объединенный
институт
ядерных
исследований
дубна

E2-88-553

A.T.Filippov

A GAUGE MODEL DESCRIBING N RELATIVISTIC PARTICLES BOUND BY LINEAR FORCES

Submitted to "Modern Physics Letters A"

1988

1. Introduction

The recent development of superstring theory has led to reconsidering basic ideas on the quantization of relativistic particle theories and even to reformulating classical ones. In the modern approach (see Refs.[1,2]) one starts from a gauge-like formulation of the relativistic particle theory[3,4] in which constraints, such as $p^2 + m^2 = 0$, $p \cdot \xi = 0$, etc., play a role of (super)gauge symmetry generators.[1] In the hamiltonian formalism, which is equivalent to the first-order lagrangian one, the constraints are introduced by use of Lagrange multipliers, and the Hamiltonian for one particle is a linear combination of the constraints.

For example, the Lagrangian describing relativistic spinless particles may be written as[1]

$$L_a = p^{\mu}\dot{q}_{\mu} - \tfrac{1}{2}l(t)(p^2 + m^2), \qquad (1)$$

where t is an evolution parameter, $0 \le t \le 1$, the dot denotes the parameter derivative, ∂_t; $q^{\mu} \equiv q^{\mu}(t) = (q^0, q^1, \ldots, q^{(D-1)})$ are the D-dimensional Minkowski space coordinates of the particle, p_{μ} are the conjugate momenta, and $l(t)$ is the Lagrange multiplier. This Lagrangian is equivalent to the reparametrization invariant one,[3]

$$L = \tfrac{1}{2}(\dot{q}^2/l - lm^2),$$

from which one obtains the usual Lagrangian $L = -m(-\dot{q}^2)^{1/2}$. All three Lagrangians give the equivalent equations of motion.

---

[1] In our notation Greek/Latin characters are used for Fermi/Bose variables; $q^{\mu}(t)$ are the coordinates; $p^{\mu}(t)$, momenta, $\xi^{\mu}(t)$ are anticommuting space-time vectors; $\mu = 0, 1, \ldots, (D-1)$, and we usually suppress these space-time indices.

However, the first one is the simplest to quantize, because powerful methods for quantization of systems with constraints in extended phase spaces are available (see, e.g., Refs.[1,2,4,5]).

Instead of reparametrization symmetry this Lagrangian has a gauge-like symmetry

$$\delta p = 0, \quad \delta q = f(t)p, \quad \delta l = \dot{f}(t), \tag{2}$$

where the transformation of the canonical variables is generated by the constraint $g = (p^2 + m^2)$,

$$\delta p = [f(t)g, p]_{PB} = 0, \quad \delta q = [f(t)g, q]_{PB} = f(t)p. \tag{3}$$

For fixed t, this defines the one-dimensional (translation) subgroup, T_1, of the linear canonical group which preserves Lorentz invariance and leaves g unchanged. The transformation of $l(t)$ is of different nature, it compensates the terms in δL_G resulting from the t-depedence of f. Thus, it is natural to view $l(t)$ as an abelian gauge potential over the base $0 \le t \le 1$. As the base is one-dimensional there is no corresponding gauge field but there exists a natural gauge invariant. As $\delta L_G = \frac{1}{2}[f(p^2 - m^2)]^{\cdot}$, the action corresponding to the Lagrangian (1) is invariant if $f(0) = f(1) = 0$. It follows that the integral

$$l_0 \equiv \int_0^1 dt\, l(t) \tag{4}$$

is invariant (l_0 is obviously the proper-time interval betwen the final and initial positions of the particle, $q(1)$ and $q(0)$). Thus the abelian gauge group T_1 is subdivided into gauge orbits enumerated by the parameter l_0.

