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NONLOCALITY AND STOCHASTIC QUANTIZATION OF FIELD THEORY

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1. Introduction

In recent years, interest has significantly increased in the study of stochastic processes and nonlocal (or extended) objects - fields; this is due to the fact that it has been possible, first, to establish an intimate connection between the theory of stochastic processes and quantum physics /1-5/, where earlier references can be found, and second, to construct unified theory of all types of elementary particle interactions including gravitational force /6-10/. The former is known under the general name of stochastic quantization of systems. There are different approaches to description of stochastic processes, which formally coincide with quantum phenomena. Among these the attraction of the stochastic quantization method proposed by Parisi and Wu /11/ is that it has succeeded in reducing quantum field theory to a gaussian stochastic process called the Langevin equation, which usually runs in an auxiliary "fifth-time".

Other directions are being developed in the investigation of nonlocal-extended objects. Some of them have been originally arisen from intrinsic problems of local quantum field theory like the ultraviolet divergences, the problems of electron self-energy, etc. To solve these problems it is usually assumed that idealized concept of the locality may be violated at small distances and some static characteristics of elementary particles must be described by nonlocal values with distributions over space, for example, charge and mass of the particle may be presented in the form

$$e=\int d\vec{r}\,\rho_e(\vec{r}) \qquad , \qquad m=\int d\vec{r}\, m(\vec{r}) \, .$$

On the other hand, mathematically it means that Dirac δ-function distribution should be changed by nonlocal distribution of the types (for detail, see Efimov /12/)

$$\delta^{(4)}(x) \Rightarrow K(x)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}(\Box l^2)^n\,\delta^{(4)}(x) \tag{1.1}$$

or for the wave function of the particle

$$\phi(x) \Rightarrow \varphi(x)=\int d^4y\, K(x-y)\,\phi(y) \tag{1.2}$$

($\phi(x)$ is local field), i.e., elementary particles may be understood as a spread-out (or nonlocal) objects with some dimension ℓ of length (see Fig.1).

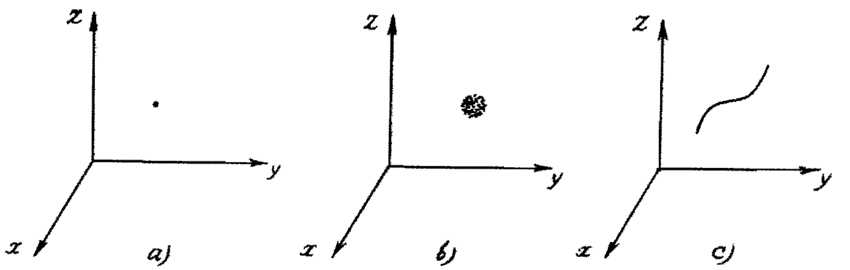


Figure 1.

Illustration of local and nonlocal objects depending on the dimension of space: a) local object; b) spread-out (extended) object (ball, bag, etc.) in the three dimensional case; c) extended object (string) in the one-dimensional case.

It should be noted that from pure geometrical point of view, relativistic invariant description of extended objects is possible only in the one dimensional case, i.e., relativistic dynamics for string may be successfully constructed. Nevertheless, from field point of view, relativistic invariant construction of interaction picture between nonlocal objects of types (1.2) is also achieved due to relativistic invariant properties of nonlocal distributions (1.1). In the last case, basic peculiarity of introducing nonlocality (1.1) is that it leads to change of the particle propagator, for example, for scalar particle:

$$\Delta(x-y) = \langle 0| T\{\phi(x)\phi(y)\}|0\rangle \Longrightarrow \tag{1.3}$$

$$\Longrightarrow D(x-y) = \langle 0| T\{\varphi(x)\varphi(y)\}|0\rangle = \frac{1}{(2\pi)^4 i}\int d^4p\, e^{-ip(x-y)} \frac{V(p^2\ell^2)}{m^2-p^2-i\varepsilon}$$

where $V(p^2\ell^2)$ is the Fourier transform of nonlocal distribution $K(x)$.

In this paper, we present method of introducing nonlocality (1.1)-(1.3) into stochastic quantization scheme within the framework of Langevin and Schwinger-Dyson formalisms (for detail, see Bern et al./13,14/). These two equivalent formulations describe quantum field theory in d- dimensions by means of markovian stochastic processes in (d+1) dimensions via a regularized Parisi-Wu-Langevin equation and by d-dimensional prescription via regularized Schwinger-

Dyson equations, respectively. We assume that the noise term in these equations plays double role in the theory; it controls the quantum behaviour of the theory and at the same time it carries nonlocality in stochastic equations. Further, we show that scheme obtained by such a way is equivalent to the nonlocal theory with regularized propagator of the type of (1.3).

An outline of the present paper is as follows. Sec.2 introduces the nonlocality into the (d+1)-dimensional Langevin formulation for the scalar theory. In Sec.3 we discuss the equivalent d-dimensional regularized Schwinger-Dyson equations, and their more-or-less conventional weak coupling expansion. Sec.4 is devoted to introduction of nonlocality into gauge theory and to reformulation of gauge-covariant Langevin systems in (d+1)-dimensions, for which we derive the regularized Langevin-Feynman rules. These rules are applied in Sec.6 to a computation of the one-loop gluon mass in QCD_4. As sketched in ref. due to Bern et al./14/, the mass is zero, providing an explicit check of gauge-invariance of this order for entire analytic regulators. Sec.7 deals with the simplest gauge theory scalar electrodynamics. This last section has preparative character in order to generalize our prescription to the nonabelian theory and the serious scholar may be advised to begin with this case.

2. Nonlocal Gaussian Noise and Regularized Langevin Systems for the Scalar Theory

2.1. Nonlocal Noise

We consider the markovian Parisi-Wu Langevin system for a d-dimensional theory of a scalar local field $\phi(x)$ with Euclidean action S

$$\dot{\phi}(x,t) = -\frac{\delta S}{\delta \phi}(x,t) + \eta(x,t), \tag{2.1}$$

where t is additional fictitious "fifth-time" variable, x are d-dimensional Euclidean coordinates and η (x,t) is the usual local Gaussian noise satisfying the following condition

$$\langle \eta(x,t)\,\eta(y\,\tau)\rangle_\eta = 2\cdot\delta(t-\tau)\,\delta^d(x-y). \tag{2.2}$$

Now question arises how to introduce nonlocality into this stochastic equation in order to obtain equivalent stochastic formulation for the nonlocal field φ(x) (1.2) with propagator (1.3) in the Euclidean metric. We assume that the noise term in (2.1) carries nonlocality only and by analogy with (1.2), in this case, it takes the form

$$\eta(x,t) \Rightarrow \Lambda(x,t)=\int(dy)\,K(x-y)\,\eta(y,t), \qquad (2.3)$$

where $(dy)=d^dy$, and $K(x)$ is nonlocal distribution investigated in detail by Efimov /12,15/. The nonlocal distribution $K(x-y)=K_{xy}(\Box)$ that multiplies the noise is a function of the Laplacian

$$\Box_{xy}=\int(dz)(\partial_\mu)_{xz}(\partial_\mu)_{zy}, \qquad (2.4)$$

$$(\partial_\mu)_{xy} \equiv \partial^x_\mu\, \delta^d(x-y)$$

which guarantees that $K_{xy}(\Box)=K_{yx}(\Box)$. We will choose here a wide class of distributions

$$K_{xy}(\Box)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}(\Box\ell^2)^n\,\delta^d(x-y) \qquad (2.5)$$

for which the ordinary Parisi-Wu equation is regained in the limit $\ell\to 0$, i.e., $K_{xy}(\Box)\xrightarrow[\ell\to 0]{}\delta^d(x-y)$.

2.2. Nonlocal Distributions

We see that the function (2.5) is the generalized form of the well-known local Dirac δ-function. As usually, its space-time properties are investigated in the Minkowski space-time with metric $g_{\mu\nu}=(g_{00}=-g_{11}=-g_{22}=-g_{33}=1\ ;\ g_{\mu\nu}=0\ ,\mu\neq\nu)$ and depends essentially on the sequence of coefficients C_n (generally speaking, they are compex numbers). We say that the generalized function (2.5) is given in some test function space if for any $f\in\mathcal{U}$ the functional

$$(K,f)=\int d^4x\,K(x)f(x)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}\ell^{2n}\Box^n f(x)\Big|_{x=0}<\infty \qquad (2.6)$$

is well-defined, i.e., the obtained series converges absolutely. Passing to the momentum space in (2.6), we obtain

$$(K,f)=\int d^4p\,\tilde{K}(p^2\ell^2)\tilde{f}(p)<\infty, \qquad (2.7)$$

where

$$\tilde{K}(p^2\ell^2)=\sum_{n=0}^{\infty}\frac{C_n}{(2n)!}\ell^{2n}(p^2)^n$$

and $\tilde{f}(p)$ is the Fourier transform of $f(x)$. In other words, the generalized function (2.5) is given on $\mathcal{U}$ if series (2.8) defines

the function $\tilde{K}(p^2\ell^2)$ for all p^2 and the integral (2.7) converges for any $f(x)\in\mathcal{U}$. Both conditions (2.6) and (2.7) are equivalent.

As shown by Efimov /15/, basic physical principles such as unitarity, causality dictate that as a Fourier transform of (2.5) entire analytic function should be chosen. Further, we are interested only in the class of distributions $K(x)$ for which $\tilde{K}(z)$ (2.8) are entire functions of the variable z with a finite order of growth $\infty>\rho\geqslant 1/2$ and which decrease rapidly enough when $z=p^2\to-\infty$ (in the Euclidean direction).

In the Euclidean domain of the variable p^2 for the Fourier transform (2.8), the Mellin representation

$$\tilde{K}(-p_E^2\ell^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{w(\xi)}{\sin\pi\xi}\,\ell^{2\xi}(m^2+p_E^2)^{\xi} \qquad (2.9a)$$

or

$$V(-p_E^2\ell^2)=[K(-p_E^2\ell^2)]^2=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{v(\xi)}{\sin\pi\xi}\,\ell^{2\xi}(m^2+p_E^2)^{\xi} \qquad (2.9b)$$

$$(1<\beta<\eta)$$

is valid. The form of functions $w(\xi)$ and $v(\xi)$ depends on the form of the function $K(-p_E^2\ell^2)$. For example, if

$$V_1=\frac{(m^2\ell^2)^2}{(\sin m\ell/m\ell-\cos m\ell)^2}(\sin b/b-\cos b)^2 b^{-4},\qquad K_2=(\sin b/b)^2, \qquad (2.9c)$$

$$V_2=(\sin b/b)^4,\qquad V_3=\exp(-b^2)$$

$$V_4=2^s\,\Gamma(1+s)\,J_s(b)/b^s,$$

where $J_s(u)$ is the Bessel function for some given value $s>0$ and $b=[(m^2+p_E^2)\ell^2]^{1/2}$, then

$$v_1(x)=9\cdot 2^{4+2x}(2x^2+7x+5)/\Gamma(7+2x), \qquad (2.9d)$$

$$w_2(x)=2^{1+2x}/\Gamma(3+2x),$$

$$v_2(x)=2^{3+2x}(2^{2x+1}-1)/\Gamma(5+2x),$$

$$v_3(\xi)=1/\Gamma(1+\xi),$$

$$v_4(x)=\frac{\Gamma(1+s)}{2^{2x}\,\Gamma(1+x)\,\Gamma(1+s+x)}.$$

The physical meaning of form factors $V(-p_E^2\ell^2)$ consists of changing the form of potentials between interacting fields (for example, the Coulomb and Yukawa laws) at small distances and in making the theory finite in each order of the perturbation series of the theory of coupling constant (Efimov/15/ and Namsrai/4/). The question about a possible unique choice of the form-factors was discussed by Efimov/15/ (see also Papp,/16/). Efimov/15/ has shown that the objects constructed by distributions $K(x)$ (2.5) are spread out (nonlocalized) over space. Thus, the relativistic invariant distributions $K(x)$ give a correct description of extended objects. In this case, roughly speaking, the parameter ℓ may be identified with the size of an extended object (a particle).

Our next goal is in introducing such type of the nonlocality into stochastic equations. We now turn to this problem.

2.3. Regularized Langevin Systems for the Scalar Theory

With the assumption (2.3), equation (2.1) acquires now the following form

$$\dot{\phi}(x,t) = -\frac{\delta S}{\delta\phi}(x,t) + \int(dy)\,K(x-y)\,\eta(y,t). \qquad (2.10)$$

Such type of expression (2.10) gives rise to realize our programme mentioned in a previous work (Namsrai/4/). We notice that our stochastic prescription using entire analytic regulators including exponential ones may be technically superior and useful for nonperturbative analysis, which appeared already in a paper due to Doering /17/ using the scalar prototype regulator described by Bern et al./13/. As in the usual local stochastic formulation, our prescription for the nonlocal Euclidean Green functions of the theory

$$\langle F[\phi(\cdot)]\rangle = \lim_{t\to\infty}\langle F[\phi(\cdot,t)]\rangle_\eta \qquad (2.11)$$

completes the computational scheme.

According to Bern et al./13/ the method expounded in this section is easy to be generalized for a local symmetry, which will be discussed in Section 5. In this case, the only change in the scheme is the replacement of Laplacian by covariant Laplacian in Eqs. (2.1)-(2.5) and (2.10).

We will further follow Bern et al./13,14/ everywhere and obtain explicit weak coupling expressions for the equation (2.10). First consider simpler case

$$S = \int(dx)\left[\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) + \tfrac{1}{2}m^2\phi^2 + \lambda(\phi)\right]. \qquad (2.12)$$

To solve the equation (2.10) with (2.12) and calculate correlation functions in the free case, it is convenient to introduce the free Green function $G(x,t)$ which satisfies

$$\frac{\partial}{\partial t}G(x,t) - (\Box - m^2)\,G(x,t) = \delta^d(x)\delta(t)$$

with the initial condition

$$G(x,t) = 0 \quad , \quad t<0 .$$

This equation is easily solved to give the explicit expressions for G:

$$G(x,t) = \theta(t)\int(dp)\,\exp\left[-ipx - (p^2+m^2)t\right], \qquad (2.13)$$

where $(dp) = d^dp/(2\pi)^d$. Thus, for (2.12) the integral formulation of the system (2.10) is

$$\phi(x,t) = \int(dy)\int_{-\infty}^{t}dt'\,G(x-y,t-t')\left[\int(dx_1)\,K_{yx_1}(\Box)\,\eta(x_1t') - \lambda'(\phi(y,t'))\right]. \qquad (2.14)$$

Here λ' is the first derivative of the potential and we have employed the technical device of choosing $t_0 = -\infty$, so that the system has equilibrated at any finite fifth-time. The integral equation may be iterated to any desired order (Parisi and Wu/11/) as

$$\phi(x,t) = \int_1 G_{x1}(K\eta)_1 - \int_1 G_{x1}\lambda'\Big(\int_2 G_{12}(K\eta)_2 - \int_2 G_{12}\lambda'\Big(\int_3 G_{23}(K\eta)_3 - \ldots\Big)\Big) \qquad (2.15)$$

where it is used compact notation

$$G_{x1} \equiv G(x-x_1\,,\,t-t_1)\,, \qquad (2.16)$$

$$(K\eta)_1 \equiv \int(dy)\,K_{x_1y}(\Box)\,\eta(y,t_1)\quad ,$$

$$\int_1 \equiv \int(dx_1)\int dt_1 .$$

According to Bern et al./13/ for concrete calculation purpose it is convenient to represent this iteration by Langevin "tree diagrams", as shown in Fig.2 for the explicit choice $\lambda = g\phi^3/3!$. In these diagrams, each line corresponds to a Langevin Green function (2.13), and its arrow represents its retarded property, while the cross at the end of a line represents a nonlocal form-factor (or regulator) times a noise factor

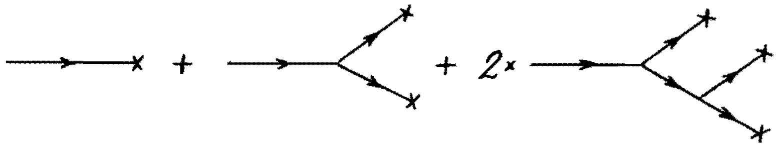
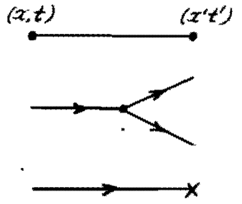


Fig.2.

Langevin tree diagrams through $O(g^2)$ in the nonlocal stochastic scheme.

