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THE LOW-ENERGY LIMIT OF A BILOCAL MESON LAGRANGIAN FROM QCD

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I. Introduction

It is generally accepted that there are two main difficulties in constructing the theory of strong interactions. The first one is connected with the large quark-gluon coupling constant for a small momentum transfer. The second one concerns the quantization of non-Abelian infrared-diverging theories and the definition of the structure of their physical vacuum. In [1,2] there was perceived one more problem in establishing a theory of hadrons. It consists in the non-understanding of the principles for constructing an S-matrix in which the asymptotic states are bound states.

In describing the atomic spectra in QED one explicitly supposes an advantage of the Coulomb gauge and the time axis of quantization chosen in the atom rest frame. A change of the gauge or of the quantization axis affects the theoretical results [3,4,1] which turn out to be invariant only with respect to simultaneous gauge and relativistic transformations [1,5].

In [5,6] there was proposed a minimal quantization method for gauge theories based on an explicit solution of the constraints. It gives a motivation for the technique of describing bound states in QED and leads to an unambiguous construction of the relativistic S-matrix for bound states [1]. The principles for the construction of such an S-matrix are the explicit solution of the Gauss equation for the gauge field time component $A_0 = (A \cdot \eta)$ as well as the choice of the quantization axis η_μ parallel to the differentiation operator for the total coordinate of the bound state. An extension of these quantization principles to QCD with increasing potential was suggested in [2]. Such a potential occurs naturally in the lowest perturbation order if one takes the non-normalizable solution of the Gauss equation.

The relativistic generalization of the potential approach consists in changing the "direction" of the Coulomb field so that it moves together with the particles whose bound states it forms. This is achieved by transforming the nonrelativistic Bethe-Salpeter kernel into a relativistic one according to

$$K(x) = \gamma_0 \cdot \gamma_0 \, V(\vec{x}) \, \delta(x_0) \longrightarrow K^{\eta}(x^{\perp}|x^{\parallel}) = \gamma^{\parallel}_{\mu} \cdot \gamma^{\parallel \mu} \, V(x^{\perp}) \, \delta(|x^{\parallel}|)$$

with

$$\gamma^{\parallel}_{\mu} = \eta_{\mu}\not\eta\ , \quad x^{\parallel}_{\mu} = \eta_{\mu}(x\eta)\ , \quad x^{\perp} = x_{\mu} - x^{\parallel} \quad \text{and} \quad \eta^2 = 1 .$$

It is shown in [2] that the low-energy limit of bound state interaction corresponds to a localization of the bound state wave functions with respect to the relative coordinate, and that such a localization is equivalent (for the properties of the solutions of the Bethe-Salpeter equation) to a localization of the increasing potential. Therefore, in the low-energy limit the latter is replaced by a 4-quark Nambu - Jona - Lasinio potential [7] with definite dependence on the vector η_{μ} :

$$K(x) = \frac{1}{\mu^2}\gamma^{\parallel}_{\nu}\delta^4(x)\gamma^{\parallel\nu} = \frac{1}{\mu^2}\not\eta\,\delta^4(x)\not\eta\ , \tag{I}$$

where μ is a parameter which is fixed by the masses of the low-lying resonances.

This paper is devoted to the investigation of the Nambu - Jona - Lasinio model of the type (I). Thereby the difference to other modern treatments of the Nambu - Jona - Lasinio model [8,9] lies not only in the matrix structure (1) but also in an exact calculation of the energy dependence in order to investigate the reasons for the appearance of tachions in the QCD low-energy expansion [10]. Furthermore, the P-A, V-T, and S-V mixings have been exactly taken into account. They occur automatically by solving the Bethe - Salpeter equation with the help of projection operators on the particle and antiparticle states.

The paper is organized as follows. In Sect.2 we define our model and solve the corresponding Bethe - Salpeter equation. The mass spectrum for the low-lying mesons is discussed in Sect.3. This is followed in Sect.4 by the determination of the Pion decay constant. Sect.5 contains the conclusion. Some of the details concerning the rewriting of the Bethe - Salpeter equation by means of projection operators and the normalization of the Bethe - Salpeter wave function are shifted to the Appendix.