To completely specify the dynamics one has to add a proper gauge-fixing condition. For example one may identify the parameter t with the time coordinate of the particle, $q^0 = t$, or choose the

Fradkin-Vilkovisky[5] gauge condition, $\dot{l} = 0$. In the first gauge all variables are physical but the Lorentz invariance is lost; in the second one, ghost variables should be added allowing for compensating unphysical variables while preserving manifest Lorentz invariance.[1-5]

For massless particles one can easily rewrite the Lagrangian (1) in a manifestly gauge form,[6]

$$L_G = \frac{1}{2}\Psi^T_\mu C(\partial_t - A)\Psi^\mu, \tag{5}$$

$$\Psi^T_\mu = [p_\mu, q_\mu], \quad C = i\sigma_2, \quad A = l(t)\sigma_-, \quad \sigma_- = \frac{1}{2}(\sigma_1 - \sigma_2),$$

where T means transposition, and σ_i are the Pauli matrices. In Eq.(5) we have dropped the term $\frac{1}{2}(pq)^{\cdot}$, which does not influence the equations of motion. It determines only the boundary coditions $f(0) = 0$, $f(1) = 0$, and including them in the definition of the gauge group we may leave it aside. Now the gauge transformation (3) can be written in the standard form

$$\delta\Psi = F\Psi, \quad \delta A = \dot{F} + [F, A] \equiv \dot{F} + FA - AF, \tag{6}$$

where the matrix F is obtained from A by simply substituting $f(t)$ for $l(t)$. Due to the abelian nature of the gauge group T_1 the term $[F,A]$, in this simple model, is zero and is written for later use.

Having a gauge theory, it is natural to ask what is its ungauged version. In this case the answer is very simple: take the Lagrangian (we will call it "rudimentary")

$$L_R = \frac{1}{2}\Psi^T C(\partial_t - H_R)\Psi = \frac{1}{2}(p\dot{q} - q\dot{p}) - \frac{1}{2}p^2, \tag{7}$$

where $H_R = \sigma_-$, and find its rigid linear symmetries. These obviously are given by

$$\delta\Psi = F\Psi; \quad CF + F^T C = 0, \quad [F, H_R] = 0, \tag{8}$$

and so the matrix F coincides with the matrix of the gauge transformation considered above. Now the general procedure of gauging can be applied. One substitutes $l(t)$ for $f(t)$ in F, call this new matrix A, and substitutes $(\partial_t - A)$ for ∂_t in Eq.(7). Absorbing the constant matrix H_R into A by redefining A, $(A + H_R) \to A$, one obtains the gauge Lagranian (5).

In constructing gauge theories by using the gauging procedure we usually have some freedom - one may add to the gauged Lagrangian some gauge invariant terms. In this model the term $-\frac{1}{2}l(t)m^2$ can be added to the Lagrangian (5) as it gives the gauge invariant term $-\frac{1}{2}l_o m^2$ in the action. A more elegant way of introducing the mass term consists in adding one more euclidean dimension to the phase space. Constructing the Lagrangian (5) in (D+1) dimensions and performing the dimensional reduction simply by setting $q^{(D+1)} = 0$ one observes that $\dot{p}^{(D+1)} = 0$. Denoting $p^{(D+1)} \equiv m$ we obtain the massive theory (1).

The final question is: what is the meaning of the ungauged Lagrangian? With the Minkowski metric it has no reasonable physical interpretation, at least at the quantum level. However, if we reduce the phase space to the euclidean one by setting $q^0 = 0$, $p^0 = 0$ the resulting Lagrangian will make sense. It is the Lagrangian of the $m = 1$ nonrelatitivistic particle with the coordinates q^n and momenta p^n, $n = 1,\dots,D-1$.

The crucial observation of Ref.6 is that the above reasoning relating the gauge Lagrangian to the nonrelativistic one,

$$L_{NR} = p_n\dot{q}^n - \tfrac{1}{2}p^n p_n,$$

can be reversed to construct the relativistic particle Lagrangian corresponding to a given nonrelativistic Lagrangian. One starts with the galilean invariant Lagrangian L_{NR}, extends the nonrelativistic phase-space coordinates q^n,p_n to the relativistic

ones, q^μ,p_μ, and writes the rudimentary Lagrangian L_R which is the Poincare invariant extension of L_{NR}. Then all linear canonical (Lorentz invariant) symmetries of L_R can easily be found by using Eq.(8), and the gauge potential A can be constructed. The usefulness of this observation consists in the possibility of performing all these steps for more complicated theories. As it has been demonstrated in Refs.[7-9], this way one can obtain theories of spinning relativistic particles[3,4], bosonic and fermionic strings, as well as some other relativistic theories, starting from corresponding nonrelativistic theories[10] and then applying the described procedure.

