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DOUBLE LOGARITHMIC ASYMPTOTICS IN QCD

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1. Introduction

Many years ago Sudakov discovered [1] that there are processes in gauge theories which being calculated in perturbation theory contain double logarithmic corrections [2]. Investigation of these Sudakov effects in QCD began from multiloop perturbative calculations of leading double logarithmic corrections to the electromagnetic quark form factor F in the following kinematics [3]:

$$Q^2 = -(p-k)^2 \gg \lambda^2 \gg m^2 \ , \ p^2 = k^2 = m^2, \tag{1}$$

where p and k are respectively momenta of an initial and a final quark with mass m; λ^2 is a parameter in the infrared regularization (*e.g.*, fictitious gluon mass). It was established that [3,4]:

$$F = \exp\left[-\frac{\alpha_s}{4\pi} C_F \ln^2 \frac{Q^2}{\lambda^2}\right] + O\left[\alpha_s^n \ln^{2n-1} \frac{Q^2}{\lambda^2}\right],$$

where $C_F = \frac{N^2-1}{2N}$ for the gauge group SU(N) and $\alpha_s = \frac{g^2}{4\pi}$. Since this expression is a rapidly decreasing function of the transferred momentum Q^2, the natural question arises: could the leading asymptotics of the form factor F be drastically changed by neglected nonleading logarithmic terms? Investigations of Sudakov effects in QED - for the electron form factor - allow one to give a negative answer [5]. Nevertheless, in QCD the algorithm of calculation of nonleading logarithmic corrections to the quark form factor has been formulated [6] but the important question stated above remains unanswered.

In the present paper we calculate the electromagnetic quark form factor in the kinematics (1) and in a covariant gauge when quarks may be thought as massless. We sum up all logarithmic corrections to the form factor and examine its nonleading asymptotics.

2. Factorization of the quark electromagnetic form factor

The quark electromagnetic form factor (denoted by F) is expressed through the amplitude $\mathfrak{M}^\mu$ of quark elastic scattering in the following manner [2]:

$$\mathfrak{M}^{\mu} = \bar{u}(k)\, \gamma^{\mu} F\, v(p) + O(\lambda^2/Q^2),$$

where $\bar{u}(k)$ and $v(p)$ are the wave functions of quarks. The form factor F is determined by the contributions of the diagrams pictured in fig.1(a); among them the leading contributions to F, *i.e.*, those not suppressed by powers of λ^2/Q^2 are given only by the diagrams of fig.1(b) /7,8/. These diagrams involve four subgraphs: H, S, J_p and J_k. The momentum l_μ carried by the internal lines of these subgraphs may belong to any of the following regions /7/:

$$\begin{aligned}
&\text{(a) hard subgraph H: } |l_+, l_-, l_T| = O(Q) \\
&\text{(b) collinear subgraph } J_p\text{: } l_+ = O(Q)\ , l_- = O\left(\frac{\lambda^2}{Q}\right)\ , l_T = O(\lambda) \\
&\text{(c) collinear subgraph } J_k\text{: } l_+ = O\left(\frac{\lambda^2}{Q}\right)\ , l_- = O(Q)\ , l_T = O(\lambda) \\
&\text{(d) soft subgraph S: } |l_+, l_-, l_T| = O(\lambda)
\end{aligned} \qquad (2)$$

where $l_\pm = \frac{l_0 \pm l_3}{\sqrt{2}}$, $l_T = (l_1, l_2)$ and $p_- = k_+ = 0, p_T = k_T = 0, p_+ = k_- = Q = \left(Q^2/2\right)^{1/2}$.

The contribution to the form factor of a diagram of fig.1(b) is expressed in a complicated way through the quantities of four involved subgraphs since there are interactions of gluons emitted within collinear subgraphs J_k and J_p with particles belonging to hard (H) and soft (S) subgraphs. However, the expression for the quark form factor is essentially simplified by summing over all possible configurations of the diagrams of fig.1(b) with the use of the properties of particles belonging to subgraphs J_k, J_p and S /8,9/. The result of this summation is shown in fig.1(c) where double lines denote operators to be determined by equations (5),(6). Diagrammatically, fig.1(c) represents a factorization but with the momenta in the subgraphs restricted to particular regions (2). The contribution of these diagrams to the form factor may be expressed in the following form:

$$\bar{u}(k)\, \gamma^{\mu}\, v(p)\, F = F_S\, F_H\, F_{J_k} \gamma^{\mu}\, F_{J_p}, \qquad (3)$$

where by F_i we denoted the contribution from a certain subgraph.

