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исследований
дубна

E2-88-706

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PHASE SPACE IN SINGULAR THEORIES

Submitted to "International Journal
of Modern Physics A"

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1988

I. Introduction

In describing the elementary particle dynamics in the framework of the field theory, singular or degenerated Lagrangians[1-4] are mainly used. Usually, the singularity of a Lagrangian is caused by the invariance of the action with respect to the transformations of field functions which depend on an arbitrary function of the coordinates and time. Such transformations defined in the tangent bundle are often called the gauge transformations; and the corresponding theories, gauge theories.

A general method to obtain the Hamiltonian dynamics for singular Lagrangians was described by Dirac[1].

The equation of motion for an arbitrary dynamic variable g has the following form in Dirac's approach[1]:

$$\dot{g} = \{g, H_T\}; \quad H_T = H_c + u_k^1 \varphi_k^1, \quad k = 1, \ldots, m. \qquad (1)$$

Here H_c is the canonical Hamiltonian, u_k^1 are arbitrary multipliers, φ_k^1 are the primary constraints of the 1st class. Summation runs over the repeated indices here and throughout the paper.

The function H_T is called the total Hamiltonian. Note that in this paper we are interested in gauge degrees of freedom and in problems related to gauge fixation; so, we shall assume that there are only relations of the 1st class in the theory. Th s assumption simplifies some formulae but the interpretation remains general (see Appendix A).

Primary constraints of the 1st class produce gauge transformations in the phase space. Secondary constraints of the 1st class can also produce gauge transformations. Dirac assumed that all constraints of the 1st class produced gauge transformations [1] and proposed to replace H_T by the generalised Hamiltonian:

$$H_E = H_T + u_k^{m_k} \varphi_k^{m_k} ; \quad m_k = 2, \dots, M_k ; \quad k = 1, \dots, m, \qquad (2)$$

where $u_k^{m_k}$ are arbitrary factors, $\varphi_k^{m_k}$ are secondary constraints, $M_k - 1$ is the maximum number of the secondary constraints obtained under the requirement for the k-th primary constraint being stationary.

Generally speaking, Dirac's assumption was wrong. There are examples where secondary constraints of the 1st class do not produce gauge transformations [3,5].

Dirac's iteration procedure provides no reasons for adding secondary constraints to the total Hamiltonian. The global and geometric generalization of Dirac's approach throws no light on this problem. Being a result of these algorithms, the total Hamiltonian describes dynamics of the system but it does not contain all gauge degrees of freedom. Therefore, it is often more convenient to employ the generalised Hamiltonian [3] in order to eliminate nonphysical degrees of freedom from the theory by using additional or gauge conditions.

A general method of applying gauge conditions within singular theories was proposed by Dirac [6]. Later on, this method was reproduced many times (e.g. see Ref. [7]). New limits

are imposed on the coordinates q and momenta p

$$\chi_i(q,p) \approx 0 ; \quad i = 1, \dots, \sum_{k=1}^{m} M_k \equiv I \qquad (3)$$

and the function $\chi_i(q,p)$ must obey the following conditions:

$$\det \| \{ \chi_i , \varphi_k^{m_k} \} \| \neq 0 ; \quad k = 1, \dots, m ; \quad m_k = 1, \dots, M_k , \qquad (4)$$

$$\{ \chi_i , \chi_{i'} \} = 0 ; \quad i, i' = 1, \dots, I . \qquad (5)$$

Note that for the functions χ_i these conditions are necessary but insufficient for being gauge conditions. This is due to the fact that constraints (3) together with equations of motion can lead to new relations of dynamic variables and physical degrees of freedom will be lost. Such examples are considered in Ref. [8].

It is easy to establish a relation between the functions $\chi_i(q,p)$ and Lagrange factors. The required stationarity of gauge conditions (3) yields

$$\dot{\chi}_i = \{ \chi_i , H_c \} + u_k^{m_k} \{ \chi_i , \varphi_k^{m_k} \} \approx 0 . \qquad (6)$$

Owing to condition (4), eq.(6) allows determination of multipliers $u_k^{m_k}$.

