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COSMIC HADRONISATION TRANSITION WITHIN THE STRING-FLIP MODEL OF QUARK MATTER

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1. INTRODUCTION

One interesting problem of quantum chromodynamics (QCD) is the derivation of the equation of state of dense, strongly interacting matter. According to the general belief, and also to lattice calculations, there appears a confined (hadronic) phase and a deconfined phase which is composed of quarks and gluons. Details of the transition are still under debate. Especially, it has been conjectured[1] that full QCD predicts two transitions: (i) from a phase of massless quarks and gluons, obeying asymptotic freedom, to massive constituent quarks, interacting via a confinement potential (chiral phase transition); (ii) the formation of bound states (hadronisation or confinement transition). However, rigorous results from QCD are at present not available, and SU(2) lattice QCD calculations[2] might yield only a crude description of the quark-gluon system, so that the interplay between the deconfinement and the chiral phase transition is poorly understood up to now.

Phenomenological models have been elaborated to describe the properties of the quark-gluon system under various conditions. Very popular is the bag model which is thought to describe the equation of state for high densities and/or temperatures. Within the bag model approach, a three-phase description of dense, strongly interacting matter has also been given recently[3,4]. Alternatively, a string-flip model has been worked out for the treatment of the many-particle system of constituent quarks interacting via a confinement potential[6]. This model allows one to describe the properties of hadrons as well as of a massive quark matter phase on the same footings.

It is the aim of the present paper to compare these two phenomenological models (bag and string-flip) and to discuss the parameter values which lead to a three-phase description of dense, strongly interacting matter. Furthermore, we apply the derived equation of state to the hadronisation transition during the evolution of the early universe. For this goal it is sufficient to consider the special case of vanishing baryon-chemical potential.

2. EQUATION OF STATE FROM THE STRING-FLIP MODEL

A quantum statistical approach to the many-quark system on the basis of the phenomenological quark potential was given within the string-flip model/6/. This approach is based on the concept of the saturation of interaction within nextneighboured color-neutral clusters. According to this model, it has been shown that there exists a hadronic phase where quarks are bound in color-neutral clusters. The interaction is governed by the Pauli exclusion principle, and with increasing density the bound states break off forming a phase of quasi-free quarks. Evaluating the interaction with the surroundings in Hartree approximation, a first-order phase transition has been found in/6/ at zero temperature for nuclear matter densities of about $5\,n_o$ ($n_o \simeq 0.17\ \mathrm{fm}^{-3}$), whereas a phase transition at zero baryon number density was obtained in/5/ at temperature of about 200 MeV.

The density of quarks and antiquarks in the quasi-free quark matter phase in Hartree approximation is given by/5,6/

$$n_Q(T) = n_{\bar Q}(T) = g_Q (2\pi)^{-3} \int d^3p\, [\exp\{\tilde E_Q(p)/T\}+1]^{-1} \quad (1)$$

with the quasi-particle energies

$$\tilde E_Q(p) = E_Q(p) + \Delta^H = (p^2 + m_Q^2)^{1/2} + \Delta^H. \quad (2)$$

From this, we find the following expression for the pressure of inteacting constituent quarks

$$P_Q(T) = P_Q^{id}(T) - n_Q(T)\,\Delta^H, \quad (3)$$

where P_Q^{id} is the ideal pressure contribution of massive quarks according to

$$P_Q^{id}(T) = 2 g_Q\, T\, (2\pi)^{-3} \int d^3p\, \ln[1 + \exp\{-E_Q/T\}] \quad (4)$$
$$= g_Q\, \pi^{-2} m_Q^2 T^2 \sum_{k=1}^{\infty} (-1)^{k+1} k^{-2} K_2(k m_Q/T).$$

