объединенный
институт
ядерных
исследований
дубна

E2-88-541

E.A.Ivanov, S.O.Krivonos, V.M.Leviant

QUANTUM N=3,4 SUPERCONFORMAL WZW SIGMA MODELS

Submitted to "Physics Letters B"

1988

1. **Introduction.** N=3,4 d=2 superconformal algebras(SA)[1] are expected to have important physical applications both in the string models[2-6] and statistical systems[7]. These SA's are the last ones (among those listed in [1]) still admitting central extensions[3] and so having a chance to give rise to nontrivial quantum physics. Their most characteristic feature is that they naturally combine the Virasoro algebra with the SO(3) or SO(4) Kac-Moody algebras.

The d=2 field representations of N=3,4 SA's found in[2,4] exist only quantum-mechanically. On the other hand, lately we have constructed the d=2 Lagrangian models respecting the N=3 and N=4 superconformal symmetries already at the classical level[5]. These models result from nonlinear realizations of d=2 superconformal symmetries and necessarily incorporate conformally invariant SO(3) and SO(4) WZW σ models. Their actions may or may not involve the Liouville terms. When the latter terms are absent[1], the models in question can be analyzed by the familiar methods of conformal field theory[9].

In this letter we present full quantum realization of N=3 and N=4 SA's in the above field-theoretic models. New nonlinear Goldstone-type representations of these SA's are found. The representation discovered earlier by Schoutens[4] turns out a particular case of ours. It emerges in the purely quantum limit where the WZW fields completely decouple. The N=4 SA is shown to be modified by a non-zero operator central charge when realised on the representations constructed. We list various cases when the fermionization of bosonic WZW currents is possible.

---

[1] This special case of our models has been recently re-discovered on entirely different grounds in [8] as a part of more general class.

2. N=3 σ model (with the Liouville terms omitted) is characterized by the action[5]

$$S=\int d^2x \left[\tfrac{1}{2}\partial_+ u\, \partial_- u + \tfrac{i}{2}\psi^k_-\partial_+\psi^k_- + \tfrac{i}{2}\psi^k_+\partial_-\psi^k_+ + \tfrac{i}{2}\chi_-\partial_+\chi_- + \tfrac{i}{2}\chi_+\partial_-\chi_+\right]$$
$$+ \tfrac{2}{k}\pi \left[\int d^2x\, W^i_+ W^i_- + \tfrac{2\pi}{3k} \int d^3x\, \varepsilon^{\alpha\beta\gamma} \varepsilon_{ijk} W^i_\alpha W^j_\beta W^k_\gamma \right], \qquad (1)$$

where

$$W^i_\pm = \tfrac{k}{8\pi} \varepsilon^{ijk} (q^{-1}\partial_\pm q)^{jk} \qquad (2)$$

and $q^{ij}(x)$ is the matrix field in vector representation of SO(3). The action (1) is invariant under the d=2 N=3 superconformal group[5] which can be realized off-shell by adding a proper set of auxiliary fields. For our purpose, it is sufficient to consider the realization of N=3 SA on the left(right) movers, in which case the corresponding transformations are guaranteed to close.

All the information about the classical superconformal properties of fields involved in (1) is encoded in the Noether currents which are covariantly split into the left- and right-moving sets. For the left movers these currents are(we omit the light cone indices of currents)

$$T(x^+) = \tfrac{1}{2}(\partial_+ u)^2 + \tfrac{1}{2}\sqrt{\tfrac{k}{4\pi}}\, \partial_+\partial_+ u + \tfrac{i}{2}\, \psi^i_+\partial_+\psi^i_+ + \tfrac{i}{2}\, \chi_+\partial_+\chi_+ + \tfrac{2\pi}{k} W^i_+ W^i_+$$
$$G^i(x^+) = -\psi^i_+\partial_+ u - \sqrt{\tfrac{k}{4\pi}}\, \partial_+\psi^i_+ - \sqrt{\tfrac{4\pi}{k}}\, \varepsilon^{ijk}\psi^j_+ W^k_+ - \sqrt{\tfrac{4\pi}{k}}\, \chi_+(W^i_+ - \tfrac{i}{2}\varepsilon^{ijk}\psi^j_+\psi^k_+)$$
$$V^i(x^+) = W^i_+ - \tfrac{i}{2}\, \varepsilon^{ijk}\, \psi^j_+\, \psi^k_+ \qquad (3)$$
$$\Gamma(x^+) = \sqrt{\tfrac{k}{4\pi}}\, \chi_+ .$$

