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EQUATIONS OF MOTION FOR THE NEW D = 10 N = 1 SUPERGRAVITY-YANG-MILLS THEORY

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Recently, the geometrical approach to the ten-dimensional supersymmetric Einstein-Yang-Mills theories draw much attention[1-5]. While studying the supersymmetric string theories this approach is so powerful, that it can be considered as a new method for obtaining the effective action[2]. The two complete sets of constraints[1-3] are considered in the usual version of D=10 N=1 supergravity. By solving Bianchi identities after having imposed these constraints we are led to the on-shell formulation. In the papers[1,6] all the equations of mation and supersymmetry transformations were derived in zero order of the string-tension parameter.

In a series of papers[4-6] a superspace formalism was established for the dual version of the D = 10 N = 1 supergravity-Yang-Mills theory. Now we know that for the massless fields in type I or heterotic superstring, corrections up to $O(\alpha'^{3})$ (α' is the slope parameter) to the D = 10 N = 1 superspace supergravity-Yang-Mills theory can be embedded in the Λ-tensor and $\bar{F}$ - tensor supercurrents. These supercurrents appear in the constraints for torsions and Yang-Mills field strength tensors, respectively and, after solving the Bianchi identities, show up in the field equations and supersymmetry transformations. However, the complete solution to these Bianchi identities are still lacking. In this short paper we'll make up this gap.

In the papers[4], for the set of Bianchi identities (see[6] for notation)

$$\nabla_{[A} T_{BC)}{}^{D} - T_{[AB|}{}^{E} T_{E|C)}{}^{D} - R_{[ABC)}{}^{D} \equiv 0 \tag{1}$$

$$\nabla_{[A} R_{BC)D}{}^{E} - T_{[AB|}{}^{F} R_{F|C)D}{}^{E} \equiv 0 \tag{2}$$

$$\nabla_{[A} F_{BC)}{}^{\hat{I}} - T_{[AB|}{}^{E} F_{E|C)}{}^{\hat{I}} \equiv 0 \tag{3}$$

$$\nabla_{[A_1} N_{A_2 \ldots A_8)} - T_{[A_1 A_2|}{}^{B} N_{B|A_3 \ldots A_8)} \equiv 0 \tag{4}$$

the on-shell constraints were presented

$$T_{\alpha\beta}{}^{c} = i\,\sigma^{c}_{\alpha\beta}\,, \quad T_{ab}{}^{c} = 0\,, \quad T_{\alpha b}{}^{c} = 0, \tag{5}$$

$$T_{\alpha\beta}{}^{\delta} = -\frac{1}{2\sqrt{2}}\left[\delta^{\delta}_{(\alpha}\delta^{\varepsilon}_{\beta)} + (\sigma^{d})_{\alpha\beta}(\sigma_{d})^{\delta\varepsilon}\right]\chi_{\varepsilon}\,, \tag{6}$$

$$T_{\alpha b}{}^{\gamma} = -\frac{1}{24}(\sigma_b)_{\alpha\delta}(\sigma^{[3]})^{\delta\gamma}\left[e^{-\Phi}\tilde{N}_{[3]} + \frac{i}{8}(\sigma_{[3]})^{\varepsilon\eta}\chi_\eta\chi_\varepsilon\right] -$$

$$-\frac{1}{48}(\sigma^{[3]})_{\alpha\delta}(\sigma_b)^{\delta\gamma}\left[e^{-\Phi}\tilde{N}_{[3]} + \frac{i}{16}(\sigma_{[3]})^{\varepsilon\eta}\chi_\eta\chi_\varepsilon - \frac{1}{8}A_{[3]}\right], \tag{7}$$

$$N_{\alpha\beta[5]} = \frac{i}{2}e^{\Phi}(\sigma_{[5]})_{\alpha\beta}, \qquad N_{\alpha\beta\gamma\ldots} = 0, \tag{8}$$