In the nonlocal stochastic scheme, the tree diagrams may be succinctly summarized in a simple set of Langevin tree rules, as shown for this case in Fig.3.



$$= G(x-x', t-t')$$

$$= -\tfrac{1}{2} g$$

$$= K\eta$$

Fig.3.

Langevin tree rules for the nonlocal stochastic quantization theory.

Using Eqs. (2.2),(2.13) and (2.15), we easily obtain correlation functions for the free case $g=0$:

$$D(x-y, t_1-t_2) = \langle \phi(x,t_1)\phi(y,t_2)\rangle_\eta =$$

$$= 2\iint (dx_1)(dy_1)\int_{-\infty}^{\min(t_1,t_2)} d\tau\, G(x-x_1, t_1-\tau)\, G(y-y_1, t_2-\tau)\cdot\int (dz_1)\, K_{x,z_1}(\Box) K_{y,z_1}(\Box).$$

Taking into account the following obvious equalities

$$\int (dz_1)\, K_{xz_1}(\Box) K_{z_1 y}(\Box) = \int (dq)\, V(-q^2\ell^2) \exp[-iq(x-y)]$$

and

$$\int_{-\infty}^{\min(t_1,t_2)} d\tau \exp\{-(t_1-\tau)(p^2+m^2)-(t_2-\tau)(p^2+m^2)\} = \frac{\exp[-(t_1-t_2)(p^2+m^2)]}{2(m^2+p^2)}$$

we get

$$D_E(x-y) = \lim_{t_1\to t_2} D(x-y, t_1-t_2) = \int (dp)\, e^{-ip(x-y)} \frac{V(-p^2\ell^2)}{m^2+p^2} \qquad (2.17)$$

which is just nonlocal Euclidean Green function (1.3) for the scalar theory. Here we have used definitions

$$K_{xy}(\Box) = \int (dp) e^{-ip(x-y)} K(-p^2\ell^2) \quad , \qquad V(-p^2\ell^2) = [K(-p^2\ell^2)]^2.$$

This result may be also obtained by using diagrammatic representation for the Langevin system. Thus, as a specific example, the zeroth order momentum space nonlocal two-point function, shown in Fig.4, contains two local Langevin Green functions in the combination

$$D^\ell_{12}(p) = 2\cdot V(-p^2\ell^2)\int_{-\infty}^{t_1} dt_3 \int_{-\infty}^{t_2} dt_4\, G_{13}(p) G_{24}(p)\, \delta(t_3-t_4) = \qquad (2.18a)$$

$$= V(-p^2\ell^2)\Delta_p^{-1} \exp[-|t_1-t_2|\Delta_p] = D(p) \exp[-|t_1-t_2|\Delta_p] .$$

where we have introduced

$$D(p) = V(-p^2\ell^2)\Delta_p^{-1} \quad , \quad \Delta_p = p^2+m^2 \quad , \qquad (2.18b)$$

$$G_{ij}(p) = \theta(t_i-t_j) \exp[-|t_i-t_j|\Delta_p] .$$

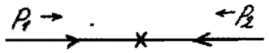


Fig.4.

Langevin line with a contraction in the nonlocal case.

The result for the nonlocal free propagator is therefore

$$\langle \phi(x_1)\phi(x_2)\rangle^{(0)} = \int (dp)\, e^{-ip(x_1-x_2)} D^\ell_{00}(p) = \int (dp)\, e^{-ip(x_1-x_2)} \frac{V(-p^2\ell^2)}{m^2+p^2}$$

or

$$\langle \phi_{p_1} \phi_{p_2} \rangle^{(0)} = V(-p_1^2 \ell^2)\Delta_{p_1}^{-1} \bar{\delta}^d (p_1 + p_2) = D(p_1)\, \bar{\delta}^d (p_1 + p_2)\,, \qquad (2.18c)$$

where

$$\bar{\delta}^d (p_1 + p_2) \equiv (2\pi)^d \delta^d (p_1 + p_2) \;\; ; \;\; \phi(x) = \int (dp)\, \phi_p\, e^{-ipx}. \qquad (2.18d)$$

In general, each line with a cross (contraction) in a Langevin diagram is represented by a factor $D_{12}^{\ell}(p)$ which includes a factor $V(-p^2\ell^2)$. In this connection, it should be noted that product of generalized functions $K_{zy}(\Box)$ may be understand as contraction operation only. For example,

$$K_{zy}^2(\Box) = \int (dz)\, K_{zx}(\Box)\, K_{xy}(\Box) \qquad (2.19)$$

or

$$\Box_{xy}^2 = \int (dz)\, \Box_{xz}\, \Box_{zy}\,,$$

etc.

For further assimilation of calculation experience, we consider ϕ^3-theory and calculate the nonlocal first-order three-point function (Fig.5)

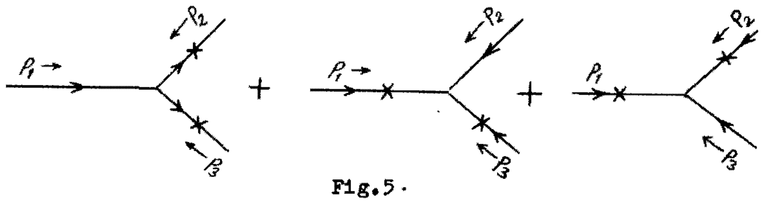


Fig.5.

Langevin three-point diagrams in nonlocal stochastic case.

Let

$$\lambda(\phi) = g\phi^3/3!$$

In this concrete case, iteraction solution (2.15) takes the form in the momentum representation

$$\phi(x,t) = \int (dp)\, e^{-ipx}\, \tilde{\phi}_p(t)\,,$$

where

$$\tilde{\phi}_p(t) = \int (dx') \int_{-\infty}^{t} dt'\, e^{ipx'} G_{tt'}(p) \Big\{ \int (dy)\, K_{x'y}(\Box)\, \eta(y,t') - \frac{\lambda}{2} \int (dx_1) \times$$
$$\times \int_{-\infty}^{t'} dt_1\, G(x'-x_1, t'-t_1) \int (dy_1)\, K_{x_1 y_1}(\Box)\, \eta(y_1, t_1) \int (dx_2) \times \qquad (2.20)$$
$$\times \int_{-\infty}^{t'} dt_2\, G(x'-x_2, t'-t_2) \int (dy_2)\, K_{x_2 y_2}(\Box)\, \eta(y_2, t_2) \Big\}.$$

To calculate $\langle \phi_{p_1} \phi_{p_2} \phi_{p_3} \rangle^{(1)}_{connec}$ for connected diagrams we use the following approximation

$$(a_x - \tfrac{g}{2} b_x)(a_y - \tfrac{g}{2} b_y)(a_z - \tfrac{g}{2} b_z) = a_x a_y a_z - \tfrac{g}{2}(b_x a_y a_z + b_y a_x a_z + b_z a_x a_y)$$

and the Gaussian noise property

$$\langle \eta(x_1 t_1)\eta(x_2 t_2)\eta(x_3 t_3)\eta(x_4 t_4) \rangle = 4\,[\delta^d(x_1 - x_2)\,\delta(t_1 - t_2)\,\delta^d(x_3 - x_4)\,\delta(t_3 - t_4) + \qquad (2.21)$$
$$+ \delta^d(x_1 - x_3)\,\delta(t_1 - t_3)\,\delta^d(x_2 - x_4)\,\delta(t_2 - t_4) + \delta^d(x_1 - x_4)\,\delta(t_1 - t_4)\,\delta^d(x_2 - x_3)\,\delta(t_2 - t_3)\,].$$

After integration over t_i and x_i variables, we have

$$\langle \phi_{p_1} \phi_{p_2} \phi_{p_3} \rangle^{(1)}_{connec} = -g \int dt_1 [\, G_{01}(p_1) D_{01}^{\ell}(p_2) D_{01}^{\ell}(p_3) + D_{01}^{\ell}(p_1) G_{01}(p_2) D_{01}^{\ell}(p_3) +$$
$$+ D_{01}^{\ell}(p_1) D_{01}^{\ell}(p_2) G_{01}(p_3)\,]\; \bar{\delta}^d (p_1 + p_2 + p_3)\,. \qquad (2.22)$$

Taking into account explicit forms (2.18a) and (2.18b) for $D_{ij}^{\ell}(p)$ and $G_{ij}(p)$ functions and carrying out some algebraic operations, we get

$$\langle \phi_{p_1} \phi_{p_2} \phi_{p_3} \rangle^{(1)}_{connec} = -g\, \frac{\sum_{i=1}^{3} (V\Delta^{-1})_{p_i}}{\sum \Delta_{p_i}} \prod_{i=1}^{3} (V\,\Delta^{-1})_{p_i}\; \bar{\delta}^d (p_1 + p_2 + p_3)\,. \qquad (2.23)$$

We note that in the presence of the form factor, the loop in Fig.6

$$\langle \phi_p \rangle^{(1)} = -\tfrac{1}{2} g \int dt_1\, G_{01}(p) \int (dk)\, D_{11}^{\ell}(k)\, \bar{\delta}^d (p) = \qquad (2.24)$$
$$= -\tfrac{1}{2} g\, \Delta_p^{-1}\, \bar{\delta}^d (p) \int (dk)\, D(k)\,, \qquad D(k) = V(-k^2\ell^2)\,\Delta_k^{-1}$$

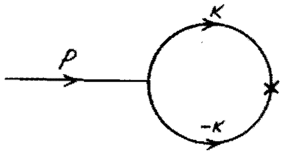


Fig.6.

Langevin tadpole diagram in the nonlocal stochastic scheme.

is not the proper vertex of (2.23) times a nonlocal propagator. This indicates some peculiarity of the effective d-dimensional action of the theory, which will be discussed in Secs. 3,4.

3. Nonlocal Schwinger-Dyson Equations

3.1. Derivation of the SD Equations

The regularized Schwinger-Dyson (SD) equations with meromorphic regulators were used in stochastic quantization scheme due to Bern et al.[13,14]. We generalize here their results for a wide class of nonlocal distributions, Fourier transforms of which are entire analytic functions of the type (2.9a). It is shown that a simple d-dimensional SD formulation depends crucially on the Markovian property of the scheme at the stochastic level. It turns out that this property does not change in our case.

We begin with the Langevin system (2.10) and (2.12). Let $F[\phi]$ be any equal fifth-time functional of the field ϕ, then its η- average evolves in fifth-time according to

$$\frac{d\langle F[\phi]\rangle_\eta}{dt} = \langle \int (dx) \frac{\partial\phi(x,t)}{\partial t} \frac{\delta F[\phi]}{\delta\phi} \rangle_\eta . \tag{3.1}$$

To transform this equation, we use the local white noise identity

$$[\eta(y,t) + 2\frac{\delta}{\delta\eta(y,t)}] \exp[-\tfrac{1}{4}\int d\tau \int (dx)\, \eta^2(x,t)] = 0 \tag{3.2}$$

which expresses the Markovian property of our scheme and is easily verified by taking differentiation of $\exp[-\frac{1}{4}\iint d\tau (dx)\, \eta^2(x,t)]$ with respect to $\eta(y,t)$. Thus, multiplying (3.2) by any functional $F[\phi]$ and integrating it over η, we get

$$\int_{-\infty}^{\infty} d\eta [\eta(y,t) + 2\frac{\delta}{\delta\eta(y,t)}] \exp[-\tfrac{1}{4}\iint d\tau (dx)\, \eta^2(x,t)] F[\phi] = 0 .$$

Integration by parts in η gives

$$\int_{-\infty}^{\infty} d\eta \exp[-\tfrac{1}{4}\iint d\tau (dx)\, \eta^2(x,t)] [\eta(y,t) - 2\frac{\delta}{\delta\eta(y,t)}] F[\phi] = 0 ,$$

from which it follows the formal definition

$$\eta(y,t) = 2\frac{\delta}{\delta\eta(y,t)} = 2\int (dz) \frac{\partial\phi(z,t)}{\partial\eta(y,t)} \frac{\delta}{\delta\phi(z,t)} \tag{3.3}$$

for any functional $F[\phi]$. Now it is necessary to define $\partial\phi(x,t)/\partial\eta(y,t)$ $\partial\phi(x,t)/\partial\eta(y,t)$. For this, using the Langevin equation and its free solution, we obtain

$$\frac{\partial\phi(x,t)}{\partial\eta(y,t)} = \frac{\delta}{\delta\eta(y,t)} \int (dx') \int_{-\infty}^{t'} dt'\, G(x-x';t-t') \int (dz)\, K_{x'z}(\Box)\, \eta(z,t') = \tag{3.4}$$

$$= \int (dx') \int_{-\infty}^{t} dt' \int (dp)\, e^{-i(x-x')p} G_{tt'}(p) K_{xy}(\Box)\, \delta(t-t') = \theta(0) K_{xy}(\Box) = \tfrac{1}{2} K_{xy}(\Box) .$$

Further, according to equalities (3.3) and (3.4) we get a chain rule into $\delta/\delta\phi$

$$\int (dy) K_{xy}(\Box)\, \eta(y,t) = 2\int (dy) K_{xy}(\Box) \int (dz) \frac{\partial\phi(z,t)}{\partial\eta(y,t)} \frac{\delta}{\delta\phi(z,t)} =$$

$$= \int (dy)\, K_{xy}(\Box) \int (dz) K_{yz}(\Box) \frac{\delta}{\delta\phi(z,t)} = \int (dz)\, K^2_{xz}(\Box) \frac{\delta}{\delta\phi(z,t)} , \tag{3.5}$$

where by definition (2.19)

$$K^2_{xy}(\Box) = \int (dz)\, K_{xz}(\Box)\, K_{zy}(\Box)$$

or

$$\int dz\, K_{xz}(\Box) K_{zy}(\Box) = \int (dp)\, V(-p^2\ell^2)\, e^{-ip(x-y)} \equiv K^2_{xy}(\Box) .$$

Finally, taking into account (2.10), (3.1)-(3.5) we arrive at the definition for the regularized SD equations

$$\frac{d}{dt}\langle F[\phi]\rangle_\eta = \langle \int (dx) [-\frac{\delta S}{\delta\phi(x)} + \int (dy) K^2_{xy}(\Box) \frac{\delta}{\delta\phi(y)}] \frac{\delta F[\phi]}{\delta\phi(x)} \rangle_\eta \tag{3.6}$$

or, at equilibrium

$$\langle \int (dx) [-\frac{\delta S}{\delta\phi(x)} + \int (dy) K^2_{xy}(\Box) \frac{\delta}{\delta\phi(y)}] \frac{\delta F[\phi]}{\delta\phi(x)} \rangle = 0 . \tag{3.7}$$

Further, following Bern et al.[13] and choosing

$$F[\phi] = \exp[\int(dx)\mathcal{J}(x)\phi(x)]$$

the Schwinger form of these equations may be easily obtained

$$\int(dx)\mathcal{J}(x)[-\frac{\delta S}{\delta\phi(x)}\Big/_{\phi\to\frac{\delta}{\delta J}} + \int(dy)K^2_{xy}(\Box)\mathcal{J}(y)]Z(J) = 0, \qquad (3.8)$$

where $Z(J) = \langle\exp(\int(dx)\mathcal{J}(x)\phi(x))\rangle$ is the vacuum-to-vacuum generating functional.

As shown below, the Schwinger-Dyson equations, plus some boundary condition which requires the permutation symmetry of Euclidean Bose time-ordered product, e.g.,

$$\langle\phi_1\phi_2\rangle = \langle\phi_2\phi_1\rangle \qquad (3.9)$$

$$\cdots\cdots\cdots$$

are equivalent (at least in weak coupling limit) to the Langevin formulation at equilibrium.

