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M.B.Paranjape *

1+1-d AXIAL ANOMALY, BERRY's PHASE AND INDUCED CHARGE, STATISTICS AND WESS-ZUMINO TERM

*Permanent address: Laboratoire de physique nucléaire, Département de physique, Université de Montréal, C.P. 6128 succ. "A", Montréal, Québec, Canada, H3C 3J7.

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We study the second quantization of single particle Dirac hamiltonians of the form

$$h(x) = -i\gamma_5 \partial_x + V(x)\gamma_5 + a(x) + m\gamma_0 e^{i\gamma_5\theta(x)} \tag{1}$$

with $\gamma_5 = \sigma^3$, $\gamma_0 = \sigma^1$, defined on a circle of radius 2π. $V(x)$ and $a(x)$ are periodic functions while

$$\theta(0) = \theta(2\pi) + 2n\pi \tag{2}$$

which defines the n soliton sector. We will work in the Hilbert space of second quantized, free, massive Dirac fermions. We define this Hilbert space as that constructed by the action of $a_p^\dagger$ and $b_p^\dagger$, $p \in \mathbb{Z}$, which are fermionic creation and annihilation operators, acting on the vacuum. The vacuum satisfies

$$a_p|0\rangle = b_p|0\rangle = 0 \quad , \quad p \in \mathbb{Z} \tag{3}$$

while

$$\{a_p, a_q^\dagger\} = \{b_p, b_q^\dagger\} = \delta_{p,q}, \text{ all others zero} \tag{4}$$

The second quantization of (1) proceeds in two steps. First we construct the hamiltonian for

$$\theta(x) = nx \quad , \quad V(x) = V \quad , \quad a(x) = a \quad , \quad V, a \text{ constants.} \tag{5}$$

Then we consider deformations later, as perturbations.

The fermionic field operator valued distribution is defined as

$$\Psi(x) = \sum_{p\in\mathbb{Z}} \left(\psi_+^0(p,x)\, a_p + \psi_-^0(p,x)\, b_p^\dagger\right) \tag{6}$$

where

$$\psi_\pm^0(p,x) = \frac{e^{ipx}}{\sqrt{2\pi}\sqrt{2E_\pm^0(p)(E_\pm^0(p)-p)}} \begin{pmatrix} m \\ E_\pm^0(p)-p \end{pmatrix} \tag{7}$$

with $E_\pm^0(p) = \pm\sqrt{p^2+m^2}$, which are the eigenmodes of the free Dirac hamiltonian $h_0(x)$. The eigenmodes for conditions (5), $h(x)$, are given by

$$\psi_\pm(p,x) = \frac{e^{-in\left(\frac{1+\gamma_5}{2}\right)x}\, e^{ipx}}{\sqrt{2\pi}\sqrt{2(E_\pm(p)-\tilde{a})(E_\pm(p)-\tilde{a}-(p+\tilde{V}))}} \begin{pmatrix} m \\ E_\pm(p)-\tilde{a}-(p+\tilde{V}) \end{pmatrix} \tag{8}$$

with $E_\pm(p) = \tilde{a} \pm \sqrt{(p+\tilde{V})^2+m^2}$ and $\tilde{a} = a - \frac{n}{2}$, $\tilde{V} = V - \frac{n}{2}$.

The field operator (6) is equally well given by

$$\Psi(x) = \sum_{p\in\mathbb{Z}} \left(\psi_+(p,x)\, \bar{a}_p + \psi_-(p,x)\, \bar{b}_p^\dagger\right) \tag{9}$$

which defines a new set of annihilation and creation operators $\bar{a}_p$ and $\bar{b}_p$. We assume that m is sufficiently large so that $E_+(p) > 0$ and $E_-(p) < 0$. These are related to the free ones by the Bogoliubov transformation

$$\begin{aligned} \bar{a}_p &= \sum_{p' \in \mathbb{Z}} \left((\psi_+(p,x) | \psi^0_+(p',x))\, a_{p'} + (\psi_+(p,x) | \psi^0_-(p',x))\, b^\dagger_{p'} \right) \\ \bar{b}^\dagger_{-p} &= \sum_{p' \in \mathbb{Z}} \left((\psi_-(p,x) | \psi^0_+(p',x))\, a_{p'} + (\psi_-(p,x) | \psi^0_-(p',x))\, b^\dagger_{p'} \right) \end{aligned} \tag{10}$$

