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UNIFICATION OF A POTENTIAL MODEL WITH CHIRAL LAGRANGIANS

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Introduction

New accelerators of hadrons are oriented, in general, to experimental investigation of heavy quarkonia, in particular, their decays into light mesons. As is known, the phenomenology of heavy quarkonia is based on nonrelativistic potential quark models /1/, while light mesons are described by the chiral phenomenological Lagrangians /2/.

Our work is devoted to unifying the potential model with the chiral Lagrangian approach.

Since a heavy quarkonium can be represented as a hydrogen-like atom, it is natural to use the analogy with QED to construct the S-matrix of the interaction. In QED, the spectrum of an atom is calculated only in the radiative gauge, in the rest frame. As fermions in an atom are off their massshells, the obtained results indeed are invariant only with respect to both gauge and relativistic unified transformations, but no separately (this fact has been noted in ref. /3/ by Heisenberg and Pauli).

The diagrammatic technique of the radiative gauge with the Heisenberg-Pauli transformation group is found, in refs. /4,5/, in the framework of the so-called "minimal" quantization method of gauge theories with the explicit solution of the Gauss equation (the equation to a temporal component of the gauge field, $A_0 = \eta \cdot A$, where η is the time-like vector - the quantization axis, $\eta^2 = 1$).

In the "minimal" quantization (unlike the Dirac one) the concepts of both gauge invariance and relativistic covariance have to be changed. The requirement of gauge invariance should be imposed not on the matrix element but on the very physical variables (which depend on the initial fields as functionals) /4,5/. The usual Lorentz transformations for the initial fields correspond just to the Heisenberg--Pauli ones for the physical variables with the gauge rotation. As is shown in refs. /4,5/, taking into account this rotation one restores relativistic covariance of the radiative gauge. Relativistic covariance, in its manifest form, can be achieved only by a special choice of the quantization axis (η) depending on boundary conditions of a physical problem. Particularly, for a bound state, atom, the axis is to be chosen so that the Coulomb field, linking the fermions in the atom, moves together with the atom (otherwise, the relativistic dispersion law $\mathcal{P}^2 = M_H^2$ would not be valid [6] where $\mathcal{P}$ and M_H are the 4-momentum and mass of the atom).

For interacting atoms, which are described by the bilocal fields $\mathcal{M}(x,y)$ the relativistic covariance can be achieved if one chooses the quantization axis (η) parallel to the operator of differentiation of the fields with respect to the total coordinate $(X = \frac{1}{2}(x+y))$, i.e.

$$\eta_\mu \sim -i\,\partial/\partial X_\mu .$$

As is known, all conventional quantization methods for gauge theories are oriented to description of S-matrix with asymptotical states of free particles. Generalization of these methods to bound states in QED requires introduction of the above pointed additional principles, namely, the "minimal" quantization[4,5] and choice of the quantization axis[4,5,7].

We shall use just these principles to generalize the potential quark model in order to describe interactions of mesons. In this model a bound state is formed due to the Coulomb and "confinement" potentials.

The paper is organized as follows. In Section 1 we obtain the effective bilocal Lagrangians. In Section 2 the quark spectrum in a hadron is considered, and for the meson spectrum, in Section 3, the three-dimensional relativistic equation is obtained. Sections 4-6 deal with quantization of the bilocal fields, their normalization and the S-matrix elements.

1. From Quarks to Mesons

We start with the following effective action having the four-quark interaction[6,8]

$$S_{eff} = \int d^4x \left\{ \bar{q}(x) \left[G^{-1}_{\hat{m}^0}(x) + L(x) \right] q(x) - \right. \qquad (1)$$

$$\left. - \frac{1}{2} \int d^4y \; q_{\alpha_2}(y)\, q_{\beta_1}(x) \left[K^{\eta}(x-y) \right]_{\alpha_1\beta_1;\alpha_2\beta_2} q_{\alpha_1}(x)\, \bar{q}_{\beta_2}(y) \right. .$$

Here $G^{-1}_{\hat{m}^0} = -i\not{\partial} + \hat{m}^0$ is the Dirac operator for free quarks with the bare masses $\hat{m}^0 = \mathrm{diag}(m^0_1, \ldots, m^0_{n_f})$, α or β are the short symbols for the Dirac and flavour indices, $L(x)$ is the external local operator (for example, the leptonic current), K^η is the instantaneous interaction (kernel) with the defined quantization axis η ,

$$K^{\eta}_{\alpha_1\beta_1;\alpha_2\beta_2}(x) = K_{\alpha_1\beta_1;\alpha_2\beta_2}(x^{\parallel}, x^{\perp}) = \not{\eta}_{(a)} V(x^{\perp}) \not{\eta}_{(b)} \; , \qquad (2)$$

$$\left(x_\mu = x^{\parallel}_\mu + x^{\perp}_\mu , \; x^{\parallel}_\mu = \eta_\mu (\eta \cdot x), \; \not{\eta}_{(a)} = \gamma^\mu_{(a)} \eta_\mu , \; \eta^2 = 1, \; (a) = (\alpha_1\beta_1), (b) = (\alpha_2\beta_2) \right)$$

where $V(x)$ is the sum of the Coulomb and "confinement" (for example, the oscillator) potentials, i.e.,

$$V(r) = \frac{4}{3}\left(-\frac{\alpha_s}{r} + V_0 r^2 \right) , \quad r = |x^{\perp}| , \qquad (3)$$

α_s and V_0 are the parameters[6]. For our purpose it is enough to consider only the colour-singlet part of K^η (for this reason the factor $\frac{4}{3} I \sim \frac{1}{4} \lambda^n_{(a)} \lambda^n_{(b)}$ is included in (3)).

Note that the action (1) with only the Coulomb (colourless) potential, i.e., when $V_0 = 0$, is derived in QCD (as well as in QED) if in the expansion of the gluon (photon) field over temporal $(A_0 = \eta \cdot A)$ and transversal $(A^{\perp})$ components one uses the classical solution to the Gauss equation instead of A_0 , and takes $A^{\perp} = 0$ [4].

More thoroughly, the four-quark interaction with increasing (confinement) potential for $\eta_\mu = (1,0,0,0)$ has been considered in ref. [6], where the spectra of light mesons and quarks with zero bare mass $(\hat{m}^0 = 0)$ are obtained. However, the authors could not solve correctly the problem of relativisation of bound state wave functions.