$$N_{\alpha[6]} = -\frac{1}{2\sqrt{2}}e^{\Phi}(\sigma_{[6]})_\alpha^{\ \beta}\chi_\beta, \tag{9}$$

$$F_{\alpha\beta}^{\hat{I}} = -i\frac{1}{5!}(\sigma^{[5]})_{\alpha\beta}\,f_{[5]}^{\hat{I}}, \tag{10}$$

$$R_{\alpha\beta de} = \frac{i}{4}(\sigma^c)_{\alpha\beta}\left\{3e^{-\Phi}\tilde{N}_{cde} + \frac{i5}{16}(\sigma_{cde})^{\delta\gamma}\chi_\gamma\chi_\delta - \frac{1}{8}A_{cde} + \right.$$

$$\left. + \frac{i}{24}(\sigma^{[3]}{}_{de})_{\alpha\beta}\left\{e^{-\Phi}\tilde{N}_{[3]} + i\frac{3}{16}(\sigma_{[3]})^{\delta\gamma}\chi_\gamma\chi_\delta + \frac{1}{8}A_{[3]}\right\}\right., \tag{11}$$

$$R_{\alpha cde} = -\frac{i}{2}\left[(\sigma_e)_{\alpha\gamma}T_{cd}{}^{\gamma} - (\sigma_d)_{\alpha\gamma}T_{ce}{}^{\gamma} - (\sigma_c)_{\alpha\gamma}T_{de}{}^{\gamma}\right], \tag{12}$$

$$\nabla_\alpha\Phi = -\frac{1}{\sqrt{2}}\chi_\alpha, \tag{13}$$

$$\nabla_\alpha\chi_\beta = -\frac{i}{12\sqrt{2}}(\sigma^{[3]})_{\alpha\beta}\left\{e^{-\Phi}\tilde{N}_{[3]} - \frac{3}{8}A_{[3]}\right\} - \frac{i}{\sqrt{2}}(\sigma^a)_{\alpha\beta}\nabla_a\Phi. \tag{14}$$

Antisymmetric tensor field $\tilde{N}_{abc}$ is essentially dual to the seven-form $N_{a_1\ldots a_7}$

$$\tilde{N}_{[3]} = \tilde{N}_{abc} \equiv \frac{1}{7!}\varepsilon_{abc}{}^{[7]}N_{[7]} \equiv \frac{1}{7!}\varepsilon_{abc}{}^{a_1\ldots a_7}N_{a_1\ldots a_7} \tag{15}$$

and our normalization convention is such that

$$R_{AB\gamma}{}^{\delta} = \frac{1}{4}R_{ABab}(\sigma^{ab})_\gamma^{\ \delta}. \tag{16}$$

In deriving these constraints Bianchi identities up to the engineering dimension D = 1 [7] are utilized and only one D = 3/2 identity

is used in computing curvature (12) (this is equation (1) with indices (α, b, c; d)).

In the rest of this work we'll deal with the D = 3/2 and D = 2 Bianchi identities for (1) and (4) which lead to the equations of motion. The pure Yang-Mills sector, i.e. equations (3), was treated in the previous paper [7], where the role of the f-tensor supercurrent in (10) was clarified. On the other hand, (2) is a consequence of (1), and hence, does not lead to new information.

An interesting point is that the only way a string correction alters the gravitational sector is the modification of the A-tensor supercurrent in (14). In (7) and (11) A - tensor appears after solving the D = 1 Bianchi identities. We start with D = 3/2 Bianchi identities

$$\nabla_\alpha N_{a_1\ldots a_7} - \frac{1}{6!}\nabla_{[a_1|}N_{\alpha|a_2\ldots a_7]} - \frac{1}{6}T_{\alpha[a_1|}{}^{E}N_{E|a_2\ldots a_7]} -$$

$$-\frac{3}{6!}T_{[a_1a_2|}{}^{E}N_{E\alpha|a_3\ldots a_7]} = 0, \tag{17}$$

$$\nabla_{(\alpha}T_{\beta)b}{}^{\delta} + \nabla_b T_{\alpha\beta}{}^{\delta} - T_{\alpha\beta}{}^{\gamma}T_{\gamma b}{}^{\delta} - T_{\alpha\beta}{}^{a}T_{ab}{}^{\delta} -$$

$$-T_{b(\alpha|}{}^{\varepsilon}T_{\varepsilon|\beta)}{}^{\delta} - R_{b(\alpha\beta)}{}^{\delta} = 0. \tag{18}$$