The hard subgraph H describes interaction of particles at short distances. All internal lines of H are off-shell. Therefore, the contribution of hard subgraph to the form factor depends only on Q^2

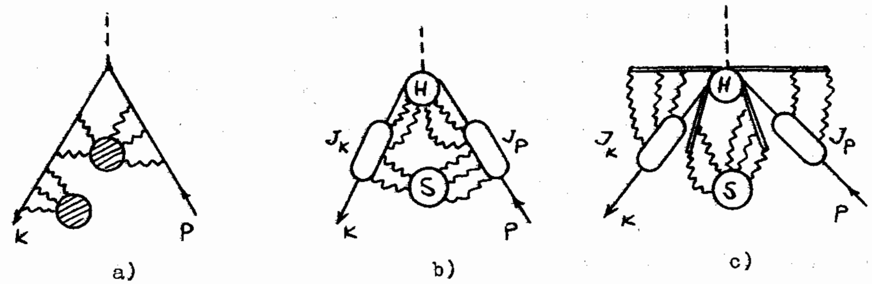

Fig.1.(a) General structure of the diagrams for the quark electromagnetic form factor. (b) Diagrams determining the leading contribution to the form factor. (c) Basic factorization of the form factor. The dashed line represents an external electromagnetic field.

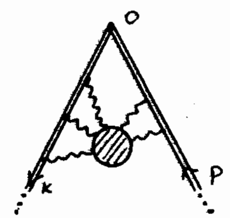

Fig.2. A typical diagram arising from the expansion of the path - ordered exponential. The double line denotes the contour of integration.

and the scale parameter μ which determines the lower boundary of off-shell momentum in (2a):

$$F_H = F_H\left(\frac{Q^2}{\mu^2}\right) = 1 - \frac{\alpha_s}{4\pi} C_F\left\{\ln^2\left(\frac{Q^2}{\mu^2}\right) - 3\ln\left(\frac{Q^2}{\mu^2}\right) + 6 - \frac{\pi^2}{6}\right\}. \quad (4)$$

The contribution of the collinear subgraph J_p is given by the expression:

$$F_{J_p} = \langle 0|T\, E_k(0,\infty)\, \Psi(0)|p\rangle_{J_p}, \quad (5)$$

where $\Psi(0)$ is the quark field operator, $|p\rangle$ is one quark state with momentum p. The operator $E_k(0,\infty)$ has appeared in this equation as a result of summation of fig.1(b) over a number of collinear gluons emitted from J_p and absorbed by H. It is equal to a path-ordered exponential:

$$E_k(0,\infty) = P\exp\left[\, ig\int_0^\infty ds\, k_\mu\, \hat{A}^\mu(ks)\, e^{-\varepsilon s}\right], \quad \varepsilon \to 0.$$

The subscript J_p in equality (5) reminds us that the momenta of all particles from F_{J_p} are to be restricted by the collinear region (2b). Contribution of the collinear subgraph J_k may be expressed as follows:

$$F_{J_k} = \langle k|T\, \bar{\Psi}(0)\, E^+_{-p}(0,\infty)|0\rangle_{J_k},$$

where "+" denotes hermitian conjugation. The meaning of the subscript J_k is the same as in equation (5).

For the contribution to the form factor of the soft subgraph we derive [9,10]:

$$F_S = \langle 0|T\, E^+_{-p}(0,\infty)\, E_k(0,\infty)|0\rangle_S, \quad (6)$$

where the subscript S points out that the momenta of all internal particles of F_S belong to the soft momentum region (2d). P - exponentials originate in this equation from summation over soft gluons of fig.1(b) absorbed by the collinear subgraphs J_p and J_k. A typical term arising from the expansion of the P - exponentials in (6) is pictured in fig.2.

