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M.K.Volkov, A.N.Ivanov[1], M.Nagy[2], N.Troitskaya[1]

EFFECTIVE CHIRAL LAGRANGIANS, THE QUARK MODEL OF SUPERCONDUCTIVITY TYPE AND THE PROBLEM OF P-A DIAGONALIZATION

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[1]Leningrad Polytechnical Institute, USSR
[2]Institute of Physics of the Electro-Physical Research Centre, Slovak Academy of Sciences, Bratislava, Czechoslovakia

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Introduction

It is well-known that hadron low-energy interactions are quite well described by the effective chiral Lagrangians (ECL)/1/. The ECL represent the Lagrange form of an approximate chiral-symmetry realization which is also the case for quantum chromodynamics (QCD). The ECL method allows a deeper insight into the current algebra results, the low energy theorems as well as the hypotheses of the vector dominance (VD) and partial conservation of axial current (PCAC). The attractive peculiarity of the ECL formalism is the possibility to work directly with observable (physical) particles and describe observable quantities. The study of the ECL properties is an important and urgent problem.

The derivation of ECL from "the first principles" in QCD is a problem not solved so far. Its solution needs joint description of such effects as bosonization, spontaneous chiral symmetry breaking (SCSB) and confinement. Unfortunately at the contemporary level of development of quantum field theory the joint description "from first principles" of these complicated nonperturbative effects has not be achieved. That is why for the study of the ECL properties one frequently uses the effective quark model based on QCD.

One of these models is the quark model of superconductivity type (QMST) /2/ which is a quark version of the modified Nambu - Jona - Lasinio model/3/ with the effective four-quark interaction. The QMST describes the bosonization effects and SCSB but it does not describe the quark confinement. The requirement of confinement in the QMST is an external condition. Limitation arising as a result of this requirement is in agreement with the PCAC and VD hypotheses supposing the weak dependence of matrix elements on the meson masses up to their zero values/4/.

In the present paper we consider ECL with the $SU(3) \times SU(3)$ chiral group describing low-energy interactions of four nonets of mesons (scalar - Σ, pseudoscalar - $\mathcal{P}$, vector - V and axial-vector - A) composed from light (u,d,s) quarks.

The purpose of this work is, first, to analyse the problem of appearance of nonphysical vertices of the low energy meson interaction in ECL which arise as a result of SCSB and P-A diagonalization and second to discuss possible solution of this problem in the framework of the QMST. First, let us remind the standard procedure of the ECL construction for the system of four meson nonets /1/.

The effective Lagrangian of the meson system in question can be given in the following form

$$\mathcal{L}_{eff} = \mathcal{L}_{ch} + \mathcal{L}_{WZ} + \mathcal{L}_{NWZ}. \qquad (1)$$

The Lagrangian $\mathcal{L}_{ch}$ includes meson kinetic terms and local interaction vertices connected with each other by chiral and vector transformations of the SU(3)xSU(3) group. The whole information about SCSB and the origin of nondiagonal P–A transitions caused by SCSB and leading to necessity of P–A diagonalization, is contained only in $\mathcal{L}_{ch}$. The Lagrangians $\mathcal{L}_{WZ}$ and $\mathcal{L}_{NWZ}$ describe the strong low-energy meson interaction vertices caused by chiral anomalies (quark loop anomalies). The quark loop anomalies with an odd number of γ^5-vertices (Adler-Bell-Jackiw anomalies /5/) reproduce all the vertices of the effective Wess-Zumino Lagrangian $\mathcal{L}_{WZ}$ /6/, e.g. PVV, APP, VPPP, etc. The Lagrangian $\mathcal{L}_{NWZ}$ is defined by quark loop anomalies with an even number of γ^5-vertices. The Lagrangian $\mathcal{L}_{NWZ}$ describes effective vertices with higher derivatives of the non-Wess-Zumino type /7/, e.g., VPP, ΣVV, etc. [1)]

For description of SCSB and P–A diagonalization it is necessary to know the explicit form of $\mathcal{L}_{ch}$. Under the standard construction of $\mathcal{L}_{ch}$ there exist essential arbitrariness /1/. However, for discussion of the SCSB and P–A diagonalization problems, it is enough to consider the "minimal" expression for $\mathcal{L}_{ch}$ /1/

$$\begin{aligned}\mathcal{L}_{ch} = {} & tr_F\left(D_\mu M^+ D^\mu M\right) + tr_F\left(\mu_1^2\, M^+ M\right) \\ & - tr_F\left(g_1^2 (M^+ M)^2\right) - tr_F\left(\mu_0 (M^+ + M)\right) \\ & - \frac{1}{2} tr_F\left(F_{\mu\nu} F^{\mu\nu} + G_{\mu\nu} G^{\mu\nu}\right) + tr_F\left(\mu_2^2 \left(V_\mu V^\mu + A_\mu A^\mu\right)\right).\end{aligned} \qquad (2)$$

Here $X = t_F^a X^a$, where $X = M\,(M^+), V, A, F$ or G and $t_F^a = \frac{1}{2}\lambda_F^a$ are the generators of the flavour group $SU(3)_F$. Further, $D_\mu M = \partial_\mu M - i g_2 [V_\mu, M]_- + i g_2 \{A_\mu, M\}_+$ is the covariant derivation of the field $M = \Sigma + iP$, where Σ and P are nonets of scalar and pseudoscalar fields, respectively;

1) The vertex VPP with first derivative is contained in L_{ch}.

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i g_2 [V_\mu, V_\nu]_- - i g_2 [A_\mu, A_\nu]_- ,$$
$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_2 [V_\mu, A_\nu]_- + i g_2 [V_\nu, A_\mu]_-$$

are the strength tensors of nonets of the vector V and axial vector A fields, g_1, g_2, μ_0, μ_1 and μ_2 are the model parameters, the numerical values of which in the standard ECL construction can be obtained only from comparison of the calculated results with experimental data. At $\mu_1^2 > 0$ vacuum expectation values of the scalar fields (diagonal nonet component) differ from zero: $\langle\Sigma\rangle = \Sigma_0 \neq 0$, which leads to SCSB. In consequence of SCSB the term $tr_F(D_\mu M^+ D^\mu M)$ in (2) results in the following interaction:

$$2\, tr_F\left(g_2\, D^\mu P \{A_\mu, \Sigma_0\}_+\right), \qquad (3)$$

that leads to nondiagonal P–A transitions. The appearance of interaction (3) means that P- and A-states in (2) are nonphysical. The transition to the physical states is carried out by the shift /1/ [2)]

$$(A_\mu)_{ij} = (A'_\mu)_{ij} + \xi_{ij} (\bar{D}_\mu P)_{ij} , \qquad (4)$$

where $\bar{D}_\mu P = \partial_\mu P - i g_2 [V_\mu, P]_-$ is covariant derivation of the P-field with respect to local vector transformations of the SU(3) group /1/. The parameters ξ_{ij} are determined from the condition of lack of direct P–A transitions. The exclusion of P–A transitions leads to: 1) change of a coefficient at the kinetic term of the P-fields, 2) appearance of new vertices of strong low-energy meson interaction. These vertices can be divided into physical and nonphysical ones. The latter do not influence the amplitudes of physical processes, which will be discussed later.

