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GREEN-SCHWARZ SUPERSTRING AS AN ASYMMETRIC CHIRAL FIELD SIGMA MODEL

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Introduction

For the last decade, the two-dimensional chiral field models (sigma models), in particular the principal chiral field ones, attract a permanent interest. This is tightly related to the progress in understanding completely integrable D=2 systems to which the chiral field models belong. One more reason is that these models share some features specific for an actual strong interaction. Nowadays, the chiral field models have received additional attention inspired by studying quantum anomalies. A novel class of such models has been formulated, viz. the principal field models with the Wess – Zumino – Witten (WZW) action /1,2/. Under a certain relation between the coupling constants, these models respect conformal symmetry /2,3/ and so can be analyzed by conventional techniques of conformal field theory /3,4/.

Recently, it has been realized that the conformally-invariant WZW σ-models have profound implications in theories of strings on group manifolds (see, e.g. /5/). An important step in understanding the parallels between the WZW models and string theories has been made by Henneaux and Mezincescu /6/. They have shown that the covariant action of Green – Schwarz (GS) superstring /7/ can be interpreted as the action of the WZW σ-model with the D=10 N=2 superspace as a target manifold. An analogous interpretation is possible as well for the heterotic superstring /8/. Later on, the attempts were undertaken /8/ to reformulate the superstring theories in manifestly geometric terms of Cartan's 1-forms which are of common use in nonlinear σ-models (see, e.g., /9/). However, as concerns the GS superstring, this program was not accomplished in full /8/.

In the present paper we fill up this gap by adopting a more general view at the σ-model interpretation of superstrings. We formulate a new class of WZW σ-models on supergroups which has certain features in common with the so-called asymmetric chiral field σ-models /10/. The GS superstring theory proves to be a representative of this class of models.

Our approach makes it possible to analyze the superstring theories in the language of superinvariant Cartan's 1-forms. In particular, this provides an algorithmic way of constructing the relevant WZW terms starting from the commutation relations of the underlying supergroup, in close analogy with the construction for ordinary WZW σ-models. As a by-product, we deduce a zero-curvature representation for the field equations of the GS superstring and its generalizations. A fresh look at the origin of the Siegel supersymmetry /7,11/ is presented. This symmetry naturally appears as a gauge freedom of the zero-curvature representation just mentioned. We believe that the observations made in this article may turn out to be helpful in circumventing the difficulties of canonical formalism of superstrings /12/.

I. Asymmetric chiral field models on supergroup

Here we collect the necessary information about the D=2 models of principal and asymmetric chiral fields in application to supergroups.

Let G be a supergroup with the algebra formed by the generators $T_M = (R_\mu, S_\alpha)$

$$[R_\mu, R_\nu] = t^{\lambda}_{\mu\nu} R_\lambda, \quad [R_\mu, S_\alpha] = C^{\beta}_{\mu\alpha} S_\beta, \quad \{S_\alpha, S_\beta\} = -\Gamma^{\mu}_{\alpha\beta} R_\mu \qquad (1.1)$$

Hereafter, we reserve the letters $\mu, \nu, \lambda, \rho, \ldots$ for the indices of even generators and $\alpha, \beta, \gamma, \ldots$ for those of odd generators (in the particular case of Poincaré superalgebra these indices are, respectively, the vector and spinor Lorentz ones). The structure constants $t^{\lambda}_{\mu\nu}, C^{\beta}_{\mu\alpha}, \Gamma^{\mu}_{\alpha\beta}$ satisfy the Jacobi identities

$$\Gamma^{\mu}_{\alpha\beta} C^{\delta}_{\mu\gamma} + \Gamma^{\mu}_{\beta\gamma} C^{\delta}_{\mu\alpha} + \Gamma^{\mu}_{\gamma\alpha} C^{\delta}_{\mu\beta} = 0, \qquad (1.2a)$$

$$\Gamma^{\nu}_{\alpha\beta} t^{\lambda}_{\nu\mu} + C^{\gamma}_{\mu\alpha} \Gamma^{\lambda}_{\gamma\beta} + C^{\gamma}_{\mu\beta} \Gamma^{\lambda}_{\gamma\alpha} = 0, \qquad (1.2b)$$

$$t^{\lambda}_{\nu\mu} C^{\gamma}_{\lambda\alpha} + C^{\beta}_{\nu\alpha} C^{\gamma}_{\mu\beta} - C^{\beta}_{\mu\alpha} C^{\gamma}_{\nu\beta} = 0, \qquad (1.2c)$$

$$t^{\lambda}_{\mu\nu} t^{\sigma}_{\lambda\rho} + t^{\lambda}_{\nu\rho} t^{\sigma}_{\lambda\mu} + t^{\lambda}_{\rho\mu} t^{\sigma}_{\lambda\nu} = 0. \qquad (1.2d)$$

Also the following identity holds ($g_{\mu\nu} \sim \mathrm{str}(R_\mu R_\nu)$)

$$\{\Gamma^{\mu}_{\alpha\beta} \Gamma^{\nu}_{\gamma\delta} + \Gamma^{\mu}_{\gamma\alpha} \Gamma^{\nu}_{\beta\delta} + \Gamma^{\mu}_{\beta\gamma} \Gamma^{\nu}_{\alpha\delta}\} g_{\mu\nu} = 0 \qquad (1.3)$$

as a consequence of cyclic property of $\mathrm{str}(\;)$

$$\mathrm{str}[\{S_\alpha, S_\beta\}\{S_\gamma, S_\delta\} + \mathrm{cycl}(\alpha,\beta,\gamma)] = 0$$

If there exists an invariant bilinear form in the adjoint representation of G (this is the case for any semisimple Lie superalgebra), then it is always possible to define an invariant nondegenerate metric ($\eta_{\mu\nu}$, $X_{\alpha\beta}$) normalized so that

$$\eta_{\mu\nu} = -\,\mathrm{str}\left(R_\mu^{ad} R_\nu^{ad}\right), \quad X_{\alpha\beta} = \mathrm{str}\left(S_\alpha^{ad} S_\beta^{ad}\right). \tag{1.4}$$

Using the evident identity

$$\mathrm{str}\left(\{S_\alpha^{ad}, S_\beta^{ad}\} R_\mu^{ad}\right) = \mathrm{str}\left([R_\mu^{ad}, S_\alpha^{ad}] S_\beta^{ad}\right)$$

one easily derives the important relation between the constants $C_{\mu\alpha}^{\beta}$ and $\Gamma_{\alpha\gamma}^{\mu}$

$$C_{\mu\alpha}^{\beta} = \Gamma_{\alpha\gamma,\mu} X^{\gamma\beta} \tag{1.5}$$

that expresses the property of invariance of the metric (1.4).

