

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА

A 95

E2-88-519

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**POLARIZABILITY OF  $\pi$ -MESONS  
IN THE QUARK CONFINEMENT MODEL**

Submitted to "Ядерная физика"

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**1988**

## I. Introduction

The study of electromagnetic mesonic interactions allows one to obtain further information about the inner structure of hadrons. Electromagnetic properties of pions at low energies are characterised by the finite number of parameters which are determined from the expansion of the pion-photon amplitudes over the  $\gamma$ -quanta frequencies. In particular, the two-photon hadronic interactions are characterised by the coefficients of electric  $\alpha_\pi$  and magnetic  $\beta_\pi$  polarizabilities<sup>/1/</sup>. The experimental data on the electric polarizability of a charged  $\pi^-$ -meson are available<sup>/2/</sup>:

$$\alpha_\pi = 6.8 \pm 1.4 \pm 1.8 \cdot 10^{-43} \text{ cm}^3$$

while those on the neutral one are to be measured in future<sup>/3/</sup>.

Theoretical values of  $\alpha_\pi$  and  $\beta_\pi$  are extracted from the amplitude of  $\pi\gamma \rightarrow \pi\gamma$ . The knowledge of the nature of strong interactions is needed for calculation of the amplitude of this process. However, the theory of strong interactions is not yet available, so one has to recourse to different low-energy approximations and models<sup>/1,4-7/</sup> for calculation of  $\alpha_\pi$  and  $\beta_\pi$ . The polarizability of  $\pi^-$ -mesons has been calculated by means of soft-pion technique and PCAC hypothesis in<sup>/4/</sup>. The electric polarizability of a neutral  $\pi^0$ -meson turned out to be equal to zero in this case, while that of a charged pion was connected with the  $\gamma$ -parameter for  $\pi^+ \rightarrow e^+ \gamma$  decay. Paper<sup>/5/</sup> is devoted to the investigation of  $\alpha_\pi$  and  $\beta_\pi$  in the framework of quantum field theory with the chiral-invariant Lagrangians. The contributions from all possible intermediate states to  $(\alpha_\pi + \beta_\pi)$  and  $(\alpha_\pi - \beta_\pi)$  have been taken into account in the approach based on the dispersion sum rules<sup>/6/</sup>. The values of  $\alpha_\pi$  and  $\beta_\pi$  have been extracted from the above sum and difference. Electric  $\alpha_\pi$  and magnetic  $\beta_\pi$  polarizabilities of pions have been calculated in the superconductivity-type Quark Model<sup>/7/</sup>. The results of this work indicate that the scalar mesons play an important role in the description of

the  $\pi\gamma \rightarrow \pi\gamma$  process. The values of  $\alpha_\pi$  and  $\beta_\pi$  obtained in the framework of the approaches mentioned above are listed in Table 1.

The present work is aimed at calculating the electric  $\alpha_\pi$  and magnetic  $\beta_\pi$  polarizabilities of charged and neutral  $\pi$ -mesons in the Quark Confinement Model (QCM)<sup>/8/</sup>. The article is organized in the following way:

The main concepts of the QCM are given in section II.

Section III is concerned with the description of the scalar-meson physics. The main parameters characterizing scalars are determined and the width of strong and radiative decays of scalar mesons are calculated.

The amplitude of the  $\pi\gamma$ -scattering and the relevant  $\alpha_\pi$  and  $\beta_\pi$  coefficients are calculated in section IV. The contributions from scalar, vector and axial intermediate states to the electric and magnetic polarizabilities of pions are studied. The intermediate  $\mathcal{E}(730)$ -meson turns out to play the main role in the description of  $\pi\gamma \rightarrow \pi\gamma$ .

## II. The main concepts of QCM

The QCM<sup>/8/</sup> is based on the definite hypothesis about hadronization and confinement. Hadrons appear as colourless collective variables in the quark-gluon interactions. The confinement is realized by averaging over gluonic vacuum.

The hadronization hypothesis is connected with the development of the QCD beyond a perturbation theory. We use the approach<sup>/9/</sup> in which the phenomenological Lagrangians were obtained in a heuristic way from the diagrams analogous to that with one-gluon exchange under some assumptions about the gluonic propagator behaviour in a low energy region. The meson interactions are described by closed quark loops in this approach. It is more convenient to use the following mathematical realization of the mentioned above approach for the calculation of hadron amplitudes. Quark currents with the quantum numbers of hadrons are constructed. For instance, in the case of two-quark mesons

$$J_Q = \bar{q}_a \Gamma_Q \lambda_Q q_a,$$

where the matrices  $\Gamma_Q, \lambda_Q$  provide necessary quantum numbers.  $\Gamma_Q = i\gamma^5, \gamma^\mu, \gamma^\mu \gamma^5$  for pseudoscalar, vector and axial-vector mesons.