For relativistic description of the spectrum of mesons and their interactions we apply to a bound-state wave function the principle of choosing the quantization axis, as has been stated in the Introduction, i.e.

$$\eta_\mu \sim \frac{1}{i} \frac{\partial}{\partial X^\mu} \quad ; \quad X^\mu = \frac{1}{2}\left(x^\mu_{(a)} + x^\mu_{(b)} \right) , \qquad (4)$$

(where $x_{(a)}$ is the coordinate of the quark (a)).

Using the identical Legendre transformation for the bilocal field $\mathcal{M}(x,y)$ (that linearizes the four-fermion term) and after quantization of the fermion fields the action (1) takes the form [9]

$$S_{eff}[\mathcal{M}] = N_c\left\{ \frac{1}{2}(\mathcal{M},[K^{\eta}]^{-1}\mathcal{M}) - i\,\mathrm{Tr}\,\mathrm{Ln}\left[-G^{-1}_{\hat{m}^0}+\mathcal{M}+L\right]\right\}, \quad (5)$$

where Tr means both the integration over continuous variables and the trace over discrete indices, and N_c is the colour number.

Extremum condition for the action (5), $\delta S_{eff}/\delta\mathcal{M}=0$, coincides with the Dyson-Schwinger equation for the quark mass operator Σ (when $\mathcal{M}=\Sigma$)

$$\Sigma(x-y) = \hat{m}^0\delta^{(4)}(x-y) - iK^{\eta}(x-y)\,G_{\Sigma}(x-y), \quad (6)$$

where $G^{-1}_{\Sigma}(x) = -i\not\partial\,\delta^{(4)}(x)+\Sigma(x)$. This equation defines a spectrum of quarks and, in particular, the spontaneous generation of the dynamical quark mass /6/.

Expansion of the action (5) around the classical solution (6) over fluctuation $\mathcal{M}'=\mathcal{M}-\Sigma$ gives the free part of the action

$$S_{free}[\mathcal{M}'] = \frac{N_c}{2}\left\{(\mathcal{M}',[K^{\eta}]^{-1}\mathcal{M}') + i\,\mathrm{Tr}(G_{\Sigma}\mathcal{M}')^2\right\}, \quad (7)$$

(where we put $L(x)=0$), and one obtains the other term describing the interaction of the bilocal fields

$$S_{int}[\mathcal{M}'] = iN_c\sum_{n=3}^{\infty}\frac{1}{n}\,\mathrm{Tr}(G_{\Sigma}\mathcal{M}')^n \equiv iN_c\sum_{n=3}^{\infty}\frac{1}{n}\,\mathrm{Tr}\,\Phi^n \quad (8)$$

where the field

$$\Phi(x,y) \equiv \int d^4z\; G_{\Sigma}(x,z)\,\mathcal{M}'(z,y)$$

is introduced for convenience, and $\mathrm{Tr}\,\Phi^n$ is to be understood as

$$\mathrm{Tr}\,\Phi^n \equiv \mathrm{tr}\int d^4x_1 d^4x_2\ldots d^4x_n\;\Phi(x_1,x_2)\,\Phi(x_2,x_3)\ldots\Phi(x_n,x_1). \quad (9)$$

Variation of (7) over $\mathcal{M}'$ leads to the homogeneous Bethe-Salpeter equation in the ladder approximation to the vertex function $\Gamma_{(ab)}(x,y)$ of the bound state

$$\Gamma_{(ab)}(x,y) = -iK^{\eta}(x-y)\int d^4z_1 d^4z_2\; G_{\Sigma_{(a)}}(x-z_1)\,\Gamma_{(ab)}(z_1,z_2)\,G_{\Sigma_{(b)}}(z_2-y) \quad (10)$$

that must be considered with equation (6).

2. Quark Spectrum in a Meson

Let us consider the Dyson-Schwinger equation (6) for a quark (a) in the momentum space

$$\Sigma_{(a)}(k^{\perp}) = m^0_{(a)} - i\int\frac{d^4q}{(2\pi)^4}\,\underset{\sim}{V}(k^{\perp}-q^{\perp})\,\not\eta_{(a)}\,G_{\Sigma_{(a)}}(q)\,\not\eta_{(a)} \quad (11)$$

where

$$G_{\Sigma}(q) = \int d^4x\,\exp(-iqx)\,G_{\Sigma}(x),$$

$$\underset{\sim}{V}(k^{\perp}) = \int d^4x\,\exp(-ikx)\,V(x^{\perp})\,\delta(x^{\parallel}).$$

Separating the integration variable in (11) into longitudinal and transversal components $q_{\mu}=(q^{\mathcal{P}},q^{\perp})$ and carrying out integration over $q^{\mathcal{P}}$ with

$$\frac{d^4q}{(2\pi)^4} = \frac{dq^{\mathcal{P}}}{2\pi}\,\frac{d^3q^{\perp}}{(2\pi)^3}, \quad q^{\mathcal{P}} = \sqrt{q^{\parallel\mu}q^{\parallel}_{\mu}}, \quad q^{\parallel}_{\mu} = \mathcal{P}_{\mu}\,\frac{\mathcal{P}\cdot q}{\mathcal{P}^2} \quad (12)$$

one can easily see that the mass operator depends only on the transversal momentum $(k^{\perp})$ in the form

$$\Sigma_{(a)}(k^{\perp}) = E_{(a)}(k^{\perp})\left[\sin\varphi_{(a)}(k^{\perp}) - \hat{\not k}^{\perp}\cos\varphi_{(a)}(k^{\perp})\right] + \not k^{\perp} \quad (13)$$

where $\hat{k}^{\perp}$ is the unit vector $(k^{\perp}\cdot\mathcal{P}=0)$. Then, the Green function $G_{\Sigma_{(a)}}$, after substituting (13), takes the form

$$G'_{\Sigma_{(a)}}(q^{\mathcal{P}},q^{\perp}) = \frac{1}{\not q - \Sigma_{(a)}(q^{\perp})} =$$

$$= -S_{(a)}(q^{\perp})\left\{\frac{\Lambda^{\mathcal{P}}_{+}}{E_{(a)}(q^{\perp})-q^{\mathcal{P}}-i\varepsilon} + \frac{\Lambda^{\mathcal{P}}_{-}}{E_{(a)}(q^{\perp})+q^{\mathcal{P}}-i\varepsilon}\right\}S_{(a)}(q^{\perp}) \quad (14)$$