Now let us define a matrix superfield $U(\xi^0, \xi^1)$ which lives in the two-dimensional space (ξ^0, ξ^1) with the Minkowski space signature and takes values in the supergroup G

$$U(\xi^0,\xi^1) = \exp\{ i x^\mu(\xi^0,\xi^1) R_\mu + i\theta^\alpha(\xi^0,\xi^1) S_\alpha \}, \tag{1.6}$$

x^μ and θ^α being even and odd supergroup parameters. The Cartan 1-forms built out of this field

$$\omega = U^{-1} dU = U^{-1}\partial_a U\, d\xi^a = \omega_a\, d\xi^a \tag{1.7}$$

are invariant under the left action of G: $U \to AU$, $A \in G$. In terms of the left-invariant 1-forms, the multivalued action of the principal chiral field on G is written, as in the case of ordinary groups /1-3/, as follows [1]:

$$A = -\frac{1}{4\gamma^2}\int_{\partial V} d^2\xi \sqrt{-g}\, g^{ab}\,\mathrm{str}(\omega_a \omega_b) + \frac{N}{24\pi}\int_V d^3\xi\, \varepsilon^{abc}\,\mathrm{str}(\omega_a\omega_b\omega_c) \tag{1.8}$$

Here γ^2 and N (integer) are the parameters of the action, the theory being conformally-invariant if $N\gamma^2/4\pi = 1$ /2,3/; $g^{ab}(\xi)$ is a Riemannian metric on the two-dimensional space ∂V ($g = \det(g_{ab}(\xi))$) which is identified with the boundary of a three-dimensional ball V. Variation of the integrand in the second (WZW) term is a closed three-form

[1] For brevity, we denote by the same letters $a, b, c, \ldots$ the vector indices of D=2 and D=3 spaces.

$$\delta\, \mathrm{str}(\omega\wedge\omega\wedge\omega) = d\Omega_2, \tag{1.9}$$

Ω_2 being a two-form. Owing to this property, variation of the second term is defined on ∂V and so gives rise to a correct contribution to the equations of motion.

Owing to the cyclic property of the operation $\mathrm{str}(\ldots)$ the action (1.8) is also invariant with respect to the right shifts $U \to UB$ and can thus be equivalently written via the right-invariant 1-forms $dU U^{-1}$. This amounts to the fact that the action (1.8) describes a nonlinear sigma-model on the homogeneous space $G_L \otimes G_R / G_{diag}$ /9/, G_L, G_R being two isomorphic supergroups with the algebra (1.1) and G_{diag} their diagonal subgroup.

In paper /10/ a more general class of chiral field models has been considered, in which the invariance under right shifts is broken. The corresponding action is

$$A = -\frac{1}{4\gamma^2}\int_{\partial V} d^2\xi \sqrt{-g}\, g^{ab}\,\mathrm{str}(\omega_a\omega_b J_{I}) + \frac{\beta}{24\pi}\int_V \mathrm{str}(\omega\wedge\omega\wedge\omega J_{II}), \tag{I.10}$$

where J_{I} and J_{II} are some constant matrices which are different in general. We will call these models the asymmetric chiral field ones. The reason for introducing matrices J_I and J_{II} is as follows. The most general bilinear Lagrangian invariant only under the left shifts can be constructed as an arbitrary bilinear form composed of the coefficients of the decomposition of the Cartan 1-form ω_a in the G generators Γ_M. The first term in (I.10) is just the convenient compact notation for this Lagrangian in terms of unexpanded quantities ω_a (1.7). This notation is always possible: one chooses $\mathrm{str}(\;)$ in the adjoint representation and picks up an appropriate matrix J_I. Analogous reasoning applies to the second term in (1.10). In general, $\det(J_{I,II}) \neq 0$. Picking up proper degenerate matrices J_I and J_{II}, one can construct invariant actions for chiral fields on various homogeneous spaces G/H of G. The matrices J_I, J_{II} are to be chosen so as to commute with the transformations from the stability subgroup H, which results in invariance of the action under gauge right shifts generated by H. Thus, the generic action (1.10) encompasses all the possible nonlinear σ-models on (super)-group G.

The action considered in /10/ corresponds to choosing $\beta = 0$ in (1.10) i.e., to the case where the WZW term is lacking. For (1.10) to be meaningful also at $\beta \neq 0$, i.e., to give rise to the equations of motion defined on the two-dimensional space ∂V, a variation of 3-form $\omega^3 = \mathrm{str}(\omega\wedge\omega\wedge\omega J_{II})$ should be closed, just as in the case of $J_{II} = I$, eq. (1.9)

$$\delta\, \mathrm{str}(\omega\wedge\omega\wedge\omega J_{II}) = d\Omega_2 . \qquad (1.11)$$

This condition severely constrains an admissible choice of matrices J_{II}. In the case of ordinary groups, the examples of WZW functionals defined on the target manifolds different from the group itself have been given in /13/. To all them there correspond nontrivial matrices J_{II} obeying the condition (1.11). Recall that for the manifolds with nontrivial third homotopy group the parameter β is quantized. No such a quantization occurs in the case of superstrings and their generalizations considered below. The reason is that the corresponding manifolds display a trivial topology.