2) Indices i and j denote the quark structure of mesons ($i(j) = u, d, s$); there is no summation over i, j indices. The use of quark indices simplifies the presentation of problems of unitary symmetry breaking. Let us note that the standard ECL construction does not take into account the quark structure of mesons.

The change of the coeffioients of the kinetio term of the P-fields leads to the complementary P-field renormalization /1/

$$P_{ij} = Z_i^{1/2} P'_{ij} \ , \qquad (5)$$

where Z_{ij} is the renormalization constant (no summation over quark structure indioes). There exists the following relation between the ξ_{ij} and Z_{ij} parameters /1/:

$$(g_2)_{ij}\, \xi_{ij}\, Z_{ij}^{1/2} = \frac{1}{F_P}\left(1 - \frac{1}{Z_{ij}}\right), \qquad (6)$$

where F_P is the PCAC P-meson constant (e.g., PCAC π-meson constant F_π = 93 MeV).

Now let us pass to the problem of appearance of new nonphysical vertices of the low-energy interaction caused by shift (4). As an example, we consider the contact $\omega 3\pi$ vertex /8/. The effective Lagrangian describing the contact $\omega 3\pi$ interaction is caused by the corresponding chiral anomaly and it gets the form /9/

$$(\mathcal{L}_{\omega 3\pi})_{WZ} - \frac{i}{6}(G_\omega)_{WZ}\, \varepsilon^{abc}\, \varepsilon^{\mu\nu\alpha\beta} \cdot \qquad (7)$$

$$\cdot\, \omega_\mu\, \partial_\nu \pi^a\, \partial_\alpha \pi^b\, \partial_\beta \pi^c .$$

Here $(G_\omega)_{WZ} = -N g_\rho / 4\pi F_\pi^3$, where N is the number of oolour degrees of freedom of quarks (real N=3), g_ρ is the $\rho \to 2\pi$ decay constant. The constant $(G_\omega)_{WZ}$ determines the contribution of the contact $\omega 3\pi$ interaction to the $\omega \to 3\pi$ decay amplitude /9/.

The vertices $\omega A\pi\pi$, $\omega AA\pi$ and ωAAA possess the struoture analogous to that given in (7). Under the shift (4) the vertices $\omega A\pi\pi$, $\omega AA\pi$ and ωAAA produce the vertex $\omega 3\pi$. As a result, the effective Lagrangian desoribing the $\omega 3\pi$ vertex takes the following form:

$$(\mathcal{L}_{\omega 3\pi})_{eff} = (\mathcal{L}_{\omega 3\pi})_{WZ}\left[1 - 3\left(1 - \frac{1}{Z}\right) + \right. \qquad (8)$$

$$\left. + 3\left(1 - \frac{1}{Z}\right)^2 - \left(1 - \frac{1}{Z}\right)^3\right] = \frac{1}{Z^3}(\mathcal{L}_{\omega 3\pi})_{WZ} .$$

Hence we get

$$(G_\omega)_{eff} = \frac{1}{Z^3}(G_\omega)_{WZ} . \qquad (9)$$

Thus the contribution of the contact $\omega 3\pi$ interaction to the $\omega \to 3\pi$ amplitude decreases Z^3 - times ($Z \simeq \sqrt{2}$, /10/ and $Z^3 \simeq 2.8$), which is in contradiction with experimental data. This is easily seen by studying the amplitude of the process $\gamma \to 3\pi$.

In the framework of the current algebra and PCAC hypothesis the following low-energy theorem has been proven /11/

$$e A(\gamma \to 3\pi) = \frac{1}{F_\pi^2} A(\pi^0 \to \gamma\gamma) = -\frac{N\alpha}{3\pi F_\pi^3} , \qquad (10)$$

$$\left(\alpha = \frac{e^2}{4\pi} = \frac{1}{137}\right)$$

where $A(\gamma \to 3\pi)$ and $A(\pi^0 \to \gamma\gamma)$ are the invariant amplitudes of the prooesses $\gamma \to 3\pi$ and $\pi^0 \to \gamma\gamma$, respectively, calculated at zero 4-momenta of interacting particles. The theoretical value $A(\gamma \to 3\pi)_{theor}$ = 9.5 (GeV)$^{-3}$ calculated by formula (10) at N=3 is in agreement with the experimental one $A(\gamma \to 3\pi)_{exp}$ = = 12.9±0.9 (GeV)^3 /12/.

According to /9/, in the low-energy limit $A(\gamma \to 3\pi)$ can be expressed through the coupling constant of the contaot $\omega 3\pi$ interaotion

$$e A(\gamma \to 3\pi) = \frac{4\pi\alpha}{g_\omega} G_{\omega 3\pi} \qquad (g_\omega = 3 g_\rho) . \qquad (11)$$

At $G_{\omega 3\pi} = (G'_\omega)_{WZ}$ (7) we have agreement with the low-energy theorem (10) /9/, whereas at $G_{\omega 3\pi} = (G_\omega)_{eff}$ (8) the theoretical value $A(\gamma \to 3\pi)_{theor}$ is almost 4 times lower than the experimental one /12/. Thus, in describing the $\omega 3\pi$ interaction vertex the inclusion of new interaction vertices caused by P-A diagonalization leads to evident contradiction with experimental data.

We can give other examples of the discrepancy between theoretical and experimental results caused by P–A diagonalization (4). Nevertheless, there exist a sufficient number of counter examples demonstrating the necessity of taking account of local interaction vertices caused by shift (4). For example, in the decays $a_1 \to \pi\rho$ and $a_1 \to \pi\gamma$ supplementary contributions from $\pi \leftrightarrow a_1$ transitions allow one to get the gauge invariant expression for the decay amplitudes. The inclusion of P–A transitions plays a very important role also in calculating the axial formfactors of the processes $\pi \to e\nu_e\gamma$ and $K \to e\nu_e\gamma$ [13], etc.

Then, for the description of strong low-energy interactions in weak kaon decays the inclusion of local vertices at a low-energy interaction caused by P–A diagonalization (4), allows one to get agreement between theoretical and experimental results [14].

Thus, the problem is : Why in P–A diagonalization (4), arising as a consequence of SCSB, there appear both physically justified local interaction vertices necessary for correct description of low-energy meson interaction processes and nonphysical vertices whose inclusion contradicts the experimental data? What is the reason for appearance of the latter and how to get rid of them?

In the framework of the standard ECL construction these questions cannot be answered. Therefore, we analyse the nature of nonphysical interaction vertices in the QMST.