3.Collinear subgraph

The contribution of the collinear subgraph J_p, for instance, is determined by equation (5) and it depends on the dimensionless

variables $\frac{\mu^2}{\lambda^2}$ and $\frac{Q^2}{\mu^2}$. We will show that

$$\frac{d\ln F_{J_p}}{d\ln Q^2} = 0 \quad (7)$$

to all orders of perturbation theory. Let us first transform equality (5) as follows:

$$\langle p|\Psi(0)|0\rangle\gamma^\mu F_{J_p} = \lim_{s\to\infty}\langle p|\bar{\Psi}\left(\frac{2ks}{\mu^2}\right)|0\rangle\langle 0|\gamma^\mu P\exp\left[ig\int_0^{2s/\mu^2} dt\, k_\nu \hat{A}^\nu(kt)\right]\Psi(0)|p\rangle_{J_p} * \exp\left[-i\frac{Q^2}{\mu^2}s\right]. \quad (8)$$

The P - exponential entering in this relation is ordered along the light-like direction of vector k_ν. It allows one to apply the light cone operator product expansion [11,12] to the right-hand side of (8). Introducing as usual the reduced matrix elements

$$\langle p|\Psi(0)|0\rangle\langle 0|\gamma^\mu D^{\mu_1}\ldots D^{\mu_n}\Psi(0)|p\rangle_{J_p} = p^\mu p^{\mu_1}\ldots p^{\mu_n} O_n\left(\frac{\mu^2}{\lambda^2}\right),$$

one finds from (8):

$$\langle p|\Psi(0)|0\rangle\gamma^\mu F_{J_p} = \lim_{s\to\infty} p^\mu \sum_{n=0}^\infty \frac{1}{n!}\left[i\frac{Q^2}{\mu^2}s\right]^n O_n\left(\frac{\mu^2}{\lambda^2}\right)\exp\left[-i\frac{Q^2}{\mu^2}s\right]. \quad (9)$$

A one-loop calculation of O_n gives for $n \ll \frac{\mu^2}{\lambda^2}$:

$$O_n\left(\frac{\mu^2}{\lambda^2}\right) = 1 - \frac{\alpha_s}{4\pi}C_F\left\{\left[2\sum_{k=0}^{n-1}\frac{1}{k+2} + \frac{1}{2}\right]\ln\left(\frac{\mu^2}{\lambda^2}\right) + \text{const}\right\}$$

that coincides with the well-known expression [13], but for $n \gg \frac{\mu^2}{\lambda^2}$

$$O_n\left(\frac{\mu^2}{\lambda^2}\right) \to O_\infty\left(\frac{\mu^2}{\lambda^2}\right) = 1 - \frac{\alpha_s}{4\pi}C_F\left\{\ln^2\left(\frac{\mu^2}{\lambda^2}\right) - \frac{3}{2}\ln\left(\frac{\mu^2}{\lambda^2}\right) + \frac{1}{2} + \frac{\pi^2}{2}\right\}$$

and does not depend on n. Substituting this relation into (8) we find the one-loop expression for F_{J_p}:

$$F_{J_p}\left(\frac{\mu^2}{\lambda^2}\right) = \left[1 - \frac{\alpha_s}{4\pi}C_F\left\{\ln^2\left(\frac{\mu^2}{\lambda^2}\right) - \frac{3}{2}\ln\left(\frac{\mu^2}{\lambda^2}\right) + \frac{1}{2} + \frac{\pi^2}{2}\right\}\right]v(p)$$

that satisfies (7). Using the results of investigation of multiloop properties of O_n [14] it may be shown that to all orders of perturbation theory the matrix elements O_n do not depend on n. Hence we conclude from (9) that $F_{J_p} = O_\infty\left(\frac{\mu^2}{\lambda^2}\right)v(p)$, in accordance with statement (7).

4. Soft subgraph

The contribution of soft subgraph to the form factor (6) may be rewritten as the so-called contour functional /10/:

$$F_S = \langle 0|TP\exp\left[ig\int_C dz_\mu \hat{A}^\mu(z)\right]|0\rangle_S,$$

where the contour C is denoted by a double line in fig.2. Contour C lies on the light-cone and therefore F_S possesses additional (as compared with nonlight-like C) logarithmic peculiarities. To study its structure, let us shift momenta p_μ and k_μ from the light-cone into the time-like direction: $p^2 = k^2 = M^2$ and consider the limit $\frac{M^2}{Q^2} \to 0$ of the regularized contour functional $F_S^{(reg)}$. It follows from the definition of P - exponentials that $F_S^{(reg)}$ is not changed under the following scale transformations: $p_\mu \to \lambda_1 p_\mu, k_\mu \to \lambda_2 k_\mu$. That is why $F_S^{(reg)}$ depends only on the invariant combination $\frac{(pk)^2}{p^2k^2}$, *i.e.*, on the cusp angle γ:

$$\gamma = \ln\frac{Q^2}{M^2}$$

of the contour C shown in fig.2. Therefore the remaining dimensional arguments of $F_S^{(reg)}$, *i.e.*, μ and λ, give the momentum scales for the Feynman integrals arising from the expansion of $F_S^{(reg)}$. As a result, the parameter μ may be thought as an ultraviolet cut-off for $F_S^{(reg)}$ and the dependence of $F_S^{(reg)}$ on μ is closely related with the renormalization properties of contour functional. Renormalization properties of contour functionals are well known /15,16/ and for the particular contour of fig.2 they may be expressed as follows /16/:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \Gamma_{cusp}(\gamma,g)\right] F_S^{(reg)}\left[\gamma,\frac{\mu^2}{\lambda^2}\right] = 0, \qquad (10)$$

where $\Gamma_{cusp}(\gamma,g)$ is the cusp anomalous dimension. In the limit $\frac{Q^2}{M^2} \to \infty$ (or $\gamma \to \infty$) $\Gamma_{cusp}(\gamma,g)$ has the asymptotics /17/:

$$\Gamma_{cusp}(\gamma,g) = \ln\frac{Q^2}{M^2}\,\Gamma_{cusp}(g) + O\left[\ln^0\frac{Q^2}{M^2}\right] \qquad (11)$$

$$\Gamma_{cusp}(g) = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F\left\{N\left(\frac{67}{36} - \frac{\pi^2}{12}\right) - n_f\frac{5}{18}\right\},$$

where n_f equals the number of quark flavors. $F_S^{(reg)}$ obeys condition /9,10/: $F_S^{(reg)}\left[\gamma,1\right] = 1$ allowing one to solve equation (10):

$$F_S^{(reg)}\left[\gamma,\frac{\mu^2}{\lambda^2}\right] = \exp\left[-\ln\frac{Q^2}{M^2}\int_{\lambda^2}^{\mu^2}\frac{dt}{2t}\,\Gamma_{cusp}(g(t)) + O\left[\ln^0\frac{Q^2}{M^2}\right]\right],$$

where equality (11) is used. The r.h.s. of this relation depends explicitly on M. This dependence, being singular in the limit $\frac{Q^2}{M^2} \to \infty$, becomes regular in the derivative: $\frac{d\ln F_S^{(reg)}}{d\ln Q^2}$. This means that $\frac{d\ln F_S^{(reg)}}{d\ln Q^2}$ does not depend on the position of the contour C (fig.2) with respect to the light-cone. Therefore we find:

$$\frac{d\ln F_S}{d\ln Q^2} = \lim_{M\to 0}\frac{d\ln F_S^{(reg)}}{d\ln Q^2} = -\int_{\lambda^2}^{\mu^2}\frac{dt}{2t}\,\Gamma_{cusp}(g(t)). \qquad (12)$$

Two-loop calculation points out /17/ that $\Gamma_{cusp} > 0$ and therefore $F_S \to 0$ for $\lambda/\mu \to 0$. This property of Γ_{cusp} is valid to all orders of α_s since it is a consequence of the following estimate of the magnitude of the contour functionals calculated within the framework of perturbation theory /18/: $F_S \le \exp(-k\,P(C))$ where P(C) is a perimeter of the contour C (fig.2) being equal to infinity.

5. Calculation of the quark form factor

Let us calculate the quark electromagnetic form factor with factorization relation (3) and with the established properties of the subgraphs. Differentiating both the sides of equation (3) with respect to Q^2 we obtain:

$$\frac{d\ln F}{d\ln Q^2} = \frac{d\ln F_H}{d\ln Q^2} - \int_{\lambda^2}^{\mu^2}\frac{dt}{2t}\,\Gamma_{cusp}(g(t)), \qquad (13)$$

where (7) and (12) have been taken into account. The l.h.s. of (13) should not depend on μ since the latter is arbitrary momentum scale dividing regions (2). As a result, F_H obeys the renormalization group equation:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} \right] \frac{d \ln F_H\left(\frac{Q^2}{\mu^2}\right)}{d \ln Q^2} = \Gamma_{cusp}(g) \cdot$$