2. Bethe - Salpeter equation and its solution

Let us consider the action [2]

$$S = S_{free} + S_{int} \tag{2}$$

with

$$S_{free} = \frac{N_c}{2}\, tr \int d^4x \left[\mu^2\gamma^{\parallel}_{\nu}\Phi_{ij}(x)\gamma^{\parallel\nu} - i\int d^4y\, G_{m_i}(x-y)\Phi_{ij}(y)G_{m_j}(y-x)\right]\Phi_{ji}(x) \tag{3}$$

and

$$S_{int} = iN_c\, Tr \sum_{n=3}^{\infty}(-1)^{n+1}n^{-1}(G_m\cdot\Phi)^n . \tag{4}$$

Here the following notations have been introduced. The meson field is given by

$$\Phi_{ij} = \sum_{a=1}^{N_f^2-1}\frac{\lambda^a_{ij}}{2}\Phi^a\ ,$$

where λ^a are the Gell-Mann matrices satisfying

$$tr(\lambda^a\lambda^b) = 2\delta^{ab}$$

and N_f denotes the number of flavours. N_c is the colour number. G_{m_i} means the Green function for a quark with constituent mass m_i :

$$(i\not\partial - m_i)\,G_{m_i} = \delta^4(x-y) .$$

Furthermore the short-hand notation

$$Tr\,(G_m\cdot\Phi)^n = tr\int d^4x_1 d^4x_2 \dots d^4x_n\, G_{m_{i_1}}(x_n - x_1)\Phi_{i_1i_2}(x_1)\times$$
$$\times G_{m_{i_2}}(x_1-x_2)\Phi_{i_2i_3}(x_2)\ \dots\ G_{m_{i_n}}(x_{n-1}-x_n)\Phi_{i_ni_1}(x_n)$$

has been used.

The Schwinger - Dyson equation corresponding to the action (2)-(4) reads

$$m_i = m_{oi} + \frac{1}{2\mu^2}\int^L \frac{d^3q}{(2\pi)^3}\frac{m_i}{\sqrt{\vec{q}^{\,2}+m_i^2}}$$

with m_{oi} as the bare quark mass and L - the ultraviolet cut-off parameter. Performing the integral leads to

$$8\pi^2\mu^2 = \frac{m_i}{m_i - m_{oi}}\left(L^2 - m_i^2\, ln\frac{2L}{m_i}\right). \tag{5}$$

This relation linking the parameters μ and L with the masses m_i and m_{oi} will be of importance in our further considerations.

We will calculate the bound state masses with the help of the Bethe - Salpeter equation in the ladder approximation [12]. For further purposes we need the plane wave expansion of the bound state field [2]:

$$\phi(x) = \sum_H \int \frac{d^3 \mathcal{P}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_H}} \left[e^{i\mathcal{P}_H x} a_H^{+}(\mathcal{P}) \Gamma^H(\mathcal{P}) + e^{-i\mathcal{P}_H x} a_H^{-}(\mathcal{P}) \bar{\Gamma}^H(\mathcal{P}) \right],$$

where

$$\omega_H = \sqrt{M_H^2 + \vec{\mathcal{P}}^2}$$

is the bound state energy and

$$\mathcal{P}_H = (\omega_H, \vec{\mathcal{P}})$$

the total momentum. On the other hand the Fourier transform of $\phi(x)$ is given by the relation

$$\phi(x) = \int \frac{d^4 \mathcal{P}}{(2\pi)^4} \underset{\sim}{\phi}(\mathcal{P}) e^{i\mathcal{P}x} . \tag{6}$$

Then the Bethe – Salpeter equation for the wave function Γ^H has the form (cf. appendix)

$$\mu^2 \Gamma^H(\mathcal{P}_H) = i \int \frac{d^4 k}{(2\pi)^4} \gamma^{\parallel}_{\mu} \underset{\sim}{G}_{m_1}\left(k + \frac{\mathcal{P}_H}{2}\right) \Gamma^H(\mathcal{P}_H) \underset{\sim}{G}_{m_2}\left(k - \frac{\mathcal{P}_H}{2}\right) \gamma^{\parallel\mu} \tag{7}$$

with the quark propagator

$$\underset{\sim}{G}_{m_i}(p) = \frac{1}{\not{p} - m_i + i\varepsilon} .$$

Using standard methods [11] this equation for the 16-component spinor Γ^H decouples into equations for lower-component spinors (for more details look to appendix). It can be solved by means of the decomposition

$$\Gamma^H = \Gamma_1^H + \frac{\not{\mathcal{P}}_H}{M_H} \Gamma_2^H ,$$

$$\Gamma_\ell^H = 1\!\!1 S_\ell + \gamma_5 P_\ell + \left(\gamma_\mu - \mathcal{P}_{H\mu} \frac{\not{\mathcal{P}}_H}{M_H^2}\right) V_\ell^\mu + \gamma_5 \left(\gamma_\mu - \mathcal{P}_{H\mu} \frac{\not{\mathcal{P}}_H}{M_H^2}\right) A_\ell^\mu ,$$