Then, one constructs the Lagrangians of a hadron-quark interaction

$$\mathcal{L}_Q^I = \frac{g_Q}{\sqrt{2}} M_Q J_Q, \quad (2.1)$$

Table 1

Approach	$\alpha_{\pi^+} \cdot 10^{-43} \text{ cm}^3$	$\alpha_{\pi^0} \cdot 10^{-43} \text{ cm}^3$
Experiment /2/	$6.8 \pm 1.4 \pm 1.8$	-
Experiment /3/	-	35
Soft pion technique ( $\gamma_{\pi^0 \nu \gamma} = 0.41$ ) /4/	3.1	0
Chiral Theory /5/	5.3	-0.7
Dispersion sum rules /6/	$5.5 \pm 2$	$0.8 \pm 2$
Quark Model of Superconductor Type /7/	6.81	-1.0
QCM	4.06	-0.18

Table 2

$h_J D_J(0) = -\frac{1}{\pi^2(0)}$
$\Pi'_P(0) = -\frac{1}{2} m_P^2 \left( \int_0^1 du \mathcal{B}(u) + \frac{m_P^2}{4\Lambda^2} \int_0^1 du \mathcal{B}(1-u) \frac{m_P^2}{4\Lambda^2} \sqrt{1-u} \right)$
$\Pi'_{V_T}(0) = \frac{1}{3} m_V^2 \left( \int_0^1 du \mathcal{B}(u) + \frac{m_V^2}{4\Lambda^2} \int_0^1 du \mathcal{B}(1-u) \frac{m_V^2}{4\Lambda^2} \sqrt{1-u} \left(1 + \frac{u}{2}\right) \right)$
$\Pi'_A(0) = \frac{1}{3} m_A^2 \left( \int_0^1 du \mathcal{B}(u) + \frac{m_A^2}{4\Lambda^2} \int_0^1 du \mathcal{B}(1-u) \frac{m_A^2}{4\Lambda^2} (1-u)^{3/2} \right)$
$\Pi'_S(0) = -\frac{1}{2} m_S^2 \left[ \int_0^1 du \mathcal{B}(u) + \frac{m_S^2}{4\Lambda^2} \int_0^1 du \mathcal{B}(1-u) \frac{m_S^2}{4\Lambda^2} (1-u)^{3/2} + \right.$ $+ 4H \left( \int_0^1 du a(u) + \frac{m_S^2}{4\Lambda^2} \int_0^1 du a(1-u) \frac{m_S^2}{4\Lambda^2} (1-u)^{3/2} - \right.$ $\left. \left. - 4H^2 \left( \int_0^1 du u \mathcal{B}(u) - \left(\frac{m_S^2}{4\Lambda^2}\right)^2 \int_0^1 du \mathcal{B}(1-u) \frac{m_S^2}{4\Lambda^2} u(1-u)^{3/2} \right) \right]$

where  $M_Q$  is a free meson field describing a meson with quantum numbers  $Q$  and mass  $m_Q$ . The coupling constant  $g_Q$  is defined from the condition that the wave function renormalization constant of a meson is equal to zero<sup>/8/</sup>:

$$Z_Q = 1 - g_Q^2 \tilde{\Pi}'_Q(m_Q^2) = 0. \quad (2.2)$$

where  $\tilde{\Pi}'_Q(p^2)$  is the mass operator of the meson  $M_Q$ . It is more convenient to use  $h^{-12}(g_Q/4\pi)^2$  instead  $g_Q$  in calculations.

The confinement hypothesis means, that the averaging of quark loops, describing hadronic interactions over the gluonic vacuum fields has to provide absence of singularities corresponding to the quark-pair production.

Quark diagrams describing hadronic interactions are generated by the  $S$ -matrix in the form

$$S = \int d\sigma_{VAC} T \exp \{ i \int d^4x \mathcal{L}_Q^I(x) \}.$$

Here, the time-ordered product is supposed to be the ordinary Wick's  $T$ -product of the hadron and quark fields with the quark propagator

$$\begin{aligned} \overline{q_{f'q'}(x)} q_{fq'}(x') &= \delta_{ff'} \delta_{qq'} (m_f + \hat{B}_{VAC}(x) \cdot i\hat{\partial})^{-1} S(x, x') = \\ &\equiv \delta_{ff'} \delta_{qq'} S(x, x' | B_{VAC}). \end{aligned}$$

The quark fields must be equal to zero after the normal ordering.

The measure  $d\sigma_{VAC}$ , i.e. the way of averaging of quark diagrams over gluonic vacuum fields is defined as follows<sup>/8/</sup>:

$$\int d\sigma_{VAC} Sp [M(x_1) S(x_1, x_2 | B_{VAC}) \dots M(x_n) S(x_n, x_1 | B_{VAC})] \rightarrow$$

$$\rightarrow \int d\sigma_\lambda Sp [M(x_1) S_\lambda(x_1, x_2) \dots M(x_n) S_\lambda(x_n, x_1)],$$

where

$$S_\lambda(x_1, x_2) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x_1-x_2)}}{\lambda \Lambda_f - \hat{p}}.$$

The mass dimensional parameter  $\Lambda_f$  characterises the confinement region of quarks with flavour  $f$ .

The measure of integration  $d\sigma_\lambda$  is defined by

$$\int d\sigma_\lambda \frac{1}{\lambda - z} = G(z) = a(-z^2) + z b(-z^2).$$

where  $G(z)$  is called the confinement function.

The explicit form of  $G(z)$  is yet unknown, so, one can suppose the functions  $a(-z^2)$  and  $b(-z^2)$  to be independent. The functions  $a(-z^2)$  and  $b(-z^2)$  are assumed to be<sup>/8/</sup> independent of colour and flavour

and to be unique for all quark loops; to be entire, analytical functions in complex  $z$ -plane and to decrease rather quickly both in the Euclidean and pseudo-Euclidean directions

$$\lim_{z^2 \rightarrow \infty} |z^2|^N |a(-z^2)| = \lim_{z^2 \rightarrow \infty} |z^2|^N |b(-z^2)| = 0 \quad \forall N > 0.$$

In this paper we use the functions  $a(u)$  and  $b(u)$  of the following form:

$$\begin{aligned} a(u) &= 2 \exp(-u^2 - u) \\ b(u) &= 2 \exp(-u^2 + 0.4u). \end{aligned} \quad (2.3)$$

