma ,$$

the solution of which can be expanded over the solutions (17)

$$m'(x,y) = m'(z|X) = iK(z)\int \frac{d^4\mathcal{P}}{(2\pi)^4} \sum_H \delta^+(\mathcal{P}^2 - \mu_H^2)\lambda_H \times$$
$$\times\left\{e^{-i\mathcal{P}X} \chi^H_{\mathcal{P}}(z_\perp) a_H(\mathcal{P}) + e^{i\mathcal{P}X}\tilde{\chi}^H_{\mathcal{P}}(z_\perp) a^+_H(\mathcal{P})\right\}, \tag{23}$$

where

$$K(z)\chi^H_{\mathcal{P}}(z_\perp) = \delta(z_{\parallel})\, V(z_\perp)\gamma_\mu \int \frac{d^4q}{(2\pi)^4} e^{iq_\perp z_\perp} \underset{\sim}{\chi}^H_{\mathcal{P}}(q)\, \gamma_{\parallel},$$

and $(x+y)/2 = X$, $x - y = z$ are the absolute and relative coordinates. λ_H is the normalization factor, a^+_H and a_H are, respectively, the creation and annihilation operators, $\chi^H_{\mathcal{P}}(q)$ means the solution of the Bethe-Salpeter equation and μ_H is the hadron mass.

The expressions (21)-(23) are useful to investigate the low--energy interactions of hadrons, i.e. the limit $V_0 \to \infty$, for which the wave function of the bound state as a consequence of the normalization condition reduces to a δ-function. For example,

$$\chi(z_\perp) \sim V_0^{1/2}\, e^{-|z_\perp^2|/V_0^{2/3}} \longrightarrow \delta^3(z_\perp).$$

In that limit the bilocal field in terms of the relative coordinate behaves as a δ-function (see (23))

$$m'(x,y) \sim \delta^4(x-y).$$

That limit can be got from the very beginning if the potential $K(z)$ is itself taken as a δ-function. It is well-known that the four--quark interaction with that sort of a potential was first considered as a starting point to formulate the spontaneous chiral symmetry breaking. Now there exists a large number of papers /10,11/ showing the transformation of a bilocal Lagrangian with a δ-like potential into a chiral Lagrangian.

In ref. /8,10,12/ it has been shown that the perturbative theory over bilocal fields with the propagator $D(x_1,x_2|x_3,x_4) = \overline{m'(x_1,x_2)\, m'(x_3,x_4)}$ containing an infinite number of resonances describes dual-resonance amplitudes.

To summarize, in the approach above the chiral Lagrangian is a low-energy limit of a generating functional for self-dual amplitudes (21) and (22) which describe the coherent interaction of hadrons in that region of small momenta where the form factors of hadrons are large /15/. With decreasing distances and increasing momenta the form factors (and together with them the coherent interactions) vanish and the Green functions of the quasiparticles become the usual ones of "bare" particles of the standard QCD perturbation theory used for the description of the quark-parton phenomenology of deep-inelastic processes.

The essence of this phenomenology consists in the following: the sum over all probabilities of final hadron states of processes $e^+e^- \to$ hadrons, etc. is described as an imaginary part of the corresponding elastic amplitude, constructed from quark-gluon diagrams of the QCD perturbation theory

$$\sum_H T_{iH} T^*_{Hj} = 2\, Jm\,(T_{ij})_{Pert.\,Theory}\,. \tag{24}$$

This relation is called the quark-hadron duality and is used in the local form to define quantum numbers of the quarks and gluons. That definition makes essentially use of the perturbation theory in the Minkowski space where the condition $Jm\, T_{ij} \neq 0$ for $|\vec{P}| \sim P_0 \to \infty$ contradicts the experimental nonobservation of the quarks in the same energy region.

In the considered PPT (for which, as stressed above, the creation amplitudes of colour particles (C) equal zero) this contradiction can be explained with the help of the unitary relation $SS^+ = 1$, $S = 1 + iT$

$$\sum_H T_{iH} T^*_{Hj} + \sum_c T_{ic} T^*_{cj} = 2\, Jm\, T_{ij} \quad , (i,j \notin c).$$

In QCD the left-hand side of the equality is fully fixed by the hadron channels (because $T_{ic} = 0$), at the same time, in QED the main contribution comes from the second term (H denotes the bound states and now C - describes charged particles and photons).