Using constraints (5), (7), (8), (9), (12) and (13) in (17) we derive the equation

$$\nabla_\alpha\tilde{N}_{abc} = \frac{e^{\Phi}}{2\sqrt{2}}(\sigma_{abc}{}^{d})_\alpha^{\ \beta}\left[\chi_\beta\nabla_d\Phi + \nabla_d\chi_\beta\right] +$$

$$+\frac{i}{4}e^{\Phi}(\sigma_{abc}{}^{d_1d_2})_{\alpha\beta}T_{d_1d_2}{}^{\varepsilon} + \frac{1}{16\sqrt{2}}e^{\Phi}(\sigma_{[bc})_\varepsilon^{\ \beta}(\sigma^{d_1d_2})_\alpha^{\ \varepsilon}\times$$

$$\times\left[e^{-\Phi}\tilde{N}_{a]d_1d_2} + \frac{i}{8}(\chi\,\sigma_{a]d_1d_2}\chi)\right] + \tag{19}$$

$$+\frac{1}{48}e^{\Phi}(\sigma_{abc})^{\beta\delta}(\sigma^{[3]})_{\delta\alpha}\left\{\frac{5}{2}e^{-\Phi}\tilde{N}_{[3]} + \frac{i3}{32}(\chi\,\sigma_{[3]}\chi) - \frac{7}{16}A_{[3]}\right\}\chi_\beta.$$

This equation should be used in deriving the equations of motions from the Bianchi identity (18).

Multiplying equation (18) by δ_δ^α, $(\sigma^b)^{\alpha\beta}$ and $(\sigma^a)_{\lambda\delta}(\sigma_a)^{\alpha\beta}$, three different projections can be extracted

By pluging in our set of constrainty and producing a rather involving algebra, finally we derive the subgravitino equation of motion

$$(\sigma^d)^{\gamma\delta}\nabla_d\chi_\gamma = \frac{1}{96\sqrt{2}}(\sigma^{[3]})^{\delta\beta}\nabla_\beta A_{[3]} - \frac{1}{48}(\sigma^{[3]}\chi)^\delta A_{[3]}, \qquad (20)$$

the Rarita-Schwinger equation

$$(\sigma_a{}^{d_1d_2})_{\beta\alpha}T_{d_1d_2}{}^\alpha = \frac{3}{56\sqrt{2}}(\sigma_a\sigma^{[3]}\chi)_\beta A_{[3]} + \frac{1}{21\sqrt{2}}(\sigma^{[3]}\sigma_a\chi)_\beta \times$$

$$\times\left[\frac{7}{2}e^{-\Phi}\tilde{N}_{[3]} + \frac{3}{16}A_{[3]}\right] - \sqrt{2}(\sigma^d)_{\beta\eta}(\sigma_a)^{\eta\varepsilon}\chi_\varepsilon\nabla_d\Phi - \qquad (21)$$

$$-\frac{1}{24\cdot 14}\left[5(\sigma_a\sigma^{[3]})_\beta^\gamma + 2(\sigma^{[3]}\sigma_a)_\beta^\gamma\right]\nabla_\gamma A_{[3]}$$

and slightly different projection of this equation

$$(\sigma^{d_1d_2})_\varepsilon^\alpha T_{d_1d_2}{}^\varepsilon = \frac{i}{12\sqrt{2}}(\sigma^{[3]}\chi)^\alpha\left\{e^{-\Phi}\tilde{N}_{[3]} - \frac{3}{4}A_{[3]}\right\}. \qquad (22)$$

But still we are not finished with the D = 3/2 Bianchi identities.

