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TO THE K°_{L}, K°_{S} MESON MASS DIFFERENCE

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Explanation of the effects arising in the nonleptonic processes with $\Delta S = 1,2$ is one of the problems of the standard model. The main difficulty consists in the description of the weak and strong interactions at large distances. At present, it is common practice to study this problem by using the weak effective Hamiltonians /1/ obtained from the Weinberg-Salam model by taking into account the gluonic corrections due to strong interactions at small distances /2,3/. In the four-quark scheme the effective Hamiltonians are

$$H_{eff}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} \sin\theta_c \cos\theta_c \sum_{i=1}^{6} c_i O_i \qquad (1)$$

$$H_{eff}^{\Delta S=2} = \frac{G_F^2}{16\pi^2} \sin^2\theta_c \cos^2\theta_c \, m_c^2 \, \eta \, O^{\Delta S=2}, \qquad (2)$$

where G_F is the Fermi constant, θ_c is the Cabibbo angle, m_c is the c-quark mass; O_i in (1) and $O^{\Delta S=2}$ in (2) are the local four-quark operators /2,3/. The coefficients C_i in (1) and η in (2) are calculated by the perturbative QCD methods and are defined by the W-boson and c-quark masses as well as the QCD parameters μ and α_s /2,3/.

The main problem when using the effective Hamiltonians (1), (2) is the calculation of the matrix elements of the O-operators. As the theory of strong interactions is incomplete yet, the calculations of these matrix elements need various models to be used /4/.

Among the effects arising in nonleptonic processes of particular interest is the K_L, K_S meson mass difference equal to $\Delta m^{exp} = (3.52 \pm 0.014) \cdot 10^{-15}$ GeV /5/. It is known /1/ that this mass difference Δm_{LS} is related with the matrix element of the $K^0 - \bar{K}^0$ transition by $\Delta m_{LS} = Re\, M(K^0 \to \bar{K}^0)/2m_K$.

As a rule, Δm_{LS} is represented in the form of $\Delta m_{LS} = \Delta m^{SD} + \Delta m^{LD}$ /6/ where Δm^{SD} is the K_L^0, K_S^0 meson mass difference calculated by means of $H^{\Delta S=2}$ (2) (the contribution of "small" distances), and Δm^{LD} takes into account the contribution of "large" distances.

The results for Δm^{SD} obtained in various models are usually compared with the result of the so-called vacuum saturation method and are characterized by the parameter

$$B = \langle \bar{K}^0 | H_{eff}^{\Delta S=2} | K^0 \rangle / \langle \bar{K}^0 | H_{eff}^{\Delta S=2} | K^0 \rangle_{VAC} .$$

The B -factor has been calculated in various approaches /8-12/. It was found that the results essentially depended on the calculation methods. The "large"-distance contribution is characterized by the parameter $D = \Delta m^{LD} / \Delta m_{LS}$ /6/. In /9/ Δm^{LD} has been obtained from the calculation of matrix element being the product of two operators $H_{eff}^{\Delta S=1}$. It is stated /10/ that the way of taking into account "large" distances is equivalent to the calculation of the diagrams with intermediate states. This program has been realized, for example, in refs. /13,14/ where the one-, two- and three-particle intermediate states were considered by using the dispersion relations.

Unfortunately, this consideration is connected with large theoretical uncertainties; therefore, the value of D changes in the wide intervals: $D = 0.1 \pm 0.41$ in /13/ and $D = -2 \div 2.8$ in /14/.

Thus, the calculation of the mass difference Δm_{LS} by taking into account the contributions of both the "small" and "large" distances is a good checking for any model that claims the description of the low-energy hadron processes. One of these models is the quark confinement model (QCM) /15/ based on the definite assumptions of confinement and hadronization. A wide range of the low-energy phenomena has been described in the framework of this model /15,16/.

The nonleptonic decays of kaons have been considered in our paper /16/. The widths of the decays $K_S^0 \to \pi^+ \pi^-$, $K^+ \to \pi^+ \pi^0$ and $K_{L,S}^0 \to \gamma\gamma$ have been calculated by using the effective Hamiltonian (1). It was found that the intermediate states, particularly $\mathcal{E}(730)$-meson, were very important for the description of these decays.