It is convenient to study the SD equations (3.7) in momentum space. Making use of the definitions (2.18b), (2.18d) and simple relations

$$\frac{\delta}{\delta\phi(x)} = \int(dp)e^{ipx}\frac{\delta}{\delta\phi_p}\ ;\quad \frac{\delta F[\phi]}{\delta\phi(x)} = \int(dq)e^{iqx}\frac{\delta F[\phi]}{\delta\phi_q}\ ;\quad \frac{\delta\phi_p}{\delta\phi_q} = \bar{\delta}^d(p+q)$$

we have the following identities

$$\int(dx)(\partial^2 - m^2)\phi(x)\frac{\delta F[\phi]}{\delta\phi(x)} = -\int(dp)(p^2+m^2)\phi_p\frac{\delta F[\phi_p]}{\delta\phi_p}\quad ;$$

$$\int(dx)\int(dy)K^2_{xy}(\Box)\frac{\delta^2 F[\phi]}{\delta\phi(y)\delta\phi(x)} = \int(dp)V(-p^2\ell^2)\frac{\delta^2 F[\phi]}{\delta\phi_p\delta\phi_{-p}},$$

etc. From which it is easily verified by a functional chain rule

$$\langle\int(dp)\Delta_p\phi_p\frac{\delta F}{\delta\phi_p}\rangle = \langle\int(dp)V(-p^2\ell^2)\frac{\delta^2 F}{\delta\phi_p\delta\phi_{-p}} - \qquad (3.10)$$

$$-\frac{g}{(N-1)!}\int\prod_{i=1}^{N-1}(dk_i)\int(dp)\bar{\delta}^d(\sum_{i=1}^{N-1}k_i - p)\phi_{k_1}\cdots\phi_{k_{N-1}}\frac{\delta F}{\delta\phi_p}\rangle,$$

where we have chosen the interaction

$$\lambda(\phi) = g\frac{\phi^N}{N!}.$$

As a first trivial example, with the boundary condition (3.9) we compute the regularized free two-point function. Setting and choosing $F=\phi_{p_1}\phi_{p_2}$ Eq. (3.10) becomes

$$\langle\phi_{p_1}\phi_{p_2}\rangle^{(0)} = \bar{\delta}^d(p_1+p_2)D(p_1)\quad ;\quad D(p) = V(-p^2\ell^2)\Delta_p^{-1}. \qquad (3.11)$$

This result is the correct nonlocal free propagator, in agreement with the Langevin result (2.18c).

3.2. Iterative Procedure for the Nonlocal SD Equations

To compute some n-point functions for any desired order of coupling constant g within the SD equations terative method of Eq. (3.10) should be given. This procedure was done by Bern et al. /13,14/. In our case with nonlocal form factors, their result is automatically transmitted. For example, it is not difficult to check in analogy with the formula (3.11) that $\langle\phi_{p_1}\phi_{p_2}\ldots\phi_{p_N}\rangle^{(0)}$ yields the usual Wick expansion, as products of nonlocal free propagators (3.11). Moreover, in the first order of g it corresponds to the regularized vertex

$$\Gamma(p_1\ldots p_N) = \langle\phi_{p_1}\ldots\phi_{p_N}\rangle^{(1)} = \bar{\delta}^d(\sum_{i=1}^{N}p_i)\prod_{i=1}^{N}D(p_i)\frac{(-g)\sum_{i=1}^{N}[D(p_i)]^{-1}}{\sum_{j=1}^{N}\Delta_{p_j}}. \qquad (3.12)$$

For N=3 the result agrees with Eq.(2.23).

Iterative chain rule may be obtained using Eq.(3.10). For illustration of this, we consider ϕ^3- theory (N=3). First, setting $F(\phi)=\phi_p$ in Eq. (3.10), we get

$$\langle\phi_p\rangle = -\frac{g}{2}\Delta_p^{-1}\int\int(dk_1)(dk_2)\bar{\delta}^d(p-k_1-k_2)\langle\phi_{k_1}\phi_{k_2}\rangle, \qquad (3.13)$$

in turn $\langle\phi_{k_1}\phi_{k_2}\rangle$ is given by the formula

$$\langle\phi_{k_1}\phi_{k_2}\rangle = \bar{\delta}^d(k_1+k_2)D(k_1) - \frac{g}{2}(\Delta_{k_1}+\Delta_{k_2})^{-1}\int\int(dq_1)(dq_2)\times \qquad (3.14)$$

$$\times[\bar{\delta}^d(k_1-q_1-q_2)\langle\phi_{k_2}\phi_{q_1}\phi_{q_2}\rangle + \bar{\delta}^d(k_2-q_1-q_2)\langle\phi_{k_1}\phi_{q_1}\phi_{q_2}\rangle].$$

Further, assuming $F[\phi]=\phi_{p_1}\phi_{p_2}\phi_{p_3}$ in Eq.(3.10), we obtain

$$\langle\phi_{p_1}\phi_{p_2}\phi_{p_3}\rangle=\left[2(\Delta_{p_1}+\Delta_{p_2}+\Delta_{p_3})^{-1}\bar{\delta}^d(p_1+p_2)\,V(-p_1^2\ell^2)\langle\phi_{p_3}\rangle+\text{cyclic perm. in }\{p\}\right]- \tag{3.15}$$

$$-\frac{1}{2}g(\Delta_{p_1}+\Delta_{p_2}+\Delta_{p_3})^{-1}\iint(dk_1)(dk_2)\cdot\left[\bar{\delta}^d(p_1-k_1-k_2)\,\langle\phi_{p_2}\phi_{p_3}\phi_{k_1}\phi_{k_2}\rangle+\right.$$

$$\left.+\text{cyclic perm. in }\{p\}\right]+\ldots$$

where definition $\langle\phi_{p_1}\phi_{p_2}\rangle=\bar{\delta}^d(p_1+p_2)\Delta_{p_1}^{-1}V(-p_1^2\ell^2)$ is used.

Using the zeroth order result (3.11) for D_p, the first order tadpole graph (Fig.6) may be immediately obtained from (3.13) and (3.14),

$$\langle\phi_p\rangle^{(1)}=-\frac{1}{2}g\frac{\bar{\delta}^d(p)}{m^2}\int(dk)\,D(k) \tag{3.16}$$

in agreement with the Langevin result (2.24). After taking next approximation in Eq.(3.13), expression (3.16) acquires the form

$$\langle\phi_p\rangle^{(2)}=-\frac{1}{2}g\left\{\Delta_p^{-1}\bar{\delta}^d(p)\int(dk_1)\,D(k_1)-g\Delta_p^{-1}\int(dk_1)(dq_1)(dq_2)\times\right.$$

$$\left.\times\bar{\delta}^d(p-k_1-q_1-q_2)(\Delta_{k_1}+\Delta_{p-k_1})^{-1}\langle\phi_{k_1}\phi_{q_1}\phi_{q_2}\rangle\right\}.$$

Finally, in order to compute the $O(g^2)$ one-loop, contribution to the two-point function (Fig.7) we take into account second term in (3.14) and put in it the disconnected part of (3.15) with

$$\langle\phi_{q_1}\phi_{q_2}\phi_{k_1}\phi_{k_2}\rangle^{(0)}_{q,k}=\langle\phi_{q_1}\phi_{k_1}\rangle\langle\phi_{q_2}\phi_{k_2}\rangle+\langle\phi_{q_1}\phi_{k_2}\rangle\langle\phi_{q_2}\phi_{k_1}\rangle=$$

$$=D(q_1)D(q_2)\left[\bar{\delta}^d(q_1+k_1)\bar{\delta}^d(q_2+k_2)+\bar{\delta}^d(q_1+k_2)\bar{\delta}^d(q_2+k_1)\right],$$

where the subscript on the right q,k means to keep only those contributions in which q's contract with k's.

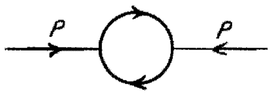


Fig.7.
One-loop two-point function in the nonlocal stochastic scheme.

As a result of a little algebra we obtain

$$\Pi(p)=\frac{1}{2}g^2\frac{D(p)}{\Delta(p)}\int(dk)\,D(k)\,D(p-k)\,\frac{(V^{-1}\Delta)_k+(V^{-1}\Delta)_{p-k}+(V^{-1}\Delta)_p}{\Delta_k+\Delta_{p-k}+\Delta_p} \tag{3.17}$$

which is the usual local loop when $\ell\to 0$.

Thus, the SD equations (3.7) or (3.13)-(3.15) may be solved iteratively, in this manner, to any desired order of g. However, the procedure is increasingly tedious. To simplify this prescription, Bern et al.[13,14,18] have developed a systematic set of Schwinger-Dyson-Feynman rules instead. We mention that construction of any expressions of the type of (3.17) according to these rules, requires more efforts than the usual Feynman diagrammatic correspondence.

Finally, for further computational purpose we present here concrete method of calculation of the expression (3.17). Explicit form of which is

$$\Pi(p)=\frac{1}{2}g^2(m^2+p^2)^{-2}\int(dk)\left[3m^2+k^2+p^2+(p-k)^2\right]^{-1}\left\{V(-p^2\ell^2)V(-(p-k)^2\ell^2)\times\right.$$

$$\times(m^2+(p-k)^2)^{-1}+V(-p^2\ell^2)V(-k^2\ell^2)(m^2+k^2)^{-1}+ \tag{3.18}$$

$$\left.+(m^2+p^2)(m^2+k^2)^{-1}(m^2+(p-k)^2)^{-1}V(-k^2\ell^2)V(-(k-p)^2\ell^2)\right\}.$$

First, consider the second term of (3.18) in the case of d=6 dimensions. By using the Mellin representation (2.9b) for $V(z)$ and the general Feynman parametric formula

$$b_1^{-\mu_1}\dots b_n^{-\mu_n} = \frac{\Gamma(\mu_1+\dots+\mu_n)}{\Gamma(\mu_1)\dots\Gamma(\mu_n)}\int_0^1 d\alpha_1\dots\int_0^1 d\alpha_n\,\delta\Big(1-\sum_{i=1}^{n}\alpha_i\Big)\times$$

$$\times\,\alpha_1^{\mu_1-1}\alpha_2^{\mu_2-1}\dots\alpha_n^{\mu_n-1}\Big[\sum_{j=1}^{n}\alpha_j b_j\Big]^{-\mu_1-\mu_2-\dots-\mu_n}$$

we get

$$\Gamma^{(2)}(p)=\frac{g^2}{4}\frac{V(-p^2\ell^2)}{(p^2+m^2)^2}\frac{\pi^3}{(2\pi)^6}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}dy\,\frac{v(y)}{\sin\pi y}\,\frac{\Gamma(-1-y)}{\Gamma(1-y)}\,\ell^{2y}f(y), \qquad (3.19)$$

where

$$f(y)=\int_0^1 dx\,(1-x)^{-y}A^{1+y},$$

$$A=-\frac{1}{4}p^2x^2+p^2x+\frac{3}{2}m^2x+m^2(1-x).$$

Further, by shifting the contour of integration to the right we can reduce this integral to series and taking into account the main asymptotics we have

$$\Gamma^{(2)}(p)=\frac{g^2}{8}(m^2+p^2)^{-2}\frac{\pi^3}{(2\pi)^6}\Big[\sigma\ell^{-2}+\Big(\frac{5}{6}p^2+\frac{5}{2}m^2\Big)v(0)\ln\mu^2\ell^2\Big], \qquad (3.20)$$

$$\sigma=\lim_{x\to-1} v(x)/(1+x)$$

here we have assumed that function $v(x)$ has zero at the point $x=-1$ and $V(-p^2\ell^2)/_{p^2\to 0}=1$ for the external momentum variable p^2. Moreover, in (3.19) we use the Γ-function properties

$$\Gamma(1+x)=x\Gamma(x)\quad,\quad \Gamma(x)\Gamma(1-x)=\frac{\pi}{\sin\pi x}.$$

First and seoond terms in (3.20) correspond to calculations of residues at points $y=-1$ and $y=0$, respectively. It is clear that $\Gamma^{(1)}(p)=\Gamma^{(2)}(p)$. Similar oalculations can be carried out for the third term in (3.18) and the result is reduced to the following formula

$$\Gamma^{(3)}(p)=-\frac{g^2}{2^9\pi^3}\,\frac{v(0)}{m^2+p^2}\,\ln\mu^2\ell^2. \qquad (3.21)$$

In (3.20) and (3.21) $v(0)=1$ which follows from the normalization condition $V(0)=1$, and μ is an arbitrary parameter with dimension of mass.

4. Renormalization Prescription and the Three-Point Funotion in Nonlooal SD Formalism

A renormalization program in the regularized SD formalism has been first discussed by Bern et al. /13/. For the nonlocal oase, their result is immediately repeated. However, some essential differenoe appears when counterterms in the Lagrangian function are constructed. In the nonlocal stochastic theory counterterms are finite, sinoe we do not assume $\ell\to 0$ at the end of calculations. It means that parameter ℓ of the theory remains everywhere, in particular, in its action. Thus, our scheme is an action regularization, beoause at the same time for the Green functions explicit divergence does not occur in the effective d-dimensional action of the theory.

For completeness, within the SD equations we present here renormalization procedure due to Bern et al. /13/ for the nonlocal case. Thus, the nonlocal SD equations

$$\Big\langle\int(dx)\Big[\frac{\delta S_0}{\delta\phi(x)}-\int(dy)K^2_{xy}(\Box)\frac{\delta}{\delta\phi(y)}\Big]\frac{\delta F}{\delta\phi(x)}\Big\rangle=0 \qquad (4.1)$$

involve the unrenormalized field $\phi(x)$ and the bare Lagrangian $\mathcal{L}_0$ whose parameters we now denote as m_0 and g_0. The usual renormalized field is $\phi_R\equiv Z_\phi^{-1/2}\phi$ by means of which renormalized Green functions $F[\phi_R]$ are constructed. Assuming the faot that the SD equations homogeneous in $\delta/\delta\phi$, we have the nonlooal SD equations

$$\Big\langle\int(dx)\Big[\frac{\delta(S_R+S_{CT})}{\delta\phi_R(x)}-\int(dy)K^2_{xy}(\Box)\frac{\delta}{\delta\phi_R(y)}\Big]\frac{\delta F[\phi_R]}{\delta\phi_R(x)}\Big\rangle=0, \qquad (4.2)$$

where $S_0=S_R+S_{CT}$ is the usual textbook breakup into the renormalized Lagrangian and the counterterm Lagrangian. Renormalization procedure formulated as usually is based on the construction of the total Lagrangians, for example, in the case of ϕ^3 theory we have explicitly

$$\mathcal{L}_R=\frac{1}{2}\phi_R(-\Box+m^2)\phi_R+\frac{g}{3!}\phi_R^3 \qquad (4.3)$$

$$\mathcal{L}_{CT}=\frac{1}{2}(Z_\phi-1)\phi_R(-\Box+m^2)\phi_R+\frac{1}{2}\delta m^2\phi_R^2+\frac{g}{3!}(Z_g-1)\phi_R^3,$$

where

$$g = Z_\phi^{3/2} g_0 / Z_g \quad , \quad m^2 = m_0^2 - \delta m^2 / Z_\phi \, . \tag{4.4}$$

Following Bern et al.[13] we compute here three-point vertices in the nonlocal theory using the iterative method presented in the previous section for the SD equations. For this purpose, continue iterative procedure carried out in Sec.3.2 up to the $O(g^3)$ -order for $\langle \phi_{p_1}\phi_{p_2}\phi_{p_3}\rangle_{conn}$. After simple but tedious calculations, we have

$$\langle \phi_{p_1}\phi_{p_2}\phi_{p_3}\rangle = -\frac{1}{2} g \rho \iint (dk_1)(dk_2) \Big\{ -\frac{g}{2} \bar{\delta}^d(p_1-k_1-k_2)(\Delta_{k_1}+\Delta_{k_2}+\Delta_{p_2}+\Delta_{p_3})^{-1} \times$$
$$\times \big[4\Sigma_1 + 2\Sigma_2 + 4\Sigma_4 + 2\Sigma_5 + \Sigma_6 + 2\Sigma_4(p_2 \leftrightarrow p_3) + \tag{4.5}$$
$$+ 2\Sigma_5(p_2 \leftrightarrow p_3) + \Sigma_6(p_2 \leftrightarrow p_3) \big] + (p_1 \leftrightarrow p_2) + (p_1 \leftrightarrow p_3) \Big\} + M_1 + M_2 \, ,$$

where

$$\rho = \Big[\sum_{j=1}^{3} \Delta_{p_j} \Big]^{-1} \quad ;$$

$$\Sigma_i(k_1,k_2,p_2,p_3) = -\frac{1}{2} g \int (dq_1)(dq_2)(ds_1)(ds_2)\, \bar{\delta}^d(k_1-q_1-q_2)\,[\Delta_{q_1}+\Delta_{q_2}+\Delta_{k_2}+\Delta_{p_2}+\Delta_{p_3}]^{-1} \sigma_i$$
$$i = 1,2,3 \quad ; \tag{4.6}$$

$$\Sigma_j(k_1 k_2 p_2 p_3) = -\frac{1}{2} g \int (dq_1)(dq_2)(ds_1)(ds_2)\, \sigma_j\, \bar{\delta}^d(p_2-q_1-q_2) \times$$
$$\times [\Delta_{q_1}+\Delta_{q_2}+\Delta_{k_1}+\Delta_{k_2}+\Delta_{p_3}]^{-1} \quad , \qquad j = 4,5,6 \quad ;$$

here :

$$\sigma_1 = \bar{\delta}^d(q_1-s_1-s_2) \langle \phi_{s_1}\phi_{s_2}\phi_{q_2}\phi_{k_2}\phi_{p_2}\phi_{p_3}\rangle^{(0)} \quad ,$$

$$\sigma_2 = \sigma_1(q_1 \leftrightarrow k_2) \quad ; \qquad \sigma_3 = \sigma_2(k_2 \leftrightarrow p_2) \quad ,$$

$$\sigma_4 = \bar{\delta}^d(q_1-s_1-s_2) \langle \phi_{s_1}\phi_{s_2}\phi_{q_2}\phi_{k_1}\phi_{k_2}\phi_{p_3}\rangle^{(0)} \quad ,$$

$$\sigma_5 = \sigma_4(q_1 \leftrightarrow k_2) \quad ; \qquad \sigma_6 = \sigma_5(k_2 \leftrightarrow p_3) \, .$$