In the given paper we show how to construct the generators of gauge transformations at given singular Lagrangian and thereafter we find infinitesimal gauge transformations [9]. Further, using the construction method of the quasigroup finite elements [10], for singular systems we obtain the Hamiltonian equations of motion which contain the whole gauge freedom.

We establish the new limitation on the function χ_i which together with the relation (4) were the necessary and sufficient conditions for elimination of the gauge freedom.

2. Construction of gauge transformations for a given singular Lagrangian

For finding the gauge-transformation generators for a given singular Lagrangian we shall use the minimal action principle for mechanical systems. The first-class constraints must remain to be first-class quantities under the gauge transformations. Then the operator engendering the infinitesimal gauge transformations must itself be a quantity of the first class, the general form of which is given in the following way:

$$\Phi = \varepsilon_k^{m_k}\left(\frac{\partial\varphi_k^{m_k}}{\partial q_i}\frac{\partial}{\partial p_i} - \frac{\partial\varphi_k^{m_k}}{\partial p_i}\frac{\partial}{\partial q_i}\right) \equiv \varepsilon_k^{m_k}\{\varphi_k^{m_k},\ \}. \tag{7}$$

$$k = 1,\dots,m; \quad m_k = 1,\dots,M_k.$$

Here q_i and p_i are coordinates and momenta, $\varepsilon_k^{m_k}$ are infinitesimal arbitrary functions, and $\varphi_k^{m_k}$ are first-class constraints, moreover φ_k^1 are primary constraints; and if $m_k > 1$, secondary constraints. The $M_k - 1$ is the maximum number of secondary constraints obtained from the requirement of stationarity of φ_k^1.

Let us now proceed in the following way: using the operators (7) we can construct infinitesimal transformations of coordinates and momenta:

$$\delta q(t) = \Phi q(t), \quad \delta p(t) = \Phi p(t) \tag{8}$$

and require that under such transformations the action remains invariant:

$$\delta S[q,p,u_k^1] = \int dt\, \delta(p_i\dot q_i - H_T) = 0. \tag{9}$$

The relation (9) gives the restrictions on the functions $\varepsilon_k^{m_k}$. In view of this, infinitesimal transformations (8) will correspond to such changes of the coordinates and momenta, at which the physical state of the system remains unchanged.

The assumption that all constraints are first-class constraints leads to the equations:

$$\{\varphi_k^i, \varphi_j^\ell\} = f_{kjk'}^{i\ell\ell'}\varphi_{k'}^{\ell'}, \quad k,j = 1,\dots,m, \tag{10}$$

$$\{H_c, \varphi_k^i\} = g_{kj}^{i\ell}\varphi_j^\ell = \Psi_k^i, \quad i = 1,\dots,M_k;\ \ell = 1,\dots,M_j.$$

Here the coefficients $f_{kjk'}^{i\ell\ell'}$ and $g_{kj}^{i\ell}$ may be functions of q and p. Inserting (8) and (7) into (9) we find:

$$\begin{aligned}\delta S &= \int dt\,[\dot q_i\delta p_i - \dot p_i\delta q_i + \frac{d}{dt}(p_i\delta q_i) - \delta H_c \\ &\quad - \delta u_k^1\varphi_k^1 - u_k^1\delta\varphi_k^1] \\ &= \int dt\,[\dot\varepsilon_k^{m_k}\varphi_k^{m_k} - \varepsilon_k^{m_k}\{H_c,\varphi_k^{m_k}\} - \varphi_k^1\delta u_k^1 \\ &\quad - u_k^1\varepsilon_{k'}^{m_k}\{\varphi_k^1,\varphi_{k'}^{m_k}\} + \frac{d}{dt}\Big(p_i\varepsilon_k^{m_k}\frac{\partial\varphi_k^{m_k}}{\partial p_i} - \varepsilon_k^{m_k}\varphi_k^{m_k}\Big)].\end{aligned} \tag{11}$$