In what follows, it will be convenient to treat the supertraces of bilinear products of generators involving matrices J_{I}, J_{II} as certain averages

$$\langle R_\mu R_\nu \rangle_{I,II} = \mathrm{str}(R_\mu R_\nu J_{I,II}) \;, \quad \langle R_\mu S_\alpha \rangle_{I,II} = \qquad (1.12)$$

$$= \mathrm{str}(R_\mu S_\alpha J_{I,II}) \;, \quad \langle S_\alpha S_\beta \rangle_{I,II} = \mathrm{Str}(S_\alpha S_\beta J_{I,II}) .$$

In this notation, the action (I.I0) and the constraint (1.11) take the form

$$A = -\frac{1}{4\gamma^2}\int_{\partial V} d^2\xi \sqrt{-g}\, g^{ab} \langle \omega_a \omega_b \rangle_{I} + \frac{\beta}{24\pi}\int_{V} d^3\xi\, \varepsilon^{abc} \langle \omega_a \omega_b \omega_c \rangle_{II} \;,$$

$$\delta \langle \omega\wedge\omega\wedge\omega \rangle_{II} = d\Omega_2 . \qquad (1.13)$$

In precisely the same way one may write the actions for σ-models defined on superalgebras admitting no cyclic operation $\mathrm{Str}(\ldots)$. This concerns, e.g., infinite dimensional superalgebras and certain infinite dimensional representation of finite dimensional superalgebras. In these cases the averages $\langle \ldots \rangle$ of bilinear in generators can be regarded as some matrices with constant entries. A further fixing of their structure proceeds by resorting to various symmetry arguments. In the next Sect. we will demonstrate that the GS superstring belongs to the class of σ-models with the action (1.13) and nontrivial matrices $\langle \Gamma_N \Gamma_M \rangle_{I,II}$.

To close this Sect., we point out once more that the use of symbols $\langle \ldots \rangle_{I,II}$ ensures a uniform convenient notation for the invariants constructed out of the coefficients in the decomposition of the Cartan 1-forms in the generators of G. Its main advantage consists in that the basic object is the full matrix left-invariant Cartan 1-form ω while one or another pattern of breaking of right G-invariance is encoded in the structure of the averages (1.12).

2. A novel class of chiral field models with multivalued action. The GS superstring as asymmetric chiral field model.

In this Sect. we show how to consistently formulate the GS superstring model within the general scheme described above. The GS superstring proves to belong to the class of asymmetric chiral field models associated with a specific choice of supergroup G as the direct product $G^1 \otimes G^2$, G^1 and G^2 being two isomorphic supergroups with the commutative even part.

Consider the contracted superalgebra following from (1.1) by rescaling the generators as $R_\mu \to \varkappa P_\mu$, $S_\alpha \to \sqrt{\varkappa}\, Q_\alpha$ and by putting then $\varkappa \to \infty$. In the contraction limit one is left with the (anti)commutation relations

$$[P_\mu, P_\nu] = [P_\mu, Q_\alpha] = 0 \;, \quad \{Q_\alpha, Q_\beta\} = -\Gamma^\mu_{\alpha\beta} P_\mu \;, \qquad (2.1)$$

where the structure constants $\Gamma^\mu_{\alpha\beta}$ satisfy the important relation (I.13). As before, the vector and spinor indices may be raised and lowered with the help of tensors $\eta_{\mu\nu}$, $\chi_{\alpha\beta}$. The superalgebra (2.1) clearly possesses the automorphisms generated by the even subalgebra of the initial superalgebra (1.1). The tensors $\eta_{\mu\nu}, \chi_{\alpha\beta}$ are invariant under these automorphisms. The superalgebra (2.1) has an obvious interpretation as the algebra of supertranslations in the superspace (x^μ, θ^α):

$$x^\mu \to x^\mu + i\epsilon^\alpha \Gamma^\mu_{\alpha\beta}\theta^\beta \;, \quad \theta^\alpha \to \theta^\alpha + \epsilon^\alpha \;, \qquad (2.2)$$

ϵ^α being an odd supertranslation parameter. These transformations are induced by the left shifts on the supergroup corresponding to (2.1)

$$\exp\{\tfrac{i}{2} x^\mu P_\mu + i\theta^\alpha Q_\alpha\} \to \exp\{i\epsilon^\alpha Q_\alpha\}\exp\{\tfrac{i}{2} x^\mu P_\mu + i\theta^\alpha Q_\alpha\} \qquad (2.3)$$

Taking into account the relation (1.3) for structure constants, the superalgebra (2.1) is isomorphic to the algebra of N=1 supertranslations in the ten-dimensional space, provided θ_α is a 32-component Majorana - Weyl spinor and $\Gamma^\mu_{\alpha\beta}$ are connected with the D=10 Dirac γ-matrices as $\Gamma^\mu_{\alpha\beta} = (\gamma^\mu)_\alpha^\delta C_{\delta\beta}$, $C_{\delta\beta} = -C_{\beta\delta}$ being the charge conjugation matrix ($\bar\theta_\alpha = C_{\alpha\beta}\theta^\beta$). From here on, the transformations (2.2),(2.3) will be called supersymmetry transformations. It is worth noting that the N=10, D=10 supertranslation algebra cannot be obtained by above "naive" contraction from any semi-simple Lie superalgebra of the kind (1.1). Nevertheless, the identity (1.3) is still valid in this case (the Γ-matrices satisfy it in the dimensions D=3,4,6,10). The automorphism group is the

$D=10$ Lorentz group. Thus, the class of superalgebras possessing the structure relations (2.1) and respecting the condition (I.3) for $\Gamma^{\mu}_{\alpha\beta}$ is wider than the one arising as a result of a straightforward contraction of the relations (1.1).