In turn, terms M_i $(i=1,2)$ are given by the following formula

$$M_1 = -g\rho\, \bar{\delta}^d(p_1+p_2+p_3) \Big\{ \big[(\Delta_{p_2}+\Delta_{p_3})^{-1} V(p_2^2\ell^2)\, \Pi(p_3) + (p_2 \leftrightarrow p_3) \big] + (p_1 \leftrightarrow p_2) + (p_1 \leftrightarrow p_3) \Big\} ,$$

$$M_2 = \Big(-\frac{g}{2}\Big)^2 \rho \int (dk_1)(dk_2)\, \gamma_1 \Big\{ \int (dq_1)(dq_2)\, \gamma_2\, \bar{\delta}^d(p_1-k_1-k_2)\, \bar{\delta}^d(k_1-q_1-q_2) \times V(-q_1^2\ell^2) \times$$
$$\times \Big[-\frac{g}{2} \int (ds_1)(ds_2) \big(H + H(q_2 \leftrightarrow p_2) + H(q_2 \leftrightarrow p_3) + N + N(q_2 \leftrightarrow k_2) + N(q_2 \leftrightarrow p_3) +$$
$$+ L + L(q_2 \leftrightarrow k_2) + L(q_2 \leftrightarrow p_2) \big) + (q_1 \leftrightarrow q_2) \Big] + (k_1 \leftrightarrow k_2) \Big\} .$$

Here

$$\gamma_1 = (\Delta_{k_1} + \Delta_{k_2} + \Delta_{p_2} + \Delta_{p_3})^{-1} \quad ,$$

$$\gamma_2 = (\Delta_{q_1} + \Delta_{q_2} + \Delta_{k_2} + \Delta_{p_2} + \Delta_{p_3})^{-1} \quad ,$$

$$H = \bar{\delta}^d(q_1+k_2)\,[\Delta_{p_2}+\Delta_{p_3}+\Delta_{q_2}]^{-1}\, \bar{\delta}^d(q_2-s_1-s_2) \langle \phi_{s_1}\phi_{s_2}\phi_{p_2}\phi_{p_3}\rangle^{(0)} ,$$

$$N = 2\bar{\delta}^d(q_1+p_2)\,[\Delta_{k_2}+\Delta_{q_2}+\Delta_{p_3}]^{-1}\, \bar{\delta}^d(q_2-s_1-s_2) \langle \phi_{s_1}\phi_{s_2}\phi_{k_2}\phi_{p_3}\rangle^{(0)} ,$$

$$L = N(p_2 \leftrightarrow p_3) \, .$$

Main asymptotics of (4.5) may be easily calculated by the same method as it has presented in previous section. We are interested only in divergent parts in the expression (4.5). For example, term Σ_4 has the form

$$\Sigma_4 = 8\Big(-\frac{g}{2}\Big)^3 (m^2+p_3^2)^{-1}(m^2+p_2^2)^{-1} \rho\, \bar{\delta}^d(p_1+p_2+p_3) \int (dq) \frac{V(-q^2\ell^2)}{m^2+q^2} \frac{1}{2} (2m^2+p_2^2+p_3^2)^{-1} \times$$
$$\times \Big\{ [\Delta_q + \Delta_{q-p_2} + \Delta_{p_2} + \Delta_{p_3}]^{-1} - [\Delta_q + \Delta_{q-p_3} + \Delta_{p_3} + 2\Delta_{p_2}]^{-1} \Big\} ,$$

where we have used the usual Wick expansion for σ_4 in (4.6) in accordance with (3.11). Integration over d^6q is easily carried out by the same prescription presented for obtaining leading terms of two-loop function $\Pi(p)$ (3.18). After some elementary calculations, main asymptotics are reduced to the following formula

$$\Sigma_4 = -\frac{g^3}{2^4\pi^3}(m^2+p_2^2)^{-1}(m^2+p_3^2)^{-1}(2m^2+p_2^2+p_3^2)^{-1}\rho \times$$

$$\times \left\{2\sigma\ell^2 + \left[\frac{11}{6}(p_2^2+p_3^2)+7m^2\right]\ln\mu^2\ell^2\right\}.$$

Remain terms in (4.5) are calculated in the same manner. According to Bern et al./13/ obtained results may be classified within the different types of diagrams, shown in Fig.8 (for detail see Bern et al./13/).

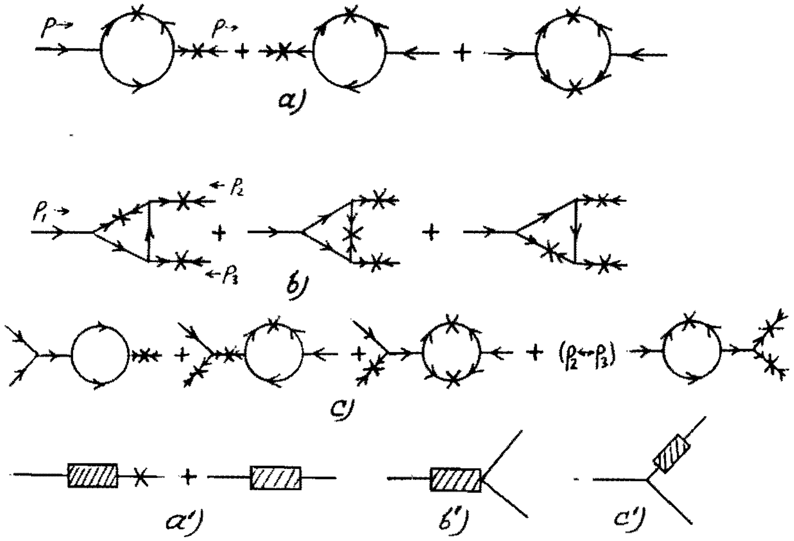


Fig.8. Nonlocal diagrams.

a) One-loop two-point functions. b) "Pure" three-point vertices that are infinite as $\ell\to 0$ c) Three-point functions with a loop on the external lines. Cyclic permutations of the external lines must also be included. a'), b') and c') correspond to their counterterm diagrams, respectively.

Final results are given in Tables 1 and 2. Comparing the sum of the loop diagrams in Table 1 with the sum of the counterterm diagrams in Table 2, we determine the renormalization constants

$$Z_\phi = 1+\frac{1}{3}\frac{g^2}{2^8\pi^3}\ln\mu^2\ell^2 \quad ;$$

$$Z_g = 1+\frac{g^2}{2^7\pi^3}\ln\mu^2\ell^2 \quad ; \qquad (4.7)$$

$$\delta m^2 = \frac{g^2}{2^8\pi^3}\left[\sigma\ell^{-2}+\frac{5}{3}\ln\mu^2\ell^2\right].$$

Table 1

Diagram	Leading terms in sum of one-loop diagrams
8a	$\frac{g^2}{2^8\pi^3}\left[\sigma\ell^{-2}+(\frac{1}{3}p^2+2m^2)\ln\mu^2\ell^2\right](p^2+m^2)^{-2}$,
8b	$\frac{g^3}{2^7\pi^3}\left[(m^2+p_1^2)(m^2+p_2^2)(m^2+p_3^2)\right]^{-1}\ln\mu^2\ell^2$,
8c	$-\frac{g^3}{2^8\pi^3}\left[(p_1^2+m^2)(p_2^2+m^2)(p_3^2+m^2)\right]^{-1}\left[\frac{\sigma\ell^{-2}+(\frac{1}{3}p_1^2+2m^2)\ln\mu^2\ell^2}{p_1^2+m^2}+\frac{\sigma\ell^{-2}+(\frac{1}{3}p_2^2+2m^2)\ln\mu^2\ell^2}{p_2^2+m^2}+\frac{\sigma\ell^{-2}+(\frac{1}{3}p_3^2+2m^2)\ln\mu^2\ell^2}{p_3^2+m^2}\right]$

It is interesting to notice that Bern et al./13/ results **are** valid for any regulators $V(-p^2\ell^2)$ if, in their final expressions for loop diagrams, coefficients $\frac{1}{3}\Lambda^2$ and $\ln(\Lambda^2/\mu^2)$ should be changed by $\sigma\ell^{-2}$ and $-\ln\mu^2\ell^2$, respectively.

The attraction of our approach is that the nonlocal scheme is unitary in the presence of the analytic regulator (for detail, see Efimov /12/). In our case, **supplementary** singularities caused by regulators do not exist and analytic properties of any diagrams are conserved at finite value of momentum variables p^2 . While for meromorphic regulators like Pauli-Villars regularization procedure, analytic properties of diagrams are broken and it in turn leads to some difficulties in proof of analyticity and unitarity of the regularized theory with these types of regulators. In last case, one expects that unitarity is regained as the regularization is removed $\Lambda\to\infty$ at which of course, singularities (poles) are displaced at infinity.

5. Nonlocal Stochastic Quantization of Gauge Fields

At first sight, majority of physicists think that stochastic quantization method appears to be no more than an amusing alternative to conventional hamiltonian, path integral and action formulations. It turns out that this method has given birth to a number of new ideas and is very useful to understand many problems of the field theory in light of its present developments. As mentioned by Bern et al./14/ these developments are Zwanziger's gauge-fixing (Zwanziger/19/ ; Floratos et al./20/), large-N quenching and large-N master fields (Alfaro and Sakita/21/, Greensite and Halpern /22/), stochastic stabilization (Greensite and Halpern/23/), stochastic regularization (Bern et al./14/; Niemi and Wijewaedhama/24/;

Table 2

Diagram	Leading terms in sum of counterterm diagrams
8a'	$-(\delta m^2+(Z_\varphi-1)(p^2+m^2))(p^2+m^2)^{-2}$
8b'	$-g(Z_g-1)[(p_1^2+m^2)(p_2^2+m^2)(p_3^2+m^2)]^{-1}$
8c'	$g[(p_1^2+m^2)(p_2^2+m^2)(p_3^2+m^2)]^{-1}[(p_1^2+m^2)^{-1}\times$ $(\delta m^2+(Z_\varphi-1)(p_1^2+m^2)+(p_2^2+m^2)^{-1}(\delta m^2+(Z_\varphi-1)(p_2^2+m^2))+$ $+(p_3^2+m^2)^{-1}(\delta m^2+(Z_\varphi-1)(p_3^2+m^2))]$

Breit et al.,/25/, Namiki and Yamanaka/26/, Bern/27/), the QCD_4 maps which run in ordinary time (Glandson and Halpern/28/; Bern and Chan/29/) and numerical applications of the Langevin equation in lattice gauge theory (Hamber and Heller/30/; Batrouni et al./31/). For review see Namsrai/4/ and Migdal/3/, where earlier references concerning this problem are cited.

To introduce nonlocality into stochastic quantization formalism

for gauge fields we follow Bern et al./14/. Our procedure is the same as it was done by these authors. However, our method is more general and deals with any form factors of the type $V(-p^2\ell^2)$.

5.1. Nonlocal Langevin Systems for Gauge Theory

Nonlocal Parisi-Wu Langevin system for SU(N) Yang-Mills theory in d-dimensions is given by

$$\dot{A}^a_\mu(x,t) = -\frac{\delta S}{\delta A^a_\mu}(x,t) + d^{ab}_\mu Z^b(x,t) + \int (dy) K^{ab}_{xy}(\Delta)\,\eta^b_\mu(y,t), \tag{5.1}$$

where local noise satisfies the following relation

$$\langle \eta^a_\mu(x,t)\eta^b_\nu(y,t')\rangle_\eta = 2\delta^{ab}\delta_{\mu\nu}\delta(t-t')\delta^d(x-y) \tag{5.2}$$

and $K^{ab}_{xy}(\Delta)$ is nonlocal distribution discussed in previous sections. According to the equilibrium hypothesis, the nonlocal Euclidean Green functions determined by vacuum expectation values of products of fields

$$\langle F[A(\cdot)]\rangle_0 = \langle A_\nu(x_1)\dots A_\mu(x_n)\rangle_0 = \prod_{i\neq j} D_{\nu\sigma}(x_i-x_j) \tag{5.3}$$

in the usual nonlocal quantum field theory (for example, see Efimov /12/ and Namsrai/4/) are now given by

$$\langle F[A(\cdot)]\rangle = \lim_{t\to\infty}\langle F[A(\cdot,t)]\rangle_\eta, \tag{5.4}$$

where $F[A]$ is any equal fifth-time functional (product) of the gauge field $A^a_\mu(\eta)$. In particular, nonlocal propagator for the photon field $A_\mu(x)$ in (5.3) takes the form

$$D^0_{\mu\nu}(x-y) = \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle = \frac{ig^{\mu\nu}}{(2\pi)^4}\int d^4p\, e^{-ip(x-y)}\frac{V_0(-p^2\ell^2)}{p^2}$$

in accordance with the nonlocal theory. Here, form factor $V_0(-p^2\ell^2)$ is given by formula (2.9b) with $m=0$.

Our notation in (5.1) is usual

$$S=\frac{1}{4}\int (dx) F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) \ , \quad F^a_{\mu\nu}\equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc}A^b_\mu A^c_\nu .$$

In this paper we use the following covariant derivative

$$d_\mu^{ab} = \delta^{ab}\partial_\mu + g f^{abc} A_\mu^c$$

In (5.1) we have chosen to add to a Zwanziger gauge-fixing term $d_\mu^{ab} Z^b$, which we will specify as $\alpha Z^a = \partial \cdot A^a$ for computational purposes. As shown below, gauge-invariant quantities do not depend on the gauge-fixing for the nonlocal case. The nonlocal distribution $K_{xy}^{ab}(\Delta)$ is a function of the covariant Laplacian

$$\Delta_{xy}^{ab} = \int (dz)(d_\mu)_{xz}^{ac}(d_\mu)_{zy}^{cb} \,, \qquad (d_\mu)_{xy}^{ab} = d_\mu^{ab}(x)\,\delta^d(x-y) \tag{5.5}$$

so that

$$K_{yx}^{ba}(\Delta) = K_{xy}^{ab}(\Delta).$$

In the weak coupling limit the Langevin equation (5.1) is the equivalent integral formulation

$$A_\mu^a(x,t) = \int_{-\infty}^{t} dt'(dy)\, G_{\mu\nu}^{ab}(x-y,t-t')\,[W_\nu^b(y,t') + \frac{1}{\alpha} Y_\nu^b(y,t') + \int (dz)\, K_{yz}^{bc}(\Delta)\,\eta_\nu^c(z,t')\,], \tag{5.6}$$

where

$$G_{\mu\nu}^{ab}(x-y,t-t') = \delta^{ab}\theta(t-t')\int (dp)\, e^{-ip(x-y)} \times [T_{\mu\nu}(p)e^{-p^2(t-t')} + L_{\mu\nu}(p)e^{-p^2(t-t')/\alpha}] \tag{5.7}$$

is the Langevin Green function, which is determined by usual procedure:

$$G_{\mu\nu}^{ab}(x,t) = \delta^{ab}[T_{\mu\nu}G^T(x,t) + L_{\mu\nu}G^L(x,t)].$$

Here $T_{\mu\nu}$ ($L_{\mu\nu}$) is the standard transverse (longitudinal) projection operators; in the momentum space they take the form

$$T_{\mu\nu}(K) \equiv \delta_{\mu\nu} - K_\mu K_\nu / K^2 \,,$$

$$L_{\mu\nu}(K) \equiv K_\mu K_\nu / K^2.$$

In (5.6) we have defined the interaction terms

$$W_\nu^b = -g f^{bcd}[\partial_\beta(A_\beta^c A_\nu^d) - (\partial_\beta A_\nu^c)A_\beta^d + (\partial_\nu A_\beta^c)A_\beta^d] - g^2 f^{bcd} f^{cne} A_\beta^n A_\nu^e A_\beta^d \;; \tag{5.8}$$

$$Y_\nu^b = g f^{bcd} A_\nu^d (\partial \cdot A^c). \tag{5.9}$$

The former arises from the action and last term is due to the Zwanziger one. In expression (5.6) we have also employed the technical device of choosing $t_0 = -\infty$, so that the system has equilibrated at any finite fifth-time