Here and in the following, $K_\nu(x)$ denotes the modified Bessel function of the second kind of order ν. Particle and antiparticle contributions are included in equation (4) and $g_Q = 12$ (= spin x color x flavor) is the quark degeneracy factor for two flavors. Neglecting quark correlations, the distribution function of the nearest neighbouring quarks with given color reads/5-7/

$$c(r) = \frac{1}{3}\, n_Q \exp\{-\frac{4}{9}\pi n_Q r^3\}. \quad (5)$$

Hence, for a given quark-quark interacting potential V the Hartree shift of the two-quark states is given by/7/

$$\Delta^H = \int d^3r\, V(r)\, c(r). \quad (6)$$

Non-relativistic quark potential models based on a QCD-motivated interaction potential of the form

$$V(r) = -\alpha_{eff}\, r^{-1} + \sigma r - \hat c \quad (7)$$

are able to describe confinement via the string tension σ and an effective coupling constant α_{eff}. With appropriately chosen parameters $\hat c$ and quark masses m_Q, properties of hadrons and hadron-hadron interactions are well reproduced/8,9/. In the following we choose the parameter values α_{eff} =0.52, σ =0.19 GeV^2, $\hat c$ =568 GeV and m_Q = 0.27 GeV/5/. Due to the potential (7) the equation (6) results in

$$\Delta^H = -\left(\frac{4}{9}\pi\right)^{1/3} \Gamma\left(\frac{2}{3}\right) \alpha_{eff}\, n_Q^{1/3} + \left(\frac{4}{9}\right)^{-1/3} \Gamma\left(\frac{4}{3}\right) \sigma\, n_Q^{-1/3} - \hat c. \quad (8)$$

One sees from this expression that the free quark formation is strongly prohibited for rather low densities $n_Q \to 0$ since $\Delta^H \to \infty$. (In principle, the Hartree approach can be improved by allowing for quark correlations in the massive quark matter phase).

In the hadronic phase at zero net baryon number density (μ = n_B = 0) the pions are supposed to dominate. The pion equation of state can be derived from

$$P_\pi(T) = -g_\pi\, T\, (2\pi)^{-3} \int d^3p\, \ln\{1 - \exp[-E_\pi(p)/T]\}, \quad (9)$$

where for non-interacting pions

$$E_\pi^2(p) = p^2 + m_\pi^2. \quad (10)$$

The pion degeneracy factor is g_π = 3, and the pion mass is chosen as m_π = 140 MeV. From equation (9) after a straightforward calculation, we obtain for the pion gas pressure

$$P_\pi(T) = g_\pi (2\pi^2)^{-1} m_\pi^2 T^2 \sum_{k=1}^{\infty} k^{-2} K_2(k m_\pi/T), \quad (11)$$

and according to the thermodynamical relation

$$\varepsilon(T) = T\,(\partial P/\partial T) - P \tag{12}$$

the energy density is given by

$$\varepsilon_\pi(T) = 3P_\pi(T) + g_\pi (2\pi^2)^{-1} m_\pi^3 T \sum_{k=1}^{\infty} k^{-1} K_1(k m_\pi/T). \tag{13}$$

In the high-temperature limit we get the well-known result

$$\varepsilon_\pi(T) = 3P_\pi(T) = g_\pi(\pi^2/90)\,T^4. \tag{14}$$

A more realistic equation of state should take into account further hadrons (e.g. nucleons and antinucleons, higher excited mesons). Furthermore, the interaction between the hadrons would lead to an energy shift due to the Pauli exclusion principle (overlapping quark wave functions).

Both the hadronic and the massive quark matter equations of state are displayed in figure 1. A first-order phase transition is predicted due to the Gibbs' criteria. The value of the critical temperature is about 175 MeV. The latent heat per unit volume in the phase change is defined by

$$L_{\pi Q} = \varepsilon_Q(T_c) - \varepsilon_\pi(T_c) = T_c \frac{\partial}{\partial T}(P_Q - P_\pi), \tag{15}$$

where the derivative is to be evaluated at $T = T_c$. For our pion versus string-flip matter phase transition we get the following value:

$$L_{\pi Q} \simeq 0.87\ \mathrm{GeV\ fm^{-3}}. \tag{16}$$

3. COMPARISON WITH THE BAG MODEL

Of course, an open question is whether or not the string-flip model is appropriate at high densities and/or temperatures. Therefore, let us compare the equation of state of the string-flip model with the results for the phenomenological bag model. For simplicity, we assume zero-mass quarks and start from the bag model equation of state including interactions up to first order in strong coupling constant/10-13/

$$P(T) = \frac{37}{90}\pi^2 T^4 - \frac{11}{36} g_s^2 T^4 - B_Q, \tag{17}$$

where B_Q is the bag constant. The running coupling constant g_s is obtained from the renormalisation group in the form/11-13/

$$g_s^2(T) = 24\pi^2/29\,\ln(M/\Lambda), \tag{18}$$

where Λ fixes the scale and M is the substraction energy which is given by/10/

$$M^2(\mu=0) = 4T^2[77\zeta(5)/17\zeta(3)] \simeq 15.622\,T^2. \tag{19}$$

At the transition point the critical temperature must satisfy ($M \approx 4T$)

$$\pi^2 T_c^2 \left[\frac{34}{90} - \frac{22}{87}\ln^{-1}(4T_c/\Lambda)\right] = B_Q. \tag{20}$$

For our value of the critical temperature we get the following result (Λ = 100 MeV):

$$B_Q \simeq (219\ \mathrm{MeV})^4. \tag{21}$$

It is worthwhile to be noticed that the equation of state for the string-flip model is fairly well reproduced in the transition region by the bag model, see figure 2. Therefore, our model supports to some extent the bag model. We emphasize moreover that, due to the relation to the non-relativistic potential model, the bag constant is accessible in our phenomenological approach. A possible fit of the critical temperature as well as of the latent heat (16) leads to the values $B_Q^{1/4} \simeq$ 178 MeV and $\Lambda \simeq$ 237 MeV. However, as is shown in the inset of figure 2, the latter fit deviates rather strongly from our string-flip curve at higher temperatures. Therefore, we prefer the value of (21).

Another question is the chiral symmetry breaking, where current quarks (massless quarks) transform into constituent quarks/1/. This transition will only shortly be discussed here. We represent the phase of massless quarks and gluons also with the help of the bag model (bag constant B), but without accounting for the first-order perturbative QCD corrections so that the asymptotic behaviour in the high-temperature limit of lattice calculations/2/ is correctly repro-