This article is further organized as follows: in section 1 we give basic assumptions of QMST. In section 2, using the $\omega\rho\pi$ vertex as an example (or, which is the same $\pi^0\gamma\gamma$) we analyse in the QMST the origin of nonphysical strong low-energy interaction vertices caused by P–A diagonalization (4) . It is shown that in the QMST the appearance of nonphysical vertices is connected with the ambiguity of calculation of integrals over virtual momenta of 2-quark loops describing the interaction vertices. The recipe is proposed to exclude nonphysical interaction vertices caused by shift (4), which is based on the use of the Pauli – Villars regularization procedure. In conclusion, the discussion of the obtained results is given.

1. Quark model of the superconductivity type

The QMST is a quark version of the Nambu – Jona – Lasinio model [2,3] with the effective four-quark interaction. The strong $q\bar{q}$ attraction caused by the effective four-quark interaction leads to appearance of $q\bar{q}$ -collective excitations(CE) with meson quantum numbers. The presence in the model of CE of certain type is defined by the initial four-quark interaction structure. For example,

to describe strong low-energy interactions of four nonets of low-lying mesons of the scalar, pseudoscalar, vector and axial vector types, the initial four-quark interaction should be [2]

$$\mathcal{L}_{int}(q) = 2G_S(\bar{q}t^a q)^2 + 2G_P(\bar{q}i\gamma^5 t^a q)^2 - 2G_V(\bar{q}\gamma^\mu t^a q)^2 - 2G_A(\bar{q}\gamma^\mu\gamma^5 t^a q)^2, \qquad (12)$$

where G_i (i=S,P,V and A) are some positive constants, $q = (u,d,s)$ are the quark fields with N degrees of freedom; $t^a = \frac{1}{2}\lambda^a_F I_c$, where λ^a_F are the Gell-Mann matrices of the quark flavour group $SU(3)_F$ (a=0,1,...8), and I_c is the unit NxN matrix. The interaction (12) is invariant with respect to the chiral U(3)x U(3) group transformation if $G_S = G_P = G_1$ and $G_A = G_V = G_2$

$$\mathcal{L}_{int}(q) = 2G_1[(\bar{q}t^a q)^2 + (\bar{q}i\gamma^5 t^a q)^2] - 2G_2[(\bar{q}\gamma^\mu t^a q)^2 + (\bar{q}\gamma^\mu\gamma^5 t^a q)^2]. \qquad (13)$$

The constants G_1 and G_2 can be determined from the pseudoscalar and vector meson mass spectrum [2,10] $G_1 = 4{,}9\ (\mathrm{GeV})^{-2}$ and $G_2 = 16(\mathrm{GeV})^{-2}$.

The total quark Lagrangian with the effective interaction (13) has the form

$$\mathcal{L}(q) = \bar{q}(i\hat{\partial} - m_0)q + \mathcal{L}_{int}(q), \qquad (14)$$

where $i\hat{\partial} = i\gamma^\mu\partial_\mu$, $m_0 = m_{0F} I_c$ is the quark mass matrix, and $m_{0F} = diag(m_{0u}, m_{0d}, m_{0s})$ with current quark masses. The mass term $\bar{q}m_0 q$ breaks "softly" the chiral symmetry. In the limit $m_{0i} \to 0$ (i=u,d,s) chiral symmetry becomes exact.

The theory with the four-quark interaction (13) is not renormalizable. Therefore, it is reasonable to use only the one-loop approximation [3]. It means that constituent quark masses, meson masses and the corresponding phenomenological coupling constants are defined only by one-loop quark diagrams [2,3]. For description of quantum corrections in the one-loop approximation it is sufficient to take ultraviolet cut-off parameter Λ , which can be identified with the scale of chiral symmetry breaking [15]. The restriction to one-loop quark diagrams provides a simple calculation of strong low-energy meson interaction characteristics with a small number of parameters. It should be noted that the one-loop approximation is in agreement with selection rules over 1/N expansion in QCD (large number of colours) with the gauge group $SU(N)_c$ as $N \to \infty$ [16].

<u>Spontaneous chiral symmetry breaking.</u> In the description of strong low-energy hadron interactions the essential role is played by SCSB. The mechanism of SCSB in a quark system with a four-quark interaction (13) is analogous to the mechanism of emergence of an energy gap in a superconductor at low temperatures [17]. In consequence of SCSB, the current quarks turn into the constituent ones. In this case the increase in quark masses ($m_i \gtrsim 0.3$ GeV) and reconstruction of low-energy vacuum take place [2,3].

In the one-loop approximation, constituent and current quark masses satisfy the equations [2,3]

$$m_{oi} = m_i \left[1 - 8 G_1 I_1(m_i) \right] , \tag{15}$$

where m_i is the mass of constituent i-quark (i=u,d,s) and $I_1(m_i)$ is the quadratically divergent integral [2,3]

$$I_1(m_i) = \frac{-iN}{(2\pi)^4} \int^{\Lambda} \frac{d^4k}{m_i^2 - k^2} = \frac{N}{16\pi^2} \left[\Lambda^2 - m_i^2 \ln\left(1 + \frac{\Lambda^2}{m_i^2}\right) \right] . \tag{16}$$

The parameters Λ and m_i (i=u,d,s) are the basic parameters of the QMST. We give their numerical values [2,10]

$$\Lambda = 1.25 \text{ GeV}, \quad m_u = 0.28 \text{ GeV}, \quad m_d = 0.284 \text{ GeV}$$
$$m_s = 0.46 \text{ GeV}. \tag{17}$$

These quantities are obtained by taking into account both the isotopic and unitary symmetry breaking. From equation (15) we can find the values of current quark masses corresponding to constituent quark masses (17)

$$m_{ou} = 3 \text{ MeV}, \qquad m_{od} = 4 \text{ MeV}, \qquad m_{os} = 80 \text{ MeV}. \tag{18}$$

Although current quark masses (18) are smaller than the generally accepted values [19]: m_{ou}=4 MeV, m_{od}=7 MeV, m_{os} = 150 MeV, there is no contraction with the Dashen mass formulae for pseudoscalar mesons [20]

$$m_\pi^2 = \frac{1}{F_\pi^2}(m_{ou} + m_{od})\sigma , \quad m_K^2 = \frac{1}{F_\pi^2}(m_{ou} + m_{os})\sigma , \tag{19}$$

where $\sigma = -\langle \bar{u}u \rangle = -\langle \bar{d}d \rangle = -\langle \bar{s}s \rangle = (0.25 \text{ GeV})^3$ is quark condensate in zero order with respect to current quark masses, i.e. at $m_{ou} = m_{od} = m_{os} = 0$ [19]. The point is that in the QMST the quark condensate $\sigma_{QMST} = m_u / 2G_1 = (0.3 \text{ GeV})^3$ [18] is larger than the generally accepted $\sigma = (0.25 \text{ GeV})^3$. Therefore, the product of current masses (18) on σ_{QMST} correctly describes the pseudoscalar meson masses: m_π = 0.15 GeV, m_K = 0.51 GeV [18]. These quantities are in satisfactory agreement with experimental values.