It is straightforward to check that, with respect to Poisson brackets, Fourier components of these currents

$$L_n = \int_0^{2\pi} dx^+ e^{inx^+} T(x^+),\quad G^i_r = \int_0^{2\pi} dx^+ e^{irx^+} G^i(x^+), \qquad (4)$$
$$T_n = \int_0^{2\pi} dx^+ e^{inx^+} V^i(x^+), \Gamma_q = \int_0^{2\pi} dx^+ e^{iqx^+} \Gamma(x^+)$$

form the N=3 SA [1]

$$[L_n, L_m] = (n-m)\, L_{n+m} + \tfrac{c}{12} (n^3-n)\, \delta_{n+m}$$

$$[L_n, G^i_k] = (\tfrac{n}{2} - k)\, G^i_{n+k},\ [L_n, T^i_s] = -s\, T^i_{n+s}$$
$$\{G^i_r, G^j_s\} = 2\, \delta^{ij} L_{r+s} + i(r-s)\, \varepsilon^{ijk} T^k_{r+s} + \tfrac{c}{3} (r^2 - \tfrac{1}{4})\, \delta_{r+s}$$
$$[L_n, \Gamma_q] = -(\tfrac{n}{2} + q)\, \Gamma_{n+q}, [T^i_n, \Gamma_q] = 0 \qquad (5)$$
$$[T^i_s, T^j_n] = i\varepsilon^{ijk}\, T^k_{s+n} + s\, \tfrac{c}{3}\, \delta^{ij}\, \delta_{s+n}$$
$$\{\Gamma_q, G^i_r\} = -T^i_{q+r},\quad \{\Gamma_p, \Gamma_q\} = \tfrac{c}{3}\, \delta_{p+q}$$
$$[T^i_r, G^j_s] = i\varepsilon^{ijk}\, G^k_{r+s} - s\, \delta^{ij}\, \Gamma_{r+s}$$

with

$$c = \tfrac{3}{2}\, k \qquad (6)$$

(we impose the standard closed string Neveu-Schwarz type boundary conditions). Note the presence of terms linear in fields in the expressions (3), which reflects the Goldstone nature of these fields[5]. This peculiarity will be important in quantum case.

Let us proceed to quantization. We follow the Witten approach [10] and replace Poisson brackets by Dirac ones

$$[W^i(x^+), W^j(x^{+\prime})] = i\, \varepsilon^{ijk}\, W^k(x^{+\prime})\delta(x^+ - x^{+\prime}) + \tfrac{ik}{4\pi}\, \delta'(x^+ - x^{+\prime})$$
$$[u(x^0, x^1), \partial_0 u(x^0, x^{1\prime})] = i\, \delta(x^1 - x^{1\prime}) \qquad (7)$$
$$\{\psi^i(x^+), \psi^j(x^{+\prime})\} = \delta^{ij}\, \delta(x^+ - x^{+\prime})$$
$$\{\chi(x^+), \chi(x^{+\prime})\} = \delta(x^+ - x^{+\prime}).$$

We will also need the explicit form of the oscillator basis decomposition for the left-moving component of u(x):

$$u(x^+) = \frac{1}{2\sqrt{\pi}} \left[q_0 + p_0\, x^+ + i\, \sqrt{2} \sum_{n\neq 0} \frac{a_n}{n}\, e^{-inx^+} \right]. \qquad (8)$$

