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THE WITTEN INDEX FOR THE SUPERMEMBRANE

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In the study of theories of extended objects as possible candidates for a unified fundamental theory of matter and forces, a central question is whether massless states are present. In string theories, it is known that massless states occur both in the bosonic string and in the various superstrings in their respective critical dimensions. For higher-dimensional extended objects, this question has been the subject of some controversy. For bosonic membranes, it has been argued using semiclassical methods [1] that the intercept of the leading mass-angular momentum trajectory is inconsistent with the existence of massless states in any integral spacetime dimension.

For supermembranes [2,3] (and more generally, for super p-branes), massless states will be present as a consequence of the spacetime supersymmetry algebra, provided that supersymmetry is neither explicitly broken (by anomalies) nor spontaneously broken. A complete study of supersymmetry anomalies has not so far been undertaken. However, in the light-cone gauge, any anomalies would be expected to manifest themselves in the Lorentz algebra and possibly through a failure of the supermultiplet structure. In fact, this can be shown to occur for all super p-branes except superstrings and the supermembrane in eleven-dimensional spacetime [4,5]. For the supermembrane in $d = 11$ spacetime, doubts concerning the presence of massless states were raised [6] as a result of applying the semiclassical methods of ref. [1], but this discussion did not take into account a treatment of the zero modes present in expansions about periodic classical solutions. Taking these into account, one can show that the perturbative partition function $\mathrm{str}(e^{-Ht})$ vanishes as a result of fermionic collective coordinate integrations [7]. As a consequence, the supersymmetric degeneracy of the energy levels is preserved to all orders in perturbation theory.

Whether supersymmetry is broken spontaneously through non-perturbative effects has not yet been established. In a Green-Schwarz formulation of a supersymmetric particle or extended-object theory, the variables transform under supersymmetry as the coordinates of superspace. Since the fermionic variables Ψ transform inhomogeneously, and Lorentz invariance requires that Ψ vanish in the vacuum, at least some of the supersymmetry generators will not annihilate the vacuum. The important question, however, is whether *all* of the supersymmetry generators are broken.

Working in the light-cone gauge, the 32-component $d = 11$ Majorana supersymmetry generators break up into two 16-component spinors α and β under the light-cone little group $SO(9)$. In this gauge, the supersymmetry transformations acquire compensating world-volume reparameterization terms [8], which have the effect that only the β-supersymmetry transforms the spinor variables inhomogeneously, thus being obviously broken by the vacuum $\Psi = 0$. Since the classical α-supersymmetry transformations do not contain an inhomogeneous shift of the spinor variables, this suggests that the α-supersymmetry is unbroken by the vacuum. Assuming that this is the case, the 16 Goldstone zero modes corresponding to the broken β-supersymmetries generate upon quantization a Clifford algebra that has an $SO(16)$ automorphism symmetry. The unique faithful irreducible representation of this algebra acts on a space of 128 boson and 128 fermion states. Under the $SO(9)$ light-cone little group, these states transform as $(\underline{44} + \underline{84})_{\rm Boson}$ and $(\underline{128})_{\rm Fermion}$, which are precisely the representations occuring in $d = 11$ supergravity [9].