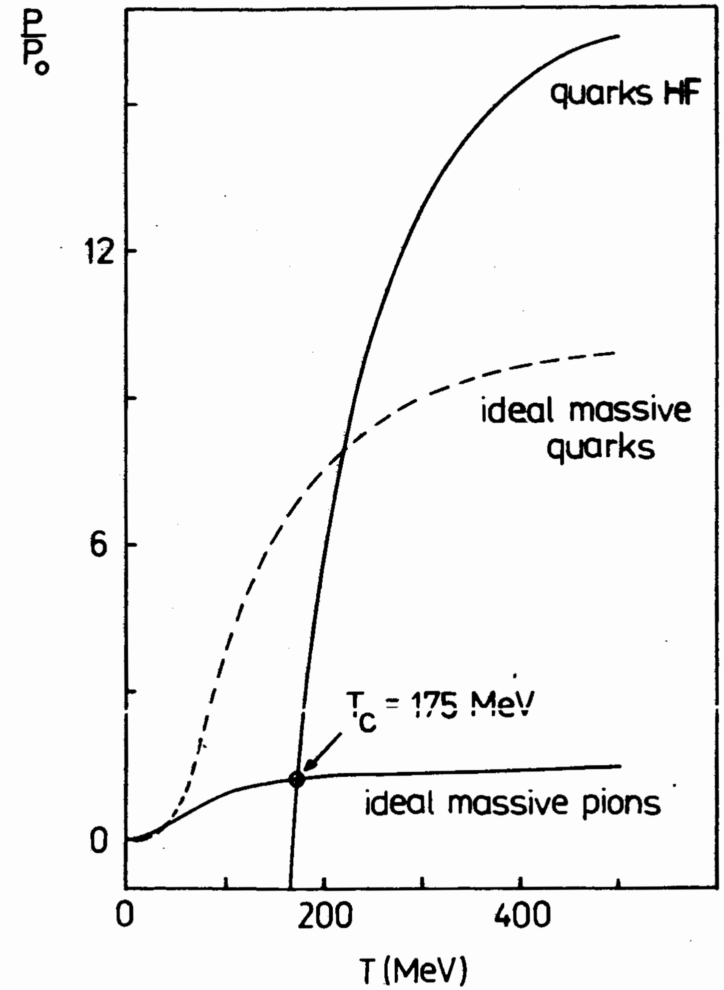

Fig. 1. The pressure of non-interacting massive pions and interacting massive quarks within the Hartree approach as a function of the temperature. The normalisation parameter for the pressure is given by $p_0 = T^4 \pi^2/45$. To show the influence of the Hartree shift the non-interacting massive quark contribution is also displayed.

duced. Obviously, depending on the bag constant $\mathcal{B}$, a phase transition from a massive quark matter phase to a massless one can be obtained. According to the simple estimate

$$\mathcal{B} \geq \mathcal{B}_a + \frac{22}{87}\pi^2 T_c^4 \ln^{-1}(4T_c/\Lambda) \qquad (22)$$

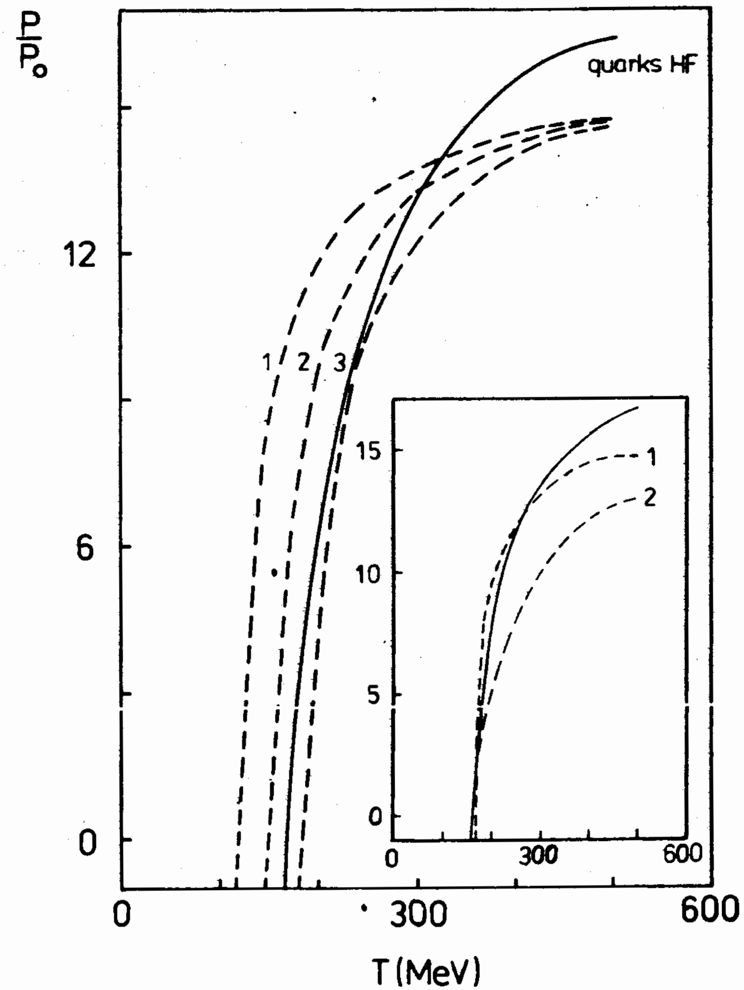

Fig. 2. Comparison between the string-flip model (full line) and the first-order perturbative QCD bag model (broken lines) with Λ = 100 MeV. The values of $\mathcal{B}_a^{1/4}$ are 145, 190 and 235 MeV for the curves 1,2 and 3, respectively. The inset shows two different fits of the string-flip model with Λ = 100 MeV, $\mathcal{B}_a^{1/4}$ = 219 MeV (curve 1) and Λ = 273 MeV, $\mathcal{B}_a^{1/4}$ = 178 MeV (curve 2), respectively.