A method of form factor expansion in powers of the coupling constant plays an important role in proof of gauge invariance of the nonlocal stochastic quantization theory. As a first step in this expansion we write in accordance with Bern et al. /14/

$$\Delta_{xy}^{ab} = \delta^{ab}\Box_{xy}\ell^2 + g(\Gamma_1)_{xy}^{ab} + g^2(\Gamma_2)_{xy}^{ab}, \tag{5.10}$$

where the regulator "vertices" Γ_1 and Γ_2 are defined as

$$(\Gamma_1)_{xy}^{ab} = \ell^2 f^{abc}(\partial_\mu^x A_\mu^c(x) + A_\mu^c(x)\partial_\mu^x)\,\delta^d(x-y) \;; \tag{5.11a}$$

$$(\Gamma_2)_{xy}^{ab} = \ell^2 f^{anm} f^{nbe} A_\mu^m(x) A_\mu^e(x)\,\delta^d(x-y). \tag{5.11b}$$

In (5.11) the derivatives ∂_μ^x act on everything to the right. Further, for any distribution of the type of (2.5) we may write down the following expansion rule

$$K_{xy}^{ab}(\Delta) = \sum_{n=0}^{\infty}\frac{C_n}{(2n)!}(\Delta_{xy}^{ab})^n = \sum_{n=0}^{\infty}\frac{C_n}{(2n)!}\ell^{2n}\delta^{ab}\Box_{xy}^n \;+$$

$$+\sum_{n=0}^{\infty}\frac{c_n \ell^{2n-2}}{(2n)!}\int(dz)\left[g(\Gamma_1)^{ab}_{xz}+g^2(\Gamma_2)^{ab}_{xz}\right]\Box^{n-1}_{zy}\cdot n +$$

$$+\frac{1}{2}\sum_{n=0}^{\infty}\frac{c_n}{(2n)!}\ell^{2n-4}\int(dz_1)(dz_2)\,g^2(\Gamma_1)^{ac}_{xz_1}(\Gamma_1)^{cb}_{z_1z_2}\,\Box^{n-2}_{z_2y}\,n(n-1)+\ldots =$$

$$= \delta^{ab}K_{xy}(\Box)+\frac{1}{2}g\int(dz_1)(dz_2)\left[K^{(1)}_{xz_1}(\Box)(\Gamma_1)^{ab}_{z_1z_2}H_{z_2y}(\Box)+\right. \qquad (5.12)$$

$$\left.+H_{xz_1}(\Box)(\Gamma_1)^{ab}_{z_1z_2}K^{(1)}_{z_2y}(\Box)\right]+\frac{1}{2}g^2\int(dz_1)(dz_2)\times$$

$$\times\left[K^{(1)}_{xz_1}(\Box)(\Gamma_2)^{ab}_{z_1z_2}H_{z_2y}(\Box)+H_{xz_1}(\Box)(\Gamma_2)^{ab}_{z_1z_2}K^{(1)}_{z_2y}(\Box)\right]+$$

$$+\frac{1}{6}g^2\int(dz_1)(dz_2)\left[K^{(2)}_{xz_1}(\Box)(\Gamma_1)^{ac}_{z_1z_2}H_{z_2z_3}(\Box)(\Gamma_1)^{cb}_{z_3z_4}H_{z_4y}(\Box)+\right.$$

$$+H_{xz_1}(\Box)(\Gamma_1)^{ac}_{z_1z_2}K^{(2)}_{z_2z_3}(\Box)(\Gamma_1)^{cb}_{z_3z_4}H_{z_4y}(\Box)+$$

$$\left.+H_{xz_1}(\Box)(\Gamma_1)^{ac}_{z_1z_2}H_{z_2z_3}(\Box)(\Gamma_1)^{cb}_{z_3z_4}K^{(2)}_{z_4y}(\Box)\right]+$$

$$+\ldots$$

Here the Fourier transforms of generalized functions are given by

$$K(-p^2\ell^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{w(\xi)}{\sin\pi\xi}\,(p^2\ell^2)^{\xi}\quad;$$

$$K^{(1)}(-p^2\ell^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{w(\xi)}{\sin\pi\xi}\,\xi\,(p^2\ell^2)^{\xi}\quad; \qquad (5.13)$$

$$K^{(2)}(-p^2\ell^2)=\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\,\frac{w(\xi)}{\sin\pi\xi}\,\xi(\xi-1)\,(p^2\ell^2)^{\xi}$$

and for the operator $H_{xy}(\Box)=(\Box_x\ell^2)^{-1}\delta^d(x-y)$ we have

$$H_{xy}(\Box)=-\int(dp)\,\widetilde{H}(p^2\ell^2)\,e^{-ip(x-y)} \qquad (5.14)$$

$$\widetilde{H}(p^2\ell^2)=1/p^2\ell^2.$$

With the form factor expansion (5.12) for any desired order it is not difficulty to iterate the integral equation (5.6) for the Langevin field

$$A_\mu[\eta]=\sum_{m=0}^{\infty}g^m A^{(m)}_\mu[\eta] \qquad (5.15)$$

up to arbitrarily high order as well. As the example, the result for the form factor $K(\Delta)$ in d=4 dimensions takes the form:

$$A^{(0)a}_\mu(x,t)=\int_{-\infty}^{t}dt'(dy)\,G^{ab}_{\mu\nu}(x-y,t-t')\int(dz)\,K_{zy}(\Box)\,\eta^b_\nu(z,t')\quad; \qquad (5.16a)$$

$$A^{(1)}_\mu(x,t)=\int_{-\infty}^{t}dt'(dy)\,G^{ab}_{\mu\nu}(x-y,t-t')\left\{W^{(0)b}_\nu(y,t')+\frac{1}{\alpha}Y^{(0)b}_\nu(y,t')+\right.$$

$$\left.+\frac{1}{2}\int(dz)\left[K^{(1)}(\Box)\Gamma_1(A^{(0)})H(\Box)+H(\Box)\Gamma_1(A^{(0)})K^{(1)}(\Box)\right]^{bc}_{yz}\eta^c_\nu(z,t')\right\}; \qquad (5.16b)$$

$$A^{(2)a}_\mu(x,t)=\int_{-\infty}^{t}dt'(dy)\,G^{ab}_{\mu\nu}(x-y,t-t')\left\{W^{(1)b}_\nu(y,t')+\frac{1}{\alpha}Y^{(1)b}_\nu(y,t')+\right.$$

$$+\int(dz)\left[\frac{1}{2}\left(K^{(1)}(\Box)\Gamma_1(A^{(1)})H(\Box)+H(\Box)\Gamma_1(A^{(1)})K^{(1)}(\Box)\right)+\right.$$

$$+\frac{1}{2}\left(K^{(1)}(\Box)\Gamma_2(A^{(0)})H(\Box)+H(\Box)\Gamma_2(A^{(0)})K^{(1)}(\Box)\right)+$$

$$+\frac{1}{6}\left(K^{(2)}(\Box)\Gamma_1(A^{(0)})H(\Box)\Gamma_1(A^{(0)})H(\Box)+\right.$$

$$+H(\Box)\Gamma_1(A^{(0)})K^{(2)}(\Box)\Gamma_1(A^{(0)})H(\Box)+ \qquad (5.16c)$$

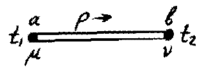
$$\left.\left.\left.+H(\Box)\Gamma_1(A^{(0)})H(\Box)\Gamma_1(A^{(0)})K^{(2)}(\Box)\right]^{bc}_{yz}\eta^c_\nu(z,t')\right\}.$$

Here, product of operators in (5.16b) and (5.16c) should be understood as contraction operation between them [see, the formula (5.12)].

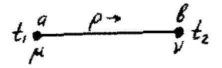
We note that more useful at arbitrary order is the equivalent description in terms of Langevin tree graphs, which are easily derived from Eqs. (5.6) or (5.16). For this purpose, the tree-graphical expansions of the form factor should be given, that is the same as it was done by Bern et al./14/ for the concrete regulator $[R(\Delta)]^{ab}_{xy} = \delta^{ab}[1-\Delta/\Lambda^2]^{-1}_{xy}$. **In the nonlocal** theory the Langevin tree graphs through $O(g^2)$ are shown in Fig.10. These diagrams may be constructed to all orders using the Langevin tree rules given in Fig.9.

Propagators:

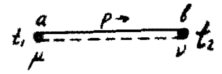
$$= G^{ab}_{\mu\nu}(p,t_1-t_2) = \theta(t_1-t_2)\delta^{ab}\left[T_{\mu\nu}(p)e^{-p^2(t_1-t_2)} + L_{\mu\nu}(p)e^{-\frac{p^2(t_1-t_2)}{\alpha}}\right]$$



$$= \delta^{ab}\delta_{\mu\nu}\,\delta(t_1-t_2)\,K(-p^2\ell^2)$$



$$= \delta^{ab}\delta_{\mu\nu}\,\delta(t_1-t_2)\,K^{(1)}(-p^2\ell^2)$$

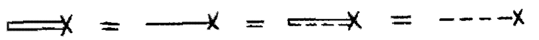


$$= \delta^{ab}\delta_{\mu\nu}\,\delta(t_1-t_2)\,K^{(2)}(-p^2\ell^2)$$

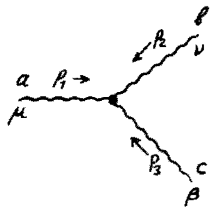
$$= \delta^{ab}\delta_{\mu\nu}\,\delta(t_1-t_2)\,H(p^2\ell^2)$$

Vertices

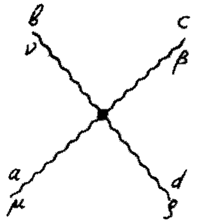
$$\equiv 1$$



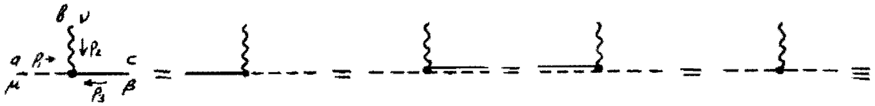
$$\equiv \eta^a_\mu$$



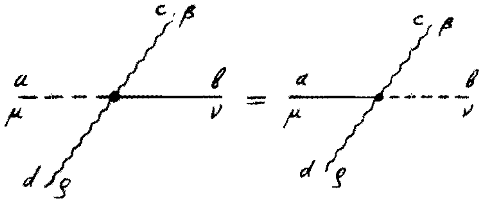
$$= -\frac{i}{2} f^{abc} g\left[(p_1-p_2)_\beta \delta_{\mu\nu} + (p_2-p_3)_\mu \delta_{\beta\nu} + (p_3-p_1)_\nu \delta_{\mu\beta}\right] - -\frac{i}{2\alpha} f^{abc} g\left[(p_3)_\beta \delta_{\nu\mu} - (p_2)_\nu \delta_{\beta\mu}\right] \equiv W^{abc}_{\mu\nu\beta}(p_1;p_2;p_3)\,;$$



$$= -\frac{g^2}{6}\Big[f^{abn}f^{cdn}(\delta_{\mu\beta}\delta_{\nu\varrho} - \delta_{\mu\varrho}\delta_{\nu\beta}) + f^{acn}f^{bdn}(\delta_{\mu\nu}\delta_{\beta\varrho} - \delta_{\mu\varrho}\delta_{\nu\beta}) + f^{adn}f^{cbn}(\delta_{\mu\beta}\delta_{\nu\varrho} - \delta_{\mu\nu}\delta_{\beta\varrho})\Big] \equiv W^{abcd}_{\mu\nu\beta\varrho}\,;$$



$$\equiv \Gamma^{abc}_{1\,\mu\nu\beta} = i g f^{abc}(p_1-p_3)_\nu\, \delta_{\mu\beta}$$



$$\equiv g^2 f^{anc} f^{nbd}\delta_{\mu\nu}\delta_{\beta\varrho} = \Gamma^{abcd}_{2\,\mu\nu\beta\varrho}$$

Fig.9.
Langevin tree rules using nonlocal form factors.

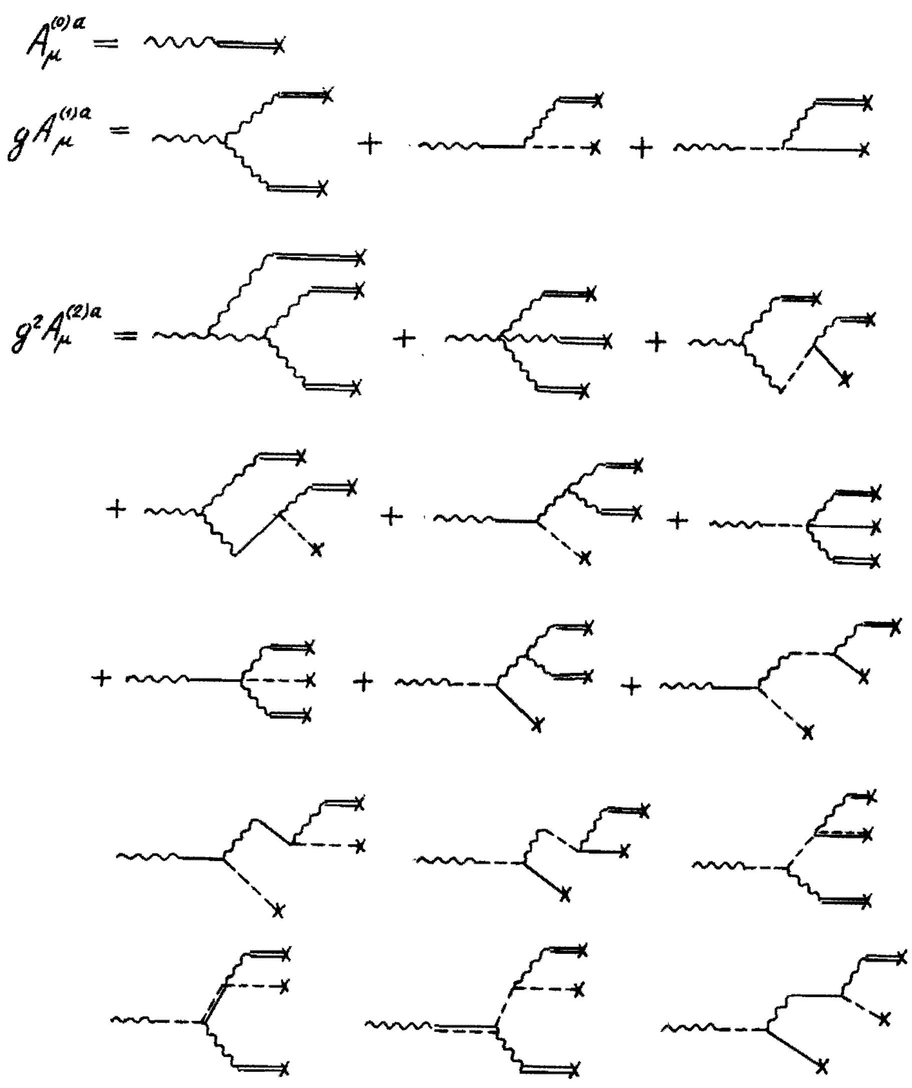


Fig.IO.

Langevin tree diagrams through $O(g^2)$ in the nonlocal stochastic scheme.

As a trivial example, we obtain the zeroth order two-point function. From the solution (5.16a) it follows in accordance with local noise property (5.2)

$$\langle A_\mu^{(0)a}(x,t)A_\nu^{(0)b}(y,t)\rangle = \delta^{ab}\int(dp)e^{-ip(x-y)}\left(T_{\mu\nu}(p)+\alpha L_{\mu\nu}(p)\right)\frac{V(-p^2\ell^2)}{p^2} \qquad (5.17)$$

or using the Langevin tree diagram shown in Fig.II

$$D_{\mu\nu}^{ab}(p;t_1,t_2) = 2\int_{-\infty}^{t_1}dt_3\int_{-\infty}^{t_2}dt_4\, G_{\mu\rho}^{ac}(p,t_1-t_3)\,G_{\nu\rho}^{bc}(p,t_2-t_4)\,\delta(t_3-t_4)\,V(-p^2\ell^2) =$$

$$= \delta^{ab}\left[T_{\mu\nu}(p)e^{-p^2(t_1-t_2)} + \alpha L_{\mu\nu}(p)e^{-p^2\frac{t_1-t_2}{\alpha}}\right]\frac{V(-p^2\ell^2)}{p^2}.$$

The result for the nonlocal free gluon propagator is just (5.17) Other free nonlocal Green functions are constructed according to the usual Wick expansion in terms of the result (5.17)

Fig.11.