Up to this step our consideration was of a general character. Now we make one suggestion, namely, we require that the Poisson bracket of the primary constraints with the first-class constraints be equal to a linear combination of the primary constraints*)

$$\{\varphi_k^1, \varphi_{k'}^{m_{k'}}\} = f_{kk'i}^{1 m_{k'} 1} \varphi_i^1 . \tag{12}$$

The requirement $\delta S = 0$ means that the sum of the coefficients of the primary and secondary constraints each turns into zero in a strong sense. Collecting the coefficients of secondary constraints and taking into account (12), from (11) we get

$$\dot{\varepsilon}_k^{m_k} - \varepsilon_{k'}^{i} g_{k'k}^{i m_k} = 0 , \; m_k > 1 . \tag{13}$$

From this equation it is seen that because of the presence of $g_{k'k}^{i m_k}$ in it, in the general case $\varepsilon_k^{m_k}$ is also a function of q and p. The relation (13) gives sufficient limitations on the functions $\varepsilon_k^{m_k}$ in order that the operators (7) give such changes of coordinates and momenta, at which the physical state of the system is not changed. For each value of the index k in (13) we choose a maximum value $M_k = max\{m_k\}$ and consider $\varepsilon_k^{M_k}$ as an arbitrary function of time $\delta\lambda(t)$. Then all other $\varepsilon_k^{m_k}$ will depend on $\delta\lambda(t)$, q and p. The form of this dependence is determined by formula (13).

*) We do not know any example when (12) is not fulfilled.

Now we rewrite formula (8) in a form more suitable for our purpose. For this, we use the identity

$$\delta\lambda^{(k)}(t) = (-1)^k \int \delta\lambda(t') \partial_{t'}^{(k)} \delta(t'-t) dt' ; \; \delta\lambda^{(k)}(t) \equiv \frac{d^k}{dt^k} \delta\lambda(t). \tag{14}$$

Then, we find the following expression for the operator Φ from formulae (7) and (14) with allowance for notation (10):

$$\Phi = (-1)^{M_k - m_k} \int \delta\lambda_{k'}(t') \{\Psi_{k'}^{m_{k'}}, \;\} \partial_{t'}^{(M_k - m_k)} \delta(t'-t) dt\, dt' . \tag{15}$$

We substitute this operator into formulae (8) and find increments of coordinates and momenta:

$$\delta q_j(t) = \int \delta\lambda_k(t') Q_{kl}(t',t'') \frac{\delta}{\delta q_l(t'')} dt' dt'' q_j(t) = \int Q_{kj} \delta\lambda_k dt' , \tag{16}$$

$$\delta p_j(t) = \int \delta\lambda_k(t') P_{kl}(t',t'') \frac{\delta}{\delta p_l(t'')} dt' dt'' p_j(t) = \int P_{kj} \delta\lambda_k dt' .$$

The following notation is introduced here

$$Q_{kl}(t,t') = (-1)^{M_k - m_k + 1} \frac{\partial \Psi_k^{m_k}}{\partial p_l} \partial_t^{(M_k - m_k)} \delta(t-t') , \tag{17}$$

$$P_{kl}(t,t') = (-1)^{M_k - m_k} \frac{\partial \Psi_k^{m_k}}{\partial q_l} \partial_t^{(M_k - m_k)} \delta(t-t') .$$

Actually, the operators Q_{kl} and P_{kl} are the generators of gauge transformations. Using these generators on the basis of the results obtained in ref.[10] for quasigroups, one can (in many ways) reconstruct finite gauge transformations.

