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SUDAKOV FORM FACTOR IN QCD

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1. Introduction

One of the old yet unresolved problems in QCD originated since the QED construction has been formulated in a pioneer paper by Sudakov [1]. It is just the problem of calculation of the asymptotics of the quark electromagnetic form factor in the following kinematics:

$$Q^2 = -(p-k)^2 \gg -p^2 = -k^2 = M^2 \gg m^2,$$

where k and p are momenta of a quark with mass m. Summation of the leading double logarithmic corrections to the Sudakov form factor in QED [1,2] and later in QCD [3,4]:

$$F = \exp\left(-\frac{\alpha_s(M^2)}{2\pi} C_F \ln^2\frac{Q^2}{M^2} + \frac{3}{4}\,\frac{\alpha_s(M^2)}{2\pi} C_F \ln\frac{Q^2}{M^2}\right) + O\left[\alpha_s^n \ln^{2n-2}\frac{Q^2}{M^2}\right]$$

(where $C_F = \frac{N^2-1}{2N}$ for the gauge group $SU(N)$ and $\alpha_s = \frac{g^2}{4\pi}$ is the running coupling constant) is the result of numerous attempts of solving this problem. The obtained expression for the Sudakov form factor is a decreasing function of Q^2 and in the limit $Q^2 \gg M^2$ its asymptotics is determined by neglected nonleading logarithmic terms.

In a preceding paper [5] we proposed the method that allowed us to calculate the electromagnetic form factor of a massless quark. In the present paper, this method is generalized to a more complicated case of the Sudakov form factor. We sum up in the Feynman gauge all logarithmic corrections to the Sudakov form factor not suppressed by powers of M^2/Q^2 and determine its asymptotic behavior for $Q^2 \gg M^2$.

2. Factorization of the Sudakov form factor

The Sudakov form factor F is related to the amplitude $\mathfrak{M}$ of quark elastic scattering in the electromagnetic field a_μ by the following relation:

$$\mathfrak{M} = \frac{1}{\not{p}}\,\gamma^\mu\,\frac{1}{\not{k}}\,F\,a_\mu(p-k)\,\left[Z_2(p)Z_2(k)\right]^{-1/2} + O\left(\frac{M^2}{Q^2}\right),$$

where $Z_2(p)$, $Z_2(k)$ are the quark wavefunction renormalization constants. The magnitude of F is determined by the set of Feynman diagrams shown in fig.1(a). To give the leading contribution to the form factor, these diagrams should have the structure pictured in fig.1(b) /6-8/. The diagrams in fig.1(b) contain five subgraphs in accordance with the values of the momentum l_μ of particles (*i.e.*,quark, gluon or ghost) belonging to them /6/:

(a) hard subgraph H:

$$|l_+,\ l_-,\ l_T| = O(Q)$$

(b) collinear subgraph J_p:

$$l_+ = O(Q),\quad l_- = O\left[\frac{M^2}{Q}\right],\quad l_T = O(M)$$

(c) collinear subgraph J_k:

$$l_+ = O\left[\frac{M^2}{Q}\right],\quad l_- = O(Q),\quad l_T = O(M) \qquad (1)$$

(d) soft subgraph S:

$$|l_+,\ l_-,\ l_T| = O(M)$$

(e) infrared subgraph IR:

$$|l_+,\ l_-,\ l_T| = O(\frac{M^2}{Q}),$$

where we use the light cone coordinates $l_\pm = \frac{l_0 \pm l_3}{\sqrt{2}}, l_T = (l_1, l_2)$ and work in the frame, where $p_T = k_T = 0$, $p_+ = k_- = Q = \left[Q^2/2\right]^{1/2}$, $p_- = k_+ = -M^2/Q$.