$\ell = 1, 2 .$

Now to study the meson mass spectrum it is enough to consider the bound states at rest, $\vec{\mathcal{P}} = 0$. In this case one has

$$\Gamma^H = \Gamma_1^H + \gamma_0 \Gamma_2^H , \tag{8}$$

$$\Gamma_\ell^H = \gamma^S L_\ell^S + \gamma^P L_\ell^P + \gamma_i^V L_{\ell i}^P + \gamma_i^A L_{\ell i}^A , \quad \ell = 1,2 , \quad i = 1,2,3,$$

where $\gamma^S = 1\!\!1$, $\gamma^P = \gamma_5$, $\gamma_i^V = \gamma_i$, $\gamma_i^A = \gamma_i \gamma_5$, and the Bethe- Salpeter equation (7) decouples into four sets of two algebraic equations for the quantities L_1^I and L_2^I :

$$\begin{cases} \mu^2 L_1^I = C^I L_1^I + B^I L_2^I , \\ \mu^2 L_2^I = D^I L_2^I + B^I L_1^I , \quad I = S, P, V, A \end{cases} \tag{9}$$

$(L_\ell^I \equiv L_{\ell i}^I$ for $I = V, A$, $i = 1,2,3)$.

The expressions for the coefficients B^I , C^I, and D^I are given in Table 1. There are employed the following short-hand notations:

Table 1. Coefficients B^I , C^I, and D^I in the system of equations (9) for I=S,P,V,A

I	B^I	C^I	D^I
S	$M_H A(M_H)$	$(m_1 - m_2) A(M_H) + \beta$	$(m_1 - m_2) A(M_H)$
P	$M_H F(M_H)$	$(m_1 + m_2) F(M_H) + \beta$	$(m_1 + m_2) F(M_H)$
V	$M_H F(M_H)$	$(m_1 + m_2) F(M_H) + 2\beta/3$	$(m_1 + m_2) F(M_H) + \beta/3$
A	$M_H A(M_H)$	$(m_1 - m_2) A(M_H) + 2\beta/3$	$(m_1 - m_2) A(M_H) + \beta/3$

$$A(M_H) = m_1 \alpha_1(M_H) - m_2 \alpha_2(M_H) ,$$
$$F(M_H) = m_1 \alpha_1(M_H) + m_2 \alpha_2(M_H) ,$$

where α_1, α_2, and β denote the integrals

$$\alpha_1(M_H) = \frac{1}{2} \int^L \frac{d^3 K}{(2\pi)^3} \frac{1}{E_1} \frac{1}{(E_1 + E_2)^2 - M_H^2 + i\varepsilon}$$

(α_2 is obtained from α_1 by interchanging the indices 1 and 2),

$$\beta(M_H) = \int^{L} \frac{d^3K}{(2\pi)^3}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)\frac{K^2}{(E_1+E_2)^2 - M_H^2 + i\varepsilon}$$

with

$$E_i = \sqrt{\vec{k}^2 + m_i^2}\;, \quad i = 1, 2.$$

β can be expressed by means of α_1 and α_2 :

$$\beta(M_H) = \frac{\mu^2}{2}\left(2 - \frac{m_{01}}{m_1} - \frac{m_{02}}{m_2}\right) + \left[(M_H^2 - 3m_1^2 - m_2^2)\,\alpha_1(M_H) + (M_H^2 - 3m_2^2 - m_1^2)\,\alpha_2(M_H)\right]/2 .$$