where $(\cdot|\cdot)$ is the inner product on the space of single particle wave functions. This Bogoliubov transformation is unitarily implementable iff the off diagonal kernels are Hilbert-Schmidt/1/. It is straight-forward to verify that this is indeed the case, therefore there exists a unitary operator $\mathcal{U}$, acting in the free fermion Hilbert space, such that

$$\bar{a}_p = \mathcal{U}^\dagger a_p \mathcal{U}, \quad \bar{b}_p = \mathcal{U}^\dagger b_p \mathcal{U}, \quad p \in \mathbb{Z} \tag{11}$$

Hence the ground state

$$|0\rangle\rangle = \mathcal{U}^\dagger |0\rangle \tag{12}$$

exists in the Hilbert space, and is annihilated by $\bar{a}_p$ and $\bar{b}_p$, $p \in \mathbb{Z}$. The free hamiltonian and the interacting hamiltonian are defined respectively as

$$H_0 = \int dx : \Psi^\dagger(x)\, h_0(x)\, \Psi(x) : = \sum_{p \in \mathbb{Z}} \left(E^0_+(p)\, a^\dagger_p a_p + |E^0_-(p)|\, b^\dagger_p b_p \right) \tag{13}$$

and

$$\bar{H}(v,a,n) = \int dx : \Psi^\dagger(x)\, \bar{h}(x)\, \Psi(x) : + C(v,a,n) = \sum_{p \in \mathbb{Z}} \left(E_+(p)\, \bar{a}^\dagger_p \bar{a}_p + |E_-(p)|\, \bar{b}^\dagger_p \bar{b}_p \right) + C(v,a,n) \tag{14}$$

where in (14) we have allowed for a normal ordering constant $C(v,a,n)$, which is the ground state energy. Its V dependence is constrained by imposing global gauge invariance. We require that there exists a unitary operator V such that

$$\bar{H}(v+1,a,n) = V^\dagger \bar{H}(v,a,n) V \tag{15}$$

Clearly $E_\pm(p+1,v) = E_\pm(p,v+1)$, hence

$$V^\dagger \bar{a}_{p+1}(v) V = \bar{a}_p(v+1), \quad V^\dagger \bar{b}_{p+1}(v) V = \bar{b}_p(v+1) \tag{16}$$

and $C(v+1,a,n) = C(v,a,n)$ implies (15).

V is obviously unitary by its definition (16). Furthermore we would like to maintain covariance under twisted chiral rotations, that is under

$$a \to a - \tfrac{r}{2}, \quad v \to v - \tfrac{r}{2}, \quad n \to n+r, \quad r \in \mathbb{Z} \tag{17}$$

the energy levels are invariant, thus

$$C(v - \tfrac{r}{2}, a - \tfrac{r}{2}, n+r) = C(v,a,n) \,. \tag{18}$$

Choosing $r = -n$, we can restrict ourselves to the zero soliton sector. Now replacing (10) into (14) we find

$$\bar{H}(v,a,0) = H_0 + v Q_5 + a Q + f(v) + C(v,a,0) \tag{19}$$

with

$$Q = \sum_{p \in \mathbb{Z}} (a^\dagger_p a_p - b^\dagger_p b_p) \tag{20}$$

$$Q_5 = \sum_{p \in \mathbb{Z}} \left(\frac{p}{\sqrt{p^2+m^2}} (a^\dagger_p a_p + b^\dagger_p b_p) + \frac{m}{\sqrt{p^2+m^2}} (a^\dagger_p b^\dagger_p + b_p a_p) \right) \tag{21}$$

$$f(v) = \sum_{p \in \mathbb{Z}} \left(\sqrt{(p+v)^2+m^2} - \sqrt{p^2+m^2} - \frac{pv}{\sqrt{p^2+m^2}} \right) \tag{22}$$

where Q and Q_5 are the usual vector and axial charges respectively. It is straight-forward to show $f(v+1) = f(v) + 2v + 1$. Thus $g(v) = f(v) - v^2$ is strictly periodic. We will now choose $C(v,a,0) = -g(v)$. This gives

$$\bar{H}(v,a,0) = H_0 + v Q_5 + a Q + v^2 \tag{23}$$

and the ground state energy is given by $-g(v)$.