The further steps are to put the currents (3) into the normally ordered form and to renormalize them in a proper way[9] so that the original N=3 SA is reproduced. Eventually, the left-moving quantum currents are as follows

$$T(x^+) = \tfrac{1}{2}\left[:(\partial_+ u)^2: + \frac{k}{\sqrt{4\pi(k+2)}}\, \partial_+\partial_+ u + i:\psi^i_+\partial_+\psi^i_+: + i:\chi_+\partial_+\chi_+: + \frac{4\pi}{k+2}:W^i_+W^i_+: \right]$$

$$G^i(x^+) = -:\psi^i_+ \partial_+ u: - \frac{k}{\sqrt{4\pi(k+2)}}\, \partial_+ \psi^i_+ - \sqrt{\frac{4\pi}{k+2}}\left[\varepsilon^{ijk}:\psi^j_+ W^k_+: + :\chi_+ (W^i_+ - \tfrac{i}{2}\varepsilon^{ijk}\psi^j_+\psi^k_+):\right]$$

$$V^i(x^+) = W^i_+ - \frac{i}{2}\,\varepsilon^{ijk} :\psi^j_+\psi^k_+: \qquad (9)$$

$$\Gamma(x^+) = \sqrt{\frac{k+2}{4\pi}}\;\chi_+ .$$

Their Foirier components defined according to (4) generate the same N=3 SA (5) but with the shifted central charge

$$c = \frac{3}{2}(k + 2). \qquad (10)$$

Of course,these generators can be explicitly expressed in terms of free oscillators associated with the left movers.It is worthwhileto quote the quantum supersymmetry transformations of fields

$$\delta\, u = i\mu^{+i}\psi^i_+$$

$$\delta\, \psi^i_+ = \frac{k}{\sqrt{4\pi(k+2)}}\,\partial_+\mu^{+i} - \sqrt{\frac{4\pi}{k+2}}\; i\mu^{+k}\varepsilon^{kij}\chi_+\psi^j_+ + \sqrt{\frac{4\pi}{k+2}}\;\varepsilon^{ijk}\mu^{+j}W^k_+ - \mu^{+i}\partial_+ u$$

$$\delta\, \chi_+ = -\sqrt{\frac{4\pi}{k+2}}\;\mu^{+i}(W^i_+ - \frac{i}{2}\,\varepsilon^{ijk}\psi^j_+\psi^k_+) \qquad (11)$$

$$\delta\, W^k_+ = i\sqrt{\frac{4\pi}{k+2}}\;\left\{\mu^{+i}\chi_+\varepsilon^{kij}W^j_+ + (\mu^{+i}\psi^k_+ - \mu^{+k}\psi^i_+)W^i_+\right\} + \frac{ik}{\sqrt{4\pi(k+2)}}\,\partial_+(\mu^{+k}\chi_+)$$

$$+ \frac{ik}{\sqrt{4\pi(k+2)}}\;\varepsilon^{kij}\partial_+(\mu^{+i}\psi^j_+) .$$

Let us point out once more the Goldstone nature of the involved fields that manifests itself as the presence of inhomogeneous terms both in the currents (9) and the transformation laws (11).Just this property is responsible for the central charge being as in eq.(10).Note that the conformal currents containing such linear pieces,were considered in [11].It is also worth mentioning that the superconformal generators with terms linear in fields appear (at the classical level) in the context of super-KdV equations[12].In the case at hand these terms are necessarily required for self-consistency of full quantum N=3 SA.

In the end of this Sect. we dwell on two supermultiplets of particular interest.

k=0,c=3. In contradistinction to the classical currents (3),the quantum ones (9) have k=0 as a well-defined limit.In this limit,all the inhomogeneous parts in the currents ,except for $\Gamma(x^+)$,vanish and the related fields lose their Goldstone character.In particular,u(x) becomes the primary field and the conformal current T coincides with the canonical energy-momentum tensor.The bosonic WZW current transforms now entirely through itself and it is consistent truncation to put it equal to zero (at least,on a proper subspace of full Hilbert space):

$$W^i_+ = 0. \qquad (12)$$

As a result,one arrives at the purely quantum realization of N=3 SA on the shortened c=3 multiplet (u, χ_+, ψ^i_+).It is just the multiplet found by Schoutens[4].The k=0 expressions for currents and the transformation laws precisely coincide with those given in [4].