These transformations may formally be written as

$$q_j'(t)=G\,q_j(t);\quad p_j'(t)=G\,p_j(t),$$
$$G=\exp\Big\{\int\lambda_k(t')\Big[Q_{ki}(t',t'')\frac{\delta}{\delta q_i(t'')}+P_{ki}(t',t'')\frac{\delta}{\delta p_i(t'')}\Big]dt'dt''\Big\}\ . \tag{18}$$

Actually, this solves the problem of construction of finite gauge transformations at a given singular Lagrangian.

Further, knowing the explicit form of coordinate transformations (16,17) in the tangent bundle we can construct Noether identities in the following form:

$$\int\frac{\delta S}{\delta q_i(t)}Q_{ki}(t',t)\,dt'=0;\quad i=1,\dots,n;\ k=1,\dots,m,$$

where

$$\frac{\delta S}{\delta q_i(t)}=\frac{\partial\mathcal{L}}{\partial q_i(t)}-\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot q_i(t)}\ .$$

3. Elimination of gauge freedom

To simplify further presentation, we shall stick to the following notation (as in formula (1,2)): $\varphi_k^1=\varphi_k$, $\varphi_k^{m_k}=\Phi_j;\ k=1,\dots,m;\ m_k=2,\dots,M_k;\ j=1,\dots,n=\sum_{k=1}^{m}(M_k-k);\quad u_k^1=\alpha_k\ ,\ u_k^{m_k}=\beta_j\ .$

Let us consider the time evolution of the system using the generalized Hamiltonian (2). We shall take an arbitrary dynamic variable g and see how it is expressed at the moment $t+\delta t$ assuming that $g(t)$ has a definite value.

According to Dirac [1], we have

$$g(t+\delta t)=g(t)+\dot g(t)\delta t=g(t)+\{g,H_E\}\,\delta t$$
$$=g(t)+(\{g,H_c\}+\alpha_k\{g,\varphi_k\}+\beta_j\{g,\Phi_j\})\,\delta t. \tag{19}$$

Let us take some other values for the coefficients α_k and β_j, e.g. α_k' and β_j'. This results in another value of $g(t+\delta t)$. We denote the difference of these two values by $\Delta g(t+\delta t)$ and write in the following way:

$$\Delta g(t+\delta t)=\delta t\big[(\alpha_k-\alpha_k')\{g,\varphi_k\}+(\beta_j-\beta_j')\{g,\Phi_j\}\big]. \tag{20}$$

On the other hand, we can choose a definite trajectory of the dynamical variable $g(t+\delta t)$ and act on it by the operator G from the expression (18). This will mean that the dynamic variable goes from one gauge to another (arbitrary). Subtracting the variable g from Gg, we obtain the gauge variation of the dynamic variable

$$\Delta g(t+\delta t)=Gg(t+\delta t)-g(t+\delta t). \tag{21}$$

On the basis of (20) and (21) we obtain the equation

$$(G-1)g(t+\delta t)=\delta t\big[(\alpha_k-\alpha_k')\{g,\varphi_k\}+(\beta_j-\beta_j')\{g,\Phi_j\}\big]. \tag{22}$$

Now let us discuss how one can use eq.(22) in the general case. Then, we shall give the corresponding examples. Naturally, we can always take the generalized coordinate q as g. Since the constraints φ_k and Φ_j are linearly and functionally independent, we can always find a situation when φ_k will contain at least one momentum variable, e.g. p_ℓ, which does not enter into Φ_j. Then, the term $\{q_\ell, \Phi_j\}$ in (22) reduces to zero for the variable q_ℓ. So we find the functinnal interdependence between gauge transformation parameters entering into formulae (18) and functions $\alpha_k - \alpha'_k$. Eq. (22) for the coordinate whose conjugate momentum is in Φ_j will connect the parameters λ_k from (18) with functions $\beta_j - \beta'_j$, $\alpha_k - \alpha'_k$ can also be included. Finally, we obtain that in the general case arbitrary fixation of the factors α_k and β_j in the generalized Hamiltonian may fail to correspond to any gauge. In other words, when choosing gauge constraints and using eq.(6) for fixation of the factors $u_k^{m_k}$, we must not break the relations between these factors as established in eq.(22). This is the only case when conditions for χ_i are the gauge constraints which, on one hand, fix the whole gauge freedom and, on the other hand, do not lead - together with the motion equation - to new constraints (relations). Thus, we have found the sufficient condition which allows functions χ_i obeying conditions (4) and (5) to be regarded as gauge functions.

