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E2-88-682

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ON QED RADIATIVE CORRECTIONS AT HERA

1988

1. Introduction

The electron-proton storage ring HERA is devoted to the study of neutral and charged current lepton-nucleon interactions in an energy range where both photon and weak boson exchange are present with comparable strength in a space-like region: E_e=30 GeV, E_p= 820 GeV. Operation is planned to be started in 1990. The physics programme[1] includes both investigation of the so-called standard physics (electroweak theory and QCD) and a search for new phenomena. In that programme, an important role will play the study of deep inelastic scattering,

$$e^{\pm} + p \longrightarrow e^{\pm} + X, \overset{(-)}{\nu} + X. \qquad (1)$$

The analysis of reactions (1) is affected by electroweak radiative corrections (EWRC), among them the numerically large are QED-contributions due to photon bremsstrahlung. These either limit the kinematical range of meaningful measurements or, if they are not too large, have to be taken into account properly. EWRC to the neutral current reaction (1), including bremsstrahlung

$$e^{\pm} + p \longrightarrow e^{\pm} + X + \gamma , \qquad (2)$$

have been first calculated in[2]. Recently, new analyses[3,4] have been performed. Whereas the genuine weak loop contributions of[3] and[4] can be reconciled with each other, there are yet some non-negligible differences of the photon bremsstrahlung part. Here, we present our results for reaction (2) in a systematic way with emphasis on formal transparency.

2. Analytical results

The cross section (2) may be parametrized as follows:

$$\frac{d^2\sigma^{QED}}{dxdy} = \frac{2\pi\alpha^2\, Sx}{Q^4} \sum_{b=e,i,q}\ \sum_{B=\gamma,I,Z}\ \sum_{q,\bar{q}} C_b\, K(B)\,\cdot \left[V(B)\cdot R_b^V(B) + S_e\cdot S_q\cdot A(B)\cdot R_b^A(B)\right], \qquad (3)$$

$$C_b = \left\{Q_e^2,\ Q_eQ_q,\ Q_q^2\right\} \qquad \text{for } b=e,i,q, \qquad (4)$$

$$K(B) = \left\{Q_e^2\, Q_q^2\ ,\ 2|Q_eQ_q|\chi\ ,\ \chi^2\right\}, \qquad (5)$$

$$V(B) = \left\{1,\ (v_e+\lambda Q_e a_e)v_q\ ,\ \ (v_e^2+a_e^2 + 2\lambda Q_e v_e a_e)\ (v_q^2 + a_q^2)\right\}, \qquad (6)$$

$$A(B) = \left\{0,\ (a_e+\lambda Q_e v_e)a_q\ ,\ 2\left[2v_e a_e + \lambda Q_e(v_e^2+a_e^2)\cdot v_q\cdot a_q\right]\right\}, \qquad (7)$$

$$\text{for } B = \gamma, I, Z$$

$$S_{e,q} = +1\ (-1) \qquad \text{for particles (antiparticles)}, \qquad (8)$$

$$Q_e = \pm 1 \qquad \text{for } e^{\pm}\ , \qquad (9)$$

$$Q_q = \left\{+\tfrac{2}{3}\ ,\ -\tfrac{2}{3}\ ,\ -\tfrac{1}{3}\ ,\ +\tfrac{1}{3}\right\} \qquad \text{for } u,\ \bar{u},\ d,\ \bar{d}, \qquad (10)$$

$$v_{e,q} = 1 - 4\sin^2\theta_W\, |Q_{e,q}|\ , \qquad (11)$$

$$a_{e,q} = 1 \qquad (12)$$

$$\chi = \frac{G_\mu}{\sqrt{2}}\ \frac{M_Z^2}{8\pi\alpha}\ \frac{Q^2}{Q^2+M_Z^2}\ , \qquad (13)$$

$$Q^2 = S\cdot x\cdot y\ , \qquad y_1 = 1-y\ , \qquad (14)$$

and λ is the longitudinal polarization of the lepton beam. The $b=e,i,\ q$ indicates radiation from electron leg(e), quark leg(q) or their interference (i), and $B = \gamma$, I, Z means photon exchange (γ), Z-boson exchange (Z), their interference (I). The dynamics is contained in the functions $R_b^{V,A}(B)$ which have been determined analytically. Only 6 of the 15 different contributions are really independent:

$$R_b^{V,A}(B) = R_b\,(B;1,y_1) + S_b^{V,A}\cdot R_b(B;\ -y_1,-1), \qquad (15)$$

$$R_b(B;a,b) = S_b(B;a,b)\ f_q(x,Q^2) + \int_x^1\left[T_b(B;a,b)\frac{\tau_b f_q(zx,Q^2)-f_q(x,Q^2)}{z-1} + U_b(B;a,b)\ \frac{f_q(zx,Q^2)}{z}\right], \qquad (16)$$

$$S_e^V = S_q^V = S_i^A = -S_i^V = -S_e^A = -S_q^A = +1\ ,$$

$$\tau_e = z\ , \qquad \tau_i = \tau_q = 1. \qquad (17)$$

The following relations are fullfilled already for squared matrix elements:

$$R_q(\gamma) = R_q(I) = R_q(Z)\ , \qquad (18)$$

$$R_i(I) = \tfrac{1}{2}\left[R_i(\gamma) + R_i(Z)\right]. \qquad (19)$$

The proofs are given in Appendix A.

As a consequence, the remaining functions to be determined are:

$$R_e(\gamma)\ ,\ R_e(I),\ R_e(Z);\qquad R_i(\gamma),\ R_i(Z);\ R_q(\gamma). \qquad (20)$$

Explicit formulae are contained in Appendix B.

The three bremsstrahlung functions for photon exchange have been published in /4/. The complete analytic expression has been used in the Addendum of /4/ and corresponds exactly to the earlier, more lengthy result on which /2/ has been based.

3. Numerical results

Numerical results obtained with the above formulae together with the genuine weak loops and vacuum polarization as introduced in /4/ are shown in Fig. 1.

The radiative corrections for electron- and positron scattering are roughly equal for small Q^2 because the dominating QED-corrections are C-even: $R_e(\gamma)$. At large x (i.e. small y for fixed Q^2) the corrections become large and negative due to soft photon radiation. It is here where the soft photon exponentiation becomes important. We applied the procedure proposed in /5/ to the corrections $S_e(\gamma)=S_e(I)=S_e(Z)$ as

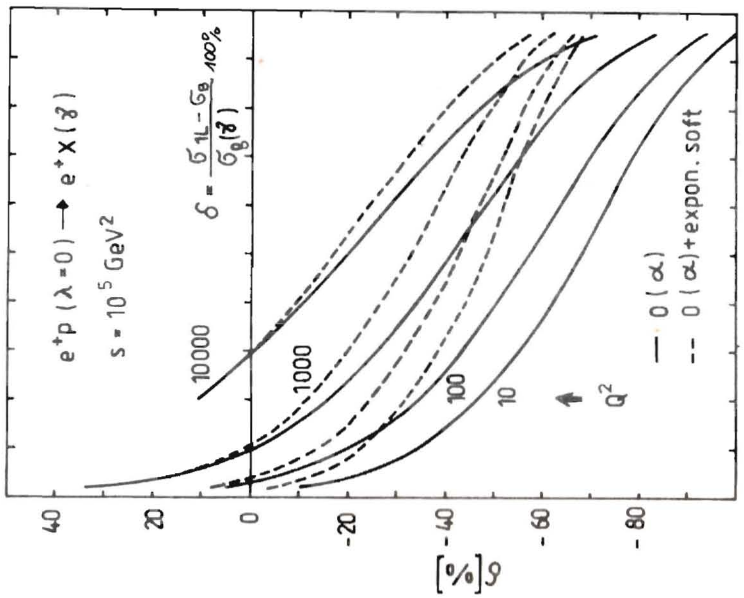


Fig. (1b): Total electroweak radiative corrections for e+p scattering. Parameters are defined in /4/.