Here we have introduced the operator of the Foldy-Wouthuysen matrix /10/ type,

$$S^{2}_{(a)}(q^{\perp}) \equiv \exp\left\{-\hat{\not q}^{\perp}\left[\varphi_{(a)}(q^{\perp}) - \frac{\pi}{2}\right]\right\} \tag{15}$$

and the projectors

$$\Lambda^{\mathcal{P}}_{\pm} \equiv \frac{1}{2}\left(I \pm \frac{\not{\mathcal{P}}}{\sqrt{\mathcal{P}^{2}}}\right) \tag{16}$$

satisfying the relations

$$S_{(a)}\not{\mathcal{P}} = \not{\mathcal{P}}\, S^{-1}_{(a)}\,, \tag{17}$$

$$\Lambda^{\mathcal{P}}_{+} + \Lambda^{\mathcal{P}}_{-} = I, \quad \Lambda^{\mathcal{P}}_{\pm}\Lambda^{\mathcal{P}}_{\pm} = \Lambda^{\mathcal{P}}_{\pm}\,, \quad \Lambda^{\mathcal{P}}_{\pm}\Lambda^{\mathcal{P}}_{\mp} = 0\,. \tag{18}$$

As a result of such a procedure for G_{Σ}, equation (11) splits into two-couple equations for $\varphi_{(a)}(k^{\perp})$ and $E_{(a)}(k^{\perp})$

$$E_{(a)}(k^{\perp})\sin\varphi_{(a)}(k^{\perp}) = m^{\circ}_{(a)} - \frac{1}{2}\int\frac{d^{3}q^{\perp}}{(2\pi)^{3}}\,\underset{\sim}{V}(k^{\perp}-q^{\perp})\sin\varphi_{(a)}(q^{\perp})\,, \tag{19}$$

$$E_{(a)}(k^{\perp})\cos\varphi_{(a)}(k^{\perp}) = |k^{\perp}| + \frac{1}{2}\int\frac{d^{3}q^{\perp}}{(2\pi)^{3}}\,\underset{\sim}{V}(k^{\perp}-q^{\perp})(\hat{k}^{\perp}\cdot\hat{q}^{\perp})\cos\varphi_{(a)}(q^{\perp}). \tag{20}$$

In the rest frame, where $\eta_{\mu} = (1,0,0,0)$, these equations coincide with ones of ref./6/. In that work the numerical solutions, in the case of the oscillator potential and $m^{\circ}_{(a)}=0$, yielding the spontaneous quark mass have been obtained. Equations (19) and (20) for the oscillator potential with $m^{\circ}_{(a)}\neq 0$ have been solved numerically in ref./11/ where it is shown that the effect of the spontaneous breakdown vanishes when $m^{\circ}_{(a)} \gtrsim \left(\frac{4}{3}V_{0}\right)^{1/3} \approx 300$ MeV (see Fig. 1). (Note that the Coulomb potential should yield renormalization of the bare mass, m_{ren}). It should be noted that the chiral symmetry breakdown has been essentially substantiated in ref./12/ where the authors have discovered this phenomenon instead of the confinement suggested due to an increasing potential.

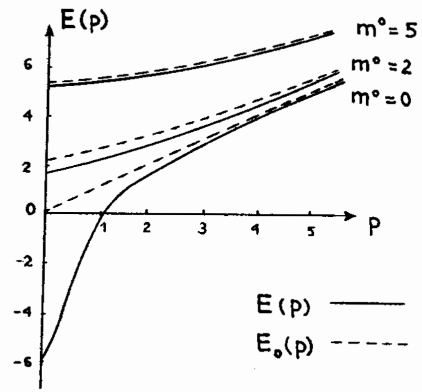

Fig. 1. The numerical solution, $E(p)$, to the Dyson-Schwinger equation ((19),(20)) for different bare quark masses m°, in units of $(\frac{4}{3}V_{0})^{1/3} = 300$ MeV (from ref. /12/). Here $E_{0}(p) = [(m^{\circ})^{2} + p^{2}]^{1/2}$.

3. Three-Dimensional Relativistic Covariance Equation for a Quarkonium

Let us consider the Bethe-Salpeter equation (10) that in the momentum space is written as

$$\Gamma_{(ab)}(k|\mathcal{P}) = -i\int\frac{d^{4}q}{(2\pi)^{4}}\,\underset{\sim}{V}(k^{\perp}-q^{\perp})\,\cdot \tag{21}$$

$$\cdot\,\not{\mathcal{P}}\left[G_{\Sigma_{(a)}}\!\left(q+\frac{\mathcal{P}}{2}\right)\Gamma_{(ab)}(q\ \mathcal{P})\,G_{\Sigma_{(a)}}\!\left(-q+\frac{\mathcal{P}}{2}\right)\right]\frac{\not{\mathcal{P}}}{\mathcal{P}^{2}}\,.$$

The special feature of this form of the equation is that it contains the choice of the quantization axis (4). Carrying out integrstion over $q^{\mathcal{P}}$, the expression in the square brackets of the equation can be written as

$$i\int\frac{dq^{\mathcal{P}}}{2\pi}\left[G_{\Sigma_{(a)}}\!\left(q+\frac{\mathcal{P}}{2}\right)\Gamma_{(ab)}(q|\mathcal{P})\,G_{\Sigma_{(b)}}\!\left(-q+\frac{\mathcal{P}}{2}\right)\right] =$$

$$= S_{(a)}(q^\perp)\Big[\frac{\Lambda_+^{\mathcal{P}} S_{(a)}(q^\perp)\Gamma_{(ab)}(q^\perp|\mathcal{P}) S_{(b)}(q^\perp)\Lambda_-^{\mathcal{P}}}{E_T(q^\perp)-\sqrt{\mathcal{P}^2}-i\varepsilon} +$$

$$+ \frac{\Lambda_-^{\mathcal{P}} S_{(a)}(q^\perp)\Gamma_{(ab)}(q^\perp|\mathcal{P}) S_{(b)}(q^\perp)\Lambda_+^{\mathcal{P}}}{E_T(q^\perp)+\sqrt{\mathcal{P}^2}-i\varepsilon}\Big] S_{(b)}(q^\perp) \equiv$$

$$\equiv \mathcal{D}_{(ab)}(q^\perp|\mathcal{P}) \otimes \Gamma_{(ab)}(q^\perp|\mathcal{P}), \qquad (22)$$

where $E_T = E_{(a)} + E_{(b)}$, $E_{(a),(b)}$ being the solutions of equations (19) and (20) for quarks (a) and (b).