k=2,c=6. In this case, the WZW σ model admits a fermionization in terms of extra SO(3) triplet of fermions ξ^k_+[10]

$$W^i_+ = -\frac{i}{2}\,\varepsilon^{ijk}\,\xi^j_+\,\xi^k_+ . \qquad (13)$$

One may check that the fermionized current G^i in the set (9) produces the following supersymmetry transformation of ξ^i_+

$$\delta\,\xi^k_+ = i\;\mu^{+i}(\;\psi^k_+\,\delta^{ij} - \delta^{ik}\psi^j_+ + \varepsilon^{kij}\chi_+)\;\xi^j_+ . \qquad (14)$$

Taking this into account,the transformation laws (11) remain unchanged upon substitution of (13) (at k=2).The multiplet $(u, \psi^k_+, \xi^k_+, \chi_+)$ has c=6 and yet certainly cannot be represented as a direct sum of two Schoutens's ones.The obvious reason is the presence of non-vanishing Goldstone-type terms in the relevant currents.

At any other value of k, our multiplets essentially involve the bosonic WZW current W^k_+. Clearly, they are by no means reduced to direct sums of the multiplets given in [4].

3. **N=4** case is most interesting as the SO(4) N=4 SA admits two independent central charges[6]

$$
\begin{aligned}
&[L_n, L_m] = (n-m)\, L_{n+m} + \frac{c_1}{12}(n^3 - n)\, \delta_{n+m} \\
&[L_n, G^i_r] = \left(\frac{n}{2} - r\right) G^i_{n+r}\ , \quad [L_n, \Gamma^i_r] = -\left(\frac{n}{2} + r\right) \Gamma^i_{n+r} \\
&[L_n, T^{ij}_m] = -\, m\, T^{ij}_{n+m}\ , \quad [L_n, \Delta_s] = -s\, \Delta_{n+s} - \frac{ic_1}{3} n^2\, \delta_{n+s} \\
&[T^{ij}_n, T^{kl}_m] = -i(\delta^{il}T^{kj}_{n+m} + \delta^{jk}T^{li}_{n+m}) + \left[(\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk})\frac{c_1}{3} - \frac{c_2}{3}\varepsilon^{ijkl}\right] n\, \delta_{n+m} \\
&\{G^i_p, G^j_r\} = 2\delta^{ij}L_{p+r} - i\,(p-r)\, T^{ij}_{p+r} + \frac{c_1}{3}\left(p^2 - \frac{1}{4}\right) \delta^{ij}\, \delta_{p+r} \qquad (15) \\
&\{G^i_p, \Gamma^j_s\} = \delta^{ij}\Delta_{p+s} + \varepsilon^{ijkl}\, T^{kl}_{p+s} - \frac{2}{3}\, i\, c_2\, p\, \delta_{p+s}\delta^{ij} \\
&[G^i_p, \Delta_s] = -\, s\, \Gamma^i_{p+s}, \quad \{\Gamma^i_p, \Gamma^j_s\} = \frac{4}{3} c_1\, \delta^{ij}\, \delta_{p+s}, \quad [\Gamma^i_p, \Delta_s] = 0 \\
&[T^{ij}_n, G^k_r] = \frac{n}{2}\, \varepsilon^{ijkl}\Gamma^l_{n+r} - i(\delta^{ik}G^j_{n+r} - \delta^{jk}G^i_{n+r}), \quad [\Delta_p, \Delta_s] = \frac{4}{3}\, p\, c_1 \delta_{p+s} \\
&[T^{ij}_n, \Gamma^k_r] = -i(\delta^{ik}\Gamma^j_{n+r} - \delta^{jk}\Gamma^i_{n+r}).
\end{aligned}
$$

The presence of two numbers c_1 and c_2 is related to the fact that the SO(4) SA contains two independent SU(2) Kac-Moody algebras (corresponding to the decomposition SO(4)~SU(2)xSU(2)) which enter with their central charges $\frac{1}{2}(c_1 + c_2)$ and $\frac{1}{2}(c_1 - c_2)$.