$$\bar{H}(v,a,0)\, |0\rangle\rangle = -g(v)\, |0\rangle\rangle \tag{24}$$

It would be illuminating to compute $g(v)$ in closed form. This choice for $C(v,a,0)$ is imposed by locality and covariance. If we now turn on $V(p)$, the non-zero momentum components of $V(x)$, as these correspond to pure gauge, we expect the spectrum of the hamiltonian to be invariant. We will show that

$$\bar{H}(V(x),a,0) = H_0 - \sum_{p \in \mathbb{Z}} (V(-p)\, j^1(p)) + v Q_5 + a Q + \sum_{p \in \mathbb{Z}} (V(-p) V(p)) \tag{25}$$

has the same spectrum as $\bar{H}(v,a,0)$. Here we use $j^1(p)$, where

$$j^\mu(p) = \sum_{q \in \mathbb{Z}} \Big((\psi^0_+(q)|\gamma^0\gamma^\mu|\psi^0_+(q+p))\, a^\dagger_q a_{q+p} + (\psi^0_+(q)|\gamma^0\gamma^\mu|\psi^0_-(q+p))\, a^\dagger_q b^\dagger_{-q-p} + (\psi^0_-(q)|\gamma^0\gamma^\mu|\psi^0_+(q+p))\, b_{-q} a_{q+p} - (\psi^0_-(q)|\gamma^0\gamma^\mu|\psi^0_-(q+p))\, b^\dagger_{-q-p} b_{-q} \Big), \tag{26}$$

which are the momentum components of the current density.

These are well defined, self-adjoint operators with dense domain, the set of finitely excited states. Now

$$\overline{H}(\mathcal{A}(x), a, 0) = e^{\sum_{p\neq 0}\left(\frac{V(-p)}{p} j^0(p)\right)}\, \overline{H}(V, a, 0)\, e^{-\sum_{p\neq 0}\left(\frac{V(-p)}{p} j^0(p)\right)} \quad . \tag{27}$$

This follows from the following commutation relations, which can be rigorously established without the need of any adhoc regularization prescriptions,

$$[j^0(p), j^1(q)] = 2p\,\delta_{p,-q}\,,\ [H_0, j^0(p)] = -p\,j^1(p)\,,\ [Q, Q_5] = [Q, j^\mu(p)] = [Q_5, j^\mu(p)] = 0 \tag{28}$$

Simply differentiating (27) with respect to $V(p)$, using (25) and the commutators (28) establishes (27). Hence the local "counter-term" $\mathcal{V}^2(x) = \sum_{p\in\mathbb{Z}} (V(-p)V(p))$ makes the hamiltonian gauge invariant. Clearly having additionally $g(V)$ is not a local counterterm. For the non-zero momentum modes, there is no corresponding term. Thus co-variance dictates that $g(V)$ should not be present, and should be subtracted in (14) by the appropriate choice of $C(V,a,0)$. We have in fact not determined the a dependence of $C(V,a,0)$. Nor have we considered its dependence on the non-zero momentum components $a(p)$, of $\mathcal{A}(x)$ and homotopically trivial deformations of $\theta(x)$. We will not investigate this further here. We have illustrated its determination in certain cases. It is in fact not important to our further investigation, to determine the normal ordering constant at all, since it shifts all energy levels equally.

It is a straight-forward but tedious computation to show, using the inverse of (10), that the charge (20) is given by

$$Q = \sum_{p\in\mathbb{Z}} (\bar{a}_p^\dagger \bar{a}_p - \bar{b}_p^\dagger \bar{b}_p) - n \quad . \tag{29}$$

This shows that there is induced charge (fermion number), the ground state has charge $-n$. We now observe a no-go theorem for induced fractional charge /2/. Once the charge operator is defined, as in (20), its spectrum is absolutely independent of the hamiltonian. If its spectrum is the integers, as in our case,

it cannot have fractional eigenvalues, for any eigenstate of the hamiltonian. Correspondingly the induced charge must be integral. More generally we can state the theorem that there can be no fractionization of the fermion number in any theory where the hamiltonian can be defined in the Hilbert space of free fermions and the fermion number operator coincides with that defined for the free fermions. Therefore, any adiabatic switching arguements /3/ for the fractionization of the fermion number must necessarily involve excursions out of the original Hilbert space.