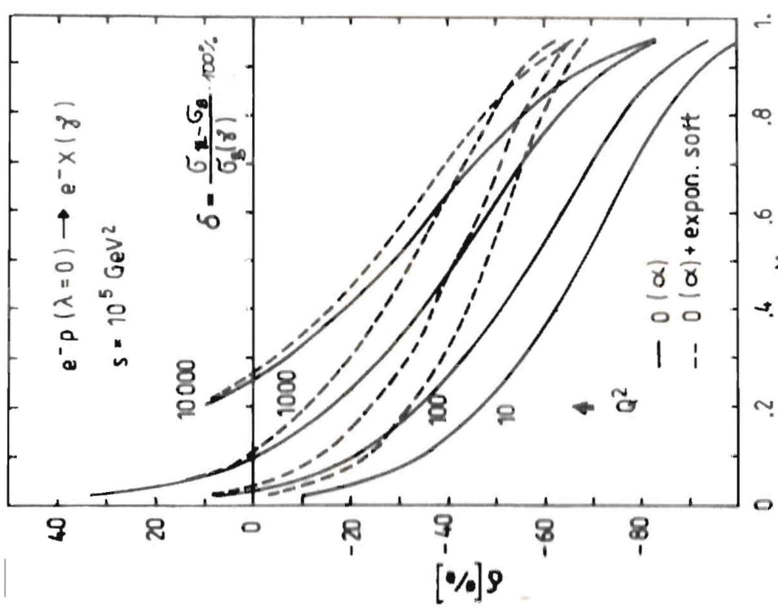


Fig. (1a): Total electroweak radiative corrections for e-p - scattering. Parameters are defined in /4/.

defined in Appendix B. Bremsstrahlung becomes also large but positive for very small x (i.e. large y for fixed Q^2). This is due to the radiation of hard, collinear photons which is concentrated there. In fact, from

$$y = 1 - \frac{E_e'}{E_e} \sin^2 \frac{\theta_e}{2} , \qquad (21)$$

where E_e (E_e') is the energy of the incoming (outgoing) electron and θ_e its scattering angle, one realizes that hard (soft) photon radiation prefers large (small) y and, for fixed Q^2, also small (large) x.

Fig. 1 also shows a pronounced dependence of the EWRC on Q^2 for given x. The origin of this behaviour is twofold. Part of the effect (at larger Q^2) is due to the normalization chosen for δ,

$$\delta = \frac{\sigma_{1L} - \sigma_B}{\sigma_B(\gamma)} \cdot 100\ \% \qquad (22)$$

From the point of view of EWRC it would be more natural to normalize the total cross-section to the full Born cross-section, including also the Z-boson exchange and photon-Z-boson interference. But a representations of (22) show the influence of EWRC on a determination of the QED-structure function $F_2(x,Q^2)$ or on a corresponding QCD-analysis quite clearly.

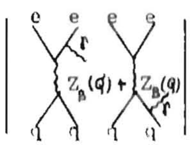
The leading effect in the Q^2 -dependence is due to the mass singularities of bremsstrahlung. It is nearly exclusive due to the terms $\ln(Q^2/m_e^2)$, with m_e - the electron mass arizing in the lepton leg radiation terms $F_e(B)$, $B = \gamma$, I, Z. One might speculate whether the Q^2 -dependence is also influenced by the quark distributions $f_Q(x, Q^2)$ which in the present approach cannot be taken into account totally correct under the integral for hard bremsstrahlung in case of lepton leg radiation (b = e) and lepton-quark interference (b=i). A numerical study of that question has been performed using the program TERAD 86 /6/ which contains for the photon-exchange bremsstrahlung (b=γ) a twofold integration over x and Q^2. We noticed that freezing out the Q^2 -dependence of the quark distributions under the integral influences the correction only negligible. This is not surprizing in view of the fact that the bremsstrahlung correction functions S, T, U depend much stronger on Q^2 than the quark distributions.

4. Summary

We have derived a closed, compact expression for the totally inclusive photon bremsstrahlung corrections to neutral-current deep inelastic scattering of longitudinally polarized electrons (positrons) off protons via photon and Z-boson exchange at HERA energies. The corrections may become large which suggests the investigation of the influence of kinematic cuts on EWRC. Soft photon exponentiation considerably changes the EWRC. That raizes the question of a more complete treatment of $O(\alpha^2)$ corrections as is available for lower energy scattering in /6/. Though after some improvement of the MC-program of the Wurzburg group the agreement with our results became much better, it would be quite desirable to get a complete numerical agreement of the bremsstrahlung results of different groups.