<u>Bozonization.</u> The bosonization procedure, i.e., the introduction of local meson fields into consideration and elimination of quark degrees of freedom, can be fulfilled in the QMST by the integral transformation method in the continual integral defined by the generating functional of the quark Green functions [2,21]:

$$W[\eta, \bar{\eta}] = W_0^{-1} \int \mathcal{D}\bar{q}\mathcal{D}q \, \exp i \int d^4x \left[\mathcal{L}(q) + \bar{\eta} q + \bar{q} \eta \right] , \tag{20}$$

where W_0 is the vacuum functional

$$W_0 = \int \mathcal{D}\bar{q}\mathcal{D}q \, \exp i \int d^4x \, \mathcal{L}(q)$$

and η and $\bar{\eta}$ are external sources of quark fields.

As a result of the standard transformations, the generating functional (20) can be expressed through the local meson fields

$$W[\eta, \bar{\eta}] = W_0^{-1} \int \mathcal{D}\mu \, \exp i \int d^4x \, d^4y \, \bar{\eta}(x) \cdot \tag{21}$$
$$\cdot S_F(x, y | \Sigma, P, V, A) \, \eta(y) \, \exp i S_{eff}[\Sigma, P, V, A] ,$$

where Σ, P, V and A are the fields of scalar, pseudoscalar, vector and axial vector nonets ($\chi = t^a \chi^a$, where $\chi = \Sigma, P, V, A$), $\mathcal{D}\mu$ is the invariant measure of integration over meson fields, $S_F(x, y | \Sigma, P, V, A)$ is the Green function of quarks in the external Σ, P, V and A fields, and finally, $S_{eff}[\Sigma, P, V, A]$ is the effective action of strong low-energy meson interactions [2,21]

$$S_{eff}[\Sigma, P, V, A] = -i \ln \frac{\mathrm{Det}(i\hat{\partial} - m + H)}{\mathrm{Det}(i\hat{\partial} - m)} + \tag{22}$$

$$+ \int d^4x \left[-\frac{g_1^2}{4G_1} tr_F (\Sigma^2 + P^2) + \frac{g_2^2}{4G_2} tr_F (V_\mu V^\mu + A_\mu A^\mu) \right],$$

where $H = g_1(\Sigma' + i\gamma^5 P) + g_2(\hat{V} + \hat{A}\gamma^5)$; moreover $g_1\Sigma' = g_1\Sigma - m_0 + m$, where g_1 and g_2 are the quark-meson interaction constants [3]) , and $m = m_F I_C$, where $m_F = \mathrm{diag}(m_u, m_d, m_s)$ is the constituent quark mass matrix. The vacuum expectation value of scalar fields Σ is different from zero $\langle \Sigma \rangle \neq 0$, while the fields Σ' are defined in such a way that $\langle \Sigma' \rangle = 0$.

From formula (22) we find the effective Lagrangian of strong low-energy meson interactions

$$\mathcal{L}_{eff}(x) = \tilde{\mathcal{L}}_{eff}(x) - \frac{g_1^2}{4G_1} tr_F (\Sigma^2 + P^2) + \tag{23}$$

$$+ \frac{g_2^2}{4G_2} tr_F (V_\mu V^\mu + A_\mu A^\mu),$$

where

$$\tilde{\mathcal{L}}_{eff}(x) = -i \langle x | \ln \frac{\mathrm{Det}(i\hat{\partial} - m + H)}{\mathrm{Det}(i\hat{\partial} - m)} | x \rangle =$$

$$= -i\, tr_{C+F+L} \langle x | \ln \left(1 - \frac{1}{m - i\hat{\partial}} H\right) | x \rangle \quad ^{4)} \tag{24}$$

The effective Lagrangian $\tilde{\mathcal{L}}_{eff}(x)$ can be expressed in the infinite series form

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[3]) These constants coincide with the constants g_1 and g_2 in the effective Lagrangian (2).

[4]) Indices C,F and L denote trace calculation over "colour", "flavour" and Lorentz indices. The wave functions $|x\rangle$ are normalized to the condition $\langle x | y \rangle = \delta^4(x - y)$.

$$\tilde{\mathcal{L}}_{eff}(x) = \sum_{n=1}^{\infty} \frac{i}{n} tr_{C+F+L} \langle x | \left(\frac{1}{m - i\hat{\partial}} H\right)^n | x \rangle = \sum_{n=1}^{\infty} \tilde{\mathcal{L}}^{(n)}_{eff}(x), \tag{25}$$

each term of which being a one-loop quark diagram

$$\tilde{\mathcal{L}}^{(n)}_{eff}(x) = \int \prod_{l=1}^{n-1} \frac{d^4x_l\, d^4k_l}{(2\pi)^4} \exp(-ik_1x_1 - \ldots - ik_nx_n) \cdot \tag{26}$$

$$\cdot \left(-\frac{1}{n} \frac{N}{16\pi^2}\right) \int \frac{d^4k}{\pi^2 i} tr_{F+L} \left\{ H(x) \frac{1}{m - \hat{k}} H(x_1) \frac{1}{m - \hat{k} - \hat{k}_1} \cdots \right.$$

$$\left. \cdot H(x_{n-1}) \frac{1}{m - \hat{k} - \hat{k}_1 - \ldots \hat{k}_{n-1}} \right\},$$

where $k_1 + \cdots + k_n = 0$. Index n corresponds to the number of one-loop quark diagram vertices. The quark diagrams with two vertices (n=2) define the kinetic terms in the effective Lagrangian, while the diagrams with n vertices (n $\geq$ 3) describe in the QMST the n-meson vertex of the strong low-energy interaction.

<u>Confinement.</u> We now turn to the discussion of the rules of one-loop quark diagram calculations determining the strong low-energy meson interaction vertices. It is quite clear that approximation of effective meson interactions by quark diagrams with constant virtual quark masses and constant quark-meson interaction vertices does not take into account the confinement. Therefore, the diagram on the whole cannot correctly describe the strong low-energy meson interaction vertex. The question occurs: What parts of the quark diagrams ought to be taken into account for obtaining effective interaction vertices in chiral Lagrangians? It is natural to assume that without knowledge of exact dynamics of quark confinement we can attach the physical meaning only to those parts of quark diagrams which weakly depend on a concrete confinement mechanism. These parts of quarks diagrams are the divergent and anomalous parts.