As compared to the standard N=4 SA [1], SA (15) includes one extra generator Δ_0. One may reduce (15) to the standard form by putting

$$\Delta_0 = 0, \Delta_n = i\, n\, \tilde{\Delta}_n \qquad (\bar{\tilde{\Delta}}_n = \Delta_{-n}, \tilde{\Delta}_n = \Delta_{-n}),$$

$\tilde{\Delta}_n$ being the subcanonical generators present in ordinary SO(4) N=4 SA[1]. However, in general Δ_0 is not obliged to be zero. This generator commutes with all the other ones and can thus be regarded as a kind of operator central charge. We will see that in the models we are considering, Δ_0 does not vanish; it generates an extra U(1) symmetry[5].

One more peculiarity of SA (15) is ununiqueness of embedding of

Virasoro subalgebra. It is easy to show that the linear combinations

$$\tilde{L}_n = L_n + \frac{1}{2}\, \alpha\, n\, i\, \Delta_n \qquad (16)$$

generate a one parameter family of Virasoro algebras, with the central charge[6]

$$c^\alpha_1 = c_1 - 4\, \alpha\, c_2 + 4\, \alpha^2\, c_1. \qquad (17)$$

One may redefine the supersymmetry generators so that they have canonical transformation properties with respect to $\tilde{L}_n$

$$\tilde{G}^i_q = G^i_q + i\, \alpha\, q\, \Gamma^i_q. \qquad (18)$$

Conformal properties of generators Δ_n also depend on the choice of $\tilde{L}_n$ [2)]

$$[\tilde{L}_n, \Delta_s] = -s\, \Delta_{n+s} - \frac{i}{3}\, c^\alpha_2\, n^2\, \delta_{n+s} \qquad (19)$$

$$c^\alpha_2 = c_2 - 2\, \alpha\, c_1.$$

It is worth mentioning that the Virasoro algebras corresponding to different α extend to different infinite dimensional subalgebras in (15). At $\alpha=0$ this is N=3 SA while at $\alpha=\pm\frac{1}{2}$ the relevant Virasoro algebras turn out to lie in two different SU(2) N=4 SA's.

The simplest N=4 WZW supermultiplet [5] has the same field content as the N=3 one. The relevant action is as follows

$$S_I = \int d^2x \left[\frac{1}{2}\partial_+ u\, \partial_- u + \frac{i}{2}\xi^{\alpha a}_+ \partial_- \xi_{+\alpha a} + \frac{i}{2}\xi^{\alpha a}_- \partial_+ \xi_{-\alpha a} + \frac{2\pi}{k_1}\, \mathcal{L}_{W.Z.}(q_{1\alpha}{}^{\beta})\right], \qquad (20)$$

α, a being isospinor indices of two SU(2)'s entering into SO(4). Note that the second SU(2) (and the corresponding Kac-Moody group) acts merely on indices a of spinors.

Another option is to consider the doubled supermultiplet involving the WZW fields for each of two SU(2)'s. The action is

$$S = S_I + S_{II}. \qquad (21)$$

---

2) For correct normalization of central terms it is necessary, as usual, to shift $\tilde{L}_0$ and Δ_0 by properly chosen constants.

$$S_{II} = \int d^2x \left[\frac{1}{2}\partial_+ v \partial_- v + \frac{i}{2}\eta_+^{\alpha a}\partial_-\eta_{+\alpha a} + \frac{i}{2}\eta_-^{\alpha a}\partial_+\eta_{-\alpha a} + \frac{2\pi}{k_2}\mathcal{L}_{W.Z.}(q_{2a}{}^{b})\right]. \qquad (22)$$

Note that just the action (21) (but not (20) or (22) separately) can be promoted to the SO(4) WZW-Liouville one [5].