It is evident that Q is globally gauge invariant,

$$V^\dagger Q V = Q \quad . \tag{30}$$

The axial charge is not globally gauge invariant, this is a manifestation of the axial anomaly /4/. It is straight-forward but tedious to show

$$V^\dagger Q_5 V = Q_5 + 2 \quad . \tag{31}$$

Thus if we define the gauge invariant axial charge $Q_5(V) = Q_5 + 2V$ we find

$$Q_5(V+1) = V^\dagger Q_5(V)\, V \quad . \tag{32}$$

If we transform to the Heisenberg representation, the usual axial anomaly equation does not exist. In this representation

$$Q_5^H(t) = e^{-it\overline{H}}\, Q_5\, e^{it\overline{H}} \tag{33}$$

however $i\frac{d}{dt} Q_5^H(t)$ does not exist. This involves the commutator $[\overline{H}, Q_5]$, which does not make sense because the range of Q_5 is disjoint from the domain of $\overline{H}$.

Our aim now is to consider the change in the fermionic ground state as we adiabatically translate the background fields. We wish to compute the Berry connection and phase /5/ for translation of the background fields around the circle. For now we keep conditions (5) and later consider the stability of the results under continuous deformations of $\theta(x)$, $N(x)$ and $a(x)$. We want to vary

$$\theta(x) \to \theta(x - x_0(t)) \tag{34}$$

as $x_0(t)$ varies adiabatically from 0 to 2π. After such a transport the ground state may obtain a geometrical phase factor as discussed by Berry /5/, in addition to the usual dynamical phase. Berry's phase is given as the integral of a U(1) connection /6/,

$$\varphi = \int_0^T A(t)\, dt \tag{35}$$

$$A(t) = \langle \Psi(t) | -i\frac{d}{dt} | \Psi(t) \rangle \tag{36}$$

where $|\Psi(t)\rangle$ is a continuously defined eigenstate of the instantaneous hamiltonian,

$$H(t)\,|\Psi(t)\rangle = E(t)\,|\Psi(t)\rangle \tag{37}$$

When $\varphi = \pi$, the state returns to minus itself. We call such states fermionic. We will look for this possibility.

$H(t)$ is given by

$$H(t) = \sum_{p\in\mathbb{Z}} \left(E_+(p)\, \bar{a}_p^\dagger(t)\, \bar{a}_p(t) + |E_-(p)|\, \bar{b}_p^\dagger(t)\, \bar{b}_p(t) \right) \tag{38}$$

where $\bar{a}_p(t)$ and $\bar{b}_p(t)$ are defined through

$$\Psi(x) = \sum_{p\in\mathbb{Z}} e^{in\left(\frac{1+\gamma_5}{2}\right)x_0(t)}\, e^{-ipx_0(t)} \left(\psi_+(p,x)\, \bar{a}_p(t) + \psi_-(p,x)\, \bar{b}_p^\dagger(t) \right) \tag{39}$$

Then

$$\bar{a}_p(t) = \left(\psi_+(p, x - x_0(t)) \middle| \Psi(x) \right), \quad \bar{b}_p^\dagger(t) = \left(\psi_-(p, x - x_0(t)) \middle| \Psi(x) \right), \tag{40}$$

hence

$$i\frac{d}{dt}\bar{a}_p(t) = -\dot{x}_0(t) \left(i\frac{d}{dx}\psi_+(p, x - x_0(t)) \middle| \Psi(x) \right)$$

$$= -\dot{x}_0(t) \left(\psi_+(p, x - x_0(t)) \middle| -i\frac{d}{dx}\Psi(x) \right) = -\dot{x}_0(t) \left[P, \bar{a}_p(t) \right] \tag{41}$$

and similarly for $\bar{b}_p(t)$, where

$$P = \sum_{p\in\mathbb{Z}} p \left(a_p^\dagger a_p - b_p^\dagger b_p \right) \tag{42}$$

is the naive translation generator. We will now show that

$$|\Psi(t)\rangle = e^{-ix_0(t)P}\, \mathcal{U}^\dagger |0\rangle \; . \tag{43}$$