Appendix A

The relation between the three lepton-quark interference bremsstrahlung contributions due to the exchange of two interfering bosons Z_1, Z_2 with masses M_1, M_2 and widths Γ_1, Γ_2 may be derived as follows:



$$= \sigma_e(Z_B) + \sigma_i(Z_B) + \sigma_q(Z_B), \qquad B = 1, 2. \tag{A.1}$$

Here we are interested only in $\sigma_i(Z_B)$:

$$\sigma_i(Z_B) = RR^*\left(\frac{Q^2}{Q^2+m_B^2}\cdot\frac{Q^2}{Q^2+m_B^{*2}} + \frac{Q^2}{Q^2+m_B^{*2}}\cdot\frac{Q^2}{Q^2+m_B^2}\right)$$

$$= |\varkappa_B|^2\cdot \mathrm{Real}\left[2RR^*\left(\frac{Q^2+m_B^2}{Q^2+m_B^{*2}}\right)\right] \tag{A.2}$$

$$= C(B)\,|\varkappa_B|^2 \;\; \mathrm{Real}\left[R_i(Z_B)\right].$$

In (A.2) all the features of the squared matrix element, which are not dependent on the propagator of the exchanged boson, we consider to be contained in the R. We use a notation which includes also the case of annihilation instead of scattering; then one has to change the indices (e,q) into initial (final) and the Q^2 is replaced by

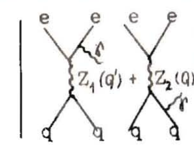
$(-s)$. Further,

$$m_B^2 = M_B^2 - iM_B\Gamma_B \;, \tag{A.3}$$

$$\varkappa_B = \frac{Q^2}{Q^2+m_B^2} \;, \tag{A.4}$$

allowing for resonance scattering as well.

The Z_1 - Z_2 interference contribution to the lepton-quark interference bremsstrahlung radiation is contained in:



$$+ (1\leftrightarrow 2) = \sigma_e(Z_1) + \sigma_i(Z_1,Z_2) + \sigma_q(Z_2) + (1\leftrightarrow 2)\;, \tag{A.5}$$

$$\sigma_i(I) = \sigma_i(Z_1,Z_2) + \sigma_i(Z_2,Z_1)$$

$$= RR^*\,\frac{Q^2}{Q^2+m_1^2}\cdot\frac{Q^2}{Q^2+m_2^{*2}}\cdot\left(\frac{Q^2+m_2^{*2}}{Q^2+m_2^{*2}} + \frac{Q^2+m_1^2}{Q^2+m_1^2}\right) +$$

$$+ RR^*\,\frac{Q^2}{Q^2+m_1^{*2}}\cdot\frac{Q^2}{Q^2+m_2^2}\cdot\left(\frac{Q^2+m_2^2}{Q^2+m_2^2} + \frac{Q^2+m_1^{*2}}{Q^2+m_1^{*2}}\right) \tag{A.6}$$

$$= \mathrm{Real}\left[2\,\varkappa_1\,\varkappa_2^*\,RR^*\left(\frac{Q^2+m_1^2}{Q^2+m_1^2} + \frac{Q^2+m_2^{*2}}{Q^2+m_2^{*2}}\right)\right]$$

$$= C(Z_1,Z_2)\;2\;\mathrm{Real}\left[\varkappa_1\,\varkappa_2^*\,R_i(I)\right].$$

Evidently

$$R_i(I) = \frac{1}{2}\left[R_i(Z_1) + R_i^*(Z_2)\right]. \tag{A.7}$$

This is true also in the special case that one of the exchanged bosons is the massless photon. Eq. (18) may be proved quite similarly.