2. Gauging Canonical Transformations

Here we present a general formulation of our approach to gauging linear canonical symmetries. We start from some reasonable nonrelativistic Lagrangian for N particles with cartesian coordinates $q^n_i(t)$ moving in the (D-1)-dimensional euclidean space $(n = 1,\dots,(D-1);\ i = 1,\dots,N))$.

To describe the spin degrees of freedom of the particles we follow the ideas of Ref.10 and introduce some Grassmann variables, $\xi^\alpha(t)$. We only treat here the Grassmann variables transforming as vectors and scalars under rotations; the relativization of theories with Grassmann spinors is a more complicated matter. Thus consider a collection

$$\xi^\alpha = (\xi^n_r,\ \xi_s), \quad r = 1,\dots,R;\ s = 1,\dots,S,$$

where n is the vector index, and ξ_s are scalars. In the one-particle case the numbers R and S specify the spin content of the particle. For example, R = 1, S = 0 corresponds to a massless particle with spin $\frac{1}{2}$; to describe a massive particle one scalar variable has to be added (R = 1, S = 1).[10] In what follows we only use the vector Grassmann variables ξ^n_r. Note that in the N-particle case the spin variables are not attached to individual particles, this is similar to the description of spin on strings.

Now we write a nonrelativistic Lagrangian

$$L_{NR} = p^n_i \dot{q}_{in} - \frac{i}{2}\xi^n_r \dot{\xi}_{rn} - \mathcal{H}_R(p,q,\xi) \qquad (9)$$

which is galilean invariant and bilinear in dynamical variables p, q, ξ.[2] Extending the variables to the D-dimensional Minkowski space we immediatly obtain the Poincare invariant rudimentary Lagrangian

$$L_R = p^\mu_i \dot{q}_{i\mu} - \frac{i}{2}\xi^\mu_r \dot{\xi}_{r\mu} - \mathcal{H}_R(p,q,\xi). \qquad (10)$$

Note that the meaning of the parameter t is essentially different for these two Lagrangians. In the nonrelativistic theory, one can interpret t as the galilean time variable while in Eq.(10) each particle has its own "time", q^0_i, and t is some arbitrarily chosen evolution parameter. Recall that we always take $0 \le t \le 1$ so that $(p_i(0), q_i(0), \xi_r(0))$ and $(p_i(1), q_i(1), \xi_r(1))$ are the initial and final positions of the system in the superspace (p_i, q_i, ξ_r).

Having a bilinear Hamiltonian $\mathcal{H}_R$ it is not difficult to rewrite L_R in the matrix form (7). One only has to extend the definitions of Ψ and C in Eq.(5),

$$\Psi^T = [p_i, q_i, \xi_r], \quad C = \begin{bmatrix} -i\sigma_2 \mathbb{1} & 0 \\ 0 & -i\hat{\mathbb{1}} \end{bmatrix}. \qquad (11)$$

where $\mathbb{1} \equiv \mathbb{1}_{ij} = \delta_{ij}$, $\hat{\mathbb{1}}$ is the unit matrix in the r-indices.

---

[2] To treat particles and strings in a background field[11] one has to relax the last requirement. We hope to discuss the corresponding extension of our approach in future.

Performing straightforward calculations one can then derive the supermatrix H_R corresponding to any given bilinear superform $\mathcal{H}_R$.

To obtain all supersymmetries of the Lagrangian L_R we may use Eq.(8), remembering the standard rule for transposing supermatrices. Denoting the independent parameters of the obtained symmetry transformation matrix F by f_a and φ_α (bosonic and fermionic, resp.), the matrix of the gauge potential, A, is obtained by substituting 1_a and λ_α for f_a and φ_α into A. Our formulation of the theory is completed by writing the gauge Lagrangian in the form (5).