The invariance of (20)-(22) under N=4 superconformal transformations and the precise form of the latter follow from the results of [5]. Here we confine ourselves to presenting explicit expressions for quantum currents. Assuming the canonical quantization rules and the Neveu-Schwarz type boundary conditions one gets for the general case (21) and arbitrary α

$$\Delta = \sqrt{\frac{k_1+2}{\pi}}\,\partial_+ u - \sqrt{\frac{k_2+2}{\pi}}\,\partial_+ v, \quad \Gamma^{\alpha a} = \sqrt{\frac{k_1+2}{\pi}}\,\xi_+^{\alpha a} - \sqrt{\frac{k_2+2}{\pi}}\,\eta_+^{\alpha a}$$

$$V^{\alpha\beta} = :W^{\alpha\beta}(q_1): - \frac{i}{4}:\xi_+^{\beta a}\xi_{+a}{}^{\alpha}: - \frac{i}{4}:\eta_+^{\beta a}\eta_{+a}{}^{\alpha}:$$

$$V^{ab} = :W^{ab}(q_2): - \frac{i}{4}:\xi_+^{\beta a}\xi_{+\beta}{}^{b}: - \frac{i}{4}:\eta_+^{\beta a}\eta_{+\beta}{}^{b}: \qquad (23)$$

$$G^{\alpha a} = \xi_+^{\alpha a}\partial_+ u + \frac{k_1}{2\sqrt{\pi(k_1+2)}}\,\partial_+\xi_+^{\alpha a} + \alpha\sqrt{\frac{k_1+2}{\pi}}\,\partial_+\xi_+^{\alpha a} + \frac{2}{\sqrt{\pi(k_1+2)}}:W_1^{\alpha\beta}\xi_{+\beta}{}^{a}: +$$

$$\frac{i}{3}\frac{1}{\sqrt{\pi(k_1+2)}}:\xi_+^{\alpha b}\xi_{+b}{}^{\beta}\xi_{+\beta}{}^{a}: + \left\{ W_1, u, \xi, k_1, \alpha \longrightarrow W_2, v, \eta, k_2, -\alpha \right\}$$

$$T = \frac{1}{2}:(\partial_+ u)^2: + \frac{k_1}{4\sqrt{\pi(k_1+2)}}\,\partial_+\partial_+ u + \frac{\alpha}{2}\sqrt{\frac{k_1+2}{\pi}}\,\partial_+\partial_+ u + \frac{i}{2}:\xi_+^{\alpha a}\partial_+\xi_{+\alpha a}: -$$

$$\frac{2\pi}{k_1+2}:Tr(W_1 W_1): + \left\{ W_1, u, \xi, k_1, \alpha \longrightarrow W_2, v, \eta, k_2, -\alpha \right\}.$$

The currents associated with the single actions S_I or S_{II} formally follow from the general expressions (23) by putting to zero the appropriate irreducible sets of fields. For instance, the currents corresponding to the choice $S = S_I$ are obtained by setting $v = \eta = W_2 = 0$. We give the expressions for T and Δ

$$T = \frac{1}{2}:(\partial_+ u)^2: + \frac{k_1}{4\sqrt{\pi(k_1+2)}}\,\partial_+\partial_+ u + \frac{\alpha}{2}\sqrt{\frac{k_1+2}{\pi}}\,\partial_+\partial_+ u + \frac{i}{2}:\xi_+^{\alpha a}\partial_+\xi_{+\alpha a}: +$$

$$\frac{2\pi}{k_1+2}:Tr(W_1 W_1):, \qquad \Delta = \sqrt{\frac{k_1+2}{\pi}}\,\partial_+ u. \qquad (24)$$

We begin with discussing this simplest case. Fourier components of corresponding currents at $\alpha = 0$ constitute the N=4 SA(15) with

$$c_1 = \frac{3}{2}(k_1+2), \quad c_2 = \frac{9}{2}k_1. \qquad (25)$$

The fact that c_1 coincides with the N=3 central charge (10) is not accidental. The subset of $\alpha=0$ currents with the indices α,a identified $(\Gamma^{\alpha}{}_{\alpha}, G^{(\alpha\beta)}, T, V_1^{(\alpha\beta)} + V_2^{(\alpha\beta)})$ generates just the N=3 SA(5) embedded as a subalgebra in (15). As a matter of fact, the actions (1) and (20) are identical, in accordance with the general conclusion of ref. [8] that N=3 supersymmetry in the models of this type implies N=4 supersymmetry.