The divergent parts of quark diagrams can be expressed through quadratically and logarithmically divergent integrals regularized by using the cut-off parameter Λ . The quadratically divergent integrals are defined by formula (16) and the logarithmically divergent integrals have the form /2/

$$I_2(m_i, m_j) = \frac{-iN}{(2\pi)^4} \int^{\Lambda} \frac{d^4k}{(m_i^2 - k^2)(m_j^2 - k^2)} = \tag{27}$$

$$= \frac{N}{16\pi^2} \frac{1}{m_i^2 - m_j^2} \left[m_i^2 \ln(1 + \Lambda^2/m_i^2) - m_j^2 \ln(1 + \Lambda^2/m_j^2) \right].$$

The divergent parts of the quark diagrams depend essentially on the cut-off parameter Λ, i.e., on the scale of chiral symmetry breaking, and therefore, depend weakly on the confinement dynamics.

The anomalous parts of the quark diagrams do not depend on virtual quark masses. Therefore, the contribution of the anomalous parts of the quark diagrams can formally be distinguished by calculating the quark diagrams in the limit of infinite masses of virtual quarks [7,22].

The restriction to the divergent and anomalous parts of the quark diagrams corresponds to quark diagram extrapolation to the zero values of mass variables of interacting mesons. This approximation is the generalization of the PCAC and VD hypotheses to the quark level [1,4]. Let us remind that in conformity with the PCAC and VD hypotheses the matrix elements of transition between hadron states, caused by strong low energy interactions, are the smooth functions of interacting meson mass variables up to zero values of the latter [4].

Thus, for a phenomenological calculation of confinement in the QMST it is enough to complete the basic hypothesis of the QMST about the description of SCSB and bosonization by effective four-quark interaction (13) by the PCAC and VD hypotheses assuming the weak dependence of vertex functions of strong low-energy meson interactions on mass variables of interacting mesons.

By keeping only the divergent and anomalous parts of the quark diagrams we can completely restore the structure of the effective Lagrangian (1). The divergent parts of the quark diagrams reproduce the minimal expression (2) of the effective Lagrangian $\mathcal{L}_{ch}$, the parameters g_1, g_2, μ_0, μ_1 and μ_2 being expressed in the QMST parameter language [2]

$$(g_1^2)_{ij} = \frac{2}{3}(g_2^2)_{ij} = I_2^{-1}(m_i, m_j),$$

$$(\mu_0)_{ij} = (g_1)_{ij}(m_{0i} + m_{0j})/8G_1, \tag{28}$$

$$(\mu_1^2)_{ii} = \frac{(g_1^2)_{ij}}{4G_1}\Big\{-1 + 4G_1\big[I_1(m_i) + I_1(m_j) + (m_i^2 + m_j^2) I_2(m_i, m_j)\big]\Big\},$$

$$(\mu_2^2)_{ij} = \frac{(g_2^2)_{ij}}{4G_2}\Big\{1 + 2G_2\big[m_i^2 I_1(m_i, m_i) + m_j^2 I_1(m_j, m_j) - (m_i^2 + m_j^2) I_2(m_i, m_j)\big]\Big\}.$$

Moreover, the Goldberger – Treiman relation takes place:

$(g_1)_{ij} = (m_i + m_j) F_p Z_{ij}^{1/2}$, where $i(j)=u,d,s$ are the quark structure indices. The anomalous parts of the quark diagrams restore $\mathcal{L}_{WZ}$ and $\mathcal{L}_{NWZ}$. Now we turn to discussion of the P–A diagonalization problem in the QMST.

2. P–A diagonalization problem in a quark model of the superconductivity type

In the QMST the effective Lagrangian including all additional vertices of a strong low-energy interaction caused by P–A diagonalization (4) and by renormalization of P- fields (5), can be given in the following form (prime over the fields is omitted)

$$\delta\mathcal{L}_{eff} = iN\,\mathrm{tr}_{F+L} \langle x | \ln\left(1 - \frac{1}{m - H - i\hat{\partial}} \delta H\right) | x \rangle, \tag{29}$$

where

$$\delta H = \frac{1}{F_p}\left(\frac{Z-1}{Z}\right) \gamma^{\mu}\gamma^5 \bar{D}_{\mu} P, \tag{30}$$

$$H = g_1(\Sigma + i\gamma^5 Z^{1/2} P) + g_2(\hat{V} + \hat{A}\gamma^5).$$

The effective Lagrangian (29) can be presented in the form of infinite series, each term of which has the form of the one-loop quark diagram and defines the corresponding vertex of a strong low-energy meson interaction. These can be both physical and non-physical interaction vertices. The procedure of eliminating non-physical vertices will be exemplified by calculating additional contribution from P–A diagonalization to the $\omega\rho\pi$ vertex [13].

It is well known [5,22] that the $\omega\rho\pi$ vertex possesses all the properties of the $\pi^0\gamma\gamma$ vertex. The effective Lagrangian of the $\pi^0\gamma\gamma$ interaction has the form

$$(\mathcal{L}_{\pi^\circ\gamma\gamma})_{eff} = (\mathcal{L}_{\pi^\circ\gamma\gamma})_{WZ} + (\delta\mathcal{L}_{\pi^\circ\gamma\gamma})_{eff} \,, \tag{31}$$

where

$$(\mathcal{L}_{\pi^\circ\gamma\gamma})_{WZ} = -iN\,tr_{F+L}\langle x | i g_1 Z^{1/2}\gamma^5\pi \frac{1}{m-i\hat{\partial}} \cdot e Q\hat{A}\frac{1}{m-i\hat{\partial}}\, e Q\hat{A}\frac{1}{m-i\hat{\partial}} | x\rangle \,, \tag{32a}$$

$$(\delta\mathcal{L}_{\pi^\circ\gamma\gamma})_{eff} = -iN\,tr_{F+L}\langle x | \frac{1}{F_\pi}\left(\frac{Z-1}{Z}\right)\gamma^\mu\gamma^5\partial_\mu\pi\frac{1}{m-i\hat{\partial}} \cdot e Q\hat{A}\frac{1}{m-i\hat{\partial}}\, e Q\hat{A}\frac{1}{m-i\hat{\partial}} | x\rangle \,. \tag{32b}$$

Here $\hat{A} = \gamma^\mu A_\mu$ is the potential of the electromagnetic field, $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$. In fig.1 the quark diagrams defining the $\pi^\circ\gamma\gamma$ - interaction vertex are depicted. The diagram depicted in fig.1a corresponds to the effective Lagrangian (32a) while the second diagram (fig.1b) is caused by P–A diagonalization and corresponds to the effective Lagrangian (32b). We write down the analytic expression for the quark diagrams (fig.1):

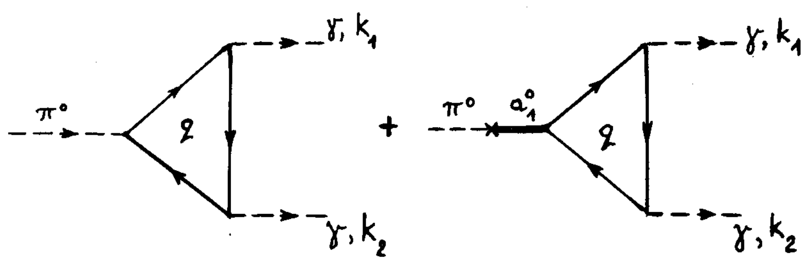

Fig.1

Quark diagrams describing in the QMST the $\pi^\circ\gamma\gamma$- interaction vertex with inclusion of the π - a_1 diagonalization .