Any theory obtained in this way is Poincare invariant by construction and can be quantized by applying one of the standard methods. However, its physics interpretation requires studying the structure of its gauge group, which depends on $\mathcal{H}_R$. To have a better insight into this matter we present another method of constructing the gauge Lagrangian corresponding to L_R.

All linear canonical symmetries of L_R can be obtained by using the generating function of supercanonical transformations,

$$\delta X = [G,X]_{SPB}, \quad G = \sum f_a g_a + \sum \varphi_\alpha \gamma_\alpha, \qquad (12)$$

where SPB means the super-Poisson brackets and g_a/φ_α is the complete set of bosonic/fermionic bilinear Poincare invariant products of the dynamical variables p,q,ξ,

$$g_a = p_i p_j,\ p_i(q_i - q_j),\ (\xi_{r_1}\xi_{r_2}), \ldots,\ \varphi_\alpha = \xi_r p_i,\ \xi_r(q_i - q_j), \ldots$$

The (super)canonical symmetries of L_R, which correspond to Eq.(8), are represented here in the form

$$\delta p = -\partial G/\partial q,\ \delta q = \partial G/\partial p,\ \delta\xi = i\partial^L G/\partial\xi,\ \delta\mathcal{H}_R = [G,\mathcal{H}_R]_{SPB} = 0, \qquad (13)$$

where L means the left derivative with respect to the grassmanian variable ξ. The last condition in Eq.(13) defines a subgroup of the linear (super)canonical group depending on our choice of $\mathcal{H}_R$.

The gauging procedure starts with considering time-dependent parameters, $f(t)$, $\varphi(t)$, of this symmetry subgroup. The variation of L_R under the localized transformation (12) is easy to calculate (remember that in our notation ∂_t is the total t-derivative)

$$\delta L_R = \partial_t \left[p\partial G/\partial p + \tfrac{1}{2}\zeta\, \partial^L G/\partial\zeta - G \right] + \dot{f}\partial G/\partial f + \dot{\varphi}\partial G/\partial\varphi \qquad (14)$$

The first term determines the boundary conditions for $f(t)$, $\varphi(t)$ and other terms can be cancelled if we add to L_R the linear combination of the generators,

$$-\sum_\alpha l_\alpha(t) g_\alpha(p,q,\zeta) - \sum_\alpha \lambda_\alpha(t)\gamma_\alpha(p,q,\zeta) = -\tfrac{1}{2}\Psi^T C A \Psi, \qquad (15)$$

where Ψ and C are defined in Eq.(11). The transformation law for the gauge potentials l, λ can be found without using the matrix representation of A; one simply requires that the variation of L_G, which is L_R plus (15), is zero. The transformations obtained in this way are identical to (6), and both constructions of the gauge theory are completely equivalent. As the theory is Poincare invariant the evolution of the centre-of-mass coordinates

$$Q = \tfrac{1}{N}\sum q_i, \quad P = \sum p_i \qquad (16)$$

is defined by Eq.(1), the gauge group being T_1. The relative motions are described by completely decoupled equations depending on N-1 relative coordinates, and Grassmann variables.

The simplest relativistic gauge theories correspond to nonrelativistic interactions which are velocity- and spin-independent. Then there exist at least N mutually commuting bosonic generators (this is shown in the next section) and R mutually commuting fermionic generators, e.g. $\varphi_r = P\zeta_r$ (they also commute with bosonic ones; "commuting" is defined through SPB in

classical theory and through graded commutators in quantum theory). Adding N+R mutually commuting gauge-fixing conditions one can in principle eliminate unphysical quantities t, y^0_k, ζ^0_r and express the time components of momenta, p^0_i, in terms of p^n_i. All models of free particles and strings treated in Refs.[6-9] are of this sort, and here we hold to this simplifying restriction.