In both (in QED and QCD) the perturbation theory for the right-hand side is correct in the energy region far from resonances. Just in this energy region of the Minkowski space the local quark-hadron duality is used.

Therefore, the PPT for gauge fields can explain why in QED and QCD different experiments to define the quantum number of the fundamental particles are used.

Conclusion

Following the QED we attempted to answer the following questions. What is the perturbation theory for bound states? What are the conditions for practical calculations within a given accuracy and for relativistic-covariance in each order of the perturbation theory? How should one modify this perturbation theory, staying in its framework, to reproduce characteristic features of the hadron physics: hadron spectrum, spontaneous chiral symmetry breaking, dual-resonance amplitudes, parton model and confinement?

To answer these questions, we treat quantization of gauge fields as a physical phenomenon needing a theoretical and phenomenological experiment but not as a strictly defined and formulated mathematical calculus.

We have seen that the standard strictly formulated scheme of quantization of gauge theories based on the Dirac method is not adequate to the relativistic description of bound states even in QED. This scheme is neither useful for calculations nor relativistic-covariant.

The answers to the above questions are in some or other way connected with the formulation of the physical perturbation theory (on spatial components of the gauge field). That perturbation theory goes out of the used standard method of quantization and the mechanism of dimensional transmutation associated with the asymptotic freedom formula. We would remind that in the region of small transfer momenta this formula turns into the hypothesis of a large coupling constant which not only eliminates the infrared divergences, but also leads to theoretical uncontrolled phenomenology. We have replaced this phenomenology by the phenomenology of chromostatics and have shown that PPT based on the chromostatics contains the asymptotic freedom formula, too (in that region where it was strongly derived).

On the other hand, the above-formulated PPT differs from the nonrelativistic potential quarkonia model /5/ because it allows at the same time on equal status to describe heavy quarks ($m_H \gg V_0^{1/3}$) and gluons, gives a concrete way to calculate radiative corrections and describes the relativistic interactions of hadrons in agreement with popular phenomenological models.

The authors thank I.M.Dremin, A.V.Efremov, V.G.Kadyshevsky, H.Leutwyler and Y.A.Smorodinsky for discussions. One of the authors (V.P.) would like to thank W.Kummer, A.D.Faddeev, E.S.Fradkin and D.V.Volkov for fruitful discussions of some aspects of the problem.

References

1. А.А.Быков, И.М.Дремин, А.В.Леонидов, УФН, I43 (I984) с. 3.
2. S.Love, Ann.Phys., 113 (1978) p. 153.
3. D.Heckathor, Nucl.Phys., B156 (1979), p. 328;
   G.S.Adkins, Phys.Rev., D27 (1983) p. 184.
4. Nguyen Suan Han, V.N.Pervushin, Mod.Phys.Lett. A., 2 (1987) p. 400.
   Илиева Н.П., Нгуен Суан Хан, В.Н.Первушин, ЯФ, 45 (I987) с.II69.
5. A.Le Yaonanc, L.Oliver, P.Pene, I.C.Raunal. Phys.Rev., D29 (1984) p. 1233; Phys.Rev., D31 (1985) p. 137.
6. V.N.Pervushin, Riv.Nuovo Cimento, 8 (1985) N10, p. 1.
7. Р.А.Азимов, В.Н.Первушин, ТМФ, 67 (I986) с. 349.
8. В.Н.Первушин, Х.Райнхардт, Д.Эберт, ЭЧАЯ, IO (I979) с.III4.
9. Y.Nambu, G.Jona-Lasinio, Phys.Rev., 122 (1961) p. 345;
   В.Г.Вакс, А.И.Ларкин. ЖЭТФ, т. 40 (I96I) с.282;
   Б.А.Арбузов, А.Н.Тавхелидзе, Р.Н.Фаустов, Докл.АН СССР I96I, I39, с. 345.

10. H.Kleinert, Proc. Erice Lecture 1976, p. 289.
11. M.K.Volkov, Ann.Phys., 157 (1984) p. 285;
D.Ebert., N.Reinhardt, Nucl.Phys., B271 (1986) p. 188.
12. G. t'Hooft, Nucl.Phys., B72 (1974) p. 461;
M.B.Einhorn, Phys.Rev., D14 (1976) p. 1451.

Received by Publishing Department
on January 29, 1988.