The system (9) of homogeneous algebraic equations obeys an unambiguous solution if

$$(C^I - \mu^2)(D^I - \mu^2) - (B^I)^2 = 0. \tag{10}$$

In general, B^I, C^I, and D^I are complex quantities. To determine the mass spectrum it is, nevertheless, sufficient to concentrate instead of (10) upon the equation for the ansolute value:

$$|C^I - \mu^2|\,|D^I - \mu^2| - |B^I|^2 = 0 . \tag{11}$$

3. Mass spectrum for low-lying mesons

In this paper we want to restrict ourselves to the calculation of the masses for the low-lying mesons, i.e. we consider the case of equal quark masses

$$m_1 = m_2 = m$$

with

$$m^2/L^2 \ll 1 .$$

Then the integral $\alpha_1 = \alpha_2 \equiv \alpha$ can be easily calculated yielding

$$\alpha(M_H) = \frac{1}{16\pi^2}\left(\ln\frac{2L}{m} - z^{1/2}\,\mathrm{arctg}\, z^{-1/2}\right), \quad z > 0, \tag{12a}$$

$$\alpha(M_H) = \frac{1}{16\pi^2}\left[\ln\frac{2L}{m} - \frac{1}{2}|z|^{1/2}\left(\ln\frac{1+|z|^{1/2}}{1-|z|^{1/2}} - 2\pi i\right)\right], \quad z<0, \tag{12b}$$

where

$$z = 4m^2/M_H^2 - 1 .$$

With the help of (12) and (5) one obtains from (11) the masses for the π, σ, ρ, and a_1 mesons.

In the case of the π - meson we suppose $M_\pi^2 < 4m^2$ and according to (12a) there occur no complexities. The corresponding mass relation reads

$$M_\pi^2 = 2\,\frac{m_0}{m}\left[L^2 - 3m^2 \ln\frac{2L}{m} + 2m^2\right]/\left(\ln\frac{2L}{m} - 1\right). \tag{13}$$

It is to be seen that the pion appears as Goldstone particle ($M_\pi = 0$ for $m_0 = 0$), and thus our theory includes spontaneous breakdown of chiral symmetry. Taking M_π = 140 MeV and m = 315 MeV as input parameters relation (13) is used to obtain values for the cut-off parameter L in dependence on the bare quark mass m_0 *).

The result is given in Table 2.

Table 2. Dependence of the cut-off parameter L on the bare quark mass m_0 for M_π =140 MeV and m=315 MeV

m_0 (MeV)	2	3	4	5	6
L (MeV)	1600	1200	1000	850	725

For the remaining masses we suppose the condition $4m^2 < M_H^2$ to be fulfilled. Like it follows from (12b), α contains an imaginary part. So the equation for the ρ - meson mass is given by

$$M_\rho^2 = \frac{|C^\rho(M_\rho) - \mu^2|\,|D^\rho(M_\rho) - \mu^2|}{4m^2\,|\alpha(M_\rho)|^2} . \tag{14}$$

Concerning the determination of M_σ and M_{a_1} let us begin with some remarks. For equal constituent quark masses there is no S-V mixing because of $B^\sigma = D^\sigma = 0$. Therefore for $I = \sigma$ the system (9) reduces to only one equation. Owing to $B^{a_1} = 0$ it decouples for $I = a_1$ into two independent equations . To obtain a consistent solution for the eigenvalue M_{a_1} we have to set $L_2^{a_1} = 0$. Then the mass relations for M_σ and M_{a_1} are of the common form

$$M_I^2 = 4m^2\,\mathrm{Re}\,\alpha(M_I) - 1 + \left[f_I\,\mu^2 - (M_I^2 - 4m^2)^2\,(\mathrm{Im}\,\alpha(M_I))^2\right]^{1/2}, \tag{15}$$

*) In deriving (13) we have already taken into account the relation $M_\pi^2 \ll 4m^2$.

with $I = \sigma, a_1$ and

$$f_I = \begin{cases} 1 & \text{for } I = \sigma , \\ 9/4 & \text{for } I = a_1 . \end{cases}$$

The results of the calculations of M_σ, M_ρ, and M_{a_1} are presented in the Figure. Taking again M_π =140 MeV and m = 315 MeV the best fit is obtained for L=1350 MeV. It corresponds to m_0=2.6 MeV and yields M_σ = = 640 MeV, M_ρ =770 MeV, and M_{a_1}= 1140 MeV. Furthermore, the equation for M_ρ has no any solution if $M_\rho^2 < 4m^2$. So M_ρ is restricted from below by M_ρ = 2m = 630 MeV what corresponds to L = 1494 MeV. This fixes the value M_{a_1}= 1225 MeV as upper limit for the axial-vector meson mass.