To understand it better consider the gauge formulation for N free spinless relativistic particles. Then we have exactly N constraints, p_i^2, generating the gauge group U_1^N and corresponding to independent reparametrizations of the individual world-line trajectories. For N coupled particles, N mutually commuting generators play the role of these reparametrizations but they are imbedded in the full gauge group as its Cartan subgroup.

3. Gauge Theory of N Harmonically Coupled Particles

Now we apply our general approach to constructing relativistic gauge models for N particles coupled by harmonic forces. To simplify the presentation we mainly treat here only spinless particles. Then the natural rudimentary Lagrangian is

$$L_R = p_i\dot{q}_i - \tfrac{1}{2}p_i p_i - \tfrac{1}{2}v_{ij}(q_i - q_j)^2; \quad v_{ij} = v_{ji}, \quad v_{ii} = 0. \qquad (17)$$

The most general Lorentz invariant linear canonical transformation is defined by the generating function

$$G = \tfrac{1}{2}a_{ij}p_i p_j + b_{ij}p_i q_j + \tfrac{1}{2}c_{ij}q_i q_j = \tfrac{1}{2}\Psi^T \begin{bmatrix} a & b \\ b^T & c \end{bmatrix} \Psi, \qquad (18)$$

where $a_{ij} = a_{ji}$, $c_{ij} = c_{ji}$. a, b, c are the NxN matrices of the corresponding parameters. The rudimentary Lagrangian is invariant under the canonical transformations generated by G,

$$\delta p_i = -\partial G/\partial q_i, \; \delta q_i = \partial G/\partial p_i; \quad \text{or,} \; \delta\Psi = C^{-1}\partial^L G/\partial\Psi, \qquad (19)$$

if $\delta\mathcal{H}_R = 0$. This condition is equivalent to the linear equations for the parameter matrices

$$[V,a] = 0, \quad [V,b] = 0, \quad b^T = -b, \quad c = -Va, \tag{20}$$

where V_{ij} is equal to v_{ij} for $i \neq j$ and $-V_{ii}$ is the sum of i-th row of v. The equations (20) leave in G not less than N mutually commuting generators as can be readily seen.

The physics content of the gauge Lagrangian, which can now be constructed by applying our general procedure, crucially depends on the coupling constants v_{ij}. If $v_{ij} = v_0$ for all i,j, the nonrelativistic version of L_R describes the system of N identical particles with pair harmonic couplings. The gauge group in that case is $T_1 \times U_1 \times SU_{N-1}$. This can be shown with the aid of the general formulae (18),(20). However, as far as we are going to gauge all linear canonical symmetries of the rudimentary Lagrangian, we are free to use any canonical coordinates related to the original ones by any linear canonical transformation. It is obviously convenient to use the centre-of-mass coordinates defined in the nonrelativistic N-particle problem. Thus we introduce the centre-of-mass coordinates Q, y_i and momenta P, z_i, $i = 1,\dots,N$, so as to diagonalize L_R:

$$L_R = P\dot{Q} - \tfrac{1}{2}P^2 + z_i\dot{y}_i - \tfrac{1}{2}z_i z_i - \tfrac{1}{2}y_i y_i, \tag{21}$$

where we set $v_0 = 1$, for brevity.

Applying now our general recipe for gauging one can obtain, after straightforward but tedious calculation, that

$$L_a = P\dot{Q} + z_i\dot{y}_i - \tfrac{1}{2}l_0(P^2 + M^2) - \tfrac{1}{2}l_1(z_i z_i + y_i y_i - P^2 - m^2) -$$
$$- \tfrac{1}{2}l^s_{ij}(z_i z_j + y_i y_j) - \tfrac{1}{2}l^a_{ij}(z_i y_j - z_j y_i), \tag{22}$$
$$l^s_{ij} = l^s_{ji}, \quad \sum_{i=1}^{N-1} l^s_{ii} = 0, \quad l^a_{ij} = -l^a_{ji}.$$