The dimensional model parameters  $\Lambda_u, \Lambda_d$  and  $\Lambda_s$  were fixed by fitting the main constants of the low-energy meson physics and turned out to be equal to<sup>/8/</sup>:

$$\Lambda = \Lambda_u = \Lambda_d = 460 \text{ MeV}, \quad \Lambda_s = 506 \text{ MeV}.$$

### III. Physics of scalar mesons

The description of the  $O^{++}$  channel is one of the problems of low-energy physics. On the one hand the introduction of scalar particles turned out to be very convenient for the construction of chiral theories<sup>/10/</sup>. The phenomenological analysis and model investigations<sup>/7,11/</sup> of the  $\pi\pi, \pi N, NN$ -scattering,  $K \rightarrow 2\pi$  and  $K \rightarrow \gamma\gamma$  decays indicate the importance of taking account of intermediate scalar states in the description of these processes. On the other hand, the quark composition of  $O^{++}$  particles is not yet clear. Scalar mesons are treated as two-quark states in<sup>/11/</sup> while the arguments in favour of their four-quark structure are given in<sup>/12/</sup>. At last, there is a number of articles, for example<sup>/13/</sup>, where scalar mesons are assumed to be hybrid states or pure glueballs.

In the QCM we treat scalar mesons ( $a_0, f_0, \epsilon, k$ ) as two-quark states described by the Lagrangian

$$\mathcal{L}_S = \frac{g_s}{\sqrt{2}} S(x) \bar{q}(x) \lambda_S (I + iH \hat{\sigma} / \Lambda) q(x), \quad (3.1)$$

where  $\lambda_S$  has the form<sup>/14/</sup>

$$\begin{aligned} \lambda_{a_0} &= \text{diag}(1, -1, 0) \\ \lambda_{f_0} &= \text{diag}(-\sin \delta_s, -\sin \delta_s, -\sqrt{2} \cos \delta_s) \\ \lambda_\epsilon &= \text{diag}(\cos \delta_s, \cos \delta_s, -\sqrt{2} \sin \delta_s). \end{aligned}$$

Let us discuss in detail the choice of a matrix  $\Gamma_S = I - iH\vec{\sigma}/\Lambda$  defining quantum numbers of  $0^{++}$  mesons. For this purpose, let us consider the diagram, fig. 1, describing a strong decay  $S \rightarrow PP$ . The corresponding structure integral calculated for zero masses of final states and normalized to unity at  $m_S = 0$  is

$$I_H(m_S) = [I_0(m_S) + 4HI_1(m_S)] / I_H(0)$$

$$I_0(m_S) = \int_0^1 du a(u) - 4 \left(\frac{m_S}{2\Lambda}\right)^2 \int_0^1 du a(-u) \frac{m_S^2}{4\Lambda^2} \left[ \frac{1}{2} \ln \frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} - \sqrt{1-u} \right]$$

$$I_1(m_S) = \int_0^1 du u b(u) + \frac{1}{2} \left(\frac{m_S}{2\Lambda}\right)^4 \int_0^1 du u b(-u) \frac{m_S^2}{4\Lambda^2} \left[ \frac{1}{2} \ln \frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} - \sqrt{1-u} \right]. \quad (3.2)$$

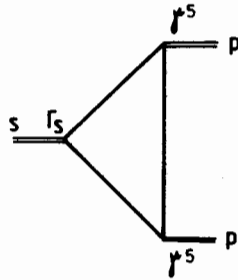


Fig. 1

The  $I_H(m_S)$  dependence on  $m_S$  with various values of  $H$  is plotted in fig. 2. One can see that  $I_0(m_S)$  becomes equal to zero at  $m_S \approx 1070$  MeV. It leads to that a theoretical value of the  $f_0(975) \rightarrow \pi\pi$  decay width is to be underestimated ( $\Gamma \sim 1$  MeV) in comparison with the experimental one  $\Gamma_{exp} = (26 \pm 5)$  MeV. This result, corresponding to the choice of two-quark current with  $\Gamma_S = I$  seems to testify in favour of a more complicated structure of scalar mesons. A four-quark component may be essential in scalar mesons. We are going to analyse this possibility in future. In the present work we use an additional two-quark interaction with a derivative in the form (3.1) and consider the parameter  $H$  as a free one. Moreover, the  $\epsilon$ -meson mass  $m_\epsilon$  and the mixing angle  $\delta_S$  are also considered as free parameters in accordance with the low-energy phenomenology.

As the basis for fitting of the parameters  $H$ ,  $\delta_S$  and  $m_\epsilon$  we take, first, the Adler condition<sup>10/</sup> which means that amplitudes of the  $\pi\pi \rightarrow \pi\pi$  and  $\pi\gamma \rightarrow \pi\gamma$  processes are equal to zero when  $m_\pi \rightarrow 0$  and, second, the experimental value of the  $f_0 \rightarrow \pi\pi$  width and  $\pi\pi$ -scat-