Let us introduce the three-dimensional covariant Salpeter wave function

$$\Psi_{\mathcal{P}}(q^\perp) \equiv \mathcal{D}_{(ab)}(q^\perp|\mathcal{P})\otimes \Gamma_{(ab)}(q^\perp|\mathcal{P}) \qquad (23)$$

where $\otimes$ means the matrix multiplication. Applying the "undressing" procedure (by means of the operator (15)) one can write (23) as

$$\Psi_{\mathcal{P}}(q^\perp) = S_{(a)}(q^\perp)\,\psi_{\mathcal{P}}(q^\perp)\,S_{(b)}(q^\perp) \qquad (24)$$

where $\psi_{\mathcal{P}}$ is the "undressed" function (as we shall see, such a procedure yields much simplicities). Substitution of (22)-(24) into (21) gives the three-dimensional relativistic covariant equation to $\psi_{\mathcal{P}}$

$$[E_T(q^\perp)-\sqrt{\mathcal{P}^2}]\Lambda_+^{\mathcal{P}}\psi_{\mathcal{P}}(q^\perp)\Lambda_-^{\mathcal{P}}+[E_T(q^\perp)+\sqrt{\mathcal{P}^2}]\Lambda_-^{\mathcal{P}}\psi_{\mathcal{P}}(q^\perp)\Lambda_+^{\mathcal{P}} = \qquad (25)$$
$$= -\hat{I}_{k^\perp}\Big\{ S^{-1}_{(a)}(q^\perp) S_{(a)}(k^\perp)\psi_{\mathcal{P}}(k^\perp) S_{(b)}(k^\perp) S^{-1}_{(b)}(q^\perp)\Big\}$$

where $\hat{I}_{k^\perp}$ is the integration operator

$$\hat{I}_{k^\perp}\{f(k^\perp,q^\perp)\} \equiv \int \frac{d^3k^\perp}{(2\pi)^3}\, \underset{\sim}{V}(k^\perp-q^\perp)\, f(k^\perp,q^\perp).$$

(This equation differs from the known quasipotential equations /13/, in particular, by the condition (4)). It is easily seen that the eigenvalues of equation (25) (the quarkonium spectrum) do not violate the dispersion law $\mathcal{P}^2 = M_H^2$.

The function $\psi_{\mathcal{P}}$, by virtue of (17), (18) and (23)-(25), satisfies the identity

$$\Lambda_\pm^{\mathcal{P}}\,\psi_{\mathcal{P}}(q^\perp)\,\Lambda_\pm^{\mathcal{P}} = 0 \;\Rightarrow\; \not{\mathcal{P}}\,\psi_{\mathcal{P}}(q^\perp) = -\psi_{\mathcal{P}}(q^\perp)\not{\mathcal{P}} \qquad (26)$$

and it can be decomposed over the γ -matrices as follows

$$\psi_{\mathcal{P}}(q^\perp) = \gamma_5\Big[L_1(q^\perp) + \frac{\not{\mathcal{P}}}{\sqrt{\mathcal{P}^2}}L_2(q^\perp)\Big] + \qquad (27)$$
$$+ \gamma_\mu\Big[N_1^\mu(q^\perp) + \frac{\not{\mathcal{P}}}{\sqrt{\mathcal{P}^2}}N_2^\mu(q^\perp)\Big]$$

where in the rest frame, $\eta_\mu = (1,0,0,0)$, $L_{1[2]}$ is the pseudoscalar, and $N^i_{1[2]}$ consists of the vector v^i , axial a^i and scalar s components, i.e.

$$N^i_{1[2]}(k^\perp) = (\delta_{ij}-\hat{k}^\perp_i\hat{k}^\perp_j)\,v^j(k^\perp) + i\epsilon_{ijn}\hat{k}^\perp_j a^n(k^\perp) + \hat{k}^\perp_i\, s(k^\perp).$$

It should be noted that such a decomposition for the "dressed" function $\Psi_{\mathcal{P}}$ is rather complicated, and one should not be surprised that in ref. /6/ the following errors have been done i) the structures of L_1, N_1^i and L_2, N_2^i are confused, ii) the equation for $N^i_{1[2]}$ is not true.

In the rest frame $\mathcal{P}_\mu = (M_H, 0,0,0)$ equation (25) by substitution of (27) turns into the following coupled equations:

$$M_L L_{2[1]}(q^\perp) = -\left[E_T(q^\perp) + \hat{I}_{k^\perp}\left(A_{(a)}A_{(b)} + C^i_{(a)}C^i_{(b)} +[-]\, B^i_{(a)}B^i_{(b)}\right)\right] L_{1[2]}(k^\perp), \qquad (28a)$$

$$M_N N^k_{2[1]}(q^\perp) = -\Big\{E_T(q^\perp) + \hat{I}_{k^\perp}\Big[\left(A_{(a)}A_{(b)} - C^i_{(a)}C^i_{(b)} +[-]\, B^i_{(a)}B^i_{(b)}\right)\delta^{kl} +$$

$$+ C^k_{(a)}C^l_{(b)} + C^l_{(a)}C^k_{(b)} -[+]\, B^k_{(a)}B^l_{(b)} -[+]\, B^l_{(a)}B^k_{(b)} + \qquad (28b)$$

$$+ i\epsilon^{kli}\left(C^i_{(b)}A_{(a)} + C^i_{(a)}A_{(b)}\right)\Big]\Big\}.$$