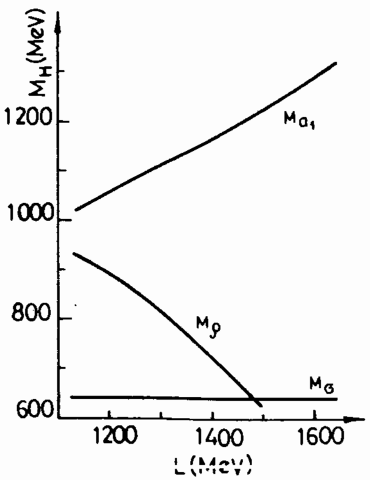

Fig. Masses of the σ, ρ, and a_1 mesons according to (14) and (15) in dependence on the cut-off parameter L.

4. Determination of the Pion decay constant

According to (8) the Bethe-Salpeter wave function for the pseudoscalar vector with the pion at rest, $P_{\pi\mu} = (M_\pi, \vec{0})$, is given by $\gamma_5 L_1^\pi + \gamma_0\gamma_5 L_2^\pi$. The corresponding Bethe – Salpeter equation has the form

$$4N_c\mu^2 L_1^\pi = C'^\pi(M_\pi)L_1^\pi + B'^\pi(M_\pi)L_2^\pi ,$$

$$4N_c\mu^2 L_2^\pi = D'^\pi(M_\pi)L_2^\pi + B'^\pi(M_\pi)L_1^\pi ,$$

where

$$B'^\pi(M_\pi) = 2m\varkappa^2(M_\pi)M_\pi ,$$

$$C'^\pi(M_\pi) = 4N_c\mu^2\left(1-\frac{m_0}{m}\right) + \varkappa^2(M_\pi)M_\pi^2 ,$$

$$D'^\pi(M_\pi) = 4m^2\varkappa^2(M_\pi)$$

and

$$\varkappa^2(M_\pi) = 4N_c\alpha(M_\pi) .$$

Here as difference to (9) we have included the constant factors N_c and $tr_\gamma 1 = 4$.

The Pion decay constant is defined by the axial-vector coupling in the second term of the action (3) which we denote by $S_{free\,2}$. The corresponding pseudoscalar part reads

$$S^\pi_{free\,2}(M_\pi) = \frac{1}{2}tr_{fl}\left[C'^\pi(M_\pi)(L_1^\pi)^2 + D'^\pi(M_\pi)(L_2^\pi)^2 + 2B'^\pi(M_\pi)L_1^\pi L_2^\pi\right] \tag{16}$$

This representation follows immediately from the equations (A5)–(A7) of the appendix. So the term of interest is

$$B'^\pi(M_\pi)\,tr_{fl}\,L_1^\pi L_2^\pi \longrightarrow m\varkappa^2 L_1^{\pi+}M_\pi L_2^{\pi+}$$

yielding

$$F_\pi = m\varkappa^2 L_1^{\pi+} .$$

To calculate F_π the normalization of the Pion wave function $\gamma_5 L_1^{\pi+}$ is required. It follows from the normalization condition (A 7) for the lower-component Bethe – Salpeter wave functions which for the pseudoscalar sector is given by

$$\frac{1}{P_\pi}\frac{\partial}{\partial P_\pi}S^\pi_{free}(P_\pi)\Big|_{P_\pi = M_\pi} = 1$$

with $P_\pi = \sqrt{P_\pi^2}$.

Then, with the help of (16) we obtain the normalization condition only for the Pion wave function:

$$\mathcal{N}^{-2}(L_1^{\pi+})^2 = 1$$

with

$$\mathcal{N}^{-2} = \frac{1}{2P_\pi}\frac{\partial}{\partial P_\pi}C'^\pi(P_\pi)\Big|_{P_\pi = M_\pi} = \varkappa^2 + \frac{N_c}{2\pi^2}\frac{m^2}{M_\pi^2}\left(z^{-1/2}\operatorname{arctg} z^{-1/2} - \frac{1}{z+1}\right),$$

so that

$$z = 4m^2/M_\pi^2 - 1 ,$$

$$L_1^{\pi+} = \mathcal{N}$$

and finally

$$F_\pi = m\varkappa^2\mathcal{N} = 4m\mathcal{N}N_c\alpha(M_\pi) .$$

With $\alpha(M_\pi)$ given by (12a) and N_c =3 we recieve for m=315 MeV and L =1350 MeV as numerical value

$$F_\pi = 93 \text{ MeV},$$

for some more values see Table 3.