Acting by ∇_β on the left-hand side of the subgravitino equation (20) and using the commutation relation for the covariant derivatives, it is easy to derive

$$\nabla_\beta(\sigma^d)^{\beta\alpha}\nabla_d\chi_\alpha = (\sigma^a)^{\beta\gamma}\left[\nabla_a\nabla_\beta\chi_\gamma + T_{\beta a}{}^\delta\nabla_\delta\chi_\gamma + R_{\beta a\gamma}{}^\delta\chi_\delta\right]. \quad (23)$$

Now one can see that by using equation (14), we are led to the dilation equation of motion

$$\nabla_a\nabla^a\Phi - \frac{1}{3!}e^{-2\Phi}\tilde{N}_{[3]}^2 =$$

$$= \frac{i}{64}(\sigma_{[3]})^{\varepsilon\eta}\chi_\eta\chi_\varepsilon\left[\frac{5}{3}e^{-\Phi}\tilde{N}^{[3]} - \frac{11}{16}A^{[3]}\right] - \frac{1}{192}\left\{13e^{-\Phi}\tilde{N}_{[3]} - \frac{3}{8}A_{[3]}\right\}A^{[3]} \quad (24)$$

$$-i\frac{11}{3072\sqrt{2}}(\sigma^{[3]}\chi)^\delta\nabla_\delta A_{[3]} - \frac{i}{1536}(\sigma^{[3]})^{\delta\beta}\nabla_\delta\nabla_\beta A_{[3]}.$$

Now we are ready to turn to the D = 2 Bianchi identities. The first one

$$\nabla_{[a_1}N_{a_2\ldots a_8]} - \frac{7}{2}T_{[a_1a_2|}{}^\varepsilon N_{\varepsilon|a_3\ldots a_8]} \equiv 0 \qquad (25)$$

gives us the equation of motion for N-field

$$e^{-\Phi}\nabla^d\tilde{N}_{abd} = -\frac{1}{4\sqrt{2}}(\sigma_{ab}{}^{a_1a_2})_\varepsilon^\beta\chi_\beta T_{a_1a_2}{}^\varepsilon \qquad (26)$$

and the second one

$$\nabla_\gamma T_{ab}{}^\delta - R_{ab\gamma}{}^\delta - T_{\gamma[a|}{}^\varepsilon T_{\varepsilon|b]}{}^\delta + T_{ab}{}^\varepsilon T_{\varepsilon\gamma}{}^\delta - \nabla_{[a}T_{b]\gamma}{}^\delta \equiv 0 \qquad (27)$$

provides us with Ricci tensor and Einstein equation.

The Ricci tensor can be calculated by multiplying (27) by $(\sigma_c)^{\delta\eta}\cdot(\sigma^b)_{\eta\delta}$. After very tedious calculations we derive

$$R_{ab} = -2\nabla_a\Phi\nabla_c\Phi + i\frac{1}{2}\chi_\alpha(\sigma_{(a})^{\alpha\beta}\nabla_{c)}\chi_\beta + \qquad (28)$$

$$+\frac{1}{8}\eta_{ac}e^{-\Phi}\tilde{N}^{[3]}\left[\frac{2}{3}e^{-\Phi}\tilde{N}_{[3]} - \frac{11}{112}A_{[3]}\right] - e^{-2\Phi}\tilde{N}_{ad_1d_2}\tilde{N}_c{}^{d_1d_2} +$$

$$+\frac{9}{28\cdot 8}e^{-\Phi}\tilde{N}_{(c}{}^{d_1d_2}A_{a)d_1d_2} - \frac{5}{128\cdot 56}\eta_{ac}A^{[3]}A_{[3]} + \frac{1}{56\cdot 16}A_a{}^{d_1d_2}A_{cd_1d_2}$$