The general solution of this equation:

$$\frac{d \ln F_H\left(\frac{Q^2}{\mu^2}\right)}{d \ln Q^2} = \Gamma(g(Q^2)) - \int_{\mu^2}^{Q^2} \frac{dt}{2t}\, \Gamma_{cusp}(g(t)) \qquad (14)$$

depends on a new function Γ. The one-loop value of Γ is found by comparing (4) with expression (14): $\Gamma = \frac{3}{4}\frac{\alpha_s}{\pi}C_F$. Substituting equation (14) into (13) we derive the final equation for the form factor:

$$\frac{d \ln F\left(\frac{Q^2}{\lambda^2}\right)}{d \ln Q^2} = \Gamma(g(Q^2)) - \int_{\lambda^2}^{Q^2} \frac{dt}{2t}\, \Gamma_{cusp}(g(t)).$$

The solution of this equation contains all the logarithmic corrections to the quark form factor not suppressed by powers of λ^2/Q^2:

$$F\left(\frac{Q^2}{\lambda^2}\right) = F_0(g(Q^2))\exp\left\{ \int_{\lambda^2}^{Q^2} \frac{dt}{t}\, \Gamma(g(t)) - \int_{\lambda^2}^{Q^2} \frac{dt}{2t} \ln\frac{Q^2}{t}\, \Gamma_{cusp}(g(t)) \right\}, \qquad (15)$$

where the functions Γ_{cusp} and Γ are defined in (11) and (14). The one-loop value F_0 may be found by comparing (15) with well-known one-loop expression for the quark form factor: $F_0 = 1 - \frac{\alpha_s}{4\pi}C_F\left(\frac{7}{2} + \frac{2}{3}\pi^2\right)$.

We cannot find the exact values of Γ_{cusp}, Γ and F_0 but we can determine the asymptotics of the quark electromagnetic form factor for $Q^2 \gg \lambda^2$. It follows from (15) that among all the logarithmic corrections to F (controlled by Γ_{cusp}, Γ and F_0) the leading ones, *i.e.*, corrections with the maximum power of logarithm per α_s, are related to Γ_{cusp} and ,consequently,

$$F\left(\frac{Q^2}{\lambda^2}\right) \xrightarrow{Q^2 \gg \lambda^2} \exp\left\{ - \int_{\lambda^2}^{Q^2} \frac{dt}{2t} \ln\frac{Q^2}{t}\, \Gamma_{cusp}(g(t)) \right\}.$$

Since $\Gamma_{cusp} > 0$ to all orders of perturbation theory ,the quark form factor is a rapidly decreasing function in the limit $Q^2 \gg \lambda^2$. Moreover, the inclusion of nonleading logarithmic corrections, *i.e.*, calculation of Γ_{cusp} to higher orders of α_s, only intensifies this asymptotics.

6. Conclusion

In the present paper, we have shown that factorization is valid for the quark electromagnetic form factor in kinematics (1). The factorization has allowed us to describe the double logarithmic asymptotics of the form factor in terms of contour functionals and matrix elements of composite twist-2 operators. Using the renormalization properties of these new objects and certain information on their structure we have derived equation (15) containing all the logarithmic corrections to the form factor. We have established that the quark form factor is a rapidly decreasing function of the transferred momentum Q^2.

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Дважды логарифмические асимптотики в КХД

В пертурбативной КХД исследуется асимптотика электромагнитного формфактора кварка. Показано, что существует связь между дважды логарифмической асимптотикой формфактора и свойствами перенормировок контурных функционалов $<0|\ TPexp[ig \int_C dz_\mu\ \hat{A}^\mu(z)]|\ 0>$. Вычислен электромагнитный формфактор безмассового кварка. Установлено, что нелидирующие логарифмические поправки к формфактору суммируются в быстро убывающую экспоненту, не изменяя лидирующую дважды логарифмическую асимптотику.

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Double Logarithmic Asymptotics in QCD

The infrared asymptotics of the quark electromagnetic form factor is investigated within the framework of perturbative QCD. The deep connection between the double logarithmic asymptotics in QCD and renormalization properties of contour functionals $<0|\ TPexp[ig \int dz_\mu \hat{A}^\mu(z)]|\ 0>$ is found. In particular, the quark electromagnetic form factor is calculated for massless quarks. It is shown that the nonleading logarithmic corrections to the form factor are summed up to give a decreasing exponential and they do not destroy the leading double logarithmic result.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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