Here we use the notation

$$A_{(a)} \equiv c_{(a)}(k^\perp)c_{(a)}(q^\perp) + \hat{k}^\perp \cdot \hat{q}^\perp\, s_{(a)}(k^\perp)\, s_{(a)}(q^\perp),$$

$$B^i_{(a)} \equiv \hat{q}^\perp_i\, s_{(a)}(q^\perp)\, c_{(a)}(k^\perp) - \hat{k}^\perp_i\, s_{(a)}(k^\perp)\, c_{(a)}(q^\perp),$$

$$C^k_{(a)} \equiv \epsilon_{ijk}\, \hat{q}^\perp_i\, \hat{k}^\perp_j\, s_{(a)}(k^\perp)\, s_{(a)}(q^\perp),$$

$$s_{(a)}(q^\perp) \equiv \sin\left[\tilde{\varphi}_{(a)}(q^\perp)/2\right],$$

$$c_{(a)}(q^\perp) \equiv \cos\left[\tilde{\varphi}_{(a)}(q^\perp)/2\right],$$

$$\tilde{\varphi}_{(a)}(q^\perp) \equiv \varphi_{(a)}(q^\perp) - \frac{\pi}{2}$$

(analogously, one can write for the index (b)), for which the following identities hold

$$A_{(a),(b)}(k^\perp)\Big|_{k^\perp = q^\perp} = 1, \quad B^i_{(a),(b)}(k^\perp)\Big|_{k^\perp = q^\perp} = 0, \quad C^i_{(a),(b)}(k^\perp)\Big|_{k^\perp = q^\perp} = 0.$$

It is easy to see that equation (28) in the limit of a small

bare quark mass $\hat{m}^\circ \to 0$ has the eigenvalue $M_L = 0$ since in this case equation (28a) coincides with the Dyson-Schwinger equation (18). The function L_1 describes the Goldstone boson

$$L_1(q^\perp) \xrightarrow[\hat{m}^\circ \to 0]{} L^0_1(q^\perp) = \frac{1}{F}\sin\varphi(q^\perp) \qquad (29)$$

where F is the energy dimension constant defined by the normalization condition (34). (It should be noted that the meson spectrum obtained in ref.[6] can be true only for the pion and its radial excitations because of the above-mentioned errors).

4. Quantization of the Bilocal Fields

Description of any particles is realized in the frame of the S-matrix formalism [14]. In quantum theory consistence can be determined for the construction of the S-matrix elements: at first, one calculates the spectrum of asymptotical stable states and then the transition matrix elements between these states considering an interaction as "a small perturbation".

The S-matrix exists if the asymptotical states are stable, i.e. they do not decay without "perturbation". As we have seen above, the stability and relativistic covariance of the asymptotical states are provided by the definite choice of the quantization axis (η). For free fields the axis is parallel to their total momenta (otherwise the "atom" and the potential forming the "atom" move in different directions). For the interacting bilocal fields $\mathcal{M}(x,y)$ the S-matrix is stable and relativistic covariant if only the quantization axis η is set parallel to the vector of differentiation with respect to the total coordinate $X = \frac{1}{2}(x+y)$ (i.e., $\eta^\mu \sim -i\,\partial/\partial X_\mu$).

In accordance with this statement, the quantized bilocal field is presented in the following form of its expansion over the creation $a^{(+)}_H(\vec{\mathcal{P}})$ and annihilation $a^{(-)}_H(\vec{\mathcal{P}})$ operators:

$$\mathcal{M}'(x,y) = \mathcal{M}'(z,X) = \sum_H \int d^3\Omega_H \int \frac{d^4q}{(2\pi)^4} \exp(iqz)\cdot \qquad (30)$$

$$\cdot\left[\exp(i\mathcal{P}X)\,\Gamma_H(q^\perp|\mathcal{P})\,a^{(+)}_H(\vec{\mathcal{P}}) + \exp(-i\mathcal{P}X)\,\bar{\Gamma}_H(q^\perp|\mathcal{P})\,a^{(-)}_H(\vec{\mathcal{P}})\right].$$

Here

$$d^3\Omega_H \equiv \frac{d^3\mathcal{P}}{\sqrt{(2\pi)^3\, 2\omega_H}}\,, \quad \omega_H = \sqrt{\mathcal{P}^2 + M_H^2}\,; \qquad \Gamma_H\,(q^\perp|\mathcal{P})$$

is the solution to (21) for an "atom" with the quantum number H. The operators $a^{(\pm)}$ satisfy the relations

$$[\,a_H^{(-)}(\vec{\mathcal{P}}),\, a_{H'}^{(+)}(\vec{\mathcal{P}}')] = \delta_{HH'}\,\delta(\vec{\mathcal{P}} - \vec{\mathcal{P}}'),$$
$$[\,a_H^{(\pm)}(\vec{\mathcal{P}}),\, a_{H'}^{(\pm)}(\vec{\mathcal{P}}')] = 0. \qquad (31)$$

The state of an "atom" (H) or a system of "atoms" $(H_1, \dots, H_n)$ is defined, as usually, by the operators $a^{(\pm)}$ and vacuum state $|0\rangle$ as follows:

$$|H; \vec{\mathcal{P}}\rangle = a_H^{(+)}(\vec{\mathcal{P}})|0\rangle\,,$$
$$|H_1, \dots, H_n; \vec{\mathcal{P}}_1, \dots, \vec{\mathcal{P}}_n\rangle = a_{H_1}^{(+)}(\vec{\mathcal{P}}_1) \dots a_{H_n}^{(+)}(\vec{\mathcal{P}}_n)|0\rangle. \qquad (32)$$

5. Normalization of Wave Functions

We shall define the relativistic covariant normalization of the bound-state wave functions as ones of a local meson field $\phi(x)$. The corresponding free action is

$$S_{free}[\phi] = \frac{1}{2}\int d^4x \left\{ [\partial_\mu \phi(x)]^2 - M^2\,\phi(x)^2 \right\} =$$
$$= \frac{1}{2}\int \frac{d^4\mathcal{P}}{(2\pi)^4}\, \tilde{\phi}(\mathcal{P})\, D^{-1}(\mathcal{P})\, \tilde{\phi}(\mathcal{P})$$

where

$$\tilde{\phi}(\mathcal{P}) = \int d^4X \exp(-i\mathcal{P}X)\,\phi(X), \quad D^{-1}(\mathcal{P}) \equiv \mathcal{P}^2 - M^2.$$