we obtain a condition for the appearance of an additional quark matter phase within a bag model approach. Hence, for values of $\mathcal{B}$ equal to the critical one (22), the chiral phase and deconfinement transition coincide. For $\mathcal{B}$ less than the critical one (22), the massless quark-gluon plasma phase is thermodynamically favoured so that in this range of parameters the deconfinement transition would proceed directly from hadrons into massless quarks and gluons (cf. fig. 3). In the following, however, we assume that the inequality (22) holds, and we consider the cosmic hadronisation under this condition.

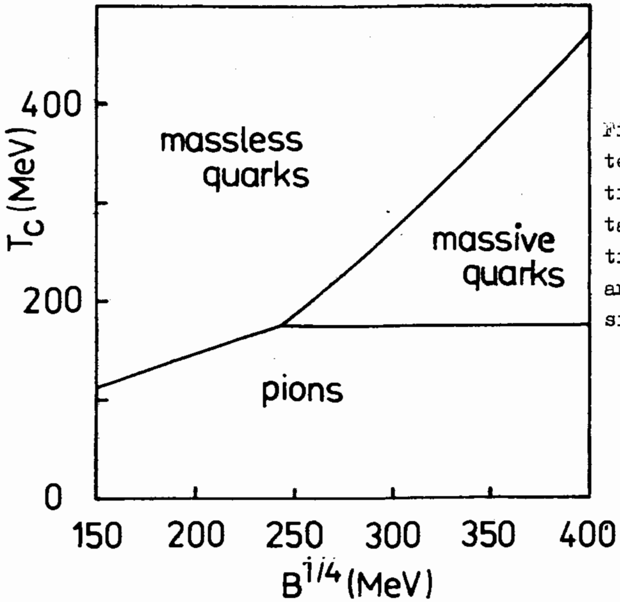

Fig. 3. The critical temperature as a function of the bag constant showing the relation of deconfinement and chiral phase transition.

4. COSMIC HADRONISATION TRANSITION

According to the hot standard model (big bang), the energy density of the universe at early times was high enough for matter to be in a quark-gluon plasma state. Due to the expansion, the energy density decreased, and there was an era when the matter has undergone a hadronisation. For the sake of simplicity, we assume that the chemical potential for baryons vanishes; the asymmetry among baryons and antibaryons is exceedingly small which is described at present by the ratio/14,16/

$$(n_B - n_{\bar{B}}) / n_\gamma \sim 10^{-10\pm 1} . \tag{23}$$

The hot standard model is based on the Einstein equations which give, for a homogeneous and isotropic universe being filled with a perfect fluid/14/

$$(dR/dt)^2 = \frac{8}{3}\pi G R^2 \varepsilon , \tag{24}$$

$$R^3 \, d\varepsilon + (\varepsilon + p) \, dR^3 = 0 . \tag{25}$$

Here, $R(t)$ is the scale factor, G the gravitational constant, and we have dropped the curvature term in (24) which is negligible at high energy densities. Equation (25) can also be written as

$$- dR / R = d\varepsilon / 3(\varepsilon + p) . \tag{25'}$$

Eliminating dR/R from (24) and (25') one obtaines

$$- \frac{d\varepsilon}{3\sqrt{\varepsilon}(\varepsilon + p)} = \left(\frac{8\pi G}{3}\right)^{1/2} dt . \tag{25''}$$