$$+i\frac{25}{96\cdot 64}\eta_{ac}\chi_\alpha(\sigma^{[3]})^{\alpha\beta}\chi_\beta A_{[3]} - i\frac{9}{64\cdot 8}\chi_\alpha(\sigma_{(a}{}^{d_1d_2})^{\alpha\beta}\chi_\beta A_{c)d_1d_2}$$

$$+\frac{33}{128\cdot 8}\chi_\alpha(\sigma_a{}^{d_1d_2})^{\alpha\beta}\chi_\beta\,\chi_\gamma(\sigma_{cd_1d_2})^{\gamma\delta}\chi_\delta - \frac{i}{56\cdot 32\cdot 8\sqrt{2}}\eta_{ac}\chi_\alpha(\sigma^{[3]})^{\alpha\beta}\nabla_\beta A_{[3]}$$

$$-i\frac{31}{56\cdot 192\sqrt{2}}\chi_\alpha(\sigma_{(a})^{\alpha\beta}(\sigma^{[3]})_{\beta\gamma}(\sigma_{c)})^{\gamma\delta}\nabla_\delta A_{[3]}$$

$$-\frac{i}{96\cdot 56}\left[(\sigma_{(c})^{\gamma\eta}(\sigma^{[3]})_{\eta\varepsilon}(\sigma_{a)})^{\varepsilon\alpha} + \frac{\eta_{ac}}{4}(\sigma^{[3]})^{\gamma\alpha}\right]\nabla_\gamma\nabla_\alpha A_{[3]}.$$

The last step we'll take here is the derivation of the Einstein equations. With the Ricci tensor in hend this is the straightforward procedure and finally we come to

$$
\begin{aligned}
R_{adb}{}^{d} - \frac{1}{2}\eta_{ab} R_{de}{}^{de} &= \eta_{ab}(\nabla_d \Phi)^2 - 2(\nabla_a \Phi)(\nabla_b \Phi) \\
&+ \frac{i}{2}\chi_\alpha (\sigma_{(a})^{\alpha\beta} \nabla_{b)} \chi_\beta \\
&+ \frac{1}{6}\eta_{ab} e^{-2\Phi} \tilde{N}_{[3]} \tilde{N}^{[3]} - e^{-2\Phi} \tilde{N}_{ad_1d_2} \tilde{N}_b{}^{d_1d_2} - \\
&- \frac{69}{4\cdot 128}\eta_{ab} e^{-\Phi} N_{[3]} A^{[3]} + \frac{9}{224} e^{-\Phi} N_{(a}{}^{d_1d_2} A_{b)d_1d_2} + \\
&+ \frac{\eta_{ab}}{448} A^{[3]} A_{[3]} + \frac{1}{112\cdot 8} A_a{}^{d_1d_2} A_{bd_1d_2} + \\
&+ \frac{3}{256} i \eta_{ab} \chi_\alpha (\sigma^{[3]})^{\alpha\beta} \chi_\beta A_{[3]} - i\frac{9}{512}\chi_\alpha (\sigma_{(a}{}^{d_1d_2})^{\alpha\beta} \chi_\beta A_{b)d_1d_2} \\
&+ \frac{33}{1024}\chi_\alpha (\sigma_a{}^{d_1d_2})^{\alpha\beta} \chi_\beta \chi_\varepsilon (\sigma_{cd_1d_2})^{\varepsilon\eta} \chi_\eta .
\end{aligned} \tag{29}
$$

This equation completes the set of equations of motion. The only thing we have to do in order to derive the explicit form for the new D = 10 N = 1 supergravity-Yang-Mills theory is to substitute for the tensor A, which implies the superstring corrections, and perform involving but straighforward calculations. The explicit form for this tensor supercurrent is given in /8/. Due to the progress achieved in constructing the four-dimensional superstring theories /9/, the compactification problem of this theory to four dimension deserves utmost interest. We'll proceed along these lines in our future investigations.

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