$$M^{(a)}_{\mu\nu} = -\frac{N\alpha}{16\pi}\, g_1 Z^{1/2}\int\frac{d^4k}{\pi^2 i}\, tr\left\{ i\gamma^5\frac{1}{m-\hat{k}+\hat{k}_2}\gamma_\nu \cdot \frac{1}{m-\hat{k}}\gamma_\mu\frac{1}{m-\hat{k}-\hat{k}_1}\right\} + \begin{pmatrix}\mu\leftrightarrow\nu\\ k_1\leftrightarrow k_2\end{pmatrix} , \tag{33a}$$

$$M^{(b)}_{\mu\nu} = \frac{N\alpha}{16\pi F_\pi}\left(\frac{Z-1}{Z}\right)\int\frac{d^4k}{\pi^2 i}\, tr\left\{ i(\hat{k}_1+\hat{k}_2)\gamma^5 \cdot \frac{1}{m-\hat{k}+\hat{k}_2}\gamma_\nu\frac{1}{m-\hat{k}}\gamma_\mu\frac{1}{m-\hat{k}-\hat{k}_1}\right\} + \begin{pmatrix}\mu\leftrightarrow\nu\\ k_1\leftrightarrow k_2\end{pmatrix} . \tag{33b}$$

The "naive" calculation of the integrals (33) leads to the result

$$M^{(a)}_{\mu\nu} = -\frac{N\alpha}{3\pi F_\pi}\left(1+\frac{p^2}{12m^2}+\ldots\right)\varepsilon_{\mu\nu\alpha\beta}\,k_1^\alpha k_2^\beta \,. \tag{34a}$$

$$M^{(b)} = -\left(\frac{Z-1}{Z}\right)M^{(a)}_{\mu\nu} , \tag{34b}$$

where $p = k_1+k_2$ is the π°– meson four–momentum. The inclusion of the $p^2/12\,m^2$ form is beyond the scope of the QMST approximation. We have calculated the p^2–term merely to show that in calculating correctly the diagram depicted in fig.1b its contribution is proportional to the p^2-term and consequently is not present in the QMST approximation. By summing the contributions (34) we get

$$M_{\mu\nu} = M^{(a)}_{\mu\nu} + M^{(b)}_{\mu\nu} = -\frac{1}{Z}\frac{N\alpha}{3\pi F_\pi}\left(1+\frac{p^2}{2m^2}+\ldots\right)\varepsilon_{\mu\nu\alpha\beta}\,k_1^\alpha k_2^\beta \,. \tag{35}$$

From expression (35) we find the effective Lagrangian of the $\pi^\circ\gamma\gamma$–interaction

$$(\mathcal{L}_{\pi^\circ\gamma\gamma})_{eff} = \frac{1}{8}\, G_{\pi^\circ\gamma\gamma}\,\varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}\left(1+\frac{\Box}{12m^2}+\ldots\right)\pi^\circ , \tag{36}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic field strength tensor and

$$G_{\pi^\circ\gamma\gamma} = -\frac{1}{Z}\frac{N\alpha}{3\pi F_\pi} \tag{37}$$

is the effective constant of the $\pi^\circ\gamma\gamma$-interaction. The constant $G_{\pi^\circ\gamma\gamma}$ calculated by formula (37) is Z times smaller than the observed one /5,22/. The decrease in the $\pi^\circ\gamma\gamma$-interaction constant is caused by the contribution of the diagram depicted in fig.1b. We analyse the reason for appearance of a nonphysical contribution. First of all we note that the calculation of diagram 1b is not unique. This diagram is linearly divergent; therefore, the calculated result depends on virtual momentum shift in the quark loop /5/. For a unique calculation of diagram 1b it is necessary to use the regularization. The most convenient is the Pauli-Villars regularization /23/

$$\left(M^{(b)}_{\mu\nu}\right)_R = \frac{N\alpha}{16\pi F_\pi}\left(\frac{Z-1}{Z}\right)\int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\left\{i(\hat{k}_1+\hat{k}_2)\gamma^5\frac{1}{m-\hat{k}-\hat{k}_2}\gamma_\nu \cdot \frac{1}{m-\hat{k}}\gamma_\mu\frac{1}{m-\hat{k}-\hat{k}_1}\right\} + \begin{pmatrix}\mu\leftrightarrow\nu\\ k_1\leftrightarrow k_2\end{pmatrix} - (m\leftrightarrow M) \tag{38a}$$

After taking off the regularization as $M\to\infty$, the contribution of diagram 1b is proportional to the p^2-term /13/

$$M^{(b)}_{\mu\nu} = \left(\frac{Z-1}{Z}\right)\frac{N\alpha}{3\pi F_\pi}\left(\frac{p^2}{12m^2}+\dots\right)\varepsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \,. \tag{38b}$$

By summing the contributions (34a) and (38b) we obtain

$$M_{\mu\nu} = -\frac{N\alpha}{3\pi F_\pi}\left(1+\frac{p^2}{12m^2 Z}+\dots\right)\varepsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \,. \tag{39}$$

Formula (39) results in the correct expression for the $\pi^\circ_\gamma\gamma$-interaction constant: /5/

$$G_{\pi^\circ\gamma\gamma} = -\frac{N\alpha}{3\pi F_\pi} \,.$$

Hence, the $\pi^\circ\gamma\gamma$-interaction (or, what is the same the $\omega\rho\pi$-interaction) exemplifies that for a correct description of low-energy interaction vertices caused by P–A diagonalization, it is necessary to use intermediate regularization providing a unique calculation of the quark diagrams describing these vertices. Therefore, instead of the effective Lagrangian (29) we shall use the Pauli-Villars regularized Lagrangian:

$$\left(\delta\mathcal{L}_{eff}\right)_R = iN\,\mathrm{tr}_{F+L}\langle x|\ln\left(1-\frac{1}{m-H-i\hat{\partial}}\delta H\right)|x\rangle - iN\,\mathrm{tr}_{F+L}\langle x|\ln\left(1-\frac{1}{M-H-i\hat{\partial}}\delta H\right)|x\rangle . \tag{40}$$

After taking off regularization $M\to\infty$ the effective Lagrangian (40) does not contain nonphysical interaction vertices. Let us consider some examples.