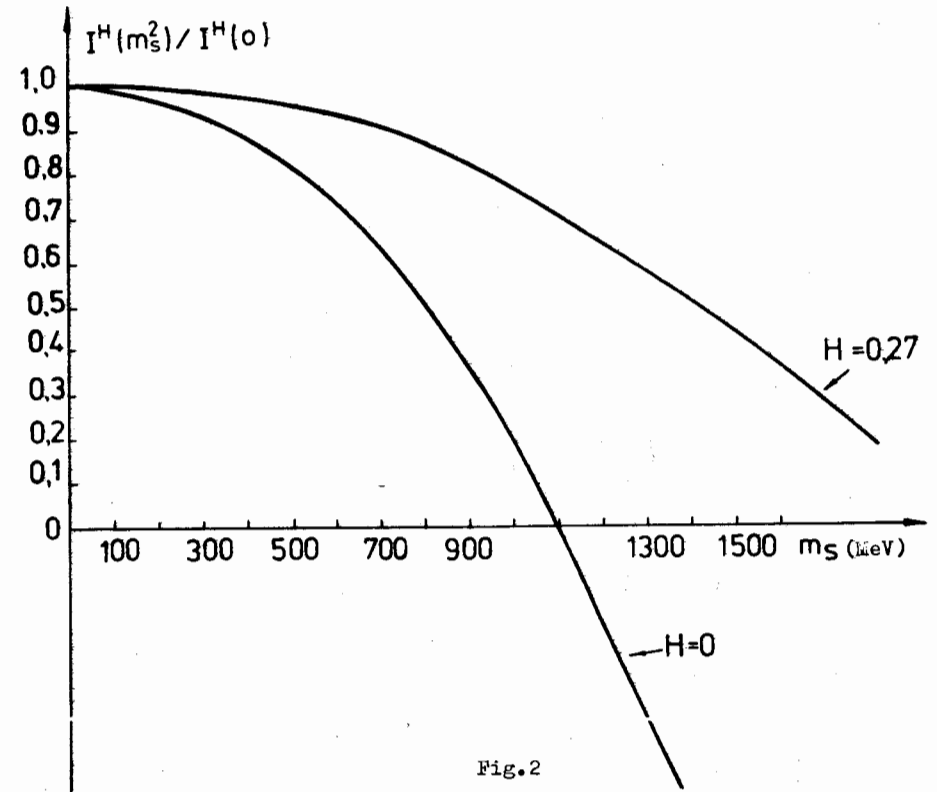


Fig. 2

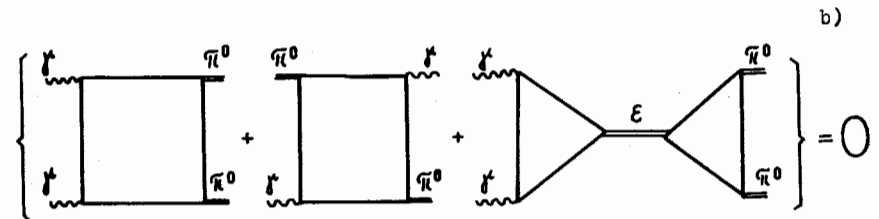
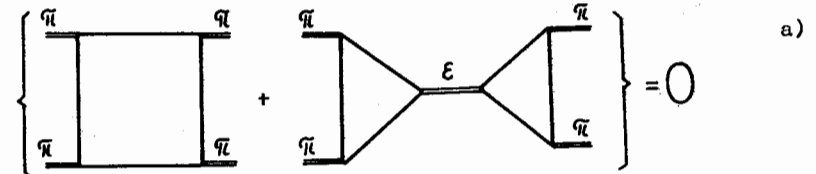


Fig. 3

tering S-wave lengths  $a_c^1$  and  $a_c^2$ . Graphically, the Adler condition can be presented as in fig. 3. On the QCM it is written in the form

$$C_0^{(0)} = 2\Lambda^2 \cos^2 \delta_s [C_0^{(0)} + 4HC_0^{(0)}]^2 h_\epsilon D_\epsilon^{(0)}, \quad (3.3)$$

$$5B(0) = 2\Lambda^2 \cos \delta_s (5\cos \delta_s - \sqrt{2} \sin \delta_s) a_c^{(0)} [C_0^{(0)} + 4HC_0^{(0)}] h_\epsilon D_\epsilon^{(0)} \quad (3.4)$$

The following notation is adopted:

$$C_n^{(m)} = \int_0^1 du u^n b(u); \quad C_n^{(m)} = \int_0^1 du u^n a(u).$$

$D_\epsilon(P')$  is the full propagator of  $\epsilon$ -meson. Due to the compositeness condition (2.2)

$$h_\epsilon D_\epsilon^{(0)} = \frac{1}{\tilde{\Pi}'_\epsilon(0)}$$

An explicit form of  $\tilde{\Pi}'_\epsilon(0)$  is given in table 2. For fitting  $H$  and  $\delta_s$  it is convenient to use the ratio of equalities (3.4) and (3.3)

$$R = \frac{5B(0) \cos \delta_s (C_0^{(0)} + 4HC_0^{(0)})}{C_0^{(0)} a(0) (5\cos \delta_s - \sqrt{2} \sin \delta_s)}$$

which is independent of  $m_\epsilon$ . The matrix element of the  $S \rightarrow PP$  decay can be written in the form:

$$\bar{g}_{SP, P_2} = \bar{g}_P \lambda_s \{ \lambda_{P_1}, \lambda_{P_2} \} \Lambda \sqrt{\tilde{h}_{P_1} \tilde{h}_{P_2} h_s^{(H)}} / 6 [L_1(m_s) + 4H L_1(m_s)] \quad (3.5)$$

where  $\lambda_{P_1}$  and  $\lambda_{P_2}$  are the isotopic matrices corresponding to pseudo-scalar mesons  $P_1$  and  $P_2$ .