A simple contraction for the nonlocal theory with form factor $V(-p^2\ell^2)$.

In the next section, we apply these Langevin equations and their rules for the nonlocal stochastic quantization theory to the computation of the one-loop gluon mass.

6. Vanishing Gluon Mass in the Nonlocal Stochastic Quantization Theory

Verification of gauge invariance in nonlocal stochastic quantization scheme with arbitrary form factors is crucial for its further developments. We will verify in this section that the QCD_4 gluon mass remains zero at the one-loop level, with any form factors $V(-p^2\ell^2)$ or $K(-p^2\ell^2)$. Our step to study this problem is following. First, we construot expressions

$$\Pi^{ab}_{\mu\nu}(x-y) = \langle A^{(1)a}_{\mu}(x,t)\, A^{(1)b}_{\nu}(y,t)\rangle_{\eta}$$

and

$$N^{ab}_{\mu\nu}(x-y) = \langle A^{(2)a}_{\mu}(x,t)\, A^{(0)b}_{\nu}(y,t)\rangle_{\eta}$$

by using equations (5.16). Second, with these obtained formulas, we sketch corresponding diagrams. It turns out that there are 47 distinct Langevin graphs in the two-point function at order $\sim g^2$ where diagrams trivially related by symmetry are not included in the count. **As a particular case (Bern's et al.**[14]**) it is seen that** only 10 make nonzero contributions to the mass renormalization, while only 2 contribute to the wave function and gauge parameter α renormalizations.

According to Bern et al.[14] we have found it convenient to **group 47 diagrams** into four classes (see diagrams sketched in Figs. 12–15) of which only the first class contributes to the wave function and α-renormalizations, and only the first two classes contribute to the mass renormalization. The third class contributes only to the finite part of the vacuum polarization, which will not be considered in this paper, while the diagrams in the fourth class vanish identically.

The structure of diagrams shown in Figs. 12–15 is similar with those considered by Bern et al.[14]. Therefore, we do not discuss them in detail and indicate only some their peculiarities. For example, the diagrams shown in Fig.12 contain only (Zwanziger gauge-fixed) Yang-Mills vertices, no form factor vertices, while the diagrams (Fig.13) contain at least one Γ_1 or Γ_2 regulator vertex, and provide the additional gluon mass contributions needed to cancel the contribution of the ordinary graphs (Fig.12). We notice that for this class of diagrams, contributions to wave function or α-renormalizations are absent. The diagrams, shown in Fig.14, also contain regulator vertices, but contribute only to the finite part of the vacuum polarization. Finally, the group of diagrams (Fig.15) vanishes identically. Some (the tadpole loops) of them vanish as usual by f^{abc} antisymmetry. The remaining diagrams vanish due to the (fifth-time) retarded property of the Langevin Green functions, which contribute a factor of $\theta(t_1-t_2)\,\theta(t_2-t_1)=0$ to each diagram.

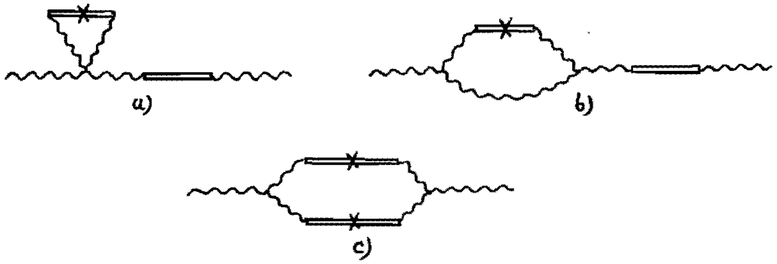


Fig.12.

"Ordinary" nonvanishing Langevin diagrams in nonlocal stochastic quantization scheme.

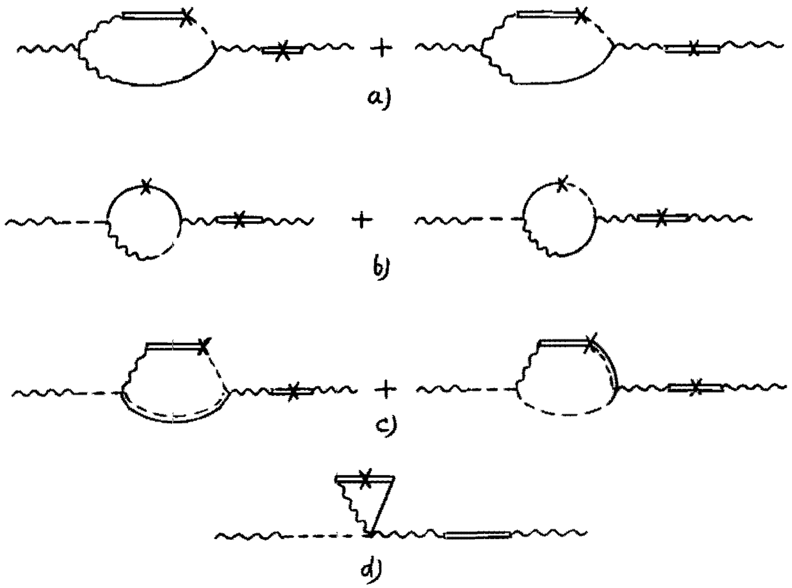


Fig.13.

Diagrams with nonlocal regulator vertices that also contribute to gluon mass.

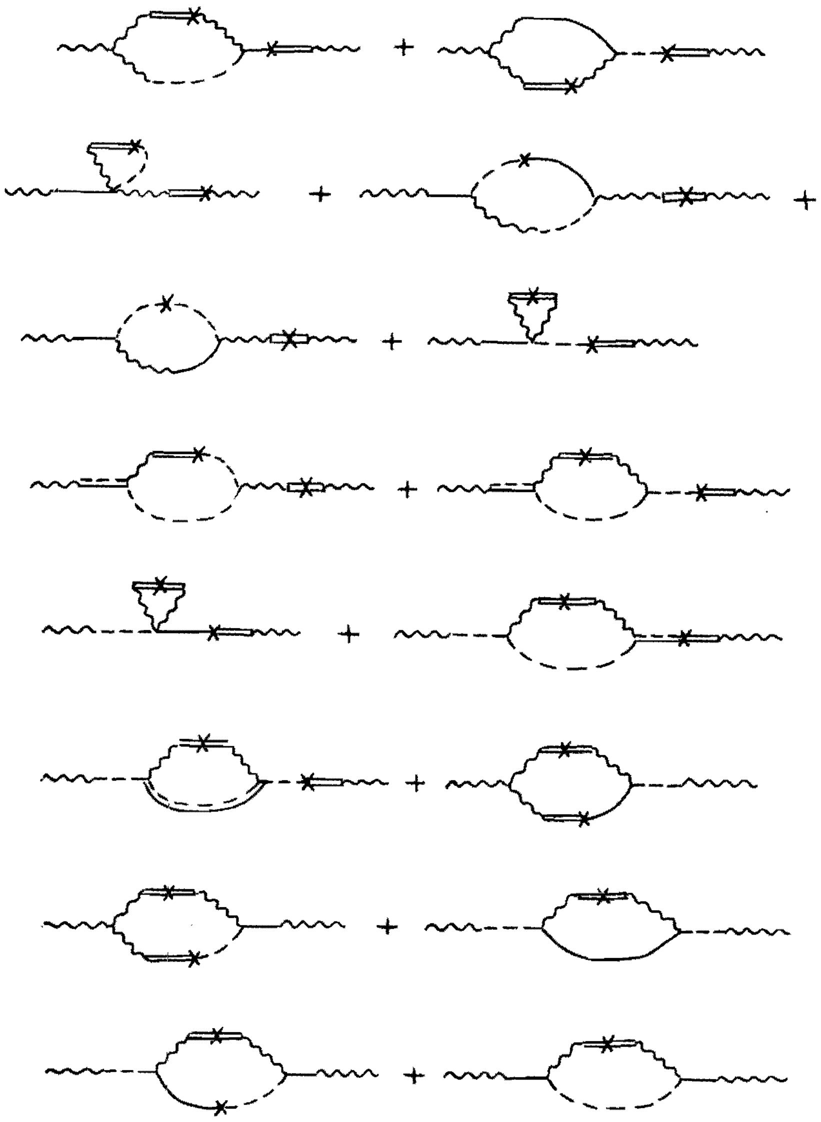


Fig.14.

Diagrams with nonlocal regulator vertices, which are finite as $\ell \to 0$.

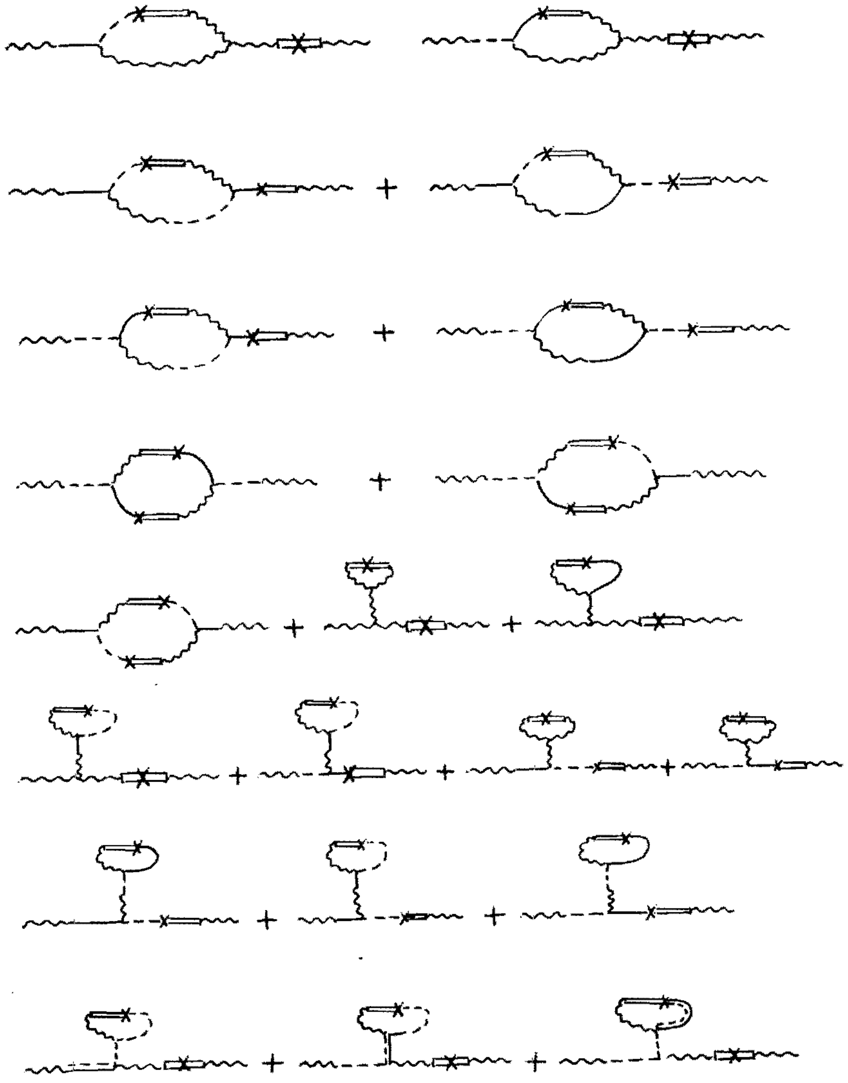


Fig.15.

Diagrams that vanish identically in nonlocal stochastic scheme .

In order to compute explicit contributions to renormalization mass correction due to diagrams shown in Figs.12 and 13 we study expressions $\Pi^{ab}_{\mu\nu}(x-y) = \langle A^{(1)a}_{\mu}(x,t)\, A^{(1)b}_{\nu}(y,t)\rangle_{\eta}$. Thus, taking into account the formula (5.16b) it is easily seen that explicit contribution from diagram 12c is calculated by using the following formula

$$\Pi^{ab}_{\mu\nu}(x-y) = \langle \Sigma^{a}_{\mu}(x,t)\, \Sigma^{b}_{\nu}(-y,t)\rangle_{\eta}, \tag{6.1}$$

where

$$\Sigma^{a}_{\mu}(x,t) = \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1 \int_{-\infty}^{t'} dt_2 \int (dp_1)(dp_2)(dp_3)\, e^{-ip_1 x}\, G^{aa_1}_{\mu\mu_1}(p_1, t-t') \times$$

$$\times G^{en}_{\beta\delta}(p_2, t'-t_1)\, K(-p_2^2 l^2)\, \bar{\delta}^{d}(p_1+p_2+p_3) \left[W^{a,ec}_{\mu_1\beta\theta}(p_1 p_2 p_3)\, G^{cs}_{\theta\rho}(p_3, t'-t_2) \times\right.$$

$$\left.\times K(-p_3^2 l^2) - \frac{1}{2}\Gamma^{a,es}_{1\,\mu,\rho\beta}\, \delta(t'-t_2) \left(H(p_1^2 l^2) K^{(1)}(-p_3^2 l^2) + H(p_3^2 l^2) K^{(1)}(-p_1^2 l^2) \right)\right] \times$$

$$\times \eta^{n}_{\delta}(p_2, t_1)\, \eta^{s}_{\rho}(p_3, t_2).$$

Here explicit form of vertices $W^{a,ec}_{\mu,\beta\theta}(p_1, p_2, p_3)$ and $\Gamma^{a,es}_{1\,\mu,\rho\beta}$ is sketched in Fig.9. Majority of terms in (6.1) corresponds to some finite and zero-diagrams shown in Fig.14 and 15. Further, according to the formula (5.2) we make noise contraction in (6.1), perform the fifth-time integrations, separate term giving contribution in accordance with diagram 12c and integrate over momentum variable with form factor $V(-p^2 l^2)$. Thus, after some tensor algebra, we obtain explicit leading value for this diagram near $p=0$ as

$$\Pi^{ab}_{1\,\mu\nu}(p) = -f^{amn} f^{bmn} \frac{g^2}{16\pi^2}\, \Delta_{\mu\beta}(p) \left(\frac{5}{2}\delta_{\beta\nu} p^2\right) \ln \mu^2 l^2,$$

where

$$\Delta_{\mu\beta}(p) = [\, T_{\mu\beta}(p) + \alpha L_{\mu\beta}(p)]\, p^{-2}.$$

Truncation near $p=0$ is accomplished by removal of the factor. We see that this term gives contribution to the wave function renormalization only. Now we study diagrams which give contribution to the gluon mass renormalization.

Contribution to mass renormalization due to diagram 12a arises from contraction result between second term for (5.16c) and $A^{(0)b}_{\nu}(x,t)$ in (5.16a):

$$\Pi^{ab}_{2\,\mu\nu}(p) = \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1 \int_{-\infty}^{t'} d\tau \int (dp_2)\, G^{aa_1}_{\mu\beta}(p, t-t')\, \theta(t-t')\, \theta(t'-\tau) \times$$

$$\times \left\{ \delta^{km}\delta^{eb} \Delta'_{\rho_1\rho_2}(p_2, t'-t_1)\, \Delta'_{\rho_3\nu}(p, t'-\tau) + \delta^{me}\delta^{kb} \Delta'_{\rho_1\rho_3}(p_2, t'-t_1) \times\right.$$

$$\left.\times \Delta'_{\rho_2\nu}(p, t'-\tau) + \delta^{mb}\delta^{kl} \Delta'_{\rho_1\nu}(p, t'-\tau)\, \Delta'_{\rho_2\rho_3}(p_2, t'-t_2) \right\} \times$$

$$\times V(-p_2^2 l^2)\, V(-p^2 l^2)\, W^{a,mke}_{\beta\rho_1\rho_2\rho_3},$$

where $\Delta'_{\rho\beta}(p,t) = T_{\rho\beta}(p)\, e^{-tp^2} + L_{\rho\beta}(p)\, e^{-tp^2/\alpha}$ and $W^{a,mke}_{\beta\rho_1\rho_2\rho_3}$ are presented in Fig.9. After some elementary calculation, we get

$$\Pi^{ab}_{2\,\mu\nu}(p) = -f^{amn} f^{bmn} g^2 \Delta_{\mu\beta}(p)\, \Delta_{\beta\nu}(p) \left(\frac{3+\alpha}{4}\right) 3 \cdot \int (dq)\, \frac{V(-q^2 l^2)}{q^2}.$$

Infraviolet divergence in this term is caused by zero mass of gluon field. Assuming $q^2 \to q^2 + \varepsilon$ result reads

$$\Pi^{ab}_{2\,\mu\nu}(p) = f^{amn} f^{bmn} g^2 \Delta_{\mu\beta}(p)\, \Delta_{\beta\nu}(p) \frac{1}{16\pi^2} \left(-\frac{3\sigma}{l^2} \frac{3+\alpha}{4} \right). \tag{6.2}$$

Here for SU(N) $f^{amn} f^{bmn} = \delta^{ab} N$.