If dynamical supersymmetry breaking occurs non-perturbatively, the above discussion would need to be ammended. A framework for addressing the non-perturbative structure of the theory has been set up in ref. [10]. This work showed that the supercharges and Hamiltonian may be separated into independently-conserved terms, $Q = Q_0 + \hat{Q}$ and $H = H_0 + \hat{H}$. Q_0 and H_0 depend only upon the zero modes, while $\hat{Q}$ and $\hat{H}$ depend only upon the non-zero modes. The important point is that the $\hat{Q}$, $\hat{H}$ system itself forms a supersymmetry algebra, which in the center of mass frame with vanishing transverse momentum coincides with the α-supersymmetry algebra. Thus the issue of non-perturbative spontaneous supersymmetry breaking may be studied by concentrating on the non-zero modes only. The other main feature of ref. [10] is a supersymmetric regularization of the theory by replacing the infinite number of degrees of freedom of a supermembrane with spherical topology by a quantum-mechanical system with a finite number of degrees of freedom. This is achieved by representing the residual reparameterization symmetry of the light-cone gauge (the group of area-preserving diffeomorphisms) as the limit of an $SU(n)$ gauge symmetry as n tends to infinity [11]. Thus the supermembrane problem may be replaced by that of an $N = 16$ supersymmetric $SU(n)$ gauge-invariant quantum-mechanical model, where the bosonic and fermionic transverse membrane variables are replaced by variables transforming in the adjoint representation of $SU(n)$. This quantum-mechanical model may also be viewed as the dimensional reduction of $d = 10$, $N = 1$ or $d = 4$, $N = 4$ super Yang-Mills with gauge group $SU(n)$.

The aim of ref. [10] was to use the above framework to construct explicitly the ground-state wave function for the $N = 16$ quantum-mechanical model. This explicit attempt was not successful, and heuristic arguments were given to the effect that normalizable zero-energy states of this system might not exist. If this conclusion were correct then all the supersymmetries would be spontaneously broken. Since the Hamiltonian $\hat{H}$ for the non-zero mode system is in fact just 1/2 times the $(\text{mass})^2$ operator for the full theory, this would imply the absence of massless states in the supermembrane spectrum.

In a supersymmetric theory with many fermionic degrees of freedom, the structure of the vacuum may be rather complicated, involving summation over all of the various sectors of the fermionic Fock space. Thus it may not be possible in practice to construct explicitly a supersymmetric vacuum wave function, even though one may have clear indications that such a state exists. In fact, there exist powerful techniques for proving the existence of zero-energy wave functions without having to construct them explicitly. It is sufficient to show that any one of a number of indices for a given theory is non-vanishing to establish the existence of a supersymmetric ground state [12]. These indices are topologically invariant, i.e. invariant under smooth deformations of the parameters of the theory. The simplest of these Witten indices is just $\mathrm{tr}(-1)^F$, which counts the difference between the numbers of bosonic and

fermionic zero-energy states (in a supersymmetric theory, the non-zero-energy states always come in equal numbers of bosons and fermions). In the case of the supermembrane, this index should be zero because the obviously-broken β-supersymmetry generators will produce equal numbers of bosonic and fermionic ground states, whether the α-supersymmetry is broken or not.

In a given theory, there may be a number of other Witten indices that reveal different aspects of the vacuum structure. The existence of such other indices relies upon the existence of further symmetries that commute with supersymmetry; typically, these further symmetries are discrete, such as charge conjugation [12]. In the following, we shall consider two such discrete symmetries that commute with the α-supersymmetry transformations. Since the β-supersymmetries are clearly broken in any case, we shall not be concerned with deriving an index that is invariant under them.

In the light-cone gauge, with $X^+ = \tau$, $g_{0a} = 0$, $g_{00} = -\det(g_{ab})$ $(a, b = 1, 2)$, the $d = 11$ supermembrane variables comprise 9 transverse bosonic coordinates X^I and a 16-component $SO(9)$ Majorana spinor S. The classical equations of motion for X^I and S are [8]

$$\begin{aligned} \ddot{X}^I &= -\{X^J, \{X^I, X^J\}\} - i\{S, \Gamma^I S\} \\ \dot{S} &= -\Gamma^I\{X^I, S\}, \end{aligned} \tag{1}$$

where the bracket notation [11.10] is defined by

$$\{A, B\} \equiv \epsilon^{ab}\partial_a A \partial_b B \tag{2}$$

and ∂_a denotes differentiation with respect to the two membrane spatial parameters σ^a and the dots in (1) denote differentiation with respect to the time parameter τ. The equations of motion (1) are supplemented by the constraint

$$\Phi \equiv \{\dot{X}^I, X^I\} + i\{\dot{S}, S\} = 0, \tag{3}$$

which ensures the invariance of the theory under the residual area-preserving diffeomorphisms

$$\begin{aligned} \delta X^I &= \{\xi, X^I\}, \\ \delta S &= \{\xi, S\}, \end{aligned} \tag{4}$$

where $\xi(\sigma^a)$ is an arbitrary function of the spatial parameters σ^a.