The properties of the first four subgraphs studied earlier /5,8-10/ in the context of investigation of the asymptotics of the electromagnetic form factor of a massless quark will be exploited below in this discussion. It is just the appearance of a new infrared subgraph that does not allow us to apply the "old" methods /8-10/ for investigation of the Sudakov form factor. For the first time the importance of its inclusion into calculation of the Sudakov form factor was stressed in ref./11/.

To calculate the subgraphs, one must first correctly define the boundaries of momentum regions (1). For that aim we introduce two arbitrary parameters μ and λ restricted by the condition /7/:

$$Q^2 > \mu^2 > M^2 > \lambda^2 > \frac{M^4}{Q^2}.$$

The parameter μ sets the lower boundary of momenta l(a) within H and, respectively, the upper boundary of momenta l(d) within S. The parameter λ sets the lower boundary of momenta l(d) within S and, respectively, the upper boundary of momenta l(e) within IR.

The form factor F does not depend on μ and λ but each subgraph does depend.

The contribution of a diagram of fig.1(b) to the form factor is expressed in a complicated way through the involved subgraphs interacting with each other. But this effects of interaction between subgraphs are cancelled /7-9,12/ when one sums the diagrams of fig.1(b) differing from each other by a number of external lines of subgraphs. With the use of properties of particles belonging to different subgraphs /8,12/ the resulting sum is defined by a set of diagrams shown in fig.1(c) where double lines denote operators to be determined below in (4). The contribution of diagrams of fig.1(c) to the form factor may be represented in the factorized form:

$$F = F_S\, F_H\, F_{J_k}\, F_{J_p}\, F_{IR}\,, \qquad (2)$$

where by F_i we denoted contribution from a certain subgraph.

The hard subgraph H describes interaction of particles at short (as compared with $1/\mu$) distances and its contribution to the form factor depends only on the variables Q^2 and μ. One-loop calculation of F_H yields the following expression:

$$F_H = F_H\left[\frac{Q^2}{\mu^2}\right] = 1 - \frac{\alpha_s}{4\pi} C_F\left\{\ln^2\left[\frac{Q^2}{\mu^2}\right] - 3\ln\left[\frac{Q^2}{\mu^2}\right] + 7 - \frac{\pi^2}{6}\right\}. \qquad (3)$$

The contribution of collinear subgraphs J_p and J_k to the form factor may be written as /5,8,12/:

$$F_{J_p} = -\frac{i}{4}\langle 0|T\ \bar{\tilde{\Psi}}(p)\ \hat{p}\ E^{\dagger}_{-\tilde{k}}(0,\infty)\ \Psi(0)|0\rangle_{J_p}\ \left[Z_2(p)\right]^{-1/2}$$

$$F_{J_k} = -\frac{i}{4}\langle 0|T\ \bar{\Psi}(0)\ \hat{k}\ E_{\tilde{p}}(0,\infty)\ \tilde{\Psi}(k)|0\rangle_{J_k}\ \left[Z_2(k)\right]^{-1/2}, \qquad (4)$$

where $\tilde{\Psi}(p) = \int dx\ e^{ipx}\ \Psi(x)$ is the quark-field operator. Subscripts J_p and J_k remind us that the momenta of particles belonging to collinear subgraphs are restricted to regions (lb) and (lc). The involved multipliers $E_{-\tilde{k}}(0,\infty)$ and $E_{\tilde{p}}(0,\infty)$ result from summation of the diagrams in fig.1(b) over the number of collinear gluon emitted within J_p and J_k and absorbed by H. They equal path-ordered exponentials:

$$E_{-\tilde{k}}(0,\infty) = P\exp\left[-ig\int_0^\infty ds\ \tilde{k}_\mu\ \hat{A}^\mu(-\tilde{k}s)\ e^{-\varepsilon s}\right] \quad \varepsilon \to 0$$

and for $E_{\tilde{p}}(0,\infty)$ analogously. The vectors $\tilde{k}$ and $\tilde{p}$ lie on the light cone and have the following components:

$$\tilde{p}_+ = \tilde{k}_- = Q,\ \tilde{p}_- = \tilde{k}_+ = 0,\ \tilde{p}_T = \tilde{k}_T = 0\ ,$$

i.e., they differ from momenta p and k by $O\left(\frac{M^2}{Q}\right)$ - terms. From one-loop calculation we find that:

$$F_{J_p} = F_{J_k} = 1 - \frac{\alpha_s}{4\pi} C_F \left\{ \ln^2\left(\frac{\mu^2}{\lambda^2}\right) - \ln^2\left(\frac{M^2}{\lambda^2}\right) - \frac{3}{2} \ln\left(\frac{\mu^2}{M^2}\right) - \frac{3}{2} + \frac{\pi^2}{6} \right\},$$

that is, the contributions of the collinear subgraphs do not depend on Q^2. These important properties of F_{J_k} and F_{J_p}, *i.e.*,

$$\frac{d \ln F_{J_p}}{d \ln Q^2} = \frac{d \ln F_{J_k}}{d \ln Q^2} = 0\ , \qquad (5)$$

are fulfilled to all orders of perturbation theory as proved in ref.[5] and they are closely related to the properties of composite twist-two operators that appear when one applies the operator product expansion on the light cone to expression (4).

The contribution of the soft subgraph S to the form factor is represented as a matrix element of a product of two P - exponentials [5,8]:

$$F_S = \langle 0 | T\ E_{\tilde{p}}(0,\infty)\ E^{\dagger}_{-\tilde{k}}(0,\infty) | 0 \rangle_S$$

which result from summation of diagrams of fig.1(b) over the number of soft gluons absorbed by J_p and J_k. The subscript S indicates that the momenta of all particles in F_S are confined to region (1d). Performing one-loop calculation of F_S we obtain:

$$F_S\left(\frac{Q^2}{\mu^2}, \frac{\mu^2}{\lambda^2}\right) = 1 - \frac{\alpha_s}{4\pi} C_F \left\{ 2 \ln\left(\frac{Q^2}{\mu^2}\right) \ln\left(\frac{\mu^2}{\lambda^2}\right) - \ln^2\left(\frac{\mu^2}{\lambda^2}\right) - 3 \right\}.$$

The study of multiloop properties of F_S has allowed us to derive the following equation [5]:

$$\frac{d \ln F_S}{d \ln Q^2} = - \int_{\lambda^2}^{\mu^2} \frac{dt}{2t}\, \Gamma_{cusp}(g(t))\ , \qquad (6)$$

where the anomalous dimension Γ_{cusp} is defined by (12).

The contribution of the infrared subgraph to the form factor may be expressed as follows [13]:

$$F_{IR} = -\int_0^\infty ds \int_0^\infty dt\ \exp(i(s+t))\ \langle 0 | T\ E_{\tilde{p}}(0,s/M^2)\ E^{\dagger}_{-\tilde{k}}(0,t/M^2) | 0 \rangle_{IR}\ , \qquad (7)$$

where P - exponentials accumulate all the interaction effects of particles belonging to infrared subgraph with particles of other

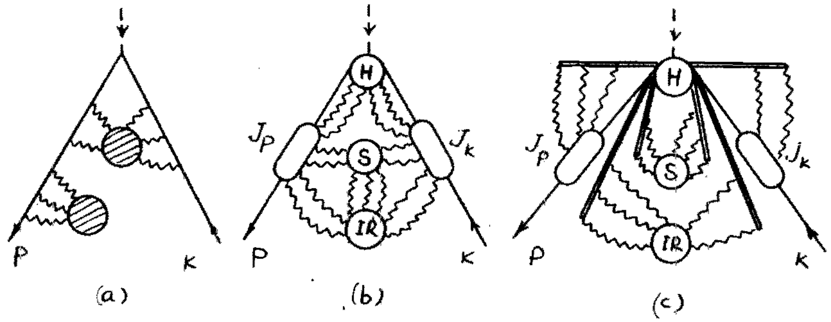

Fig.1.(a) General structure of the diagrams for the quark electromagnetic form factor. (b) Diagrams determining the leading contribution to the form factor. (c) Basic factorization of the form factor. The dashed line represents an external electromagnetic field.