Now we calculate corrections to the gluon mass renormalization due to diagrams shown in Fig.12b and Fig.13a, which are calculated by using contraction of first term in (5.16c) with $A^{(0)b}_{\nu}(x,t)$. Corresponding expressions take the form

$$\Pi^{as}_{3\,\mu\sigma}(p) = 16 \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1 \int (dp_1)\, V(-p^2 l^2)\, V(-(p-p_1)^2 l^2)\, \theta(t-t')\, \theta(t'-t_1)\, \theta(t-t_1) \times$$

$$\Delta'_{\nu\nu_2}(p_1, t'-t_1)\, \Delta'_{\mu\nu}(p, t-t')\, \Delta'_{\lambda\sigma}(p_1, t-t_1)\, \Delta_{\beta\rho_1}(p-p_1, t'-t_1). \tag{6.3}$$

$$W^{anm}_{\nu\beta\nu_1}(-p, p-p_1, p_1)\, W^{mns}_{\nu_2\rho_1\lambda_1}(-p_1, p_1-p, p)$$

and

$$\Pi^{as}_{4\mu\sigma}(p)=2g^2\delta^{as}N\ell^2\int(dq)\Big\{\frac{q_\nu q_\sigma}{2q^4}(\alpha-3)V(-q^2\ell^2)K(-p^2\ell^2)\times$$

$$\times\big[H(q^2\ell^2)K^{(1)}(-p^2\ell^2)+H(p^2\ell^2)K^{(1)}(-q^2\ell^2)\big]+\frac{4q_\nu q_{\lambda_1}}{q^2}V(-p^2\ell^2)\times \tag{6.4}$$

$$\times K(-q^2\ell^2)H(q^2\ell^2)K^{(1)}(-q^2\ell^2)\Big\}\Delta_{\sigma\lambda_1}(p)\,\Delta_{\mu\nu}(p)$$

respectively. In (6.3) integration over fifth-time variables should be carried out, after which this expression is reduced to analogous formula for $\Pi^{as}_{4\mu\nu}(p)$ in (6.4):

$$\Pi^{as}_{3\mu\sigma}(p)=\delta^{ab}Ng^2\Delta_{\mu\nu}(p)\Delta_{\lambda_1\sigma}(p)\big[\tfrac{5}{4}+\tfrac{3\alpha}{4}\big]\delta_{\nu\lambda_1}\int(dq)\,\frac{V(-q^2\ell^2)}{q^2}=$$

$$=\delta^{ab}Ng^2\Delta_{\mu\nu}(p)\Delta_{\nu\sigma}(p)\big[\tfrac{5+3\alpha}{4}\big]\frac{\sigma}{\ell^2}\,\frac{1}{16\pi^2}\,. \tag{6.5}$$

By definition (5.13) for the form factors $K^{(i)}(-p^2\ell^2)$ it is easily seen that first term with $K^{(1)}(-p^2\ell^2)$ in (6.4) goes to zero at the limit $p^2\to 0$ and main asymptotic of its second term is constant, so that third term gives the following leading term

$$\Pi^{as}_{4\mu\sigma}(p)=\frac{\delta^{as}Ng^2}{16\pi^2}\Delta_{\mu\nu}(p)\Delta_{\nu\sigma}(p)\,[-\sigma\ell^{-2}]\,. \tag{6.6}$$

Analogously, contributions to the mass renormalization in QCD_4 due to diagrams shown in Fig.13b,c,d are calculated by using contraction of third, fourth and fifth terms in (5.16c) with $A^{(0)\,b}_{\nu}(x,t)$. Corresponding result reads

$$\Pi^{as}_{5\mu\sigma}(p)=\frac{\delta^{as}Ng^2}{16\pi^2}\Delta_{\mu\nu}(p)\Delta_{\nu\sigma}(p)\,V(-p^2\ell^2)H(p^2\ell^2)\,\frac{1}{\ell^2}\times$$

$$\times\Big[\frac{w(-2)}{2}-\frac{w(-2)}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}dy\,\frac{w(y)}{\sin\pi y}\,\frac{\Gamma(-y)}{\Gamma(2-y)}\,\varepsilon^{y}\ell^{2y} \tag{6.7a}$$

$$\Pi^{as}_{6\mu\sigma}(p)=-\frac{\delta^{as}Ng^2}{16\pi^2}\Delta_{\mu\nu}(p)\Delta_{\nu\sigma}(p)\Big[\frac{w(-2)}{2\ell^2}\Big]V(-p^2\ell^2)H(p^2\ell^2) \tag{6.7b}$$

$$\Pi^{as}_{7\mu\sigma}(p)=\frac{\delta^{as}Ng^2}{16\pi^2}\Delta_{\mu\nu}(p)\Delta_{\nu\sigma}(p)\,V(-p^2\ell^2)H(p^2\ell^2)\,\frac{w(-2)}{\ell^2}\,. \tag{6.7c}$$

In obtained expressions (6.2), (6.5)-(6.7) truncation near $\ell=0$ is accomplished by removal of the **two factors** $\Delta_{\alpha\beta}(p)$, all sum of resulting in these diagram 's contributions is zero, so the gluon remains massless in this order for the nonlocal stochastic quantization theory with arbitrary form factors. This generalizes the regularized scheme proposed by Bern et al. /14/ .

Thus, nonlocal method presented here for Langevin Schwinger-Dyson formalisms of stochastic quantization gives ultraviolet finiteness to all orders for gauge theory Green functions in d dimensions and ensures its gauge invariance. The latter is achieved by using the covariant Laplacian function (in which the gauge-fixing term is absent) in the construction of the theory. In our case, the nonlocal distribution $K_{xy}(\Box)$ is translation invariant and so that a gauge-covariant parallel transport of the local noise guarantees the gauge covariance of the **regularized** Langevin system under the local d-dimensional gauge transformation (for detail, see Bern et al. /14/):

$$\dot{A}^{a}_{\mu}(x,t)\Longrightarrow\Omega^{ab}(x)\,\dot{A}^{b}_{\mu}(x,t)\qquad,$$

$$\eta^{a}_{\mu}(x,t)\Longrightarrow\Omega^{ab}(x)\,\eta^{b}_{\mu}(x,t)\qquad,$$

$$K^{ab}_{xy}(\Delta)\Longrightarrow\Omega^{aa'}(x)\,\Omega^{bb'}(y)\,K^{a'b'}_{xy}(\Delta)\,,$$

where $\Omega(x)\in SO(N^2-1)$ is the adjoint representation of SU(N).

7. Scalar Electrodynamics

For concrete computational purpose, we present **the method of electrodynamics construction of** charged spinless particles and illustrate the extension of the scheme to include mather multiplets. As in Yang-Mills, the basic idea is that gauge-invariance is maintained by choosing each form factor as a function of the covariant derivative in the relevant representation.

The nonlocal and Zwanziger gauge-fixed Langevin system for scalar electrodynamics (SED) takes the form

$$\dot{A}_\mu(x,t) = -\frac{\delta S}{\delta A_\mu}(x,t) + \partial_\mu Z(x,t) + \int (dy)\, K_{xy}(\Box)\, \eta_\mu(y,t) \tag{7.1a}$$

$$\dot{\phi}(x,t) = -\frac{\delta S}{\delta \phi^*}(x,t) + ie\phi(x,t) Z(x,t) + \int (dy)\, K_{xy}(\Delta)\, \eta(y,t) \tag{7.1b}$$

$$\dot{\phi}^*(x,t) = -\frac{\delta S}{\delta \phi}(x,t) - ie\phi^*(x,t) Z(x,t) + \int (dy)\, K_{xy}(\Delta^*)\, \eta^*(y,t)\,, \tag{7.1c}$$

where local noises satisfy the usual relations

$$\langle \eta_\mu(x,t)\, \eta_\nu(y,t') \rangle = 2\, \delta_{\mu\nu}\, \delta(t-t')\, \delta^d(x-y) \tag{7.2a}$$

$$\langle \eta^*(x,t)\, \eta(y,t') \rangle = 2\, \delta(t-t')\, \delta^d(x-y)\,. \tag{7.2b}$$

Here

$$S = \int (dx) \left[\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} + |(\partial_\mu - ieA_\mu)\phi|^2 \right] \tag{7.2c}$$

is the usual Euclidean action of SED constructed by using local fields $A_\mu(x,t)$ and $\phi(x,t)$. In contradistinction to nonlocal quantum field theory (Efimov/12,15/), interaction Lagrangian in (7.2c) is local. The appropriate covariant Laplacians for the charged scalar fields are

$$\Delta_{xy} = \int (dz) (D_\mu)_{xz} (D_\mu)_{zy}\,, \qquad (D_\mu)_{xy} = (\partial^x_\mu - ieA_\mu(x))\, \delta^d(x-y)\,,$$
$$\Delta^*_{xy} = \int (dz) (D^*_\mu)_{xz} (D^*_\mu)_{zy}\,, \qquad (D^*_\mu)_{xy} = (\partial^x_\mu + ieA_\mu(x))\, \delta^d(x-y) \tag{7.3}$$

and we will choose $\alpha Z = \partial A$ as above.

Further, to check the finiteness and gauge-invariance of system we compute, as in Sec.6, the d=4 one-loop photon mass using Langevin techniques. We first need the integral form of the Langevin system

$$A_\mu(x,t) = \int (dy) \int_{-\infty}^{t} (dt')\, G_{\mu\nu}(x-y,t-t') \big[-ie\phi^*(y,t') (\overrightarrow{\partial}_\nu - \overleftarrow{\partial}_\nu) \phi(y,t') -$$
$$- 2e^2 \phi^*(y,t') \phi(y,t')\, A_\nu(y,t') + \int (dz)\, K_{xy}(\Box)\, \eta_\nu(z,t') \big] \tag{7.4}$$

$$\phi(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G(x-y,t-t') \big[-ieA_\mu(y,t')\, \partial_\mu \phi(y,t') -$$
$$- ie\partial_\mu (A_\mu(y,t') \phi(y,t')) + ie \frac{1}{\alpha} \phi(y,t')\, \partial_\mu A_\mu(y,t') -$$
$$- e^2 \phi(y,t') A_\mu(y,t') A_\mu(y,t') + \int (dz)\, K_{yz}(\Delta)\big|_{t'}\, \eta(z,t') \big] \tag{7.5}$$

with a similar equation for ϕ^* . Here

$$G_{\mu\nu}(x-y,t-t') = \theta(t-t') \int (dp)\, e^{-ip(x-y)} \left[T_{\mu\nu}(p)\, e^{-p^2(t-t')} + L_{\mu\nu}(p)\, e^{-p^2(t-t')/\alpha} \right] \tag{7.6a}$$

$$G(x-y,t-t') = \theta(t-t') \int (dp)\, e^{-ip(x-y)} \exp\left[-(p^2+m^2)(t-t') \right] \tag{7.6b}$$

are the photon and scalar Langevin Green functions, respectively.

The first step in a weak coupling expansion of (7.4) and (7.5) is the expansion of the charged scalar form factor to the desired order which is given by the formula (5.12) in Sec.5. There $g \leftrightarrow e$

$$(\Gamma_1)^{ab}_{xy} \Rightarrow (\Gamma_1)_{xy} = -i (\partial^x_\mu A_\mu(x,t) + A_\mu(x,t) \partial^x_\mu)\, \delta^d(x-y)\, \ell^2\,, \tag{7.7}$$
$$(\Gamma_2)^{ab}_{xy} \Rightarrow (\Gamma_2)_{xy} = -A_\mu(x,t) A_\mu(x,t)\, \delta^d(x-y)\, \ell^2$$

should be changed. As usual, in (7.7) the derivatives act on everything to the right. This expression may be continued to all orders as shown in Fig.16. In the figure, in accordance with the diagrams (Fig.9) each specific lines correspond to form factors K , $K^{(1)}$, $K^{(2)}$ and $H(p^2\ell^2)$ and wavy lines correspond to gauge fields, while the three-,four- and five-point vertices represent Γ_1 and Γ_2 , respectively. The filled arrows ($\longrightarrow$) denote the retarded property of the Langevin Green functions, while the other arrows ($\longrightarrow$) track the direction of the charge flow on a scalar lines.

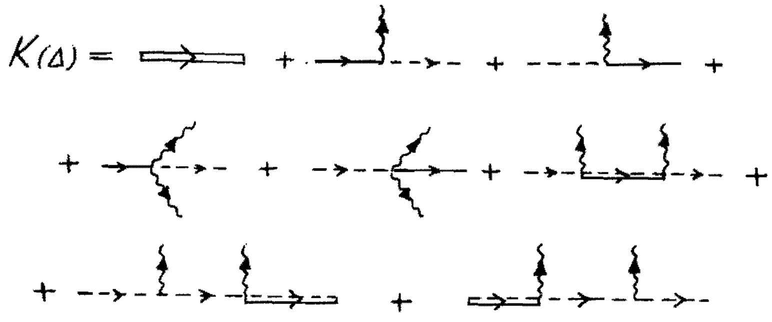


Fig. 16.

Expansion of the charged scalar form factor in the nonlocal stochastic scheme.

Having expanded the form factor, an essentially standard (Parisi and Wu[11]; Bern et al.[14]) iterative procedure allows the expansion of the Langevin solution

$$A_\mu[\eta] = \sum_{m=0}^{\infty} e^m A_\mu^{(m)} \tag{7.8}$$

$$\phi(\eta) = \sum_{m=0}^{\infty} e^m \phi^{(m)} \quad , \quad \phi^*(\eta) = \sum_{m=0}^{\infty} e^m \phi^{*(m)} \tag{7.9}$$

to any desired order. For the photon mass computation, the relevant results with the form factor $K^{(1)}(-p^2\ell^2)$ are

$$A_\mu^{(0)}(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G_{\mu\nu}(x-y,t-t') \int (dz)\, K_{zy}(\Box)\, \eta_\nu(z,t') \tag{7.10a}$$

$$e A_\mu^{(1)}(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G_{\mu\nu}(x-y,t-t') \left[-ie\phi^{*(0)}(y,t')(\vec{\partial}_\nu - \overleftarrow{\partial}_\nu)\phi^{(0)}(y,t') \right] \tag{7.10b}$$

$$e^2 A_\mu^{(2)}(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G_{\mu\nu}(x-y,t-t') \left[-ie^2\phi^{*(1)}(y,t')(\vec{\partial}_\nu - \overleftarrow{\partial}_\nu)\phi^{(0)}(y,t') - \right.$$

$$\left. -ie^2\phi^{*(0)}(y,t')(\vec{\partial}_\nu - \overleftarrow{\partial}_\nu)\phi^{(1)}(y,t') - 2e^2 A_\nu^{(0)}(y,t')\phi^{*(0)}(y,t')\phi^{(0)}(y,t') \right] \tag{7.10c}$$

and

$$\phi^{(0)}(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G(x-y,t-t') \int (dz)\, K_{yz}(\Box)\, \eta(z,t') \quad , \tag{7.11a}$$

$$e\phi^{(1)}(x,t) = \int (dy) \int_{-\infty}^{t} dt'\, G(x-y,t-t') \Big\{ -ie\Big[A_\mu^{(0)}(y,t')\partial_\mu \phi^{(0)}(y,t') +$$

$$+ \partial_\mu \left(A_\mu^{(0)}(y,t')\phi^{(0)}(y,t') \right) - \frac{1}{\alpha}\phi^{(0)}(y,t')\,\partial_\mu A_\mu^{(0)}(y,t') \Big] +$$

$$+ \frac{e}{2} \int (dz) \left[H(\Box)\Gamma_1 K^{(1)}(\Box) + K^{(1)}(\Box)\Gamma_1 H(\Box) \right]_{yz} \eta(z,t') \Big\} . \tag{7.11b}$$

Such expansions may be represented diagrammatically to all orders as Langevin tree graphs, shown in Fig.17.

According to Bern et al.[14] these Langevin tree diagrams may be constructed to all orders from the simple set of momentum space Langevin tree rules shown in Fig.18. Finally, the diagrams of the n-point nonlocal Green functions are formed by contracting the trees, as usually according to Eqs. (7.a) and (7.2b).