The equations of motion (1) may be derived from the Lagrangian density

$$L = \frac{1}{2}(\dot{X}^I)^2 + iS\dot{S} - \frac{1}{4}(\{X^I, X^J\})^2 + iS\Gamma^I\{X^I, S\}, \tag{5}$$

with the constraint (3) still to be imposed. Of course, one could also introduce a gauge field for the transformations (4), making them τ-dependent, and thus derive (3) as well from the Lagrangian by varying with respect to the gauge field in the usual way. Note that since we have been taking the membrane world-volume metric to satisfy $g_{00} = -\det(g_{ab})$, there are no further density factors needed in (5). The Hamiltonian corresponding to the Lagrangian density (5) is

$$H = \int d^2\sigma \Big[\frac{1}{2}(P^I)^2 + \frac{1}{4}(\{X^I, X^J\})^2 - i\bar{S}\Gamma^I\{X^I, S\}\Big], \tag{6}$$

for which Hamilton's equations give $P^I = \dot{X}^I$.

The supersymmetry transformations that leave the action invariant are

$$\begin{aligned} \delta X^I &= i\bar{\alpha}\Gamma^I S + i\{X^I, \bar{\alpha}\int_0^\tau S d\tau\}, \\ \delta S &= \beta - \frac{1}{2}\Big(\dot{X}^I\Gamma^I - \frac{1}{2}\{X^I, X^J\}\Gamma^{IJ}\Big)\alpha + i\{S, \bar{\alpha}\int_0^\tau S d\tau\}, \end{aligned} \tag{7}$$

where α and β are the $SO(9)$ spinor supersymmetry parameters discussed above. The β-supersymmetry is clearly spontaneously broken, as we have noted. For the α-supersymmetry, we have the supercharge

$$Q = \int d^2\sigma \Big(P^I\Gamma^I + \frac{1}{2}\{X^I, X^J\}\Gamma^{IJ}\Big)S. \tag{8}$$

Upon quantization, we have

$$\begin{aligned} [P^I(\sigma), X^J(\sigma')] &= -i\delta^{IJ}\delta^2(\sigma,\sigma'), \\ [S^\alpha(\sigma), S^\beta(\sigma')]_+ &= \delta^{\alpha\beta}\delta^2(\sigma,\sigma'), \end{aligned} \tag{9}$$

where the charge conjugation matrix has been taken to be just $\delta^{\alpha\beta}$. From (9), it follows that the supercharge and the Hamiltonian satisfy the algebra

$$[Q^\alpha, Q^\beta]_+ = 2H\delta^{\alpha\beta} + 2(\Gamma^I)^{\alpha\beta}\int d^2\sigma X^I\Phi, \tag{10}$$

which reduces to the usual result when the constraint (3) is imposed. Note that the total transverse momentum does not occur in this relation because we are considering only the algebra of the α-supersymmetry generators.

Even though we are considering just the α-supersymmetry generators here, we still have terms in (6) and (8) that depend upon the Goldstone zero modes corresponding to the broken β-supersymmetries. To reveal these, we separate X^I, P^I and S according to

$$\begin{aligned} X^I(\sigma) &= X_0^I + \tilde{X}^I(\sigma), \\ P^I(\sigma) &= P_0^I + \tilde{P}^I(\sigma), \\ S(\sigma) &= S_0 + \tilde{S}(\sigma), \end{aligned} \tag{11}$$

where the zero modes are σ^a-independent and $\int d^2\sigma \tilde{X}^I = \int d^2\sigma \tilde{P}^I = \int d^2\sigma \tilde{S} = 0$. Thus one finds [10] $Q = Q_0 + \tilde{Q}$ and $H = H_0 + \tilde{H}$, with