Here the constraint coupled to l_0 generates T_1, the one coupled to l_1 generates U_1, and the others give the algebra of SU_{N-1}. The

constraints coupled to l^s_{ii} generate its Cartan subalgebra; together with two abelian costraints related to T_1 and U_1 they form N mutually commuting generators of the full gauge group. In writing (22) we have used the freedom to add parameters M^2 and m^2 to the abelian generators; likewise, the term P^2 in the U_1-generator, commuting with all constraints can be removed or multiplied by an arbitrary number. If the pair couplings were not identical, i.e. v_{ij} depended on i,j, the SU_{N-1} symmetry would be broken.[9] Note that the gauge group for two particles is $T_1 \times U_1$. To obtain the corresponding Lagrangian from (22) one simply has to set $z_i = y_i = 0$, $i \geq 2$, and keep the first two constraints.

Spinning particles can be treated similarly. Here we only write the gauge Lagrangian for two particles with spin one half

$$L_a = P\dot{Q} + p\dot{q} - \tfrac{i}{2}\,\xi_r\dot{\xi}_r - \tfrac{1}{2}l_0 P^2 - \tfrac{1}{2}l_1(p^2 + q^2) -$$
$$- \lambda_1(P\xi_1) - \lambda_2(P\xi_2) - il_2\xi_1\xi_2, \tag{23}$$

where the notation for the relative coordinates has been changed, $p \equiv z_1$, $q \equiv y_1$. This is written for massless particles and zero total mass. To include mass dependence one should use the trick of dimensional reduction explained in Introduction. One may notice that the relative motion is completely decoupled from spin degrees of freedom. This fact is a consequence of our simplifying assumptions on the rudimentary Hamiltonian. Anyway, the model (23) is too simple to be applied to interesting physics which requires inclusion of velocity-dependent (spin-orbit) forces as well as

---

[9] For the choice $v_{ij} = \delta_{|i-j|,1} + \delta_{iN}\delta_{j1} + \delta_{i1}\delta_{jN}$, which defines a closed "discrete string", the gauge group is, for even N, $T_1 \times U_1 \times (U_1 \times SU_2)^n$, where $n = \tfrac{1}{2}(N - 2)$.[8]

spin-dependent (spin-spin) forces. Such more general models can also be treated by our method.

A most natural approach to quantizing the gauge theories of coupled particles suggested above is the covariant BRST method. Nevertheless, for physics applications, noncovariant gauges might prove preferable. For two spinless particles (Eq.(23) with $\xi \equiv 0$) one may choose the following gauge-fixing conditions

$$Q^0 = t, \quad q^0 \cos(t) - p^0 \sin(t) = 0.$$

By solving the equations of motion in this gauge one can find that it is t-independent if

$$l_0 = (P^n P^n)^{-1/2}, \quad l_1 = 1.$$

Then the system is described in terms of the space components of the coordinates and momenta and the evolution parameter is identified with the centre-of-mass time. The quantization may proceed in the standard way, and the spectrum of the internal motions is simply that of the (D-1)-dimensional oscillator.

4. Discussion

Finally we will briefly summarize the motivation and results of this letter as well as some problems for future investigations besides those mentioned above. The principal point is our proposal to derive relativistic particle Lagrangian from nonrelativistic ones by first extending the usual phase space to the relativistic phase space and then gauging the rigid canonical symmetries of the obtained (formally relativistic) rudimentary Lagrangian.

This approach has been proposed in Ref.6 using the example of one particle with some hints that it can be of more general nature. In Ref.7 it has been demonstrated to produce, in a direct and clear manner, known reparametrization invariant relativistic

theories of free particles as well as bosonic and fermionic strings. These results were presented in some detail and reviewed in Ref.9. In addition, the gauge Lagrangian for two and three particle systems coupled by linear forces has been proposed.[8] Many previous attempts to construct a consistent and tractable model for coupled relativistic particles failed, and there is a wide-spread belief that it is even impossible to do ("no-go theorems"). This can partly be explained by complexity of candidate reparametrisation invariant Lagrangians which we avoid by using gauged canonical symmetries. But the main source of difficulties certainly is the interaction potential. Most of the previous attempts were addressed to arbitrary potentials.