This paper is devoted to the calculation of the K_L^0, K_S^0 meson mass difference. As usual, the contribution of "small" distances is taken into account by using $H_{eff}^{\Delta S=2}$ and of "large" distances by using the intermediate states. The interaction Lagrangians of the mesons with quarks underlie the QCM /15/

$$\mathcal{L} = \frac{g_Q}{\sqrt{2}} M_Q \bar{q} \Gamma_Q \lambda_Q q .$$

Here, M_Q are a meson fields with quantum numbers Q and masses m_Q, g_Q are the meson-quark coupling constants determined by the compositeness condition /15/, the matrices Γ_Q, λ_Q provide the necessary quantum numbers Q.

The model parameters are Λ_f characterizing the confinement region of the quark with flavour f. They are chosen by fitting over the main decays of light mesons and equal $\Lambda_u = \Lambda_d = \Lambda = 460$ MeV, $\Lambda_s = 506$ MeV. The auxiliary parameters characterizing the scalar mesons ($H = 0.27$, $\delta_s = -27°$, $m_\mathcal{E} = 730$ MeV) are determined by the low-energy theorems /16/.

The matrix element of the transition $\langle \bar{K}^0 | H_{eff}^{\Delta S=2} | K^0 \rangle$ is defined by the diagrams of Fig. 1 and is written in the form

$$M_{SD}(K^0 \to \bar{K}^0) = \frac{G_F^2}{16\pi^2} \sin^2\theta_c \cos^2\theta_c \, m_c^2 \, \eta \, T_{K^0\bar{K}^0}^{\Delta S=2} ,$$

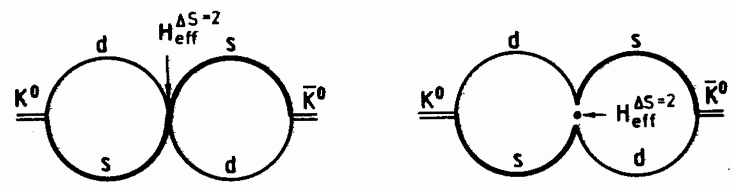

Fig. 1

where

$$T_{K^0\bar{K}^0}^{\Delta S=2} = \iiint dx_1 dx_2 dy \, e^{ip_1 x_1 + ip_2 x_2} \langle 0 | T(\mathcal{L}_{K^0}(x_1) \mathcal{L}_{\bar{K}^0}(x_2) O^{\Delta S=2}(y)) | 0 \rangle .$$

Using the Firtz transformation /1/ we have after the standard calculations /15/:

$$T_{K^0\bar{K}^0}^{\Delta S=2} = \frac{8}{3} \Big[\frac{g_K}{\sqrt{2}} \int \frac{d^4K}{(2\pi)^4 i} \int d\sigma_\lambda \, tr \Big(\gamma^5 \frac{1}{\lambda \Lambda_s - \hat{K}} \gamma^\mu (1+\gamma^5) \frac{1}{\lambda \Lambda_d - (\hat{K}+\hat{p})} \Big) \Big]^2 = \tag{3}$$

$$= \frac{8}{3} m_K^2 f_K^2 .$$

Integration over $d\sigma_\lambda$ in (3) means averaging over gluon vacuum which provides the quark confinement /15/. The decay constant f_K in the QCM is written as

$$f_K = \frac{\Lambda}{\pi} \sqrt{\frac{3}{8}} \Big\{ (\sqrt{1-\Delta} + \sqrt{1+\Delta}) \int_0^\infty du \, a(u) + \frac{m_K^2}{4\Lambda^2} \int_0^{u_\Delta} du \, a(-u \frac{m_K^2}{4\Lambda^2}) .$$

$$\cdot \sqrt{1-u+\left(\frac{u\Delta}{2}\right)^2}\left[\sqrt{1-\Delta}+\sqrt{1+\Delta}+\frac{\Delta u}{4}\left(\sqrt{1+\Delta}-\sqrt{1-\Delta}\right)\right]\Big\}\cdot$$

$$\cdot\left\{\int_0^\infty du\, b(u)+\frac{m_K^2}{4\Lambda^2}\int_0^{u_\Delta} du\, b\left(-u\frac{m_K^2}{4\Lambda^2}\right)\frac{\left(1-u+\frac{u}{4}\left(3\sqrt{1-\Delta^2}-1\right)+\frac{\Delta^2u^2}{4}\right)}{\sqrt{1-u+\left(\frac{\Delta u}{2}\right)^2}}\right\}^{-1/2}$$

Here, $a(u), b(u)$ are the confinement functions shown in [15]. The following notation is introduced: $\Lambda^2=\frac{\Lambda_s^2+\Lambda_d^2}{2}$; $\Delta=\frac{\Lambda_s^2-\Lambda_d^2}{\Lambda_s^2+\Lambda_d^2}$; $u_\Delta=\frac{2}{\sqrt{1-\Delta^2}+1}$. The numerical value of f_K in the QCM is equal to 160 MeV. As it follows from (3) our result coincides with the one of the vacuum saturation method, i.e. $B=1$.