4. Examples

To make it all clear, let us consider some simple examples. The first example is the chargeless electrodynamics.

The electrodynamics Lagrangian has the form:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

This theory has one primary constraint $\varphi \equiv \pi_0 \approx 0$ and one secondary constraint $\Phi \equiv \partial_i \pi^i \approx 0$ of the 1st class. The generalized Hamiltonian is defined by the following expression:

$$H_E = H_c + \int d^3x \,(\alpha \pi^0 + \beta\, \partial_i \pi^i),$$

where H_c is the canonical Hamiltonian.

Formula (7) in the field theory is generalized in the standard way

$$\delta A_\sigma(\vec{x},t) = \int d^3x\, \varepsilon_k^{m_k}(\vec{x},t) \Big[\frac{\delta \varphi_k^{m_k}}{\delta A_\mu(\vec{x},t)} \frac{\delta}{\delta \pi^\mu(\vec{x},t)} - \frac{\delta \varphi_k^{m_k}}{\delta \pi^\mu(\vec{x},t)} \frac{\delta}{\delta A_\mu(\vec{x},t)} \Big] A_\sigma(\vec{x},t). \tag{23}$$

Inserting the constraints $\varphi_1^1 \equiv \varphi$ and $\varphi_1^2 \equiv \Phi$, we obtain

$$\delta A_0 = \varepsilon_1^1(\vec{x},t), \tag{24}$$
$$\delta A_i = \partial_i \varepsilon_1^2(\vec{x},t).$$

From (13) we find $\varepsilon_1^1 = \dot{\varepsilon}_1^2$. Parametrizing ε_1^2 by an arbitrary function $\varepsilon(\vec{x},t)$, we finally obtain the well-known transformations

$$\delta A_\mu(\vec{x},t) = \partial_\mu \varepsilon(\vec{x},t).$$

Now we construct the operator Q defined by the relation (17):

$$Q_\mu(y,x) = \partial_\mu^y \delta(y-x).$$

Then, we find the operator G:

$$G=\exp\{-\int dy\,dz\,\lambda(y)\,\partial^y_\mu\,\delta(y-z)\frac{\delta}{\delta A_\nu(z)}\},$$

and finite gauge transformations

$$A'_\mu(x)=G A_\mu(x)=A_\mu(x)+\partial_\mu\lambda(x).$$

Replacing g in (22) by A_0 and then by A_i, we find the following relations :

$$\partial_0^2\lambda(x)\,\delta t=[\alpha'(x)-\alpha(x)]\,\delta t,$$

$$\partial_i\partial_0\lambda(x)\,\delta t=\partial_i\,[\beta'(x)-\beta(x)]\,\delta t.$$

Finally

$$\partial_0\,[\beta'(x)-\beta(x)]=\alpha'(x)-\alpha(x).$$

Now let us consider the model Lagrangian proposed in Ref.[11,12]:

$$\mathcal{L}=\frac{1}{2}[(\frac{d}{dt}-yT)\vec{x}]^2-V(\vec{x}^2),$$

$$T=\begin{pmatrix}0&1\\-1&0\end{pmatrix},\qquad \vec{x}=\begin{pmatrix}x_1\\x_2\end{pmatrix}.$$

A two-dimensional vector $\vec{x}$ and y are the generalized coordinates here. This model has one primary constraint $p_y\approx 0$ and one secondary constraint $\vec{p}\,T\vec{x}\approx 0$. It is easy to find the operator of finite gauge transformations

$$G=\exp\{\int dt'dt''\lambda(t')[\delta(t'-t'')x_1(t')\frac{\delta}{\delta x_2(t'')}-\delta(t'-t'')x_2(t')\frac{\delta}{\delta x_1(t'')}$$
$$-\delta'_{t'}(t'-t'')\frac{\delta}{\delta y(t'')}]\}.$$