We will construct the asymmetric chiral field models on the supergroup generated by the direct sum $g^1 \oplus g^2$ of two superalgebras (2.1):

$$[P^i_\mu, P^j_\nu] = [P^i_\mu, Q^j_\alpha] = 0\,,$$
$$\{Q^i_\alpha, Q^j_\beta\} = -\Gamma^\mu_{\alpha\beta} P^i_\mu \delta^{ij} = -(\delta^{ij} P^+_\mu + \sigma_3^{ij} P^-_\mu)\Gamma^\mu_{\alpha\beta}\,, \tag{2.4}$$

with $P^{\pm}_\mu = \frac{1}{2}(P^1_\mu \pm P^2_\mu)$, $\sigma_3^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The superalgebra (2.4) generates the direct product of supergroups $G^1 \otimes G^2$, where each multiplier G^1, G^2 can be parametrized as follows:

$$U_1 = \exp\left(\tfrac{i}{2} x^{1\mu} P^1_\mu + i\theta^{1\alpha} Q^1_\alpha\right) \in G^1\,,$$
$$U_2 = \exp\left(\tfrac{i}{2} x^{2\mu} P^2_\mu + i\theta^{2\alpha} Q^2_\alpha\right) \in G^2\,. \tag{2.5}$$

Let us introduce the chiral fields $U_j(\xi^0, \xi^1)$, which amounts to regarding the parameters $x^{j\mu}$ and $\theta^{j\alpha}$ as the fields given on the two-dimensional space with coordinates (ξ^0, ξ^1). The related Cartan 1-forms are

$$\omega^j = U_j^{-1} dU_j = \tfrac{i}{2}\omega^{j\mu} P^j_\mu + i\omega^{j\alpha} Q^j_\alpha = \omega^j_a d\xi^a =$$
$$= \left[\tfrac{i}{2}(\partial_a x^{j\mu} + i\partial_a \theta^j \Gamma^\mu \theta^j) P^j_\mu + i\partial_a \theta^{j\alpha} Q^j_\alpha\right] d\xi^a\,. \tag{2.6}$$

These are left-invariant with respect to two independent N=1 supersymmetries (2.3):

$$x^{j\mu} \to x^{j\mu} + i\epsilon^j \Gamma^\mu \theta^j\,, \quad \theta^j \to \theta^j + \epsilon^j\,. \tag{2.7}$$

Let us make an observation to be important in what follows. On the homogeneous space $G^1 \otimes G^2/G^-$, G^- being an abelian subgroup with the generator P^-_μ, the supersymmetry $G^1 \otimes G^2$ is realised as N=2 supersymmetry. Indeed, the space $G^1 \otimes G^2/G^-$ is parametrised by coordinates $(x^\mu = x^{1\mu} + x^{2\mu}, \theta^{j\alpha})$ which are inert under the action of generator P^-_μ. Therefore, P^-_μ is zero on $(x^\mu, \theta^{j\alpha})$ and the superalgebra $g^1 \oplus g^2$ is reduced to that of N=2 supersymmetry on this coordinate set. It is immediately seen from eq. (2.7) that

$$x^\mu \to x^\mu + i\sum_{j=1}^{2} \epsilon^j \Gamma^\mu \theta^j\,, \quad \theta^j \to \theta^j + \epsilon^j\,. \tag{2.8}$$

These transformations precisely coincide with those of N=2 supersymmetry.

An element of the coset space $G^1 \otimes G^2/G^-$ can be represented as

$$U = \exp\left\{\tfrac{i}{2}(x^{1\mu} + x^{2\mu})P^+_\mu + i\sum_{j=1}^{2}\theta^{j\alpha}Q^j_\alpha\right\} = U_1 U_2 \exp\left\{-\tfrac{i}{2}(x^{1\mu} - x^{2\mu})P^-_\mu\right\}. \tag{2.9}$$

The supertranslations (2.8) are realized on U according to the generic law of nonlinear realizations

$$U \to U' = A_1 A_2 U \exp\left\{\tfrac{1}{2}\sum_{k,j=1}^{2} \epsilon^j \Gamma^\mu \theta^k (\sigma_3)^{jk} P^-_\mu\right\}\,, \tag{2.10}$$
$$A_j = \exp(i\epsilon^{j\alpha} Q^j_\alpha)\,.$$

Like in ordinary nonlinear σ-models, a convenient way to single out the space $G^1 \otimes G^2/G^-$ from the parameter space of supergroup $G^1 \otimes G^2$ is to impose the invariance under right gauge G^--shifts

$$U_1 \to U_1' = U_1 \exp\left(\tfrac{i}{2} a^\mu(\xi^0, \xi^1) P^1_\mu\right)\,,$$
$$U_2 \to U_2' = U_2 \exp\left(-\tfrac{i}{2} a^\mu(\xi^0, \xi^1) P^2_\mu\right)\,, \tag{2.11}$$

$$\omega^j_a \to \omega^j_a + \tfrac{i}{2}\partial_a a^\mu(\xi^0, \xi^1)\sum_{k=1}^{2}(\sigma_3)^{jk} P^k_\mu\,. \tag{2.12}$$

The theory of asymmetric chiral field on $G^1 \otimes G^2$ respecting this invariance will be the theory of chiral field on the space $G^1 \otimes G^2/G^-$ and so will possess the N=2 supersymmetry (just inherent in the GS covariant action [7]).

Now we are prepared to formulate the basic principles of constructing the actions for asymmetric σ-models on $G^1 \otimes G^2/G^-$ which include the GS superstring action as a particular case.

1. The action is built up from the $G^1 \otimes G^2$ invariant Cartan 1-forms ω^j (2.6) and it is represented by the generic formula (1.11).

2. It is invariant under gauge transformations (2.12) and thus describes a nonlinear σ-model on the homogeneous space $G^1 \otimes G^2/G^-$. This requirement is equivalent to demanding the action to be N=2 supersymmetric.

3. The differential three-form Ω_3 entering into the definition of the WZW term satisfies the standard condition (1.9)

$$\Omega_3 = d\Omega_2 \tag{2.13}$$

4. The kinetic term in the action is of the correct order with

respect to time derivative, i.e., of the second order for bosons and of the first one for fermions.

It turns out that these four natural conditions fix the action up to two free parameters that is the coupling constants in front of the quadratic and cubic parts of the action (these are analogous to the parameters γ and β in eqs.(I.10) (1.13)). A further fixing of the ratio of these constants proceeds, just as in ordinary WZW σ-models[2,3], with imposing some extra local symmetries. In the present case the latter proves to be a generalization of a familiar Siegel supersymmetry [7,11].