For the field $\phi(X)$ expansion over the creation and annihilation operators is given by

$$\phi(X) = \int \frac{d^3\mathcal{P}}{(2\pi)^{3/2}} \frac{1}{\sqrt{\mathrm{Res}\, D}} \left\{ \exp(i\mathcal{P}X)\, a^{(+)}(\vec{\mathcal{P}}) + \exp(-i\mathcal{P}X)\, a^{(-)}(\vec{\mathcal{P}}) \right\}$$

where

$$\mathrm{Res}\, D = \lim_{\mathcal{P}_0 \to \omega} [(\mathcal{P}_0 - \omega)\, D(\mathcal{P}_0)] = \frac{1}{2\omega}\,,$$

$$\omega = \sqrt{\vec{\mathcal{P}}^2 + M^2}\,.$$

Substituting the expansion (31) into the action (7) and using formulas (22)-(24), (26) and (27), we arrive at the following conditions for the wave functions Ψ, $L_{1[2]}$ and $N^i_{1[2]}$:

$$\frac{N_c}{4M_H} \int \frac{d^3q^\perp}{(2\pi)^3} \mathrm{tr}\left[\Lambda_-^{\mathcal{P}} \Psi_H(q^\perp) \Lambda_+^{\mathcal{P}} \Psi_H^+(q^\perp) - \Lambda_+^{\mathcal{P}} \Psi_H(q^\perp) \Lambda_-^{\mathcal{P}} \Psi_H^+(q^\perp) \right] = 1, \qquad (33)$$

$$\frac{N_c}{M_L} \int \frac{d^3q^\perp}{(2\pi)^3} \left[L_1(q^\perp) L_2^+(q^\perp) + L_2(q^\perp) L_1^+(q^\perp) \right] = 1, \qquad (34)$$

$$\frac{N_c}{M_N} \int \frac{d^3q^\perp}{(2\pi)^3} \left[N_1^i(q^\perp) N_2^{i+}(q^\perp) + N^i(q^\perp) N_1^{i+}(q^\perp) \right] = 1. \qquad (35)$$

These conditions are derived by using the following equation:

$$(E_T \pm M_H) - (E_T \pm M_H)^2 / (E_T \pm \sqrt{\mathcal{P}^2}) =$$
$$= \frac{\omega_H}{M_H}(\mathcal{P}_0 - \omega_H) + O\left((\mathcal{P}_0 - \omega_H)^2\right),$$
$$\omega_H = \sqrt{\vec{\mathcal{P}}^2 + M_H^2}\,, \qquad (H = L, N)$$

6. S-Matrix Elements

Matrix elements for the bilocal fields can be conveniently written in terms of the functions $\Phi(x, y)$ (see (9)) for which the expansion over the annihilation and creation operators is

$$\Phi(x,y) = \Phi(z,X) = \sum_H \int d^3\Omega_H \int \frac{d^4q}{(2\pi)^4} \exp(iqz) \cdot \tag{36}$$

$$\cdot \left\{ \exp(i\mathcal{P}X)\, a_H^{(+)}(\vec{\mathcal{P}})\, \underset{\sim}{\Phi}_H(q^\perp|\mathcal{P}) + \exp(-i\mathcal{P}X)\, a_H^{(-)}(\vec{\mathcal{P}})\, \underset{\sim}{\bar{\Phi}}_H(-q^\perp|\mathcal{P}) \right\} .$$

Here $\underset{\sim}{\Phi}_H$ and $\underset{\sim}{\bar{\Phi}}_H$ are the positive- and negative-frequency components of the function Φ corresponding to an "atom"(formed of the particles (a) and (b)):

$$\underset{\sim}{\Phi}_H(q^\perp|\mathcal{P}) = G_{\Sigma_{(a)}}(q^\perp + \frac{\mathcal{P}}{2})\, \Gamma_H(q^\perp|\mathcal{P}) =$$

$$= -S_{(a)}(q^\perp)\left[\frac{(E_T - M_H)\Lambda_+^{\mathcal{P}}\Psi_H(q^\perp)\Lambda_-^{\mathcal{P}}}{E_{(a)}(q^\perp) - q^{\mathcal{P}} - \frac{M_H}{2} - i\varepsilon} + \frac{(E_T + M_H)\Lambda_-^{\mathcal{P}}\Psi_H(q^\perp)\Lambda_+^{\mathcal{P}}}{E_{(a)}(q^\perp) + q^{\mathcal{P}} + \frac{M_H}{2} - i\varepsilon} \right] S_{(b)}^{-1}(q^\perp), \tag{37}$$

$$\underset{\sim}{\bar{\Phi}}_H(-q^\perp|\mathcal{P}) = G_{\Sigma_{(a)}}(-q^\perp + \frac{\mathcal{P}}{2})\, \bar{\Gamma}_H(-q^\perp|\mathcal{P}) =$$

$$= -S_{(a)}(q^\perp)\left[\frac{(E_T - M_H)\Lambda_-^{\mathcal{P}}\Psi_H^+(-q^\perp)\Lambda_+^{\mathcal{P}}}{E_{(a)}(q^\perp) - q^{\mathcal{P}} - \frac{M_H}{2} - i\varepsilon} + \frac{(E_T + M_H)\Lambda_+^{\mathcal{P}}\Psi_H(-q^\perp)\Lambda_-^{\mathcal{P}}}{E_{(a)}(q^\perp) + q^{\mathcal{P}} + \frac{M_H}{2} - i\varepsilon} \right] S_{(b)}^{-1}(q^\perp).$$