Appendix B

The lepton leg photon bremsstrahlung correction functions are:

$$S_e(B;\,a,b) = a^2\left[\ln\left(\frac{x_1^2y^2}{abx^2}\right)(L_e-1) + \frac{3}{2}L_e - \frac{1}{2}\ln^2(ab) - 2\right]. \qquad B=\gamma,I,Z \tag{B.1}$$

The exponentiation of the contribution from soft photons has been done following[5]:

$$S_e(B;a,b)\longrightarrow \bar{S}_e(B;a,b) = S_e(B;a,b) - a^2\delta_{inf}\,, \qquad (B.2)$$

$$\delta_{inf} = (L_e-1)\,\ln\frac{x_1^2\,y^2}{(1-xy)(1-yx_1)} \qquad (B.3)$$

$$T_e(B;a,b)=a^2\left[2\left(\ln\frac{Q^2}{m_e^2}-1\right)\right], \quad B=\gamma,I,Z \qquad (B.4)$$

$$U_e(\gamma;a,b)=y^2\left[1-L_e-\frac{(a^2+b^2)(z-1)}{2ab}+\frac{1}{2z}\ln\frac{Sy(z-1)}{M_p^2\,x} - \frac{y}{2z_1}L_0-\left(a-\frac{y}{2z}-z\,\frac{a^2+b^2}{y}\right)\frac{1}{b}\ln\frac{S^2\,b^2}{m_e^2\,M_p^2}\right]. \qquad (B.5)$$

In $U_e(\gamma;a,b)$ a dependence on a quark mass m_q has been eliminated in favor of the proton mass M_p using the (formal) relation $m_q=z\,M_p$. This is justified by the analytic agreement with formulae obtained in a formalism which is not based on a quark parton model[7]. All contributions to $R_e(B)$ show the logarithmic mass singularity from the electron mass.

$$U_e(I;a,b)=y\Big\{\left[y+z(a+b)\right]\cdot(1-L_e)+\frac{z^2Ra^2}{z_1^2}-\frac{z_2^2\cdot R}{z_2'}-\frac{y^2}{2z_1}(1+R)\,L_0+\frac{y(1+R)}{2z}\ln\left(1+\frac{z}{R}\right)+\frac{z^2a^2+(zb-yR)^2}{2z_1}\,L_1-\frac{z^2a^2+z_2^2}{2z_2'}\,L_2\,, \qquad (B.6)$$

$$z_1=za-b\,,\quad z_1'=zb+Rz_1\,,\quad z_3=zb-yR\,,$$
$$z_2=zb+y\,,\quad z_2'=za+Rz_2\,,\quad x_1=1-x\,,\quad R=\frac{M_z^2}{Q^2}\,, \qquad (B.7)$$

$$L_0=\ln\frac{Q^2z_1^2}{m_e^2y^2(z-1)}\,,\qquad L_e=\ln\frac{Q^2}{m_e^2}\,, \qquad (B.8)$$

$$L_i=\ln\frac{Q^2\,z_i^2}{m_e^2y^2(z-1)R(R+z)}\,,\quad i=1,2\,. \qquad (B.9)$$

$$U_e(Z;a,b)=y\Big\{-y\,\frac{(1+R)^2}{2(z+R)}+z(a+b)+y\,(1-L_e)-\frac{(1+R)zb}{z_1^2}\left[(za)^2+(za)z_3+z_3^2\right]+\frac{(1+R)za}{z_2^2}\left[(za)^2+zaz_2+z_2^2\right]-\frac{1}{z_1'}\Big\{z_2z_3-\frac{z(1+R)}{2(z+R)}\left[(za)^2+z_3^2\right]+\frac{1}{z_2'}\Big\{(za)^2-\frac{z(1+R)}{2(z+R)}\cdot\left[(za)^2+z_2^2\right]\Big\}+y\,\frac{(1+R)^2}{2}\left[\frac{1}{z}\ln\left(1+\frac{z}{R}\right)-\frac{y}{z_1}L_0\right]+\left[(1+R)\,\frac{z^2ab}{z_1'}(za-Rz_3)+z_2z_3-a^2Rz^2-z_3y(1+R^2)\right]\cdot\frac{1}{2z_1'}\cdot L_1+\left[R-\frac{(1+R)za}{z_2}\right]\frac{(za)^2+z_2^2}{2z_2}\cdot L_2\,. \qquad (B.10)$$