We begin by writing down the action in the most general form similar to (1.13)

$$A=-\int\limits_{\partial V} d^2\xi\sqrt{-g}\,g^{ab}\langle\omega_a\omega_b\rangle_{I}-\int\limits_{V}\langle\omega\wedge\omega\wedge\omega\rangle_{II}\,, \qquad (2.14)$$

where $\omega=\omega^1+\omega^2$ and the averages have the most general structure allowed by the Grassmann parity and the condition that the metric in the N=2 superspace $(x^\mu,\theta^{1\alpha},\theta^{2\alpha})$ is induced by the invariant metric (1.4) (the last requirement follows from the desire to maintain invariance under the automorphism group operating on indices $\mu,\nu,\dots;\alpha,\beta,\dots$)

$$\langle P^i_\mu P^j_\nu\rangle_\Lambda=\eta_{\mu\nu}\,p^{ij}_\Lambda\,,\quad \langle P^i_\mu Q^j_\alpha\rangle=0\,, \qquad (2.15)$$

$$\langle Q^i_\alpha Q^j_\beta\rangle_\Lambda=X_{\alpha\beta}\,q^{ij}_\Lambda\,,\quad X_{\alpha\beta}=-X_{\beta\alpha}\,.$$

Here $\Lambda=I,II$; p^{ij}_Λ, q^{ij}_Λ are the matrices with the constant entries to be determined from the requirements 1)–4). We set at once $q^{ij}_\Lambda=0$ since only in this case the condition 4) is satisfied (see the end of this Sect.). A straightforward computation using the formulas from Appendix shows that the following relations are necessary and sufficient for the requirements 1)–4) to be fulfilled

$$p^{1j}_{I,II}=p^{2j}_{I,II}\,, \qquad (2.16)$$

$$p^{12}_{II}=-p^{21}_{II}\,. \qquad (2.17)$$

Among the original constants these relations leave two independent ones which are convenient to choose as $\frac{1}{4}p^{22}_{I}=l_{I}$, $\frac{1}{4}p^{12}_{II}=l_{II}$. Notice that eqs.(2.16),(2.17) imply degeneracy of the matrices p^{ij}_Λ: $\det(p_\Lambda)=0$. This property reflects, in particular, the fact that under the condition 2) the bilinear part of the action involves only the 1-forms on the coset $G^1\otimes G^2/G$, not on the whole supergroup $G^1\otimes G^2$.

With the constraints (2.16),(2.17) solved and in the basis (P^+_μ, P^-_μ) the action (2.14) takes the form

$$A=-4l_{I}\int\limits_{\partial V} d^2\xi\sqrt{-g}\,g^{ab}\langle\omega_a\omega_b\rangle_{I}-4l_{II}\int\limits_{V}\langle\omega\wedge\omega\wedge\omega\rangle_{II}\,, \qquad (2.14')$$

$$\langle P_\mu P_\nu\rangle_{I}=\eta_{\mu\nu}\begin{pmatrix}1&0\\0&0\end{pmatrix},\quad \langle P_\mu P_\nu\rangle=\eta_{\mu\nu}\begin{pmatrix}0&-1\\0&0\end{pmatrix}$$

$$\langle P_\mu Q_\alpha\rangle_\Lambda=\langle Q_\alpha Q_\beta\rangle_\Lambda=0$$

or

$$A=\ l_{I}\int\limits_{\partial V} d^2\xi\sqrt{-g}\,g^{ab}(\omega^{1\mu}_a+\omega^{2\mu}_a)(\omega^1_{b\mu}+\omega^2_{b\mu})+ \qquad (2.18)$$

$$+\,l_{II}\int\limits_{V} d^3\xi\,\varepsilon^{abc}(\omega^{1\mu}_a+\omega^{2\mu}_a)\partial_b(\omega^1_{c\mu}-\omega^2_{c\mu})\,,$$

where we have made use of the Maurer–Cartan equations $\partial_a\omega^j_b-\partial_b\omega^j_a=-[\omega^j_a,\omega^j_b]$. Substituting explicit expressions for $\omega^{j\mu}_a$ (2.6): $\omega^{j\mu}_a=\partial_a x^{j\mu}+i\partial_a\theta^j\Gamma^\mu\theta^j$ and passing in the second term to integration over ∂V with the help of identity (1.3), one eventually gets

$$A=l_{I}\int\limits_{\partial V} d^2\xi\Big\{\sqrt{-g}\,g^{ab}\Big(\partial_a x^\mu+i\sum_{j=1}^{2}\partial_a\theta^j\Gamma^\mu\theta^j\Big)\Big(\partial_b x_\mu+i\sum_{j=1}^{2}\partial_b\theta^j\Gamma_\mu\theta^j\Big)- \qquad (2.19)$$

$$-\frac{l_{II}}{l_{I}}\varepsilon^{ab}\,i\Big(\partial_a x^\mu+i\sum_{j=1}^{2}\partial_a\theta^j\Gamma^\mu\theta^j\Big)\Big(\sum_{k,j=1}^{2}\partial_b\theta^k(\sigma_3)^{kj}\Gamma_\mu\theta^j\Big)\Big\}\,.$$

If the superalgebra (2.1) is chosen to be that of N=1, D=10 supertranslations the action (2.19) coincides, up to a freedom in choosing l_{II}/l_{I}, with the covariant GS superstring action [7]. The genuine GS action emerges at $l_{II}/l_{I}=2$ corresponding to an additional invariance of the action under local Siegel supersymmetry transformations [7,11].

Thus, the "preaction" of superstring (with arbitrary l_{II}/l_{I}) can be algorithmically constructed by the generic scheme of construction of asymmetric chiral field σ-models, as applied to the direct product of two independent N=1 supertranslation groups. The most important point is that the relevant chiral field parametrizes the coset $G^1\otimes G^2/G$. As a result, the action depends only on the N=2 superspace coordinates x^μ, $\theta^{j\alpha}$ on which the $(N=1)\otimes(N=1)$ supersymmetry is realized as the N=2 one. Moreover, the quadratic term is such as though one starts from the beginning with the N=2 supertranslation algebra. The $G^1 \otimes G^2$ structure manifests itself only in the WZW term which essentially involves the 1-form $(\omega^1_\mu - \omega^2_\mu)$ related to the extra translation generator $P^-_\mu = \frac{1}{2}(P^1_\mu - P^2_\mu)$. It is to be mentioned that all the previous attempts to derive this term entirely within the standard σ-model framework have used somewhat artificial constructions /8/. In our scheme it appears quite naturally as a result of nontrivial choice of the supergroup one starts with. Also, it becomes clear why the GS action respects no internal SO(2) symmetry inherent in the N=2 superalgebra: the obvious reason is that the correct initial superalgebra (2.1) possesses no such a symmetry at all.