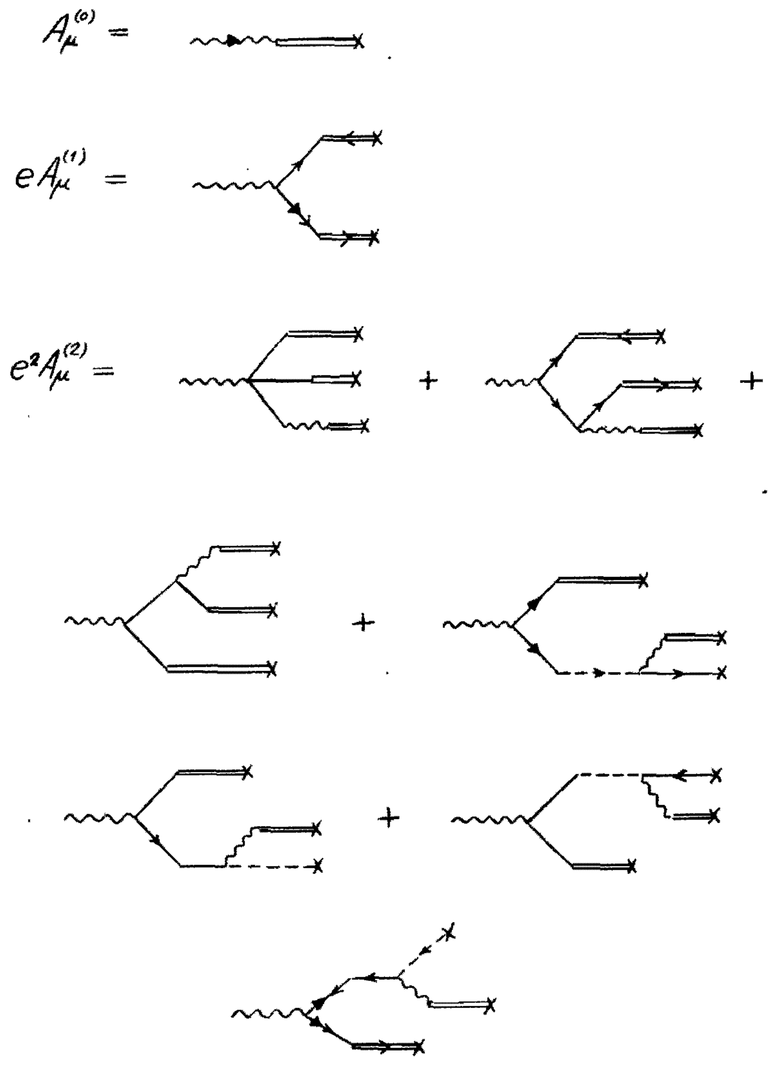
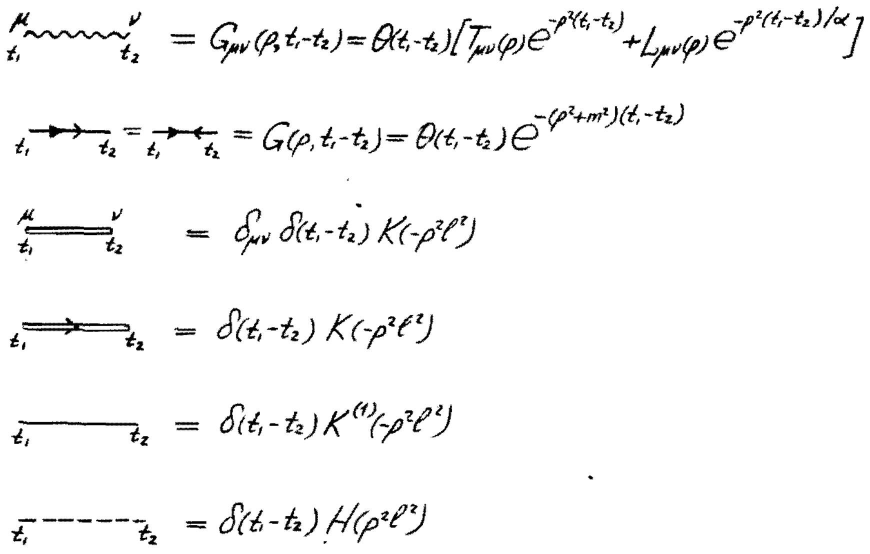


Fig.17.
Langevin tree diagrams for photon field in the nonlocal stochastic scheme.

Propagators:



$$= G_{\mu\nu}(p,t_1-t_2)=\theta(t_1-t_2)[T_{\mu\nu}(p)e^{-p^2(t_1-t_2)}+L_{\mu\nu}(p)e^{-p^2(t_1-t_2)/\alpha}]$$

$$= G(p,t_1-t_2)=\theta(t_1-t_2)e^{-(p^2+m^2)(t_1-t_2)}$$

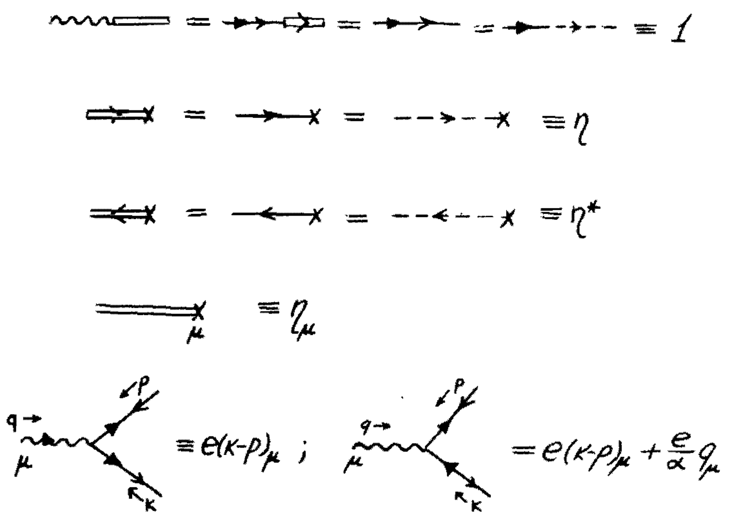
$$= \delta_{\mu\nu}\,\delta(t_1-t_2)\,K(-p^2\ell^2)$$

$$= \delta(t_1-t_2)\,K(-p^2\ell^2)$$

$$= \delta(t_1-t_2)\,K^{(1)}(-p^2\ell^2)$$

$$= \delta(t_1-t_2)\,H(p^2\ell^2)$$

Vertices



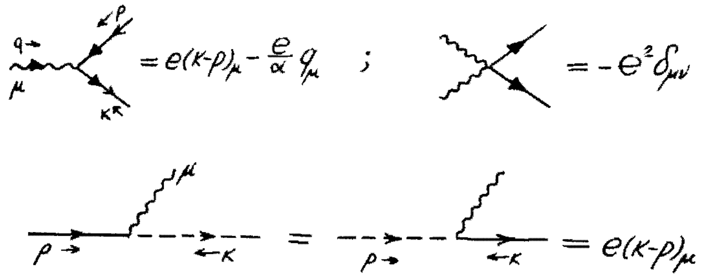


Fig.18.

Langevin tree rules for scalar electrodynamics in the nonlocal stochastic scheme.

There are three types of diagrams (Fig.19) giving nonvanishing contributions to the zero-momentum vacuum polarization, in which we do not explicitly exhibit diagrams which are trivially related by symmetry. We now go to study these diagrams. First, to calculate corresponding contributions, expression of $A^{(2)}_\mu(x,t)$ should be found. In accordance with (7.4) its value in momentum representation acquires the following form:

$$
\begin{aligned}
A^{(2)}_\mu(p,t) = {} & 2e^2 \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1 \int_{-\infty}^{t'} dt_2 \int (dp_1)(dp_2)(dp_3)\, G_{\mu\nu}(p,t-t') \times \\
& \times G(p_2, t'-t_2) K(-p_1^2 \ell^2) \Big\{ \int_{-\infty}^{t_2} dt_3 \int_{-\infty}^{t_2} dt_4 \int (dp_4)\, G(p_1, t'-t_1)(p_1+p_2)_\nu \times \\
& \times \bar{\delta}^d(p+p_1-p_2)\, \bar{\delta}^d(p_2-p_3-p_4)\, G_{\delta\sigma}(p_3, t_2-t_3)\, G(p_4, t_2-t_4) \times \\
& \times K(-p_3^2 \ell^2) K(-p_4^2 \ell^2) \Big[(p_3+2p_4)_\delta - \frac{1}{\alpha} p_{3\delta} \Big] \eta^*(p_1,t_1)\, \eta(p_4,t_4)\, \eta_\sigma(p_3,t_3) + \\
& + \int_{-\infty}^{t_2} dt_3 \int (dp_4)\, G(p_1, t'-t_1)(2p_4+p_3)_\rho (p_1+p_2)_\nu\, \bar{\delta}^d(p+p_1-p_2)\, \bar{\delta}^d(p_2-p_3-p_4) \times \\
& \times G_{\rho\beta}(p_3, t_2-t_3) K(-p_3^2 \ell^2) \frac{1}{2} \Big[H(p_2^2 \ell^2) K^{(1)}(-p_4^2 \ell^2) + K^{(1)}(-p_2^2 \ell^2) H(p_4^2 \ell^2) \Big] \times
\end{aligned}
\tag{7.12}
$$

$$
\begin{aligned}
& \times \eta^*(p_1,t_1)\, \eta_\beta(p_3,t_3)\, \eta(p_4,t_4) - \int_{-\infty}^{t'} dt_3\, G(p_3, t'-t_3) \times \\
& \times G_{\nu\beta}(p_1, t'-t_1)\, \bar{\delta}^d(p+p_2-p_1-p_3)\, K(-p_2^2 \ell^2) K(-p_3^2 \ell^2) \times \\
& \times \eta_\beta(p_1,t_1)\, \eta^*(p_2,t_2)\, \eta(p_3,t_3) \Big\}.
\end{aligned}
$$

Next, following the methods of Sec.6 and using contractions between $A^{(1)}_\mu(x,t)$ and $A^{(1)}_\nu(y,t)$ [$A^{(2)}_\mu(x,t)$ and $A^{(0)}_\nu(y,t)$] one can obtain the explicit value for diagrams, shown in Fig.19.

Thus, the diagram 19a gives the following contribution to the photon mass renormalization

$$
\begin{aligned}
\Pi^{(a)}_{\mu\nu}(p) = {} & -8e^2 \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1 \int_{-\infty}^{t'} dt_2 \int (dp_2)\, G_{\mu\rho}(p, t-t')\, G_{\rho\beta}(p, t'-t_1)\, G(p_2, t'-t_2) \times \\
& \times G(p_2, t'-t_2)\, G_{\nu\delta}(p_2, t'-t_1)\, \delta_{\beta\delta}\, K(-p^2 \ell^2) K(-p_2^2 \ell^2)
\end{aligned}
$$

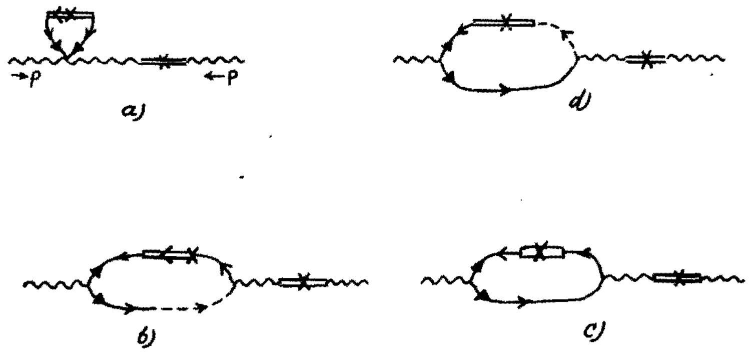


Fig.19.

Nonvanishing contributions to photon mass in the nonlocal stochastic scheme.

After integration over fifth-time variables and truncation near $p=0$ which is accomplished by removal of the two $\Delta_{\mu\rho}=(T_{\mu\rho}(p)+\alpha L_{\mu\rho}(p))p^{-2}$ factors, we obtain at equilibrium

$$\Pi^{(a)}_{\mu\nu}(0)=-2\ell^2\int(dq)\,\frac{V(-q^2\ell^2)}{m^2+q^2}=-2e^2\,\frac{\sigma+m^2\ell^2\cdot\ln\mu^2\ell^2}{16\pi^2\ell^2}\,. \tag{7.13}$$

where we have assumed $V(-p^2\ell^2)/_{p^2\to 0}=1$ by the normalization condition and notation $\sigma=\lim\limits_{x\to -1} v(x)/(1+x)$. It is easily seen that contributions corresponding to diagrams 19b,c are equal to each other, explicit value of which is given by

$$\Pi^{(b)}_{\mu\nu}(p)=\Pi^{(c)}_{\mu\nu}(p)=-4e^2V(-p^2\ell^2)\ell^2\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt_2\int_{-\infty}^{t'}dt_3\int(dp_1)\,G_{\mu\rho}(p,t-t')\times$$

$$\times G(p_1,t'-t_2)\,G(p+p_1,t'-t_2)\,G_{\delta\beta}(p,t_2-t_3)\,G_{\nu\beta}(p,t'-t_3)\,(2p_1+p)_\rho\times$$

$$\times(2p_1+p)_\delta\,K(-p_1^2\ell^2)\,K^{(1)}(-(p+p_1)^2\ell^2)\,H(p_1^2\ell^2)\,.$$

Elementary integration over fifth-time variables gives in the limit $p\to 0$

$$\Pi^{(b)}_{\mu\nu}(p)=-e^2\Delta_{\mu\rho}(p)\Delta_{\rho\nu}(p)\,\ell^2\,V(-p^2\ell^2)\int(dq)\,\frac{q^2K(-q^2\ell^2)K^{(1)}(-q^2\ell^2)H(q^2\ell^2)}{m^2+q^2} \tag{7.14}$$

or

$$\Pi^{(b)}_{\mu\nu}(0)=\frac{e^2}{16\pi^2}\,\frac{\sigma}{2\ell^2}\,.$$

Finally, contribution corresponding to diagram 19d is

$$\Pi^{(d)}_{\mu\nu}(p)=-8e^2V(-p^2\ell^2)\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt_2\int_{-\infty}^{t_2}dt_3\int_{-\infty}^{t_2}dt_4\int(dq)\,G_{\mu\rho}(p,t-t')\times$$

$$\times(2q+p)_\rho\,G(q_1,t'-t_4)\,G(p+q,t'-t_2)\left[(2q+p)_\delta-\frac{1}{\alpha}p_\delta\right]\times$$

$$\times G_{\delta\sigma}(p,t_2-t_3)\,G_{\sigma\nu}(p,t'-t_3)\,G(q,t_2-t_4)\,V(-q^2\ell^2)\,.$$

Here some integrations over fifth-time variables and d^4q should be carried out and the result reads to the limit $p\to 0$

$$\Pi^{(d)}_{\mu\nu}(0)=\frac{e^2}{16\pi^2\ell^2}\left[\sigma+2m^2\ell^2\,\ln\mu^2\ell^2\right]. \tag{7.15}$$

The reader may easily verify that the sum of all contributions is zero, so the photon remains massless to this order for the nonlocal stochastic quantization theory, as it **should be.**

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Нелокальность и стохастическое квантование физических полей

Предложен метод введения нелокальности в стохастическую формулировку полевой теории в рамках уравнений Ланжевена и Швингер-Дайсона. В этих уравнениях белый шум играет двойную роль: он контролирует квантовое поведение физических систем и одновременно вносит нелокальность в теорию. Полученная таким образом схема полностью воспроизводит результаты нелокальной квантованной теории поля. При этом лагранжиан взаимодействия и поля остаются локальными. Представлены стохастические регуляризационные процедуры для скалярных и калибровочных полей, а также подробно изучена скалярная электродинамика. В нашей схеме условия унитарности и градиентной инвариантности выполняются.

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Nonlocality and Stochastic Quantization of Field Theory

Concept of nonlocality is introduced into physics by means of stochastic context using Langevin and Schwinger-Dyson techniques. This allows us to reformulate finite theory of quantum field, free from ultraviolet divergences, based on the stochastic quantization method with nonlocal regulators. As a nonlocal regulator we choose any entire analytic function in the momentum space, which guarantees that our regularization method for any theory of interest does not violate basic physical principles such as unitarity, causality, and gauge invariance of the theory. Here we present regularization scheme for scalar, gauge and scalar electrodynamics theories. Our mathematical prescription is similar to continuum regularization method of quantum field theory with meromorphical regulators investigated by Bern and his team.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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