Recalling the definition of the bound-state system amplitude (33) and the expansion (37), it is easy to obtain the matrix element, for example, between vacuum and the state of n "atoms"

$$\langle H_1, \vec{\mathcal{P}}_1; \dots H_n, \vec{\mathcal{P}}_n | i S_{int} | 0 \rangle =$$

$$= -i(2\pi)^4 \delta^{(4)}\left(\sum_{i=1}^n \mathcal{P}_i\right) \frac{\mathcal{M}^{(n)}(\vec{\mathcal{P}}_1, \dots \vec{\mathcal{P}}_n)}{\prod_{j=1}^n \sqrt{(2\pi)^3\, 2\omega_j}} \tag{38}$$

where

$$\mathcal{M}^{(n)}(\vec{\mathcal{P}}_1, \dots \vec{\mathcal{P}}_n) = i \int \frac{d^4q}{(2\pi)^4} \sum_{\substack{\text{cycle} \\ \text{permitations}\{i_k\}}} \frac{1}{n}\, \Phi_{H_1}^{(a_1 a_2)}(q^\perp | \mathcal{P}_{i_1}) \cdot$$

$$\cdot\, \Phi_{H_2}^{(a_2 a_3)}(q^\perp - \frac{\mathcal{P}_{i_1}}{2} - \frac{\mathcal{P}_{i_2}}{2} | \mathcal{P}_{i_2})\, \Phi_{H_3}^{(a_3 a_4)}(q^\perp - \mathcal{P}_{i_2} - \frac{\mathcal{P}_{i_1}}{2} - \frac{\mathcal{P}_{i_3}}{2} | \mathcal{P}_{i_3}) \cdot$$

$$\dots\, \Phi_{H_n}^{(a_n a_1)}(q^\perp - \mathcal{P}_{i_2} - \dots - \mathcal{P}_{i_{n-1}} - \frac{\mathcal{P}_{i_1}}{2} - \frac{\mathcal{P}_{i_n}}{2} | \mathcal{P}_{i_n}) .$$

The expression for the Green function of bound states is defined in ref./10/. The result for the Green function of the bilocal fields $\mathcal{M}'(z,X)$ and $\mathcal{M}'(z',X')$ is as follows:

$$\overbrace{\mathcal{M}'(z,X)\,\mathcal{M}'}(z',X') = \frac{1}{(2\pi)^{16}} \int d^4\mathcal{P}\, d^4Q\, d^4p\, d^4q \cdot \tag{39}$$

$$\cdot \exp\left[i(\mathcal{P}X - QX' + pz - qz') \right] \mathcal{G}(pq|\mathcal{P}Q)$$

where

$$\mathcal{G}(pq|\mathcal{P}Q) = \tag{40}$$

$$= \delta^{(4)}(\mathcal{P}-Q) \sum_H \left\{ \frac{\Gamma_H(q^\perp|\mathcal{P})\, \bar{\Gamma}_H(-p^\perp|\mathcal{P})}{2\omega_H(\mathcal{P}_0 - \omega_H - i\varepsilon)} - \frac{\Gamma_H(p^\perp|\mathcal{P})\, \bar{\Gamma}_H(-q^\perp|\mathcal{P})}{2\omega_H(\mathcal{P}_0 + \omega_H - i\varepsilon)} \right\} .$$

The above expressions (36)-(40) represent the elements of diagrammatic technique for a field theory of bound states.

Let us, as a simple example, define the pion leptonic decay constant F_π. The interaction of a meson with the leptonic current described through the local operator $L(x)$ is defined by the substitution $\mathcal{M}' \to \mathcal{M}' + L$ into (7)

$$S_L^{(2)} \equiv i N_c\, \mathrm{tr} \int d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2 \cdot \tag{41}$$

$$\cdot\, \hat{\mathcal{M}}'(x_1,x_2)\, G'_{\Sigma_{(a)}}(x_2 - y_1)\, L(y_1)\, \delta^{(4)}(y_1 - y_2)\, G_{\Sigma_{(b)}}(y_2 - x_1) .$$

Substituting this expression into the left-hand side of the following equation for the decay matrix element

$$\langle \ell\nu | i S_L^{(2)} | H, \vec{\mathcal{P}} \rangle = -i(2\pi)^{\frac{5}{2}} \delta^{(4)}(\mathcal{P}-\mathcal{P}_l) \frac{F_H}{2\omega_H} \mathcal{P}^\mu \ell_\mu \tag{42}$$

(where ℓ_μ is the leptonic matrix element absorbing the factor $G_F \cos\vartheta_c/\sqrt{2}$, G_F is the Fermi constant, ϑ_c is the Cabibbo angle), and recalling (39) we arrive at the result

$$F_H = -\frac{2N_c}{M_H} \int \frac{d^3q^\perp}{(2\pi)^3} L_2(q^\perp) \sin\varphi(q^\perp). \tag{43}$$

By this expression one can calculate the leptonic decay constant for the pion with the solutions $L_2(q^\perp)$ and $\varphi(q^\perp)$ being known. If we use in (43) for the pion the solution of the Goldstone mode (29), $\sin\varphi(q^\perp) = F L_1^0(q^\perp)$, then from the normalization condition (34) we get $F = F_\pi$.

Substituting the Goldstone mode solution $L_1 \approx L_1^0$ (29) into the right-hand side of equation (28)

$$-M_\pi L_2(q^\perp) = \left[2E(q^\perp) + \hat{I}_{k^\perp}\right] L_1(k^\perp) , \quad (m_u^0 \approx m_d^0 = m^0 \sim 0)$$

and recalling the Dyson Schwinger equation,

$$E(q^\perp)\sin\varphi(q^\perp) = m^0 - \frac{1}{2}\hat{I}_{k^\perp}\sin\varphi(k^\perp)$$

we obtain the approximate solution for L_2 ,

$$L_2(q^\perp) \xrightarrow[m^0 \to 0]{} L_2^0 = \frac{2m^0}{M_\pi F_\pi} . \tag{44}$$

Then from (34), when $L_1 \approx L_1^0$ and $L_2 \approx L_2^0$, we get

$$2N_c \int \frac{d^3q^\perp}{(2\pi)^3} \sin\varphi(q^\perp) \approx \frac{F_\pi^2 M_\pi^2}{2m^0} \tag{45}$$

It is easy to note that the left-hand side of (45) coincides (up to sign) with the vacuum condensate of light quarks

$$\langle \bar{q}q \rangle = iN_c \, \mathrm{tr} \int \frac{d^4q}{(2\pi)^4} \left[G_\Sigma(q) - G_{m^0}(q)\right] \approx -2N_c \int \frac{d^3q^\perp}{(2\pi)^3} \sin\varphi(q^\perp).$$