<u>The $\omega 3\pi$ vertex.</u> To be convinced of that the effective Lagrangian (40) does not contain nonphysical contribution to the $\omega 3\pi$ vertex, it is enough to examine the contribution of the vertex with one $\pi \leftrightarrow a_1$ transition[5)]. The effective Lagrangian of the $\omega 3\pi$-interaction describing the corresponding contribution has the form

$$\begin{aligned}\left(\delta\mathcal{L}^{(1)}_{\omega 3\pi}\right)_{eff,R} = {}& -iN\,\mathrm{tr}_{F+L}\langle x|\frac{1}{m-i\hat{\partial}}g_\rho\hat{\omega}\frac{1}{m-i\hat{\partial}}ig_1(m)\cdot\\ & Z^{1/2}\gamma^5\pi\frac{1}{m-i\hat{\partial}}ig_1(m)Z^{1/2}\gamma^5\pi\frac{1}{m-i\hat{\partial}}\frac{1}{F_\pi}\left(\frac{Z-1}{Z}\right)\hat{\partial}\pi\gamma^5|x\rangle -\\ & -iN\,\mathrm{tr}_{F+L}\langle x|\frac{1}{M-i\hat{\partial}}g_\rho\hat{\omega}\frac{1}{M-i\hat{\partial}}ig_1(M)Z^{1/2}\gamma^5\pi\frac{1}{M-i\hat{\partial}}\cdot\\ & \cdot ig_1(M)Z^{1/2}\gamma^5\pi\frac{1}{M-i\hat{\partial}}\frac{1}{F_\pi}\left(\frac{Z-1}{Z}\right)\hat{\partial}\pi\gamma^5|x\rangle + (\text{permutations}) =\\ & = \int\prod_{\ell=1}^{3}\frac{d^4x_\ell\, d^4k_\ell}{(2\pi)^4}\exp\left[ik_\ell(x-x_\ell)\right]\varepsilon^{abc}\pi^a(x_1)\pi^b(x_2)\pi^c(x_3)\omega^\mu(x)\cdot\\ & \cdot M_\mu(k_1,k_2,k_3)_R \,,\end{aligned} \tag{41}$$

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5) These contributions arise from the $\omega A\pi\pi$-vertex after the shift (4) .

where $\hat\omega = \frac{1}{2}\gamma^\mu\omega_\mu$, $\pi = \frac{1}{2}\tau^a\pi^a = \frac{1}{2}\vec\tau\cdot\vec\pi$ $((\vec\pi = \pi^1,\pi^2,\pi^3)$, τ^a are the Pauli matrices of an isotopic spin (a=1,2,3)), and $M_\mu(k_1,k_2,k_3)_R$ is defined by the integral

$$M_\mu(k_1,k_2,k_3)_R = -\frac{N}{16\pi^2}\frac{g_\rho}{2}\frac{m^2}{F_\pi^3}\left(\frac{Z-1}{Z}\right)\cdot \qquad (42)$$
$$\int\frac{d^4k}{\pi^2 i}\, tr\left\{\gamma_\mu\frac{1}{m-\hat k}\gamma^5\frac{1}{m-\hat k-\hat k_1}\gamma^5\frac{1}{m-\hat k-\hat k_1-\hat k_2}\hat k_3\gamma^5\cdot\right.$$
$$\left.\frac{1}{m-\hat k-\hat k_1-\hat k_2-\hat k_3}\right\} - (m\to M) + (\text{permutations}).$$

The integrals with virtual masses m should be calculated in the QMST approximation, i.e., after extracting the Lorentz structure

$$F\,\varepsilon_{\mu\nu\alpha\beta}\,k_1^\nu k_2^\alpha k_3^\beta$$

it is necessary to extrapolate F to zero values of 4-momenta of interacting mesons ($p, k_1, k_2, k_3 \to 0$) [6]. In calculating integrals with regulators one should take into account that M is considerably larger than mass of any meson (in the final analysis $M\to\infty$). We give the result of calculations

$$M_\mu(k_1,k_2,k_3) = -3i\left(\frac{Z-1}{Z}\right)\frac{Ng_\rho}{4\pi^2F_\pi^3}\,\varepsilon_{\mu\nu\alpha\beta}\,k_1^\nu k_2^\alpha k_3^\beta\cdot \qquad (43)$$
$$\cdot\int\frac{d^4k}{\pi^2 i}\left[\frac{m^4}{(m^2-k^2)^4} - \frac{M^4}{(M^2-k^2)^4}\right] = 0 \;.$$

Hence, the contribution of the vertex with one $\pi\leftrightarrow a_1$ transition tends to zero. Disappearance of the vertices with two and three $\pi\leftrightarrow a_1$ transitions in the effective $\omega 3\pi$ interactions proceeds in an analogous way. As a result, the effective vertex of the $\omega 3\pi$ -interaction is defined only by the chiral anomaly $(\mathcal{L}_{\omega 3\pi})_{eff} = (\mathcal{L}_{\omega 3\pi})_{WZ}$.

---

6) F is a function of all invariant variables $p^2 = (k_1+k_2+k_3)^2$, k_1^2, k_2^2, k_3^2, $s_{12} = (k_1+k_2)^2$, $s_{13} = (k_1+k_3)^2$, $s_{23} = (k_2+k_3)^2$.

<u>The $\varepsilon\pi\pi$ -vertex</u>. Let us consider the vertex of the $\varepsilon\pi\pi$-interaction where ε is the scalar isoscalar meson $\varepsilon(700)$, being the chiral partner of π -meson [2] . The vertex of the $\varepsilon\pi\pi$-interaction plays an important role in describing strong low-energy meson interactions [2,14] . In distinction to the $\omega 3\pi$ vertex, the $\varepsilon\pi\pi$ vertex is not connected with the chiral anomaly and it is present only in $\mathcal{L}_{ch}$. In describing $\mathcal{L}_{ch}$ by "minimal" expression (2) up to the $\pi - a_1$ diagonalization, the $\varepsilon\pi\pi$-interaction vertex does not contain interacting meson field derivatives and it is a constant. After the $\pi - a_1$ diagonalization (4), in the $\varepsilon\pi\pi$-interaction vertex there appear higher derivatives [1]

$$(\mathcal{L}_{\varepsilon\pi\pi})_{eff} = mg_1 Z\left[\varepsilon\vec\pi^{\,2} + \left(\frac{Z-1}{Z}\right)\frac{(\Box\varepsilon)\vec\pi^{\,2} - \varepsilon\partial^\mu\vec\pi\cdot\partial_\mu\vec\pi}{2m^2} - \right. \qquad (44)$$
$$\left. - \left(\frac{Z-1}{Z}\right)^2\frac{\varepsilon\partial^\mu\vec\pi\cdot\partial_\mu\vec\pi}{4m^2}\right].$$

The effective $\varepsilon\pi\pi$- interaction (44) is described in the QMST parameter language. This is convenient for further study of the $\varepsilon\pi\pi$ - interaction in the QMST, although it is completely clear that expression (44) can be rewritten in terms of the standard ECL construction.