We use the values $\mu=0.45$ GeV, $\alpha_s=0.55$ fixed in [16] for the calculation of η . As a result, we have

$$\Delta m^{SD}=2.26\cdot 10^{-15}\ \mathrm{GeV}\cdot \qquad (4)$$

One can see that this value is 69% of the experimental one. The "large"-distance contribution Δm^{LD} is calculated by taking into account the diagrams with intermediate one-meson states by analogy with [10].

Unlike[10] we take into account not only the pseudoscalar mesons but also the scalar and axial ones using the experimental values of the mixing angles and meson masses.

The diagrams of Fig. 2 are to define the matrix element

$$M_{LD}(K^0\to\bar{K}^0)=\sum_{x=P,A,S} D_x(m_K^2)\left[M(K^0\to x)\right]^2,$$

where $D_x(m_K^2)$ are the full meson propagators[15],

$$M(K^0\to x)=\frac{G_F}{2\sqrt{2}}\sin\theta_c\cos\theta_c\sum_{i=1}^{6}c_i T^i_{Kx}.$$

Here

$$T^i_{Kx}=\iiint dx_1\,dx_2\,dy\, e^{ip_1x_1+ip_2x_2}\langle 0|T(\mathcal{L}_{K^0}(x_1)\mathcal{L}_x(x_2)O^i(y))|0\rangle\cdot$$

where c_i, O_i are the coefficients and operators as in (1), T^i_{Kx} are calculated in a standard manner [15]. Using the set $\{c_i\}$ fixed in [16] we have the following value for Δm^{LD}:

$$\Delta m^{LD}=1{,}45\cdot 10^{-15}\ GeV\cdot \qquad (5)$$

The relative contributions of the intermediate mesons to Δm^{LD} are shown in the Table. One can see that the main contribution is given by the $\pi, \eta, \mathcal{E}$ -mesons.

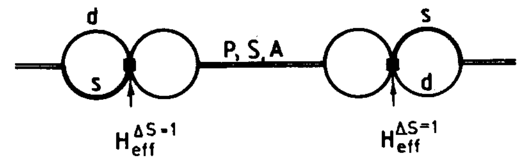

Fig. 2

Table

Meson	π	η	η'	$\mathcal{E}$	f_0	a_0	a_1	E	D
Relative contribution	-0.253	0.545	0.017	0.346	0.051	0.05	0.085	0.075	0.086

Summing up (4) and (5) we finally have

$$\Delta m_{LS}=3.71\cdot 10^{-15}\ \mathrm{GeV}\cdot$$

This value coincides quite accurately with the experimental one[5]. The value $D=0.41$ is close to [9].

Thus, the matrix element of the $K^0-\bar{K}^0$ -transition and the relevant K_L^0, K_S^0 meson mass difference are calculated in the QCM. It is found that the B -factor is equal to 1.

The contribution of the diagrams with intermediate meson states taking into account the "large" distances is found to be 41% of the total value. The obtained result for Δm_{LS} is in good agreement with experimental data.

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О разности масс K_L^0- и K_S^0-мезонов

Показано, что экспериментальное значение разности масс K_L^0- и K_S^0-мезонов может быть объяснено в рамках стандартной модели при учете промежуточных мезонных состояний, эффективно учитывающих вклад "больших" расстояний. Соответствующие расчеты проведены в модели конфайнмированных кварков. Оказалось, что вклад "малых" расстояний $\Delta m^{SD} = 2{,}26\cdot10^{-15}$ ГэВ, вклад "больших" расстояний $\Delta m^{LD} = 1{,}45 \times 10^{-15}$ ГэВ, а их суммарное значение, $\Delta m_{LS} = 3{,}71\cdot10^{-15}$ ГэВ, хорошо согласуется с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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To the K_L^0, K_S^0 Meson Mass Difference

The experimental value of the K_L^0, K_S^0 meson mass difference is shown to be explained in the standard model by taking into account the intermediate meson states, which effectively accumulate the contribution of large distances. The calculations are performed in the quark confinement model. It is found that the contributions of "small" and "large" distances are equal to $\Delta m^{LD} = 2.26 \times 10^{-15}$ GeV and $\Delta m^{SD} = 1.45\cdot10^{-15}$ GeV so that the sum $\Delta m_{LS} = 3.71\cdot10^{-15}$ GeV is in good agreement with the experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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