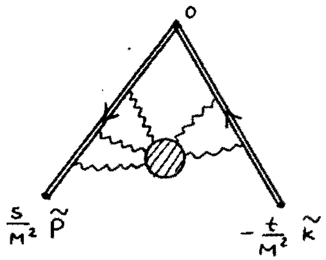

Fig.2. A typical diagram arising from the expansion of the path - ordered exponential. The double line denotes the contour of integration.

subgraphs. One-loop calculation of F_{IR} gives:

$$F_{IR}\left(\frac{\lambda^2Q^2}{M^4}\right) = 1 - \frac{\alpha_s}{4\pi} C_F \left\{ \ln^2\left(\frac{\lambda^2Q^2}{M^4}\right) + \frac{\pi^2}{2} \right\}. \qquad (8)$$

To verify the factorization relation (2), we combine one-loop values of all subgraphs and from (2) easily derive the well-known expression for the Sudakov form factor:

$$F\left(\frac{Q^2}{M^2}\right) = 1 - \frac{\alpha_s}{4\pi} C_F \left\{ 2\ln^2\left(\frac{Q^2}{M^2}\right) - 3\ln\left(\frac{Q^2}{M^2}\right) + 1 + \frac{2}{3}\pi^2 \right\}.$$

3. Infrared subgraph

The contribution of the infrared subgraph (7) to the Sudakov form factor may be represented as an integral of the contour functional /13/:

$$F_{IR} = -\int_0^\infty ds \int_0^\infty dt \, \exp(i(s+t)) \, \langle 0|TP\exp\left[ig\int_C dz_\mu \tilde{A}^\mu(z)\right]|0\rangle_{IR}, \qquad (9)$$

where the contour C is pictured by a double line in fig.2. The contour C is formed by two segments directed along vectors $\frac{\tilde{k}_\nu}{M^2}$ and $\frac{\tilde{p}_\nu}{M^2}$. That is why F_{IR} depends on the only scalar $\frac{\tilde{k}\tilde{p}}{M^4}$ formed by these vectors and the parameter λ that sets the upper boundary of momenta in the infrared subgraph:

$$F_{IR} = F_{IR}\left(\frac{\lambda^2Q^2}{M^4}\right).$$

The contour C in eq.(9) lies on the light cone and therefore F_{IR} possesses additional (as compared with the nonlight-like contour) logarithmic corrections. Let us deform slightly the contour C, *i.e.*, shift the vectors $\tilde{k}$ and $\tilde{p}$ from the light cone into the time-like direction:

$$\tilde{p}^{\,2} = \tilde{k}^{\,2} = m^2 \ll Q^2$$

and consider the resulting from (9) expression for $F_{IR}^{(reg)}$. $F_{IR}^{(reg)}$ depends on λ^2 and two scalars M^4/Q^2 and m^2/Q^2 formed by vectors $\frac{\tilde{k}_\nu}{M^2}$ and $\frac{\tilde{p}_\nu}{M^2}$:

$$F_{IR}^{(reg)} = F_{IR}^{(reg)}\left[\gamma, \frac{\lambda^2Q^2}{M^4}\right],$$

where $\gamma = \ln\frac{Q^2}{m^2}$ is the cusp angle of the contour C in fig.2. The peculiarities of F_{IR} on the light cone are revealed as a singular dependence of $F_{IR}^{(reg)}$ on m in the limit $m^2/Q^2 \to 0$. But structure of these peculiarities is such that the dependence of the derivative $\frac{d\ln F_{IR}^{(reg)}}{d\ln Q^2}$ on m^2 becomes regular in the limit $m^2 \ll Q^2$ /5/. Hence the following equality takes place:

$$\frac{d\ln F_{IR}}{d\ln Q^2} = \lim_{m\to 0} \frac{d\ln F_{IR}^{(reg)}}{d\ln Q^2} \qquad (10)$$

and both its sides do not possess light cone singularities. Feynman integrals arising in the expansion of $F_{IR}^{(reg)}$ have only two momentum scales: λ and M^4/Q^2. The parameter λ cuts off large (as compared with M^2/Q) values of momenta. Therefore the dependence of $F_{IR}^{(reg)}$ on λ may be thought of as the dependence of (9), with the omitted subscript IR (*i.e.*, without any restriction of momenta), on the renormparameter introduced to substract arising ultraviolet divergences /14-16/. Renormalization properties of the contour functional (9) with contour C (fig.2) or, equivalently, the dependence of F_{IR} on λ are described by the following renormalization group equation /15/:

$$\left(\lambda\frac{\partial}{\partial\lambda} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(\gamma,g) + 2\,\Gamma_{end}(g)\right) F_{IR}^{(reg)}\left(\gamma, \frac{\lambda^2Q^2}{M^4}\right) = 0, \qquad (11)$$

where $\Gamma_{cusp}(\gamma,g)$ and $\Gamma_{end}(g)$ are the cusp and end anomalous dimensions of contour functionals. The cusp anomalous dimension in the limit of large cusp angles $\gamma = \ln\frac{Q^2}{m^2}$ has the asymptotics /16/:

$$\Gamma_{cusp}(\gamma,g) = \ln\frac{Q^2}{m^2}\,\Gamma_{cusp}(g) + O\left(\ln^0\frac{Q^2}{m^2}\right) \qquad (12)$$

$$\Gamma_{cusp}(g) = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F\left\{ N\left(\frac{67}{36} - \frac{\pi^2}{12}\right) - n_f\frac{5}{18} \right\},$$

where n_f equals the number of quark flavors.

Differentiating both the sides of eq.(11) with respect to Q^2 and taking the limit $m^2 \ll Q^2$ using relations (10) and (12) we obtain:

$$\left(\lambda\frac{\partial}{\partial\lambda} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(g)\right) \frac{d\ln F_{IR}\left(\frac{\lambda^2Q^2}{M^4}\right)}{d\ln Q^2} = 0.$$

The solution of this equation:

$$\frac{d\ \ln F_{IR}}{d\ \ln Q^2} = \Gamma_0\left[g\left(\frac{M^4}{Q^2}\right)\right] - \int_{M^4/Q^2}^{\lambda^2} \frac{dt}{2t}\,\Gamma_{cusp}(g(t)) \qquad (13)$$

depends on a new function Γ_0. Substituting (8) into (13) we find the one-loop value of Γ_0: $\Gamma_0 = 0 + O(\alpha_s{}^2)$.

4. Calculation of the Sudakov form factor

Let us differentiate both the sides of the factorized expression (2) for the form factor with respect to Q^2 using the properties (5), (6) and (13) of subgraphs. The result is as follows:

$$\frac{d\ \ln F}{d\ \ln Q^2} = \frac{d\ \ln F_H\left(\frac{Q^2}{\mu^2}\right)}{d\ \ln Q^2} + \Gamma_0\left[g\left(\frac{M^4}{Q^2}\right)\right] - \int_{M^4/Q^2}^{\mu^2} \frac{dt}{2t}\,\Gamma_{cusp}(g(t)).$$

We are convinced that the dependence on λ is eliminated in the r.h.s. of this relation. On the other hand, the independence of the form factor of μ implies F_H to obey the following equation /5/:

$$\frac{d\ \ln F_H\left(\frac{Q^2}{\mu^2}\right)}{d\ \ln Q^2} = \Gamma(g(Q^2)) - \int_{\mu^2}^{Q^2} \frac{dt}{2t}\,\Gamma_{cusp}(g(t)).$$

The one-loop value of function Γ: $\Gamma = \frac{3}{4}\frac{\alpha_s}{\pi}C_F$ can easily be found by substituting (3) into this equation.