The decay width for  $f_c \rightarrow \pi\pi$  is

$$\Gamma(f_c \rightarrow \pi\pi) = \frac{3}{32\pi} \sqrt{1 - \frac{4m_\pi^2}{m_{f_c}^2}} \frac{g_{f_c \pi\pi}^2}{m_{f_c}}$$

The dependences of  $R(H, \sin \delta_s)$  and  $Q(H, \sin \delta_s) = \frac{\Gamma(f_c \rightarrow \pi\pi)}{\Gamma_{\text{total}}(f_c \rightarrow \pi\pi)}$

on  $H$  for different values of  $\delta_s$  are plotted in fig. 4. One can see the values  $R$  and  $Q$  to be close to unity if  $H$  and  $\delta_s$  are equal

$$H = 0.27, \quad \sin \delta_s = -0.45 \quad \text{or} \quad \delta_s = -27^\circ.$$

The mass of  $\epsilon$ -meson is fixed by the condition of the best coincidence of  $\pi\pi$ -scattering S-wave lengths  $a_c^1$  and  $a_c^2$  with the experimental data<sup>15,16/</sup>. In the QCM the  $\pi\pi$ -scattering matrix element is defined by the diagrams, given in fig. 5 and is calculated in a standard way. The  $a_c^1$  and  $a_c^2$  dependence on the  $\epsilon$ -meson mass is plotted in fig. 6.

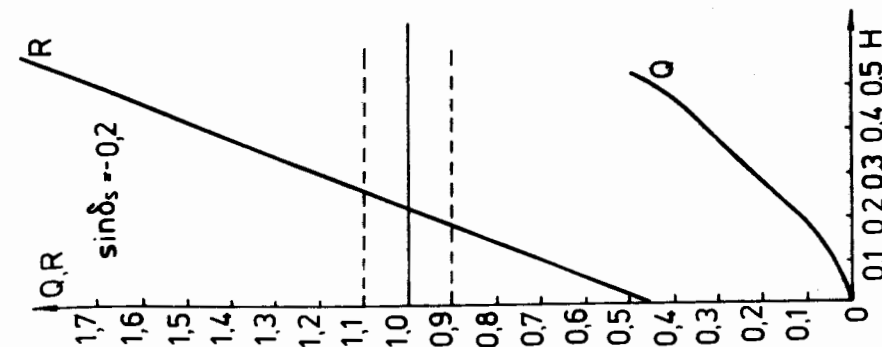
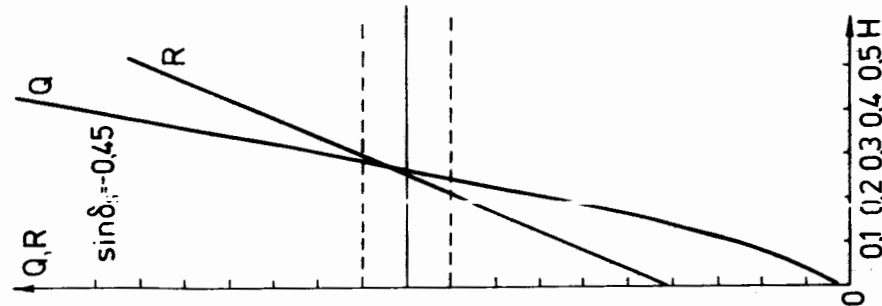
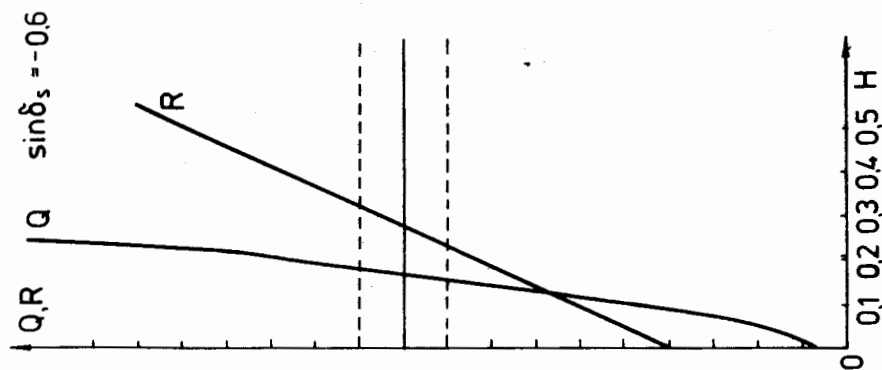


Fig. 4

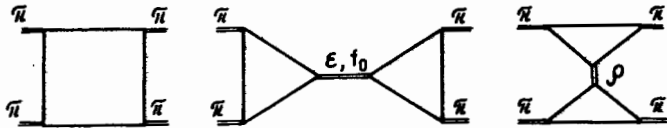


Fig. 5

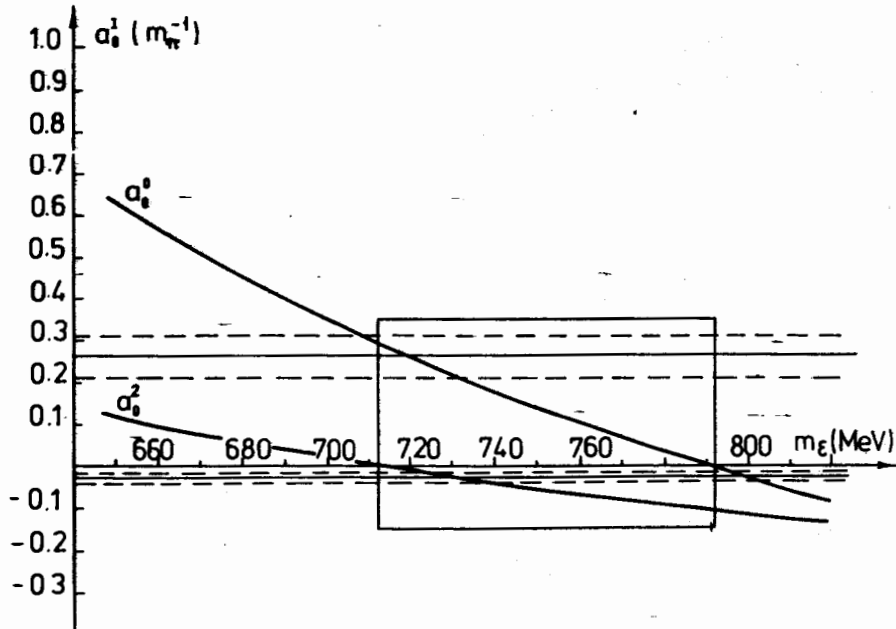


Fig. 6

One can see that the best agreement with the experimental data /15/ is achieved if