In conclusion of this Sect., we discuss the case with the nonzero matrix $\langle Q^i_\alpha Q^j_\beta \rangle$ in formulas (2.15). The WZW term does not change while the quadratic term acquires an adding $\tilde{A}$

$$\tilde{A} \sim \int_{\partial V} d^2\xi \sqrt{-g}\, g^{ab} \sum_{j,k=1}^{2} \partial_a \theta^{j\alpha} \langle Q^j_\alpha Q^k_\beta \rangle_{\bar{1}} \partial_b \theta^{k\beta} .$$

Such an adding would contribute to the fermionic kinetic energy by the terms containing two derivatives in time which is unacceptable from the standpoint of quantization. So, in accordance with the requirement 4) one is led to put $\langle Q^j_\alpha Q^k_\beta \rangle_{\bar{1}} = 0$.

3. The equations of motion and Siegel supersymmetry in terms of Cartan's forms. The zero-curvature representation

Here we rewrite the equations of motion of the considered class of σ-models and the transformations of Siegel supersymmetry in the language of superinvariant Cartan's forms. It will be shown that at the critical point $\ell_{II}/\ell_I = 2$ corresponding to invariance under the Siegel supersymmetry there arises a zero-curvature representation for these equations.

The equation of motion may be deduced by varying the actions (2.18) or (2.19) with respect to the fields $g^{ab}(\xi), x^\mu(\xi), \theta^{j\alpha}(\xi)$. It is convenient to use the superinvariant variations $\omega^j = U_j^{-1}\delta U_j$

$$\omega^j = \frac{i}{2}(\delta x^{j\mu} + i\,\delta\theta^j \Gamma^\mu \theta^j) P^j_\mu + i\,\delta\theta^{j\alpha} Q^j_\alpha$$
$$= \frac{i}{2}\omega^{j\mu} P^j_\mu + \omega^{j\alpha} Q^j_\alpha \qquad (3.1)$$

Simple calculations (see Appendix) yield the following equations:

$$\delta g^{ab}: \quad T_{ab} = 0 ; \qquad (3.2a)$$

$$\tilde{\omega}^{j\alpha}: \quad P^{ab}_+ (\omega^1_{a\mu} + \omega^2_{a\mu}) \Gamma^\mu_{\alpha\beta} \omega^{1\beta}_b = 0 ,$$
$$P^{ab}_- (\omega^1_{a\mu} + \omega^2_{a\mu}) \Gamma^\mu_{\alpha\beta} \omega^{2\beta}_b = 0 ; \qquad (3.2b)$$

$$\tilde{\omega}^{j\mu}: \quad \partial_a (P^{ab}_+ \omega^{2\mu}_b + P^{ab}_- \omega^{1\mu}_b) = 0 . \qquad (3.2c)$$

Here T_{ab} is the energy-momentum tensor

$$T_{ab} = (\omega^{1\mu}_a + \omega^{2\mu}_a)(\omega^1_{b\mu} + \omega^2_{b\mu}) - \frac{1}{2} g_{ab} g^{cd} (\omega^{1\mu}_c + \omega^{2\mu}_c)(\omega^1_{d\mu} + \omega^2_{d\mu}) \qquad (3.3)$$

and

$$P^{ab}_\pm = \sqrt{-g}\, g^{ab} \pm \frac{1}{2} \frac{\ell_{II}}{\ell_I} \varepsilon^{ab} . \qquad (3.4)$$

One has also to add to equations (3.2) the Maurér – Cartan equations

$$\partial_a \omega^{j\mu}_b - \partial_b \omega^{j\mu}_a = 2i\, \omega^{j\alpha}_b \Gamma^\mu_{\alpha\beta} \omega^{j\beta}_a , \qquad (3.5)$$
$$\partial_a \omega^{j\alpha}_b - \partial_b \omega^{j\alpha}_a = 0 .$$

Put together, equations (3.2) and (3.5) are equivalent to those written in terms of $x^\mu, \theta^{j\alpha}$ and following from the action (2.19). Note that for ordinary chiral fields, the Cartan form representation of the equations of motion has been given in /14,9/ (in the latter paper – with the WZW term taken into account).

Equation (3.2c) can be rewritten in the two-fold way

$$\partial_a \left[P^{ab}_+ (\omega^1_{b\mu} + \omega^2_{b\mu}) - \frac{\ell_{II}}{\ell_I} \varepsilon^{ab} \omega^1_{b\mu} \right] = 0 , \qquad (3.6a)$$

$$\partial_a \left[P^{ab}_- (\omega^1_{b\mu} + \omega^2_{b\mu}) + \frac{\ell_{II}}{\ell_I} \varepsilon^{ab} \omega^2_{b\mu} \right] = 0 , \qquad (3.6b)$$

after that equations (3.2b) and (3.2c) are divided into the two sets

$$\begin{cases} \partial_a \left[P^{ab}_+ (\omega^1_{b\mu} + \omega^2_{b\mu}) - \frac{\ell_{II}}{\ell_I} \varepsilon^{ab} \omega^1_{b\mu} \right] = 0 , \\ P^{ab}_+ (\omega^1_{b\mu} + \omega^2_{b\mu}) \Gamma^\mu_{\alpha\beta} \omega^{1\beta}_a = 0 ; \end{cases} \qquad (3.7a)$$

$$
\begin{cases}
\partial_a [P_-^{ab}(\overset{1}{\omega}_{b\mu} + \overset{2}{\omega}_{b\mu}) + \dfrac{l_{II}}{l_I}\varepsilon^{ab}\overset{2}{\omega}_{b\mu}] = 0, \\
P_-^{ab}(\overset{1}{\omega}_{b\mu} + \overset{2}{\omega}_{b\mu})\Gamma^{\mu}_{\alpha\beta}\,\omega_a^{2\beta} = 0 .
\end{cases} \qquad (3.7b)
$$