Combining the last two expressions we derive the final equation for the Sudakov form factor:

$$\frac{d\ \ln F}{d\ \ln Q^2} = \Gamma(g(Q^2)) + \Gamma_0\left[g\left(\frac{M^4}{Q^2}\right)\right] - \int_{M^4/Q^2}^{Q^2} \frac{dt}{2t}\,\Gamma_{cusp}(g(t)).$$

It is just this equation that we solve to determine the Sudakov form factor:

$$F\left(\frac{Q^2}{M^2}\right) = F_0(g(Q^2))\exp\left\{\int_{M^2}^{Q^2}\frac{dt}{t}\,\Gamma(g(t)) + \int_{M^4/Q^2}^{M^2}\frac{dt}{t}\,\Gamma_0(g(t)) - \int_{M^2}^{Q^2}\frac{dt}{2t}\,\ln\frac{Q^2}{t}\,\Gamma_{cusp}(g(t)) - \int_{M^4/Q^2}^{M^2}\frac{dt}{2t}\,\ln\frac{tQ^2}{M^4}\,\Gamma_{cusp}(g(t))\right\}. \qquad (14)$$

This expression contains all logarithmic corrections to the Sudakov form factor not suppressed by powers of M^2/Q^2. The involved functions Γ, Γ_0 are known in the one-loop but function Γ_{cusp} in the two-loop approximation. The one-loop value of F_0: $F_0 = 1 - \frac{\alpha_s}{4\pi}C_F(1 + \frac{2}{3}\pi^2)$ may be found by comparing (14) with the one-loop expression for the Sudakov form factor. To determine the asymptotic behavior of the Sudakov form factor, we should not know the exact values of these functions. It follows from (14) that the asymptotics of F is controlled by Γ_{cusp}:

$$F\left(\frac{Q^2}{M^2}\right) \xrightarrow{Q^2 \gg M^2} \exp\left\{-\int_{M^2}^{Q^2}\frac{dt}{2t}\,\ln\frac{Q^2}{t}\,\Gamma_{cusp}(g(t)) - \int_{M^4/Q^2}^{M^2}\frac{dt}{2t}\,\ln\frac{tQ^2}{M^4}\,\Gamma_{cusp}(g(t))\right\}. \quad /5/$$

The cusp anomalous dimension is a positive definite function $\Gamma_{cusp} > 0$ which is confirmed by two-loop calculation (12). Therefore we conclude that the Sudakov form factor, with all logarithmic corrections being included, is a rapidly decreasing function in the limit $Q^2 \gg M^2$.

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Судаковский формфактор в КХД

В пертурбативной КХД исследуется дважды логарифмическая асимптотика электромагнитного формфактора кварка в судаковской области. Она описывается в терминах контурных функционалов $\langle 0|TP\exp(ig\int_C dz_\mu \hat{A}_\mu(z))|0\rangle$ и матричных элементов составных операторов твиста-2. Использование этих новых объектов позволило суммировать все нелидирующие логарифмические поправки к судаковскому формфактору в быстро убывающую функцию переданного импульса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Korchemsky G.P., E2-88-628
Sudakov Form Factor in QCD

The double logarithmic asymptotics of the quark electromagnetic form factor in the Sudakov region is investigated within the framework of perturbative QCD. It is described in terms of the contour functionals $\langle 0|TP\exp(ig\int_C dz_\mu \hat{A}_\mu(z))|0\rangle$ and matrix elements of composite twist-2 operators. Using the renormalization properties of these new objects, the nonleading logarithmic corrections to the Sudakov form factor are summed to give a rapidly decreasing function of the transferred momentum.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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