$$m_\varepsilon = 730 \text{ MeV}.$$

In this case

$$a_\omega^c = 0.227 m_\pi^{-1} \quad a_\omega^2 = -0.017 m_\pi^{-1}.$$

So, we have fixed all the parameters necessary for the description of scalar mesons. It is of interest to calculate widths of strong and radiative decays of  $0^{++}$  mesons. We should like to note that just the radiative decay width  $a \rightarrow \gamma\gamma$  in the framework of the two-quark scheme is one of an arguments in favour of the four-quark structure of  $a_0(975)$ -meson /12/.

The diagrams describing strong and radiative decays of scalar mesons are presented in figs. 1 and 7. The matrix element of strong

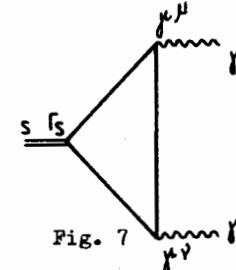


Fig. 7

decays  $S \rightarrow pp$  is of the form (3.5) while that for the radiative one  $S \rightarrow \gamma\gamma$  is

$$M(S \rightarrow \gamma\gamma) = [g^{\mu\nu} q_1^\nu q_2^\mu - q_1^\mu q_2^\nu] g_{S\gamma\gamma},$$

where

$$g_{S\gamma\gamma} = \alpha \sqrt{6} h_s \frac{1}{\Lambda} \text{Sp} Q^2 \lambda_s \left[ \int_0^1 du a \left( -u \frac{m_s^2}{4\Lambda^2} \right) (1+u) \ln \frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} \right. \\ \left. + H \frac{m_s^2}{4\Lambda^2} \int_0^1 du u \delta \left( -u \frac{m_s^2}{4\Lambda^2} \right) \ln \frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} \right], \quad (3.6)$$

with  $\alpha = e^2/4\pi^2$  and  $Q = 1/3 \text{diag}(2, -1, -1)$  being the charge matrix. The widths of strong and radiative decays are given in Table 3.

#### IV. Polarizability of $\pi^-$ -meson

Now we pass to calculation of electric and magnetic polarizabilities of  $\pi^+$  and  $\pi^0$  mesons. As it has been mentioned in the introduction, these parameters are extracted from the amplitude of Compton scattering on  $\pi^-$ -meson.

Table 3

Process	QCM	Experiment
$f_c \rightarrow \pi\pi$	26 MeV	$26 \pm 5$ MeV /18/
$Q_c \rightarrow \pi\eta$	59 MeV	$54 \pm 7$ MeV /18/
$E \rightarrow \pi\pi$	300 MeV	-
$f_c \rightarrow \gamma\gamma$	0.03 keV	0.8 keV /18/
$Q_c \rightarrow \gamma\gamma$	0.5 keV	$0.19 \pm 0.07^{+0.10}_{-0.07}$ /19/
$E \rightarrow \gamma\gamma$	0.95 keV	-

Table 4

Diagram	$\alpha_{\pi^+}$ $10^{-43} \text{cm}^3$	$\alpha_{\pi^0}$ $10^{-43} \text{cm}^3$	$\beta_{\pi^+}$ $10^{-43} \text{cm}^3$	$\beta_{\pi^0}$ $10^{-43} \text{cm}^3$
	-0.615	-6.15	0.615	6.15
	4.51	4.51	-4.51	-4.51
	-	-	+0.26	+1.7
	0.163	1.46	-0.17	-1.42
Result	4.06	-0.18	-3.8	1.92

The matrix element of  $\pi\gamma - \pi\gamma$  on the mass-shell of photons in a general case is /17/

$$M_{\mu\nu}(\pi\gamma - \pi\gamma) = -\sum_{\alpha=1}^2 T_{\mu\nu}^{\alpha} F_{\alpha}(\xi, t). \quad (4.1)$$

where

$$T_{\mu\nu}^1 = (q_1, p_1) g_{2\mu} p_{2\nu} + (q_2, p_2) g_{1\nu} p_{1\mu} - g_{\mu\nu} (p_1, q_1)(p_2, q_2) - p_{1\mu} p_{2\nu} (q_1, q_2)$$

$$T_{\mu\nu}^2 = g_{\mu\nu} q_1 q_2 - q_{1\nu} q_{2\mu}$$

$F_{1,2}(\xi, t)$  - are the form factors defined by inner structure of  $\pi$ -meson,  $q_1, q_2$  are momenta of initial and final photons,  $p_1, p_2$  are momenta of initial and final  $\pi$ -mesons.

$$\xi = (p_1 + q_1)^2 = (p_2 + q_2)^2$$

$$t = (p_1 - p_2)^2 = (q_1 - q_2)^2$$

Electric and magnetic polarizabilities of  $\pi$ -mesons can be connected with the form factors  $F$  by the following relations /17/:

$$\alpha_{\pi} = -\frac{F_1(m_{\pi}^2, 0) + F_2(m_{\pi}^2, 0)}{m_{\pi}} \quad (4.2)$$

$$\beta_{\pi} = \frac{F_2(m_{\pi}^2, 0)}{m_{\pi}}$$

The diagrams defining the  $\pi\gamma - \pi\gamma$  amplitudes are presented in table 4. We take into account the diagrams with intermediate scalar, vector and axial-mesons along with the box ones.