Surprisingly, at the point $l_{II}/l_I = 2$, where the operators $P_\pm^{ab}$ become projectors, $P_\pm^{1a} = (g^{01}/g^{00} \mp 1/(g^{00}\sqrt{-g}))P_\pm^{0a}$, the equations (3.7a),(3.7b) amount to the zero-curvature condition

$$
[L_a^+, L_b^+] = [L_a^-, L_b^-] = 0 . \qquad (3.8)
$$

Here the differential operators $L_a^{\pm}$ are given by

$$
L_a^{\pm} = \partial_a \mp i\lambda^2 \varepsilon_{ab} P_\pm^{bc}(\omega_c^{1\mu} + \omega_c^{2\mu})R_\mu^{\pm} - 2\lambda\,\omega_a^{\frac{1}{2}\alpha} S_\alpha^{\pm}, \qquad (3.9)
$$

where λ is a spectral parameter and the generators $R_\mu^{\pm}$, $S_\alpha^{\pm}$ form the two mutually (anti)commuting superalgebras

$$
[R_\mu^{\pm}, R_\nu^{\pm}] = R^{\pm}_{[\mu\nu]}, \quad [R_\mu^{\pm}, S_\alpha^{\pm}] = C^{\beta}_{\mu\alpha}S_\beta^{\pm}, \quad \{S_\alpha^{\pm}, S_\beta^{\pm}\} = -\Gamma^{\mu}_{\alpha\beta}R_\mu^{\pm}, \qquad (3.10)
$$

the constants $C^{\beta}_{\mu\alpha}$, $\Gamma^{\mu}_{\alpha\beta}$ being related by the equality (1.6). The structure of the commutator $[R_\mu^{\pm}, R_\nu^{\pm}]$ is noncritical for deducing equations (3.7) from the representation (3.8). However, in general (including the case of the GS superstring) the generator $R^{\pm}_{[\mu\nu]}$ should be nonzero for the consistency between the Jacobi identities and the relation (1.5). When commuted with R_μ, S_α, the generator $R_{[\mu\nu]}$ may produce new generators so the full zero-curvature representation superalgebra may turn out to be infinite dimensional (this is definitely so for the GS case). Fortunately, it is just the structure of the relations $[R_\mu, S_\alpha]$ and $\{S_\alpha, S_\beta\}$ that is crucial for deriving equations (3.7). It is interesting that the superalgebra (2.1) can be regarded as a contraction of (3.10).

Let us specially take notice of the fact that the condition $l_{II}/l_I = 2$, which is necessary for the representation (3.8) to exist, is just the one making the action (2.18),(2.19) invariant under local Siegel supertransformations. We now discuss the implementation of this supersymmetry in the general case when the structure constants $\Gamma^{\mu}_{\alpha\beta}$ are not obliged to coincide with the Dirac γ-matrices. Given a set of matrices $(\tilde\Gamma^{\mu})^{\beta\gamma}$ such that

$$
\Gamma^{\mu}_{\alpha\beta}(\tilde\Gamma^{\nu})^{\beta\gamma} + \Gamma^{\nu}_{\alpha\beta}(\tilde\Gamma^{\mu})^{\beta\gamma} = \eta^{\mu\nu}\delta_\alpha^\gamma , \qquad (3.11)
$$

the action (2.19) is invariant under the local supertransformations of the fields $x^\mu(\xi)$ and $\theta^{j\alpha}(\xi)$ compactly expressed via the variations (3.1) as

$$
\tilde\omega^{1\mu} + \tilde\omega^{2\mu} = 0 , \qquad (3.12)
$$
$$
\tilde\omega^{\frac{1}{2}\alpha} = P_\pm^{ab}(\overset{1}{\omega}_{b\mu} + \overset{2}{\omega}_{b\mu})(\tilde\Gamma^{\mu})^{\alpha\beta}\,\varkappa^{\frac{1}{2}}_{a\beta}(\xi^0, \xi^1) .
$$

Here $\varkappa^{i}_{a\beta}(\xi)$ are fermionic parameters which are the world-sheet vectors. These transformations should be accompanied by appropriate transformations of g^{ab}, but for simplicity we assume equation (3.2a) to be fulfilled and, respectively, the action to be completely written in terms of x^μ and $\theta^{j\alpha}$. Making use of a general formula for variation of the action (see Appendix), one easily obtains that the conditions for the action to be invariant under (3.12) are as follows:

$$
P_+^{ab}P_+^{cd}(\overset{1}{\omega}_{b\mu} + \overset{2}{\omega}_{b\mu})(\overset{1}{\omega}_{d\nu} + \overset{2}{\omega}_{d\nu})\omega_a^{1\alpha}\Gamma^{\mu}_{\alpha\beta}(\tilde\Gamma^{\nu})^{\beta\gamma} = 0, \qquad (3.13)
$$
$$
P_-^{ab}P_-^{cd}(\overset{1}{\omega}_{b\mu} + \overset{2}{\omega}_{b\mu})(\overset{1}{\omega}_{d\nu} + \overset{2}{\omega}_{d\nu})\omega_a^{2\alpha}\Gamma^{\mu}_{\alpha\beta}(\tilde\Gamma^{\nu})^{\beta\gamma} = 0 .
$$

Taking into account the projection properties displayed by $P_\pm^{ab}$ at $l_{II}/l_I = 2$

$$
P_+^{ab}g_{bc}P_-^{cd} = P_-^{ab}g_{bc}P_+^{cd} = 0 , \qquad (3.14)
$$
$$
P_\pm^{ab}P_\pm^{cd} = P_\pm^{cb}P_\pm^{ad} .
$$

and equations (3.2a), it is a simple exercise to show that (3.13) are satisfied providing the relations (3.11) hold. In the case of the GS superstring the transformations (3.12) are precisely the Siegel ones [7]. In this case $(\tilde\Gamma^{\mu})^{\alpha\beta} = C^{\alpha\delta}(\gamma^{\mu})_{\delta}^{\beta}$, $C^{\alpha\delta}C_{\delta\beta} = \delta^\alpha_\beta$ and the condition (3.11) turns out to be trivially satisfied.