Now we restrict our consideration to the case where the phase transition occurs smoothly and adiabatically (i.e. without supercooling) and refer to the works/15-17/. Therefore, equation (25) may be written as

$$R^3 d(\varepsilon + p(T_c)) + (\varepsilon + p(T_c)) \, dR^3 = 0 \tag{26}$$

which implies that

$$(\varepsilon + p(T_c)) R^3 = const \tag{27}$$

at $T = T_c$. For the duration of the quark-hadron phase transition we obtain from (25'')

$$\Delta t = \frac{2}{3}\left(\frac{3}{8\pi G p(T_c)}\right)^{1/2} \arctan \frac{\sqrt{\varepsilon_p(T_c)} - \sqrt{\varepsilon_h(T_c)}}{\sqrt{p(T_c)}\,[1 + \sqrt{\varepsilon_p(T_c)\varepsilon_h(T_c)}/p(T_c)]} \tag{28}$$

or, due to the equation (15),

$$\Delta t \simeq \left(\frac{1}{6\pi G p(T_c)}\right)^{1/2} \arctan \sqrt{3}\, \frac{[1 - L_{\pi Q}/3p(T_c)]^{1/2} - 1}{1 + 3[1 + L_{\pi Q}/3 \cdot p(T_c)]^{1/2}} . \tag{28'}$$

In order to calculate this duration we have to specify the relevant degrees of freedom of the matter. At the epoch of the quark-hadron phase transition, photons (g_γ = 2), electrons (g_e = 4) muons (g_μ = 4) and neutrinos (g_ν = 6) are taken into account as a ultra-relativistic background, for which the partial statistical factor yields

$$g = g_\gamma + \frac{7}{8}(g_e + g_\mu + g_\nu) = 57/4 \tag{29}$$

so that the background pressure from these particles is

$$P_{BG} = g(\pi^2/90) T^4 \simeq 1.563 \, T^4 . \tag{30}$$

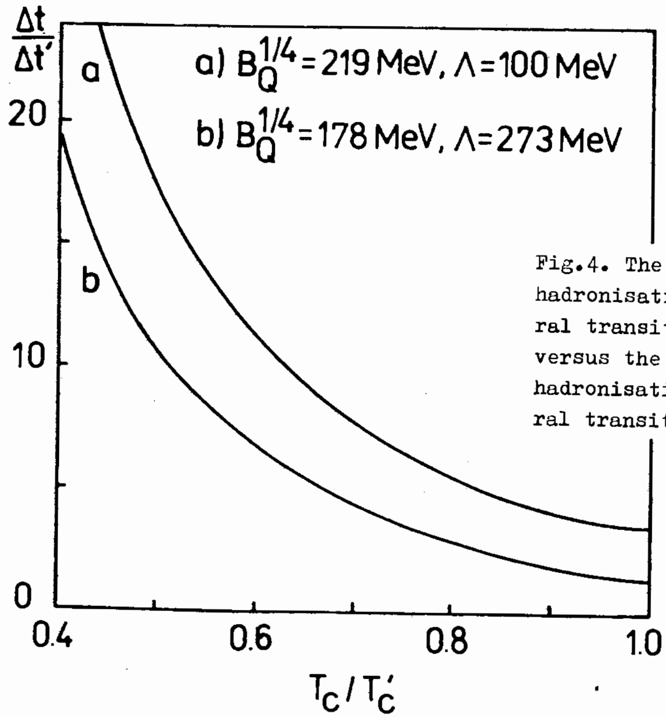

Fig.4. The ratio of the hadronisation to the chiral transition duration versus the ratio of the hadronisation to the chiral transition temperature.

Hence, for the massive quark and hadronic phase, respectively, the critical values of the total energy density and pressure are obtained by

$$\varepsilon_p(T_c) = \varepsilon_Q(T_c) + 3\, p_{BG}(T_c),$$
$$\varepsilon_h(T_c) = \varepsilon_\pi(T_c) + 3\, p_{BG}(T_c), \qquad (31)$$
$$p(T_c) = p_c + p_{BG}(T_c),$$

where p_c is to be numerically determined by the cross over of the pion and string-flip equation of state (cf. fig. 1). Using (28),(30) (31), or equivalently (2''), (16) and (31), now the duration of the hadronisation transition can be evaluated, yielding

$$\Delta t \simeq 5.3\ \mu\text{sec}. \qquad (32)$$

This value is in the same order of magnitude as the age of the universe at T_c . In fact, this finding agrees fairly well with earlier results/11,15,16/.