As a result, we obtain the following low-energy relation between F_π and $\langle \bar{q}q \rangle$

$$-2m^0 \langle \bar{q}q \rangle \approx M_\pi^2 F_\pi^2 . \tag{46}$$

7. Low-Energy Limit of the Bilocal Lagrangians for Light Quarkonia and the Lagrangian for Heavy Quarkonia

As is shown, in the low-energy limit ($q^\perp \to 0$, with $\hat{m}^0 \sim 0$) the solution to the Salpeter equation is defined by the Goldstone mode $L_1^0 = \frac{1}{F_\pi} \sin\varphi(q^\perp)$). Since the solution to the Dyson-Schwinger equation has the asymptotics /6/ $\varphi(q^\perp \sim 0) \sim \mathrm{const}$, it is easily seen that the zero limit of the vertex function in the momentum space is a constant. Then, the corresponding bilocal field in this low-energy limit is the δ -function,

$$\mathcal{M}'(x,y) \sim \delta^{(4)}(x-y). \tag{47}$$

On the other hand, this limit can be obtained if one suggests that the Bethe-Salpeter kernel is a δ -function, i.e., $K(x-y) \sim \sim \delta^{(4)}(x-y)$. Just such a potential for the four-fermion interaction was an initial approximation in the original formulation of the spontaneous breakdown of the chiral symmetry /15/.

There is great number of literature (for example, see /16-18/) where it is shown that the δ -type potential (kernel) leads to the phenomenological chiral Lagrangians. (Light mesons in the framework of our approach but with the δ -type potential is studied in ref./18/).

Thus, the limit (47) due to the asymptotics of a quarkonium wave function (but not the δ -type potential) leads to the phenomenological chiral Lagrangians too.

As regards heavy quarkonia in the zero limit of the internal momentum $(q^\perp \to 0)$, the coupled equations of the Dyson-Schwinger (11) and Bethe-Salpeter (21) for the spectra reduce to the Schrödinger equation with the renormalized quark mass ($m_{ren} = \Sigma$) /19/

$$\left[M_H - 2m_{ren} - \frac{(q^\perp)^2}{m_{ren}} \right] \Psi^\circ_H(q^\perp) = \alpha \int \frac{d^3k^\perp}{(2\pi)^3} \frac{1}{|k^\perp - q^\perp|^2} \Psi^\circ_H(k^\perp)$$

where, in accordance with equations (23) and (24),

$$\Psi^\circ_H(q^\perp) \equiv \frac{I+\gamma^\circ}{2} \psi_H(q^\perp) \frac{I-\gamma^\circ}{2} .$$

Interactions of heavy quarkonia are described by the S-matrix just in the same way as an atom in QED because in this case the (colourless) Coulomb potential dominates.

Conclusion

The results of the paper are the relativization of the potential model and the generalized description of the interactions of heavy and light mesons.

For constructing the model giving a unified description of the spectrum of quarkonia and their interactions, we started from the analogy between a heavy quarkonium and a hydrogen-like atom described by QED. Definition of the relativistic covariant S-matrix for asymptotical bound states in QED requires the use of the principles of the "minimal" quantization and the choice of the quantization axis.

The "minimal" quantization of a non-Abelian theory (QCD_{min}) has been formulated in refs./4,5/. QCD_{min} compared with the usual formulation in the radiative gauge (based on the Dirac method) contains additional physical information: i) the non-normalizable solutions of the Gauss equation (which lead to infrared redefinition of the Coulomb potential, i.e., to appearance of the increasing potential): ii) the topological degeneration of the physical variables (which lead to the mechanism of confinement - as destructive interference of the phase of the topological degeneration) /5,20/.

In the present work we have in fact considered a lower order of QCD_{min} on the transversal gluon fields ($A^\perp$) with the above stated redefinition of the Coulomb potential in the infrared region.

In ref./21/, just the principles used for quarkonia have been applied to gluons and their bound states. It has been shown that the

explicit solution to the equation for the gluon spectrum, in the increasing potential, leads to the dynamical mass of gluons, and modifies the asymptotical freedom formula in the infrared region. As a result, the effective quark-gluon coupling keeps its small value, and the vacuum is stable. These results allow one to hope that the radiative corrections give a small contribution to the relativistic potential model considered in this work.

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Объединение потенциальной модели с киральными лагранжианами

Предлагается новый подход к релятивизации потенциальной модели, позволяющий описать спектры легких и тяжелых мезонов и их взаимодействия. Этот подход основан на применении принципов "минимального" квантования калибровочных теорий /с явным решением уравнения Гаусса/ и выбора оси квантования /параллельной вектору оператора дифференцирования по полной координате связанного состояния/, которые используются для релятивистского описания атомов в КЭД. Выведено уравнение Дайсона-Швингера, описывающее спектр кварков и возникновение их динамической массы, а также получена новая релятивистски-ковариантная форма уравнения Солпитера на спектр легких и тяжелых мезонов. Для описания взаимодействия мезонов предлагается использовать формализм S-матрицы с асимптотическими состояниями кварккониев, являющихся решениями уравнения Солпитера. Показано, что соответствующий билокальный лагранжиан /в бесцветном канале/ взаимодействия в пределе малых переданных импульсов для легких мезонов переходит в феноменологические киральные лагранжианы, а для тяжелых мезонов совпадает с лагранжианом КЭД для взаимодействия атомов.

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Pervushin V.N. et al. E2-88-643

Unification of a Potential Model with Chiral Lagrangians

A new approach to relativization of the potential model, which allows one to describe the spectra of light and heavy mesons and their interaction, is proposed. This approach is based on application of the principles of "minimal" quantization of the gauge theories (with explicit solutions of the Gauss equation) and choice of the quantization axis (by setting one parallel to the vector of differential with respect to the total bound state coordinate) which are used in QED for describing the relativistic atoms. We have obtained the Dyson-Schwinger equation describing the spectrum of quarks and generation of their dynamical masses and derived a new relativistic covariant form for the Salpeter equation for light and heavy meson spectra. To describe the interactions of mesons we have proposed to use the S-matrix formalism with the asymptotical states of quarkonia which are the solutions to the Salpeter equation. It is shown that the relevant in the limit of a small transfer momentum bilocal Lagrangian of the interactions (in the colourless channel), for light mesons, tends to the phenomenological chiral Lagrangians, and, for heavy mesons, it corresponds to the QED Lagrangian for interacting atoms.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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