$$Q_0 = vP_0^I\Gamma^I S_0, \tag{12}$$

$$H_0 = \frac{1}{2}v(P_0^I)^2, \tag{13}$$

$$\tilde{Q} = \int d^2\sigma \Big(\tilde{P}^I\Gamma^I + \frac{1}{2}\{\tilde{X}^I, \tilde{X}^J\}\Gamma^{IJ}\Big)\tilde{S}, \tag{14}$$

$$\tilde{H} = \int d^2\sigma \Big(\frac{1}{2}(\tilde{P}^I)^2 + \frac{1}{4}(\{\tilde{X}^I, \tilde{X}^J\})^2 - i\bar{\tilde{S}}\Gamma^I\{\tilde{X}^I, \tilde{S}\}\Big), \tag{15}$$

where $v = \int d^2\sigma$ is a normalizing factor. An important point [10] is that neither H_0 nor $\tilde{H}$ contains S_0, so S_0 is a constant of the motion and hence Q_0 and $\tilde{Q}$ are independently conserved.

The fact that $\tilde{Q}$ and $\tilde{H}$ do not involve the zero modes gives rise to a discrete symmetry G of the Q and H generators that transforms only the zero modes:

$$\begin{aligned} G: \quad (X_0^I, P_0^I, S_0) &\to (-X_0^I, -P_0^I, -S_0), \\ (\tilde{X}^I, \tilde{P}^I, \tilde{S}) &\to (\tilde{X}^I, \tilde{P}^I, \tilde{S}). \end{aligned} \tag{16}$$

There is also another discrete symmetry C that combines a sign change for X^I, P^I and S with an orientation-reversing, area-preserving diffeomorphism that changes the sign of the bracket (2):

$$\begin{aligned} C: \quad (X_0^I, P_0^I, S_0) &\to (-X_0^I, -P_0^I, -S_0), \\ (\tilde{X}^I, \tilde{P}^I, \tilde{S}) &\to (-\tilde{X}^I, -\tilde{P}^I, -\tilde{S}), \\ \{A, B\} &\to -\{A, B\}. \end{aligned} \tag{17}$$

A convenient way to view the orientation-reversing diffeomorphism is to change the sign of ϵ^{ab}. One may verify by inspection that the supercharges and Hamiltonians (12-15) are invariant under G and C.

Using the discrete symmetries G and C we may construct generalized Witten indices. For example, by projecting into the space of states invariant under the the operator G, one effectively factors out the effects of the zero modes associated with the spontaneously-broken β-supersymmetry. Thus, one could consider $\mathrm{tr}[(-1)^F(1+G)/2]$. Since the 128 bosonic zero-energy states constructed using even powers of fermionic creation operators built from S_0 are even under G, while the 128 fermionic zero-energy states are odd under G, one has $\mathrm{tr}[(-1)^F(1+G)/2] = 128\tilde{\mathrm{tr}}(-1)^F$, where $\tilde{\mathrm{tr}}$ is calculated within the subspace of states for the non-zero mode system with generators $\tilde{Q}$ and $\tilde{H}$. This means that one can focus attention on the zero-energy eigenstates of $\tilde{H}$. A non-zero value for this index would thus imply directly the existence of the zero-energy eigenstates sought for in [10].

In practice, we shall find it more convenient to discuss a different Witten index that involves projecting onto states invariant under the operator C. We recall that if any Witten index built using operators that commute with the supercharge Q is non-zero, then the α-supersymmetry will be unbroken. We thus consider

$$I = \mathrm{tr}[(-1)^F(1+C)/2]. \tag{18}$$

Without breaking the spacetime supersymmetry, we may compactify the transverse directions of spacetime on a 9-torus,

$$X^I \approx X^I + L^I. \tag{19}$$

This is equivalent to imposing boundary conditions periodic in X^I on the quantum wave functions of the system. Note that these boundary conditions are consistent with the discrete symmetry C defined in (17). Among the periodic wave functions, the projection operator in (18) selects those that are even under (17).