4 Conclusions

We have presented the interpretation of the superstring-type action (2.19) (including the GS superstring covariant action) as the action of asymmetric chiral field defined on the direct product of two Lie supergroups with the commutative even part. We succeeded in carrying out all the considerations in a close analogy with the conventional bosonic case and entirely in terms of super-invariant Cartan's forms. This ensures a manifest supersymmetry of the theory at any stage. We emphasize that the proposed construction applies not only to the N=1 supertranslation groups in proper dimensions (D =3,4,6,10) but also, as has been described in Sec.2, to any supergroup resulting by contraction from some semisimple Lie supergroup. The constants $\Gamma^{\mu}_{\alpha\beta}$ should not obligatory coincide with $\gamma^{\mu\delta}_{\alpha}C_{\delta\beta}$-matrices; what is actually important is that those satisfy the identity (1.3). It would be of interest to consider the examples of the corresponding superstring models. One may also expect that, upon an appropriate generalization, the proposed scheme will help in constructing such theories as superstring in harmonic superspace /15/, superstrings on the supergroups with nontrivial even parts, etc.

As a by-product, we have derived a zero-curvature representation (3.8) for the superstring field equations in an arbitrary gauge. Let us recall in this connection that the choice of conformal gauge $\sqrt{-g}\, g^{ab} \sim \delta^{ab}$ which is normally assumed necessary for the linearization of superstring field equations is incompatible with complicated topologies on the world sheet.

Similarly to any zero-curvature representation /16/, the set (3.8) possesses a gauge symmetry

$$L^{\pm}_{a} \to U^{\pm} L^{\pm}_{a} (U^{\pm})^{-1}, \tag{4.1}$$

where $U^{\pm} = \exp(i\, x^{\pm\alpha}(\xi)\, S^{\pm}_{\alpha})$. This choice of $U^{\pm}$ is dictated by the form of superalgebra (3.10). It turns out that the Siegel transformations (3.12) constitute a particular case of (4.1) corresponding to $x^{\pm\alpha} = \lambda\omega^{\frac{1}{2}\alpha}$ (at the infinitesimal level). It would be desirable to understand why in the present case the zero-curvature representation superalgebra and the superalgebra on which the σ-model is originally constructed do not coincide (as takes place for ordinary σ-models) but are related by contraction. Also, there arises an interesting task of setting up an infinite set of the conserved currents associated with the representation (3.8).

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Appendix

We list here a number of helpful formulas used in the process of varying the actions (2.14),(2.18) and (2.19). With their aid

one derives the conditions necessary for the invariance of the generic action (2.14) under local transformations (2.11),(2.12), gets the equations of motion and checks the invariance with respect to the Siegel supersymmetry

$$\delta(\sqrt{-g}\, g^{ab}) = \sqrt{-g}\left(\delta g^{ab} - \tfrac{1}{2}(\delta g^{cd} g_{cd})\, g^{ab}\right), \tag{A.1}$$

$$\delta\omega^{i} = d\tilde{\omega}^{i} + [\omega^{i}, \tilde{\omega}^{i}], \quad \tilde{\omega}^{i} = U^{-1}_{i}\,\delta U_{i}, \tag{A.2}$$

$$\langle [\omega^{k}, \tilde{\omega}^{k}] \wedge \omega^{k} \wedge \omega^{k} \rangle_{\underline{II}} = \langle \omega^{k} \wedge \omega^{k} \wedge [\omega^{k}, \tilde{\omega}^{k}] \rangle_{\underline{II}} = 0. \tag{A.3}$$

(Here we have used the identity (1.3) and the averaging (2.15) with $\langle Q^{i}_{\alpha} Q^{j}_{\beta} \rangle = 0$). Let us also quote the general formula of variation of the action (2.18), (2.19)

$$\delta A = (-4l_{\underline{I}}) \int_{\partial V} d^2\xi \sqrt{-g}\, \delta g^{ab} T_{ab} + (-8l_{\underline{I}}) \int_{\partial V} d^2\xi\, P^{ab}_{+} \Big\{ \partial_a (\tilde{\omega}^{1}_{\mu} + \tilde{\omega}^{2}_{\mu})\, \omega^{2\mu}_{b} +$$

$$+ (\omega^{1}_{a\mu} + \omega^{2}_{a\mu})(\omega^{2\alpha}_{b} \Gamma^{\mu}_{\alpha\beta} \tilde{\omega}^{2\beta}) \Big\} +$$

$$+ (-8l_{\underline{I}}) \int_{\partial V} d^2\xi\, P^{ab}_{-} \Big\{ \partial_a (\tilde{\omega}^{1}_{\mu} + \tilde{\omega}^{2}_{\mu})\, \omega^{1\mu}_{b} + (\omega^{1}_{a\mu} + \omega^{2}_{a\mu})(\omega^{1\alpha}_{b} \Gamma^{\mu}_{\alpha\beta} \tilde{\omega}^{1\beta}) \Big\},$$

Here $P^{ab}_{\pm}$ are defined in eqs.(3.3) and (3.4).

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Исаев А.П., Иванов Е.А. E2-88-570
Суперструна Грина - Шварца как асимметрическая модель кирального поля

Строится новый класс двумерных σ-моделей весс-зуминовского типа на однородном пространстве $G \circledast G/G^-$, где супергруппа G получается контракцией из произвольной полупростой супергруппы Ли, а G^- - некоторая абелева подгруппа трансляций в $G \circledast G$. Показано, что уравнения движения, вытекающие из неоднозначного действия для этих моделей, могут быть записаны в виде представления нулевой кривизны. Модель суперструны Грина - Шварца принадлежит к предложенному классу σ-моделей.

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Isaev A.P., Ivanov E.A. E2-88-570
Green-Schwarz Superstring as an Asymmetric Chiral Field Sigma Model

A new class of two-dimensional σ-models of the Wess - Zumino - Witten type is constructed. The target manifold of these models is coset space $G \circledast G/G^-$, where supergroup G is obtained by contraction from an arbitrary semisimple Lie supergroup and G^- is some abelian subgroup of translations in $G \circledast G$. It is shown that the equations of motion following from the Wess - Zumino - Witten type action of these models admit a zero curvature representation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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