Using this operator, we find gauge transformations

$$x_1'(t)=x_1(t)\cos\lambda(t)-x_2(t)\sin\lambda(t),$$

$$x_2'(t)=x_1(t)\sin\lambda(t)+x_2(t)\cos\lambda(t),$$

$$y'(t)=y(t)+\dot{\lambda}(t).$$

Formula (22) connecting the coefficients in the generalized Hamiltonian has the following form in this model:

$$\frac{d}{dt}[x_1\cos\lambda-x_2\sin\lambda]-\dot{x}_1=-(\beta'-\beta)x_2,$$

$$\ddot{\lambda}=\alpha'-\alpha.$$

These formulae allow determination of relations between the coefficients α and β.

In conclusion the authors express their gratitude to A.N.Tavkhelidze for the support and useful discussions. We are also thankful to V.V.Nesterenko, and N.B.Skachkov for discussions.

Appendix A

Here we consider the construction of the infinitesimal gauge transformations for spinor electrodynamics. The spinor degrees of freedom are presented in this model and the second class constraints are also arising here. Lagrangian has the form:

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\overline{\psi}\gamma_\mu(\partial_\mu-ieA_\mu)\psi-m\overline{\psi}\psi. \qquad \text{(A1)}$$

Here $A_\mu, \psi, \overline{\psi}$ are playing the role of the generalized coordinates. The generalized momenta are determined in the standard way:

$$\pi_\mu=\frac{\partial\mathcal{L}}{\partial\dot{A}_\mu}=F_{0\mu}\,;\; p_\psi=\frac{\partial\mathcal{L}}{\partial\dot{\psi}}=i\overline{\psi}\gamma_0\,;\; p_{\overline{\psi}}=\frac{\partial\mathcal{L}}{\partial\dot{\overline{\psi}}}=0. \qquad \text{(A2)}$$

From (A2) we can find the set of the primary constraints

$$\varphi_1^1=\pi_0\;;\quad \varphi_2^1=p_\psi-i\overline{\psi}\gamma_0\,;\quad \varphi_3^1=p_{\overline{\psi}}\,. \qquad \text{(A3)}$$

Canonical Hamiltonian has the form

$$H_c=\frac{1}{4}F_{ij}F^{ij}+\frac{1}{2}\pi^i\pi^i+\pi_i\partial_i A_0+iep_\psi A_0\psi$$
$$+i\overline{\psi}\gamma_i(\partial_i-ieA_i)\psi+m\overline{\psi}\psi. \qquad \text{(A4)}$$

Further we note, that in the presence of the spinor degrees of freedom we can use the Poisson brackets in the meaning of R.Casalbuoni[13].

Except the primary constraints (A3), there is one secondary constraint in the theory

$$\varphi_1^{2'}=\partial_i\pi^i+e\overline{\psi}\gamma_0\psi. \qquad \text{(A5)}$$

For separation of the constraints (A3) and (A5) into the first and second class constraints, we must construct linear combinations from them. In final we obtain, that the first class constraints are following

$$\varphi_1^1=\pi_0\,,\quad \varphi_1^2=\partial_i\pi^i-ie(p_\psi\psi+\overline{\psi}p_{\overline{\psi}})\,. \qquad \text{(A6)}$$

Other two will determine by the multipliers of the Lagrangian in the total Hamiltonian. Using (23) we find

$$\delta A_0=\varepsilon_1^1\,,\; \delta A_i=\partial_i\varepsilon_1^2\,,\quad \delta\psi=ie\varepsilon_1^2\psi\,,\quad \delta\overline{\psi}=-ie\overline{\psi}\varepsilon_1^2.$$

Expression (13) gives $\varepsilon_1^1=\dot{\varepsilon}_1^2$. At last, we obtain the well-known transformation rule:

$$\delta A_\mu=\partial_\mu\varepsilon\,,\quad \delta\psi=ie\varepsilon\psi\,,\quad \delta\overline{\psi}=-ie\varepsilon\overline{\psi},$$

where by ε we parametrized ε_1^2.