The problem of calculating the index (18) is similar to that for super Yang-Mills theories [12], in that the Hamiltonian (13,15) has zero-energy "valleys", i.e. directions in (X^I, S) space where the potential part of H is flat for non-zero values of X^I and S. These valleys now have finite extent as a consequence of our periodic boundary conditions (19), but they complicate the evaluation of expressions such as (18) since there could still be zero-energy states for each of the flat directions. The flat directions are determined by the classical equations of motion for zero energy and the structure of the Hamiltonian (13,15). Setting $\dot{S} = 0$ in the fermion equation of motion (1), we find

$$\Gamma^I\{\tilde{X}^I, \tilde{S}\} = 0. \tag{20}$$

For configurations satisfying (20), the Hamiltonian $H = H_0 + \tilde{H}$ becomes a sum of squares, requiring for zero energy that

$$P_0^I = \tilde{P}^I = 0, \tag{21}$$
$$\{\tilde{X}^I, \tilde{X}^J\} = 0. \tag{22}$$

Since zero-energy configurations must be inert under supersymmetry transformations, one obtains by varying (20) under an α-transformation, and taking (22) into account,

$$\{(\Gamma^I\tilde{S})^\alpha, (\Gamma^I\tilde{S})^\beta\} = 0, \quad \alpha = 1\cdots 16. \tag{23}$$

For symmetric bispinors $\chi^{\alpha\beta}$ of $SO(9)$, one has the Fierz decomposition

$$\chi^{\alpha\beta} = \frac{1}{16}\Big(\delta^{\alpha\beta}\chi_{\gamma\gamma} + (\Gamma^I)^{\alpha\beta}\chi_{\gamma\delta}(\Gamma^I)^{\gamma\delta} + \frac{1}{4!}(\Gamma^{IJKL})^{\alpha\beta}\chi_{\gamma\delta}(\Gamma^{IJKL})^{\gamma\delta}\Big). \tag{24}$$

Substituting (23) into (24), we find that (23) is equivalent to

$$\{\tilde{S}^\alpha, \tilde{S}^\beta\} = 0. \tag{25}$$

Zero-energy solutions satisfying (20-22) and (25) take the general form $\tilde{X}^I = \tilde{X}^I(f(\sigma^a))$ and $\tilde{S}^\alpha = \tilde{S}^\alpha(f(\sigma^a))$, where $f(\sigma^a)$ is an arbitrary function of σ^1 and σ^2. Physically, these classical solutions correspond to membranes that have collapsed to string-like one-dimensional configurations of zero area.

Promoting the classical zero-energy solutions to (20-23) to quantum operators, we can build from them the operators that map between the ground states of the quantum Hamiltonian. Physical states must also satisfy the constraint (3), which requires that the operator being applied to the ground state be an invariant under (4). This is analogous to the requirement in super Yang-Mills theories that the operators should be gauge-invariant. The operators that we can apply to a given ground state must also be restricted to those that preserve the periodic boundary conditions (19). In particular, operators involving the bosonic coordinates X^I must be periodic with periods L^I. Equation (21) then shows that the bosonic part of an operator that maps between zero-energy states must be a constant. Moreover, owing to the periodic boundary conditions and supersymmetry, the lowest-lying states of non-zero energy are lifted above zero by discrete amounts determined by the values of the L^I. Thus, starting from one ground state, the other states degenerate in energy with it can be obtained only by applying products of fermionic creation operators corresponding to the flat valley directions.

In performing the sum over states that contribute to the Witten index (18), we must now consider the projection operator $(1+C)/2$. We should like to argue that the fermionic states, which are built by applying odd numbers of fermi creation operators, are all odd under C, while the bosonic states are all even. (We are assuming that our original ground state is bosonic and invariant under C; if this state were odd, one could calculate the index with the $(1 - C)/2$ projection operator instead.) Since all the fermionic creation operators are odd under C, the only way to have an operator even under C with an odd number of fermi creation operators would be to use a bracket $\{\tilde{S}^\alpha, \tilde{S}^\beta\}$. But for the valley modes corresponding to classical solutions of zero energy, we have equation (25). Thus there is no way to construct a fermionic operator that is even under C that creates states at the same energy as that of the original ground state. All fermionic states degenerate in energy with the chosen bosonic ground state are therefore excluded in the index (18).