This is valid for the case of arbitrary background fields. The spectrum of P is just the integers, thus by the spectral theorem

$$e^{-i2\pi P} = \int d\mu\, e^{-i2\pi\mu} |\mu\rangle\langle\mu| = \int d\mu\, |\mu\rangle\langle\mu| = 1 \tag{44}$$

Therefore $|\Psi(t)\rangle$ is continuously defined. It is also non-degenerate if there are no zero modes. To prove (43) it is equivalent to show

$$H(t) = e^{-ix_0(t)P}\, \bar{H}(v,a,n)\, e^{ix_0(t)P} \; . \tag{45}$$

Integrating (41) we get the unique solution

$$\bar{a}_p(t) = e^{-ix_0(t)P}\, \bar{a}_p\, e^{ix_0(t)P}, \quad \bar{b}_p(t) = e^{-ix_0(t)P}\, \bar{b}_p\, e^{ix_0(t)P} \tag{46}$$

and therefore (45) is obvious. Thus we wish to compute

$$A(t) = -\dot{x}_0(t)\, \langle 0 | \mathcal{U}\, P\, \mathcal{U}^\dagger | 0 \rangle \; . \tag{47}$$

Using the inverse of (10), (11) and (12) we get, after some algebra

$$A(t) = -\dot{x}_0(t) \sum_{p\in\mathbb{Z}} p \sum_{q\in\mathbb{Z}} \left(\left| \left(\psi_+(p) \middle| e^{-in\left(\frac{1+\gamma_5}{2}\right)x} \middle| \psi_-(q) \right) \right|^2 - \left| \left(\psi_-(p) \middle| e^{-in\left(\frac{1+\gamma_5}{2}\right)x} \middle| \psi_+(q) \right) \right|^2 \right)$$

$$= -\dot{x}_0(t)\, \frac{1}{2} \sum_{p\in\mathbb{Z}} p \left(\frac{p+v-\frac{n}{2}}{\sqrt{(p+v-\frac{n}{2})^2+m^2}} - \frac{p+v+\frac{n}{2}}{\sqrt{(p+v+\frac{n}{2})^2+m^2}} \right) \equiv -\dot{x}_0(t)\, \zeta(v,n) \tag{48}$$

Then

$$\varphi = -\int_0^T \dot{x}_0(t)\, \zeta(v,n)\, dt = -2\pi\, \zeta(v,n) \; . \tag{49}$$

We have not succeeded in computing $\zeta(v,n)$ in closed form, however it is easy to show

$$\zeta(v+1,n) = \zeta(v,n) + n, \quad \zeta(-v,n) = -\zeta(v,n) \; . \tag{50}$$

Thus we find for $v = \frac{k}{2}$, $k \in \mathbb{Z}$.

$$\varphi\left(\tfrac{k}{2}, n\right) = -2\pi\, \zeta\left(\tfrac{k}{2}, n\right) = -\pi k n \; . \tag{51}$$

This means for v a half odd integer the states behave fermionically in the odd soliton number sector, but bosonically in the even sector. This corresponds to induced statistics for the solitons from the fermions. The result is in fact invariant under arbitrary continuous deformations of $\theta(x)$ and $a(x)$. It is clear that the constant part of $a(x)$ is irrelevant, we have included it explicitly. These other deformations do not affect φ because they respect the real structure of the Hilbert bundle. For arbitrary $\theta(x)$ and $a(x)$ the first quantized hamiltonian can be taken completely real. In the representation