Appendix B

Here we generalized the method of construction of infinitesimal gauge transformations for Lagrangians with higher derivatives. For simplicity we restrict ourselves to the case when the Lagrangian consists only of second-order derivatives

$$\mathcal{L}(x,\dot{x},\ddot{x})\,,\quad \dot{x}=\frac{dx(t)}{dt}\,,\quad x=(x_1,\dots,x_n). \qquad \text{(B1)}$$

Canonical variables for such Lagrangians are determined as follows:

$$q_{1i}=x_i\,,\qquad q_{2i}=\dot{x}_i\,, \qquad \text{(B2)}$$
$$p_{1i}=\frac{\partial\mathcal{L}}{\partial\dot{x}_i}-\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\ddot{x}_i}\,,\quad p_{2i}=\frac{\partial\mathcal{L}}{\partial\ddot{x}_i}.$$

The Lagrangian (B1) is called singular if canonical variables satisfy the relations[14]

$$\varphi_k^1(q_1, q_2, p_1, p_2) = 0 , \quad k = 1, \ldots, m \tag{B3}$$

or, which is the same, rank $\|\lambda_{ij}\| = n-m$, where the matrix λ_{ij} is determined by

$$\lambda_{ij} = \frac{\partial^2 \mathcal{L}}{\partial \ddot{x}_i \partial \ddot{x}_j} . \tag{B4}$$

The canonical Hamiltonian of the theory is constructed by the Ostrogradsky[15] method

$$H_c = p_1 \dot{x} + p_2 \ddot{x} - \mathcal{L}(x, \dot{x}, \ddot{x}) , \tag{B5}$$

which will be a function only of canonical variables. Poisson brackets are determined in the standard way:

$$\{f, g\} = \frac{\partial f}{\partial q_{ik}} \frac{\partial g}{\partial p_{ik}} - \frac{\partial f}{\partial p_{ik}} \frac{\partial g}{\partial q_{ik}} . \tag{B6}$$

Then the equation of motion for dynamical variables will take a form completely similar to (1) with the canonical Hamiltonian (B5) and primary constraints (B3).

As before, secondary constraints are obtained by the Dirac iteration method. As far as we demand that all constraints are of the first order, the relations (10) hold valid.

The action for Lagrangians with second derivatives is written in the form:

$$S = \int dt \, [p_1 \dot{x} + p_2 \ddot{x} - H_T] . \tag{B7}$$

Then following the considerations analogous to section 2 for $\varepsilon_\alpha^{m_\alpha}$ coefficients entering into the definition of the operator $\Phi(\varepsilon_\alpha^{m_\alpha} \varphi_\alpha^{m_\alpha})$ (7) we again obtain relation (13).

Thus, our method of the construction of gauge transformations can be applied to the Lagrangians depending on coordinates and velocities, as well as to the Lagrangians with higher derivatives.

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Received by Publishing Department
on September 26, 1988.

Гогилидзе С.А. и др. E2-88-706
Фазовое пространство в сингулярных теориях

Излагается метод построения генераторов калибровочных преобразований в фазовом пространстве при заданном сингулярном лагранжиане. На основе этих генераторов формируются необходимые и достаточные условия непротиворечивости калибровочных функций. Полученные результаты анализируются на конкретных примерах.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Gogilidze S.A. et al. E2-88-706
Phase Space in Singular Theories

The method is formulated for constructing the generators of Gauge transformations in the phase space for a given singular Lagrangian. These generators are used to impose necessary and sufficient conditions on gauge functions. The results are analysed for particular examples.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988