Note that  $F_1$  receives contribution only from the diagrams with intermediate vectors and axials.

The form factors  $F_{1,2}(m_{\pi}^2, 0)$  can be written in the form

$$F_1(m_{\pi}^2, 0) = F^V(m_{\pi}^2, 0) + F_1^A(m_{\pi}^2, 0) \quad (4.3)$$

$$F_2(m_{\pi}^2, 0) = F^{\text{Box}}(m_{\pi}^2, 0) + F^S(m_{\pi}^2, 0) + F_2^A(m_{\pi}^2, 0) - F^V(m_{\pi}^2, 0).$$

Disregarding the  $\pi$ -meson mass ( $m_{\pi}^2/4\Lambda^2 = 0.02$ ), contributions from different diagrams can be written as

$$F_{\pi^+}^{\text{Box}}(0, 0) = -\alpha \cdot 4\pi h_{\pi} \cdot 1/g \cdot \beta(0)/8\Lambda^2$$

$$F_{\pi^0}^{\text{Box}}(0, 0) = -\alpha \cdot 4\pi h_{\pi} \cdot 10/g \cdot \beta(0)/8\Lambda^2 \quad (4.4)$$

$$F_{\pi^+}^S(0, 0) = F_{\pi^0}^S(0, 0) = \sum_{s=f, \xi} g_{sPP}(0) g_{s\gamma\gamma}(0) D_s(0)$$



$$\begin{aligned}
F_{1\pi^+}^A(0,0) &= -\alpha \cdot 4\pi h\pi m_{\pi^+}^2 / 27 (a(0)/\Lambda)^2 h_a D_a(0) \\
F_{2\pi^+}^A(0,0) &= \alpha \cdot 4\pi h\pi 2/9 a(0) G_a^{(0)} h_a D_a(0) \\
F_{1\pi^0}^A(0,0) &= -\alpha \cdot 4\pi h\pi m_{\pi^0}^2 / 9 (a(0)/\Lambda)^2 h_f D_f(0) \\
F_{2\pi^0}^A(0,0) &= \alpha \cdot 4\pi h\pi 2/3 a(0) G_a^{(0)} h_f D_f(0) \\
F_{\pi^+}^V(0,0) &= -\alpha \cdot 4\pi h\pi m_{\pi^+}^2 / 3 (a(0)/\Lambda)^2 h_p D_p(0) \\
F_{\pi^0}^V(0,0) &= -\alpha \cdot 4\pi h\pi m_{\pi^0}^2 \cdot 2 (a(0)/\Lambda)^2 h_w D_w(0)
\end{aligned}$$

Here  $D_X(p^2)$  is a full propagator of meson  $X$ . The explicit form of  $h_X D_X(0)$  for  $SVA$ -mesons is given in Table 2. The constants  $g_{SPP}(0)$  and  $g_{S\gamma\gamma}(0)$  are defined by formulae (3.5) and (3.6) with  $m_S=0$ . The values of  $H$  and  $\delta_S$  fixed in the previous section are used in the  $g_{SPP}$  and  $g_{S\gamma\gamma}$  calculations.

Substituting (4.3) and (4.4) into (4.2), one gets the values for the electric and magnetic polarizabilities of  $\pi^+$  and  $\pi^0$ -mesons. One can see from (4.4) that  $\alpha_\pi$  and  $\beta_\pi$  depend on the  $\mathcal{E}$ -meson mass. This dependence is plotted in fig. 8.

The contributions to electric and magnetic polarizabilities of  $\pi^+$  and  $\pi^0$  mesons from different diagrams and the final value for these parameters with  $m_{\mathcal{E}} = 730$  MeV are presented in table 4. One can see (Table 4) that diagrams with intermediate scalar mesons give the main contribution to the  $\pi^+$  meson polarizability. The small value of the  $\pi^0$ -meson electric polarizability is explained by mutual cancellation of the contributions from the box diagram and the diagrams with intermediate scalar mesons in accordance with the Adler condition (3.4). The full cancellation does not take place because of the inclusion of the  $f_v(980)$ -meson along with the  $\mathcal{E}(730)$ -one. The vector and axial intermediate mesons were considered apart from the scalar ones. Intermediate  $\omega(780)$ ,  $f_1(1285)$ -mesons are seen from table 4 to play a rather important role in the description of  $\pi^0$ -polarizability.

One can see from table 4 that the obtained value for  $\alpha_{\pi^+}$  is close to that from [4]. The quantity  $(\alpha_{\pi^+} + \beta_{\pi^+}) = 0.24 \cdot 10^{-43} \text{ cm}^3$  is small and positive, which is in agreement with the results of the approach based on the dispersion sum rules [6]. Deviation of this value from zero is caused by the vector and axial vector contributions.