The presence of c_2 in the commutator $[L_n, \Delta_s]$ means that Δ_s has unconventional conformal properties with respect to Virasoro algebra $\{L_n\}$, unless k_1 is zero. It is natural to make use of the above α freedom in order to ensure normal transformation properties of the current Δ under conformal group [6]:

$$c_2^{\alpha} = 0 \quad \longrightarrow \quad \alpha = \frac{c_2}{2c_1} = \frac{k_1}{2(k_1+2)} \qquad (26)$$

$$c_1^{\alpha} = 6\,\frac{k_1+1}{k_1+2}. \qquad (27)$$

With this choice, the field u remains the Goldstone field with respect to the generator Δ_0 which produces constant shifts of this field and, as is seen from eqs. (24), (8), is nothing else as the zero mode momentum of u(x). With respect to conformal group, u(x) behaves now as a primary field. Respectively, the term $\sim(\partial_+)^2 u$ drops out from the conformal current (24) and the latter coincides with the canonical energy-momentum tensor.

Let us specialize to the two important examples of the multiplets constructed.

As in the N=3 case, upon quantization it becomes possible to consistently set $k_1 = 0$. In this limit, one arrives at the N=4 Schoutens multiplet $(u, \xi_+^{\alpha a})$ having $c_1^\alpha = c_1 = 3, c_2^\alpha = c_2 = 0$ [4].

At $k_1=2$, $c_1^\alpha = \frac{9}{2}$ one may fermionize the bosonic WZW current W_1 by a SU(2) triplet of fermions $\chi_+^{(\alpha\beta)}$

$$W_1(q_1)^{\alpha\beta} = \frac{i}{4} \chi_+^{(\alpha\rho)} \chi_{+(\rho}^{\beta)} \tag{28}$$

and realize N=4 SA on the set $(u, \xi_+^{\alpha a}, \chi_+^{(\alpha\beta)})$.

Now we turn to the general case (23). At $\alpha=0$ the relevant central charges are expressed in terms of integers k_1, k_2 as

$$c_1 = \frac{3}{2}(k_1 + k_2 + 4), \quad c_2 = \frac{3}{2}(k_1 - k_2). \tag{29}$$

In this case one cannot put the conformal currents into the canonical form by adjusting parameter α. However, one may still require the commutator (21) to have no anomalous term

$$c_2^\alpha = 0 \quad \longrightarrow \quad \alpha = \frac{c_2}{2c_1} = \frac{k_1 - k_2}{2(k_1 + k_2 + 4)} \tag{30}$$

$$c_1^\alpha = \frac{6(k_1+2)(k_2+2)}{k_1 + k_2 + 4}. \tag{31}$$

Under the choice (30) the current Δ and the surviving inhomogeneous part in T are expressed via the orthogonal combinations of fields u and v. The first combination is the Goldstone field for U(1) generator Δ_0 while the second one is the dilaton.

List, as before, some particular cases. At $k_1=k_2=0$ the multiplet in question has c = 6 and it reduces to the direct sum of two Schoutens's multiplets. Only in this case the conformal current coincides with the canonical energy-momentum tensor.

The options $k_1=2, k_2=2$ or $k_1=k_2=2$ correspond to situations when one or both of the WZW currents can be fermionized by SU(2) triplets of extra fermions. In this case, c_1^α is parametrized by one integer k

$$c_1^\alpha = 24\,\frac{k+2}{k+6}. \tag{32}$$

At $k_1=k_2=k$ one has $\alpha=0$ and

$$c_1^\alpha = 3(k+2) = c_1, \quad c_2^\alpha = c_2 = 0. \tag{33}$$

In this case the Virasoro algebra we have chosen to be basic coincides with the one entering into the N=3 subalgebra of (15). The N=4 SA[15] is realized on the direct sum of two N=3 multiplets discussed in Sect. 2.

4. Conclusions. In this paper we have described a new realization of N=3 and N=4 (Δ_0 extended) SA's on the d=2 fields. The next steps should be explicit construction of corresponding Hilbert spaces and analysis of representations of N=3,4 SA's on the states along the lines of refs.[9]. This might help in establishing a link with string theories and in checking the N=3,4 determinant formulas conjectured in [13]. Note that in the above N=4 models the states will be labelled by an additional quantum number associated with the central charge generator Δ_0. One more interesting problem is to study effects of adding Liouville terms to the actions.