$\gamma_5 = \sigma^2$, $\gamma_0 = \sigma^1$,

$$\bar{h}(x) - \gamma_5 V \tag{52}$$

is completely real. Thus the eigenmodes $\psi_I(p)$ can be chosen completely real. The same is true for the free hamiltonian $h_0(x)$ and its eigenmodes. Then the coeffecients in the Bogoliubov transformation relating the new annihilation and creation operators to the free ones are completely real. This implies that the new vacuum is a real linear combination of the free basis states. The set of all such real linear combinations of the free basis states defines a real Hilbert space, which is preserved for arbitrary $\theta(x)$ and $a(x)$. Now for $V = \frac{k}{2}$, $k \in \mathbb{Z}$, $\bar{h}(x)$ is not real, however it is related to a real hamiltonian by a fixed unitary transformation

$$\bar{h}\left(x, \frac{k}{2}, a(x), \theta(x)\right) = e^{-ik\left(\frac{1+\gamma_5}{2}\right)x}\, \bar{h}\left(x, 0, a(x) - \frac{k}{2}, \theta(x) + kx\right) . e^{ik\left(\frac{1+\gamma_5}{2}\right)x} . \tag{53}$$

This is also enough to define a (modified) real structure in the corresponding Hilbert space. Thus the adiabatic transport of the vacuum defines a section of a real line bundle over a circle. There are only two topological possibilities, the trivial bundle or the Möbius bundle. Berry's phase measures the Chern number of the bundle /6/, which is a topological invariant. Thus the result is invariant under arbitrary deformations of $\theta(x)$ and $a(x)$.

If we consider switching on $N(x)$, the ground state is simply additionally (gauge) transformed by the unitary operator in (27).

Then the corresponding gauge covariant transport would be generated by

$$P(N(x)) = e^{\sum_{p\neq 0}\left(\frac{V(-p)}{p} j^\circ(p)\right)}\, P\, e^{-\sum_{p\neq 0}\left(\frac{V(-p)}{p} j^\circ(p)\right)} \tag{54}$$

Consequently the Berry connection and phase are invariant. The loop generated by $P(N(x))$ is of course different from that generated by P . Nevertheless, it is just the appropriate gauge transform of the original loop.

There is an intuitively clear interpretation of our results, (51), the Berry phase simply corresponds to the change of phase of the wave function of a particle of charge $-n$ moving in a gauge field*. This phase is simply the charge times the Wilson loop. For the case of more general V however, the phase from the Wilson loop seems to be augmented due to non-zero curvature of the Hilbert bundle, since $\zeta(V, n)$ is not equal to just Vn .

Finally we comment that the non-zero Berry phase implies that the effective action obtained from integrating out the fermions, contains a Wess-Zumino term, in the gauge and soliton fields. This term is not invariant under translation of the fields around the circle, and gives exactly the Berry phase for the translation. For $V = \frac{k}{2}$, the Berry phase is invariant under arbitrary changes in the soliton profile $\theta(x)$, hence the induced Wess-Zumino term is also invariant. This question was examined by Niemi /7/, for the case of 3 + 1 dimensional fermions interacting with skyrmions through just such a chirally twisted mass term. Here it is known that as the skyrmion profile is

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changed, a fermion mode crosses zero /8/. It was allegedly proven in /7/ that such a crossing would result in a discontinuous change in the effective action, in that the Wess-Zumino term would cease to be induced. This is of course true, but for the following reasons. The effective action from integrating out the fermions is correctly understood as the logarithm of the ground state persistence amplitude,

$$e^{i\Gamma_{eff}} = \langle\langle 0 | T \exp\{-i\int_{-\infty}^{\infty} dt\, H(t)\} | 0 \rangle\rangle \qquad (55)$$

Here the hamiltonian is time dependent due to explicitly time dependent background fields which vary about their fixed values, that define $|0\rangle\rangle$. (This may be slightly generalized to allow for different initial and final configurations.) Now in our case Γ_{eff} contains a Wess-Zumino term, which is invariant under arbitrary deformations of $\theta(x)$. But suppose that the deformation results in a mode crossing zero. Then Γ_{eff} will change discontinuously only if we discontinuously redefine it as

$$e^{i\Gamma_{eff}} = \langle\langle 1 | T \exp\{-i\int_{-\infty}^{\infty} dt\, H(t)\} | 1 \rangle\rangle \quad , \qquad (56)$$

where $|1\rangle\rangle$ is the new ground state with the particle corresponding to the mode which crossed zero (from below), removed. The R.H.S. of (55) is clearly continuously defined. In as much as the regularized fermionic determinant is just a mnemonic for the R.H.S. of (55), it will not change discontinuously for any continuous variations of the parameters.

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