Now we treat the $\varepsilon\pi\pi$ -interaction vertex in the QMST. The first term in (44) completely coincides with the calculated result of the divergent part of the triangle quark diagram $\varepsilon\pi\pi$ [2] . The last two terms arise from the $\pi - a_1$ diagonalization. Are they present in the effective Lagrangian (40)? For the sake of simplicity we consider the terms proportional to $\left(\frac{Z-1}{Z}\right)^2$

$$\left(\delta\mathcal{L}^{(2)}_{\varepsilon\pi\pi}\right) = -iN\,tr_{F+L}\langle x|\,g_1(m)\,\varepsilon\,\frac{1}{m-i\hat\partial}\cdot \qquad (45)$$
$$\cdot\frac{1}{F_\pi}\left(\frac{Z-1}{Z}\right)\hat\partial\pi'\gamma^5\frac{1}{m-i\hat\partial}\,\frac{1}{F_\pi}\left(\frac{Z-1}{Z}\right)\hat\partial\pi\gamma^5\frac{1}{m-i\hat\partial}|x\rangle +$$
$$+ (m\to M) = \int\prod_{\ell=1}^{2}\frac{d^4x_\ell\, d^4k_\ell}{(2\pi)^4}\exp[ik_\ell(x-x_\ell)]\cdot M(k_1,k_2)_R\,,$$

where

$$M(k_1,k_2)_R = -\frac{N}{16\pi^2}\,\frac{g_1(m)}{2}\,\frac{1}{4F_\pi^2}\left(\frac{Z-1}{Z}\right)^2 \cdot \tag{46}$$

$$\int \frac{d^4k}{\pi^2 i}\, tr\left\{ \frac{1}{m-\hat{k}}\,\hat{k}_1\gamma^5\,\frac{1}{m-\hat{k}-\hat{k}_2}\,\hat{k}_2\gamma^5\,\frac{1}{m-\hat{k}-\hat{k}_1-\hat{k}_2} + (k_1 \leftrightarrow k_2)\right\}$$

$$-\ (m \rightarrow M).$$

In the QMST approximation from the integrals with virtual masses m only the divergent parts survive. In calculating integrals with regulators one should take into account that finally $M \to \infty$, i.e., M is considerably larger than mass of each interacting meson. We write the calculated result

$$M(k_1,k_2) = \frac{1}{F_\pi^2}\left(\frac{Z-1}{Z}\right)^2 \left[2m\, g_1(m)\, I_2(m,m) - \right. \tag{47}$$

$$\left. - 2M g_1(M)\, I_2(M,M)\right](k_1 k_2).$$

By removing the regularization $M \to \infty$, the term $2M g_1(M) I_2(M,M)$ tends to zero; therefore, $(\delta\mathcal{L}^{(2)}_{\varepsilon\pi\pi})_{eff}$ becomes

$$(\delta\mathcal{L}^{(2)}_{\varepsilon\pi\pi})_{eff} = m g_1(m) Z\left[-\left(\frac{Z-1}{Z}\right)^2 \frac{\varepsilon\,\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}}{4m^2}\right]. \tag{48}$$

The effective interaction (48) coincides with the corresponding term in (44). Analogously, it can be shown that the effective Lagrangian (40) contains the $\varepsilon\pi\pi$ – interaction terms proportional to $(Z-1)/Z$.

<u>The π^4 - vertex.</u> The effective π^4- interaction vertex obtained in the standard ECL construction from "minimal" construction (2) by taking account of the π-a_1 diagonalization has the form[1]

$$(\mathcal{L}_{\pi^4})_{eff} = -\frac{g_1^2(m) Z^2}{8}\left[(\vec{\pi})^2 - \left(\frac{Z-1}{Z}\right)^2 \frac{\partial_\mu\vec{\pi}^2\,\partial^\mu\vec{\pi}^2}{4m^2} + \right.$$

$$\left. + \left(\frac{Z-1}{Z}\right)^4 \frac{(\partial_\mu\vec{\pi}\times\partial_\nu\vec{\pi})\cdot(\partial^\mu\vec{\pi}\times\partial^\nu\vec{\pi})}{24m^4}\right]. \tag{49}$$

It is possible to show that the terms with higher derivatives in the effective π^4-interaction vertex (49) are completely reproduced in the QMST and kept in the effective Lagrangian (40).

Further, one can show that momentum dependences of the $\varepsilon\pi\pi$ and π^4 interaction vertices (44) and (49) are consistent. The amplitude of the low-energy $\pi\pi$ scattering evaluated in the tree approximation with the help of the effective interactions (44) and (49), satisfies all the conditions defined by the chiral symmetry and SCSB (e.g., Adler's condition, etc.).

The study of concrete low-energy interaction vertices can be, of course, continued. However, the considered examples demonstrate that the effective Lagrangian (40) correctly describes the low-energy meson interaction vertices caused by the P-A diagonalization (4).

Conclusion

Now let us discuss the obtained results. First of all we should like to note that the solution of the problem of excluding nonphysical interactions caused by the P-A diagonalization is obtained at the prescription level. It was found that for exclusion of nonphysical vertices caused by the P-A diagonalization, it is sufficient to use the Pauli-Villars regularized Lagrangian (4) instead of the effective Lagrangian (29). The latter contains all the vertices (both physical and nonphysical) of the low-energy meson interaction. These vertices appear after the shift (4) and renormalization (5). The regularization is necessitated by ambiguity of the calculation of individual quark diagrams. The above mentioned diagrams describe the new interaction vertices. The Pauli-Villars regularization procedure corresponds to subtraction of a quantity from a quark diagram. This quantity is defined by the same diagram in which all the masses of virtual quarks are replaced by the masses of fermion regulators M. The removal of regularization corresponds to the limit $M \to \infty$. The application of regularization to the whole effective Lagrangian, conditioned by the P-A diagonalization, leads as a whole to the subtraction of the corresponding regulator contribution from each vertex appearing as a result of the shift of the axial field (4). This seams to be enough for a correct description of the vertices of the low-energy interaction which appear in the P-A diagonalization.

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Волков М.К. и др. E2-88-558
Эффективные киральные лагранжианы, кварковая модель сверхпроводящего типа и проблема P-A диагонализации

В кварковой модели сверхпроводящего типа проведен анализ вершин, появляющихся при учете P-A переходов и с последующей диагонализации кирального лагранжиана. Условно такие вершины могут быть разбиты на вершины физического и нефизического типа. Показано, что при использовании регуляризации Паули – Вилларса вклады последних можно исключить из описания физических процессов. Это полностью соответствует экспериментальным данным.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Volkov M.K. et al. E2-88-558
Effective Chiral Lagrangians, the Quark Model of Superconductivity Type and the Problem of P-A Diagonalization

The vertices appearing after the inclusion of P-A transitions and a subsequent diagonalization of the chiral Lagrangian are analysed in the quark model of superconductivity type. These vertices can conditionally be divided into physical and nonphysical ones. It is shown that if the Pauli – Villars regularization is used, the contribution of the latter can be eliminated from the description of physical processes. This is in full agreement with experimantal data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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