The lepton-quark interference bremsstrahlung correction functions are:

$$S_1(\gamma;a,b)=a^2\,4\ln\frac{x_1}{x}\,\ln\frac{b}{a}+y\Big\{a\ln\frac{b}{y}-\frac{b+a}{2}\left[\ln^2\frac{b}{y}+\pi^2\theta(b)\right]\Big\}, \qquad (B.11)$$

$$T_1(B;a,b)=4a^2\,\ln\frac{b}{a}\,,\qquad B=\gamma,I,Z\,, \qquad (B.12)$$

$$U_1(\gamma;a,b)=y\left\{\frac{y^2}{z_1}-\frac{a+b}{z}+\frac{z}{z-1}(a+b)\ln\frac{z_1}{|b|}+\left(a+b+\frac{y}{z}\right)\ln\frac{a}{b(z-1)}\right\}, \qquad (B.13)$$

$$S_1(Z;a,b)=a^2\Big\{\ln\frac{b}{a}\Big[4\ln\frac{x_1}{x}+\ln\frac{y^2}{ab}-4\ln\left(1+\frac{1}{R}\right)+2\,\Phi\left(1+\frac{a}{yR}\right)-2\,\Phi\left(1-\frac{b}{yR}\right)+\pi^2\left[\theta(-b)-\theta(a)\right]\Big]\Big\}+2y(1+R)\cdot a\Big\{\ln\frac{|b|}{yR}-(1+R)\,\ln\left(1+\frac{1}{R}\right)-(a+b-yR)\cdot\Big[\Phi\left(1+\frac{1}{R}\right)-\Phi\left(1-\frac{b}{yR}\right)-\ln\left(1+\frac{1}{R}\right)\ln\frac{|b|}{y}\Big]\Big\}, \qquad (B.14)$$

$$U_1(Z;a,b)=\frac{4a^2zR}{R+z}\ln\frac{b}{a}+(1+R)\Big\{\frac{y^3}{z_1}-y\,\frac{(a+b)}{z}+\frac{2(za)^2}{(z-1)(R+z)}\ln\frac{az_1'}{bz_2'}+y\,\frac{yR-z(a+b)}{z-1}\ln\frac{z_1'}{z_1(R+z)}+$$

$$+ y\left[z(a+b)+y\right]\frac{1}{R+z}\ \ln\frac{z_2'}{R(z-1)b} + y\left[\left(y+\frac{a+b}{z}\right)\cdot\right. \qquad (B.15)$$

$$\left.\cdot\frac{R}{z} - \left(a+b+\frac{y}{z}\right)\right]\ln\left(1+\frac{z}{R}\right)\Big\}\ ,$$

$$\phi(z) = -\int_0^1 \frac{dx}{x}\ln(1-zx)\ . \qquad (B.16)$$

The interference functions $F_i(B)$ do not contain any mass singularities.
The quark leg photon bremsstrahlung correction functions are:

$$S_q(\gamma;\ a,b)=a^2\left[-\frac{5}{4}-\frac{{}^2}{6}-\frac{7}{4}\ln\frac{x_1}{x}-\frac{1}{2}\ln^2\frac{x_1}{x}+\left(\frac{3}{4}+\ln\frac{x_1}{x}\right)L_q\right] \qquad (B.17)$$

$$T_q(\gamma;a,b) = a^2\left[\ln\frac{Q^2}{m_q^2(z-1)}-\frac{7}{4}\right] \qquad (B.18)$$

$$U_q(\gamma;a,b)=a^2\left\{\frac{2z\ln z}{z-1}+1-\left(1+\frac{1}{z}\right)\left[\frac{1}{2}\ln\frac{Q^2z^2}{m_q^2(z-1)}-1\right]\right\}+$$

$$+\ ab\,\frac{1}{2z}-\frac{y}{4}\left(a+b+\frac{y}{z}\right)\ . \qquad (B.19)$$

Acknowledgement

We would like to thank the organizers of the International Symposium on Hadron Interactions - Theory and Phenomenology, Bechyne Castle, June 26 - July 1, 1988, Czechoslovakia, where the present results could be presented.

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Received by Publishing Department
on September 14, 1988.