Bosonic states constructed by applying even numbers of fermionic valley-mode creation operators are even under C, and are therefore counted in the index (18). In addition to the 127 additional bosonic states generated by applying even numbers of the creation operators constructed from S_0, we presumably also have other bosonic states generated by applying even numbers of the creation operators constructed from the $\tilde{S}^\alpha(f(\sigma^a))$ satisfying (25) that we discussed earlier. Since the C-even lowest-energy states are all bosonic, and are hence unpaired under the α-supersymmetry, they must in fact be at zero energy.

At this stage, we have established that the Witten index (18) is strictly positive. Thus supersymmetry cannot be spontaneously broken and so it is guaranteed that the massless states corresponding to $d = 11$ supergravity, discussed in [9], are indeed present in the supermembrane spectrum. We have not attempted to determine the precise value of the index (18), other than showing that it is greater than zero. Ostensibly, the number of fermionic valley modes of the supermembrane that satisfy (25) is infinite. Thus the index (18) may in fact be infinite.

In the special case of a membrane with spherical topology, one may use the supersymmetric ultraviolet regularization scheme of refs [11,10], in which the supermembrane is viewed as the $n \to \infty$ limit of an $N = 16$ $SU(n)$ gauge-invariant quantum-mechanical model. If one passes to such a regularized theory after having compactified spacetime as in equation (19), the spectrum will then be discrete and the Witten index (18) will take a definite integer value. This regularized problem is essentially equivalent to one of the approaches to calculating the Witten index for the maximal super Yang-Mills theory [13]. By "essentially equivalent", we mean that periodic boundary conditions have been imposed on all the bosonic modes of the supermembrane prior to the passage to the regularized theory. In ref. [13], the Witten index was calculated for the $N = 16$ quantum-mechanical model with $SU(2)$ gauge group, but without any periodic identification of the bosonic fields. This model is exactly the $n = 2$ version of the regularized non-zero mode sector of the supermembrane considered in [10]. While the results of [13] gave a non-zero Witten index, the specific value has been the subject of some discussion [14]. The difficulty stems from the fact that quantum-mechanical models of the type discussed in [10] may have a continuous spectrum starting at zero energy. While the Witten index can be defined within a given scheme for discretizing the spectrum, the results may not agree between one scheme and another, although if the index is non-zero for one scheme, it is presumably non-zero for all. In the case of the supermembrane, however, the imposition of the periodic boundary conditions (19) seems quite natural, and one may simply adopt this as a procedure for defining the infinite-volume theory. The essential point is that for all values of the L^I, spacetime supersymmetry is unbroken and so the theory does have massless states in its spectrum. Of course it may be that in taking the limit of the $SU(n)$ models as n tends to infinity, the Witten index will become infinite.

Given the non-zero value of the index (18), supersymmetry is unbroken and so the non-zero-mode system of ref. [10] must have either a zero-energy normalizable state or a continuum of states going to zero energy (in which case the energy eigenstates would be delta-function normalizable). A non-zero value for the index has stronger implications than finding an explicit zero-energy wave function in the regularized theory would have, since the index is an invariant under adiabatic changes in the parameters of the theory. In particular, one may be able to establish that supersymmetry remains unbroken in the spectrum even after ultraviolet renormalization, provided the C symmetry (17) is not violated by anomalies.

There are some important open questions that remain. One is certainly to establish that the supermembrane in $d = 11$ is free of anomalies. At present it is only known that the $d = 11$ theory survives a severe test for anomalies that rules out all the other super p-brane theories (for $p > 1$) in lower spacetime dimensions [4,5]. Another important question is the nature of the supermembrane spectrum, and in particular whether it is continuous or discrete.

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