Note that it is not advisable to eliminate the auxiliary variables p,l from our gauge Lagrangian. If one tried to do that one would obtain a highly nonlinear Lagrangian $L(q,\dot{q})$ which would be extremely difficult to work with, even for three-particle case.

Thus the conceptual and practical advantages of our approach are clear enough to motivate an attempt to apply it to N particles which was first presented in our preprints, Refs.8,9. In this letter we have tried to give a concise presentation of the approach and to clarify some conceptual points. We have constructed the classical Lagrangians for spinless particles and demonstrated how the spin variables could be included. The available choice of gauge-fixing conditions allows one in principle to quantize all constructed models. Up to now the author has completed the first quantization for two and three particles but presenting the results would require rather lenghty and technical considerations. A very interesting unsolved problem is the extenson of our theory to nontrivial background fields, which would allow to treat interactions of the bound states. However, even first-quantized theory in the trivial background might have interesting applications to bound states of light quarks. Developing such ideas require a careful investigation of gauge fixing and quantization of our model which is now in progress.

Acknowledgements

I am greatly indebted to J.Ambjorn, M.Fabbrichesi, V.N.Gribov, A.P.Isaev, V.V.Nesterenko, H.B.Nielsen, M.Peskin, A.S.Schwarz, A.A.Slavnov, and V.I.Tkach for discussions and critical remarks.

References

1. W.Siegel, Phys. Lett. 151B (1985) 391.
2. A.Neveu and P.West, Phys. Lett. 184B (1986) 343.
3. L.Brink, P.Di Vecchia and P.Howe, Nucl.Phys. B118 (1977) 76.
4. M.Henneaux and C.Teitelboim, Ann.Phys. 143 (1982) 127.
5. E.S.Fradkin and G.A.Vilkovisky, Phys.Lett. 55B (1975) 224.
6. A.T.Filippov, JINR Rapid Comm. 3 (1987) 5.
7. A.T.Filippov, Pis'ma v Zs.Exptl.Teor.Fiz. 46 (1987) 337.
8. A.T.Filippov, JINR preprint E2-87-771, Dubna, 1987; preprint NBI-HE-88-07, Copenhagen 1988.
9. A.T.Filippov, JINR preprint E2-87-806, Dubna, 1987.
10. F.A.Berezin and M.S.Marinov, JETP Lett. 21 (1975) 320; R.Casalbuoni, Nuovo Cim. 33A (1976) 389.
11. J.Maharana and G.Veneziano, Nucl.Phys. B283 (1987) 126; J.Maharana, preprint IP/BBSR/87-16, Bhubanesvar, 1987; S.Fubini and M.Roncadelli, Phys.Lett. 203B (1988) 433.

Received by Publishing Department
on July 22, 1988.

Филиппов А.Т. E2-88-553

Калибровочная модель, описывающая N релятивистских частиц, связанных линейными силами

Построена релятивистская модель N частиц, связанных линейными силами, основанная на локализации линейных канонических симметрий простого (рудиментарного) нерелятивистского лагранжиана, формально расширенного на релятивистское фазовое пространство. Новый (калибровочный) лагранжиан инвариантен относительно преобразований Пуанкаре, гамильтониан является линейной комбинацией связей первого рода, замкнутых относительно скобок Пуассона и порождающих локализованные канонические симметрии. Лагранжевы множители этих связей интерпретируются как "калибровочные потенциалы". Кратко обсуждается фиксация калибровки и квантование.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Filippov A.T. E2-88-553

A Gauge Model Describing N Relativistic Particles Bound by Linear Forces

A relativistic model of N particles bound by linear forces is obtained by applying the gauging procedure to the linear canonical symmetries of a simple (rudimentary) nonrelativistic N-particle Lagrangian extended to relativistic phase space. The new (gauged) Lagrangian is formally Poincaré invariant, the Hamiltonian is a linear combination of first-class constraints which are closed with respect to Poisson brackets and generate the localized canonical symmetries. The "gauge potentials" appear as the Lagrange multipliers of the constraints. Gauge fixing and quantization of the model are also briefly discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988