As already mentioned above, the chiral symmetry breaking depends on the value of $\mathcal{B}$. The corresponding critical values entering in equation (28) are

$$\varepsilon_p'(T_c') = \frac{37}{30}\pi^2 T_c'^4 + 3\, p_{BG}(T_c') + \mathcal{B},$$
$$\varepsilon_h'(T_c') = \frac{37}{30}\pi^2 T_c'^4 - \frac{11}{12} g_s^2(T_c') T_c'^4 + 3 p_{BG}(T_c') + \mathcal{B}_Q, \qquad (33)$$
$$p'(T_c') = p(T_c') + p_{BG}(T_c'),$$

where T_c' is the critical temperature of the chiral phase transition which can be iteratively evaluated by using of/13/

$$T_c'^{(n+1)} = \left[\frac{87}{22}\pi^{-2}(\mathcal{B}-\mathcal{B}_Q)\ln(4T_c'^{(n)}/\Lambda)\right]^{1/4}. \qquad (34)$$

The pressure at the ciral transition point $p(T_c')$ is determined by the equation (17). The dependence of the ratio of the hadronisation to the chiral phase transition duration on the ratio of T_c/T_c' is displayed in figure 4. Even the exact value of T_c/T_c' is unknown (/1/ claims $T_c/T_c' \lesseqgtr 1$) one observes in figure 4 that our present model predicts a duration of chiral symmetry breaking being rather short in comparison with the hadronisation transition. Therefore, the hadronisation process seems to dominate in the quark-hadron transition.

5. CONCLUSIONS

In the present paper we apply a string-flip model to describe the massive quark matter phase. Relying on a quark-quark interacting potential, suitable to reproduce individual hadron properties, we derive an equation of state, at μ = 0, which can surprisingly well be fitted by a bag model equation of state. In this sense, hadron spectroscopy supports a bag constant of $\mathcal{B}_Q^{1/4} \simeq$ 219 MeV. The transition to the hadronic phase appears as first order transition.

We also discuss the possibility of a simple three-phase model (hadrons - massive quarks - massless quarks and gluons). Furthermore, we apply the present model to the cosmic quark-hadron phase transition. The hadronisation transition appears as the most important part.

Finally, we mention that the massive quark matter phase might be responsible during the course of relativistic heavy-ion collisions for a large bulk viscosity and, therefore, might manifest itself by strong dissipative effects. Also the massive quark matter phase might modify direct photon and dilepton production rates. Further studies of such-effects are under way. Recently, the string-flip model with massive quarks has been successfully applied in explaining the J/ψ suppression observed in recent ultra-relativistic heavy-ion collisions/7/.

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Фос Х. и др. E2-88-648

Космический переход адронизации в модели стринг-флиппа кварковой материи

Получено уравнение состояния для массивной кварковой материи в рамках модели стринг-флиппа для взаимодействия кварков. Рассмотрен переход к адронной материи при конечной температуре и нулевом химическом потенциале. Уравнение состояния хорошо представлено через мешковое уравнение состояния с мешковой постоянной $B_Q^{1/4} \approx 219$ МэВ. Космический переход адронизации получен аналогично тому, как это делали другие авторы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Hadronisation Transition within the String-Flip Model of Quark Matter

The equation of state for a massive quark matter phase is obtained within a string-flip model for the quark interaction. The hadronisation transition is considered at finite temperature and vanishing baryon-chemical potential. The equation of state of the massive quark matter phase can be fairly well fitted by a bag model equation of state with $B_Q^{1/4} \approx 219$ MeV. From the latent heat and the critical pressure the time duration of the cosmic hadronisation transition is obtained in accordance with earlier estimates.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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