Finally, we remark that in the quantum formulas given above, one may put k_1 and/or k_2 equal to -1 with preserving the positiveness of total central charges of Virasoro and Kac-Moody algebras. In particular, k=-1 in eq.(10) yields a c=3/2 N=3 multiplet. It would be of interest to inquire whether the latter is equivalent to the c=3/2 multiplet built up by Schwimmer and Seiberg [2] within the vertex construction.

Acknowledgements. One of us (E.I.) thanks Jan Lacki for stimulating discussions on Kac-Moody algebras.

After this work has been completed, we have received **a preprint** of Sevrin et al.[14], where similar issues are discussed. In particular, the Δ_0 extended N=4 SA is presented.

References.

[1] M. Ademollo, L. Brink, A. D'Adda, R. D'Auria, E. Napolitano, S. Sciuto, E. Del Giudice, P. Di Vecchia, S. Ferrara, F. Gliozzi, R. Musto and R. Pettorino, Phys. Lett. B 62 (1976) 105.

[2] A. Schwimmer and N. Seiberg, Phys. Lett. B 184 (1987) 191.

[3] D. Chang and A. Kumar, Phys. Lett. B 193 (1987) 181.

[4] K. Schoutens, Phys. Lett. B 194 (1987) 75.

[5] E. Ivanov, S. Krivonos and V. Leviant, JINR preprint E2-87-357, Dubna, April 1987; Nucl. Phys. B 304 (1988) 601.

[6] K. Schoutens, Nucl. Phys. B 295[FS21] (1988) 634.

[7] M. Henkel and A. Patkos, preprint NBI-HE-87-59 (1987).

[8] Ph. Spindel, A. Sevrin, W. Troost and A. Van Proeyen, Phys. Lett. B 206 (1988) 71.

[9] V. Knizhnik and A. Zamolodchikov, Nucl. Phys. B 247 (1984) 83;
P. Di Vecchia, V. G. Knizhnik, J. L. Petersen and P. Rossi, Nucl. Phys. B 253 (1985) 701;
D. Gepner and E. Witten, Nucl. Phys. B 278 (1986) 493.

[10] E. Witten, Commun. Math. Phys. 92 (1984) 455.

[11] V. Dotsenko and V. Fateev, Nucl. Phys. B240(1984) 312, B251(1985) 691;
C. Thorn. Nucl. Phys. B 248 (1984) 551.

[12] M. Chaichian and P. P. Kulish, Phys. Lett. B 183 (1987) 169.

[13] A. Kent and H. Riggs, Phys. Lett. B 198 (1987) 491.

[14] A. Sevrin, W. Troost, A. Van Proeyen and Ph. Spindel, Preprint KUL-TF-88/8, June 1988.

Received by Publishing Department
on July 19, 1988.

Иванов Е.А., Кривонос С.О., Левиант В.М. E2-88-541

Квантовые N=3,4 суперконформные ВЗВ сигма-модели

Построены новые представления голдстоуновского типа N=3,4 суперконформных алгебр /СА/ на d=2 полях. Они существуют как на классическом, так и на квантовом уровнях и включают как существенную часть токи весс-зуминовских SO(3) и SO(4) моделей. Мультиплеты, найденные ранее Схоутенсом, являются чисто квантовым пределом мультиплетов, построенных в данной работе. Показано, что N=4CA на рассматриваемых представлениях расширяется операторным центральным зарядом.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Ivanov E.A., Krivonos S.O., Leviant V.M. E2-88-541

Quantum N=3,4 Superconformal WZW Sigma Models

New Goldstone-type d=2 field representations of N=3,4 superconformal algebras (SA) are constructed. They exist both in classical and quantum regions and essentially involve the SO(3) and SO(4) WZW currents. The multiplets found previously by Schoutens are purely quantum limiting case of ours. N=4 SA is shown to be extended by an operator central charge on the representations considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988