Thus, the electric and magnetic polarizabilities of  $\pi^+$  and  $\pi^0$  mesons have been calculated in the Quark Confinement Model. The matrix element of  $\pi\gamma \rightarrow \pi\gamma$  was calculated taking account of the diagrams with intermediate scalar, vector and axial states. Diagrams with intermediate  $\mathcal{E}(730)$ -meson turned out to play the main role in the cal-

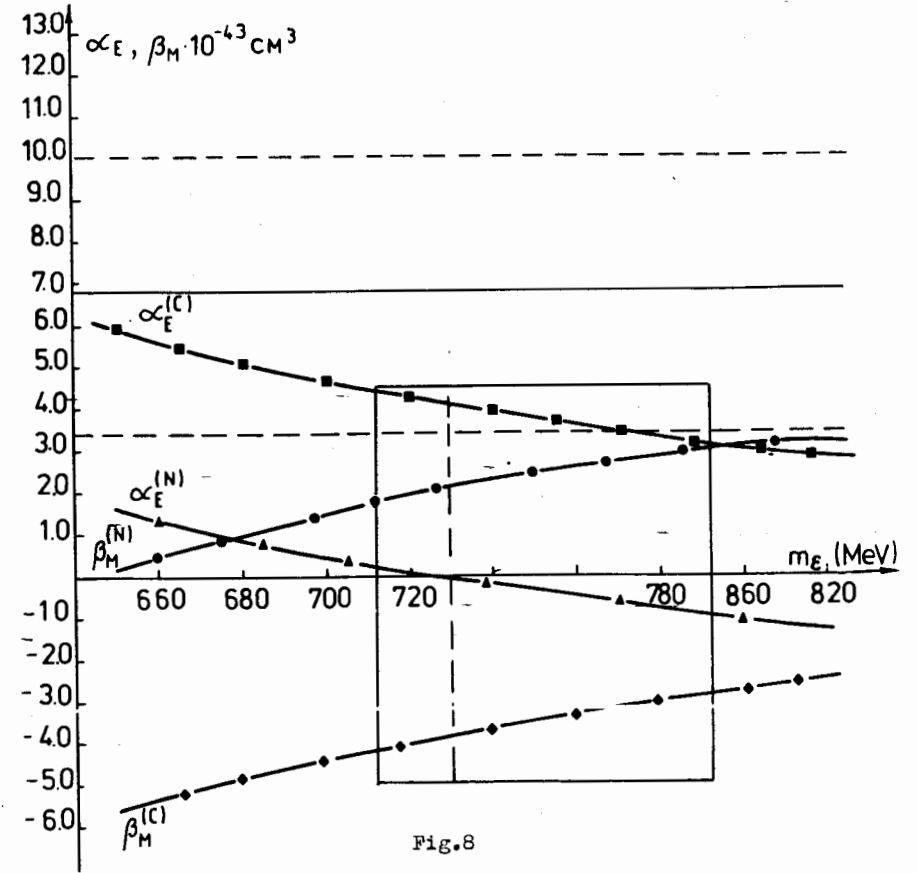


Fig.8

ulation of  $\alpha_\pi$  and  $\beta_\pi$ . The obtained results are in satisfactory agreement with experimental data and are not in contradiction with the results of other approaches.

In conclusion the authors would like to thank S.B.Gerasimov, A.B.Govorkov and M.K.Volkov for many useful discussions.

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Received by Publishing Department  
on July 13, 1988.

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E2-88-519

Поляризуемость  $\pi$ -мезонов в модели конфайнированных кварков

Электрические  $\alpha_\pi$  и магнитные  $\beta_\pi$  поляризуемости  $\pi^+$  и  $\pi^0$ -мезонов вычислены в модели конфайнированных кварков. Учтены диаграммы с промежуточными векторными, аксиально-векторными и скалярными мезонами. Оказалось, что промежуточные скалярные состояния вносят существенный вклад в электрическую магнитную поляризуемости пионов. Получены следующие значения для  $\alpha_\pi$  и  $\beta_\pi$ :

$$\alpha_{\pi^\pm} = 4.06 \cdot 10^{-43} \text{ см}^3 \quad \beta_{\pi^\pm} = -3.84 \cdot 10^{-43} \text{ см}^3$$

$$\alpha_{\pi^0} = -0.18 \cdot 10^{-43} \text{ см}^3 \quad \beta_{\pi^0} = 1.92 \cdot 10^{-43} \text{ см}^3$$

Вычислены ширины сильных ( $a_0(980) \rightarrow \pi\eta$ ,  $f_0(975) \rightarrow \pi\pi$ ,  $\epsilon(730) \rightarrow \pi\pi$ ) и радиационных ( $a_0(980)$ ,  $f_0(975)$ ,  $\epsilon(730) \rightarrow \gamma\gamma$ ) распадов скалярных мезонов. Полученные результаты удовлетворительно согласуются с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

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E2-88-519

Polarizability of  $\pi$ -Mesons in the Quark Confinement Model

The electric  $\alpha_\pi$  and magnetic  $\beta_\pi$  polarizabilities are calculated in the Quark Confinement Model (QCM). The diagrams with vector, scalar and axial intermediate states are taken into account. It is found that intermediate scalar mesons give an essential contribution to electric and magnetic polarizabilities of pions. The following values for  $\alpha_\pi$  and  $\beta_\pi$  are obtained:

$$\alpha_{\pi^\pm} = 4.06 \cdot 10^{-43} \text{ cm}^3 \quad \beta_{\pi^\pm} = -3.84 \cdot 10^{-43} \text{ cm}^3$$

$$\alpha_{\pi^0} = -0.18 \cdot 10^{-43} \text{ cm}^3 \quad \beta_{\pi^0} = 1.92 \cdot 10^{-43} \text{ cm}^3$$

The widths of strong ( $a_0(980) \rightarrow \pi\eta$ ,  $f_0(975) \rightarrow \pi\pi$ ,  $\epsilon(730) \rightarrow \pi\pi$ ) and radiative ( $a_0(980)$ ,  $f_0(980)$ ,  $\epsilon(730) \rightarrow \gamma\gamma$ ) decays are calculated. The results are obtained to be in satisfactory agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988