<u>Table 3.</u> Dependence of the Pion decay constant on the cut-off parameter L for $M_\pi = 140$ MeV, $m = 315$ MeV and $N_c = 3$

(MeV)	1000	1100	1200	1300	1400	1500	1600
(MeV)	80.0	84.1	88.2	91.5	94.5	97.2	99.7

5. Conclusion

Within the Bethe – Salpeter approach for a QCD-motivated Nambu – Jona – Lasinio model we determined the masses for the low-lying mesons. Thereby, in contrary to the usual procedure we did not expand in energy. So our results (12) for $\alpha(M_I)$ are exact in the limit $m^2 \ll L^2$ yielding an imaginary part in the case $4m^2 < M_H^2$, i.e. for the σ, ρ, and a_1 mesons. In our treatment no tachions /10/ appear at least in the Pion sector. Furthermore, we want to stress that within our approach all mixings have been taken into account from the very beginning by using an adequate decomposition of the Bethe – Salpeter wave function. For equal quark masses the S–V mixing disappears, and we have the P–A and V–T mixings in the form

$$L_2^I = \eta^I L_1^I \quad , \quad I = P, V \quad .$$

with

$$\eta^I = \frac{C^I - \mu^2}{B^I} = \frac{B^I}{D^I - \mu^2} \quad .$$

The obtained results are in good agreement with experiment. Concerning the a_1 mass they support the last data. For the best over-all fit (L=1350 MeV) the mass M_{a_1} is given by 1140 MeV and it is generally restricted from above by 1225 MeV. Furthermore, we have calculated the Pion decay constant. For the cut-off parameter with the value L=1350 MeV we obtained F_π = 93 MeV.

Appendix

1. Bethe – Salpeter equation with the help of projection operators on particle and antiparticle bound states

The Bethe – Salpeter equation for bound states in the ladder approximation corresponding to the action (2)–(4) is in momentum space represented by

$$\underset{\sim}{\Phi}(\mathcal{P}) = \frac{1}{\mu^2}\int\frac{d^4k}{(2\pi)^4}\,\gamma^I_\nu\, \underset{\sim}{G}_{m_1}\left(k+\frac{\mathcal{P}}{2}\right)\underset{\sim}{\Phi}(\mathcal{P})\,\underset{\sim}{G}_{m_2}\left(k-\frac{\mathcal{P}}{2}\right)\gamma^{I\nu}, \qquad (A\ 1)$$

where $\mathcal{P}^\mu$ denotes the total bound state momentum. Now let us introduce the Bethe – Salpeter wave function Γ^H which is connected with the field $\underset{\sim}{\Phi}$ by the relation

$$\underset{\sim}{\Phi}(\mathcal{P}) = \int\frac{d^4k}{(2\pi)^4}\,\underset{\sim}{G}^{-1}_{m_1}\left(k+\frac{\mathcal{P}}{2}\right)\Gamma^H(\mathcal{P})\,\underset{\sim}{G}^{-1}_{m_2}\left(k-\frac{\mathcal{P}}{2}\right).$$

Inserting this expression for $\underset{\sim}{\Phi}$ into (A 1) we receive the Bethe – Salpeter equation for the wave function Γ^H, formula (7).

To obtain equations for lower-component Bethe – Salpeter wave functions it is necessary to define the projection operators for the two particles (a=1,2) /11/

$$\Lambda_\pm^{(a)}(\vec{k}) = (1 \pm S_a^{-2}\gamma_0)/2$$

and

$$\Lambda_\pm^{\prime(a)}(\vec{k}) = (1 \pm S_a^{2}\gamma_0)/2$$

with

$$S_a^{\pm 2} = \frac{m_a \pm k_i\gamma_i}{E_a(\vec{k})}$$

and

$$E_a(\vec{k}) = \sqrt{\vec{k}^2 + m_a^2}\ .$$

By means of these operators the propagator $\underset{\sim}{G}_{m_a}(k)$ can be represented in the following manner:

$$\begin{aligned}\underset{\sim}{G}_{m_a}(k) &= \frac{1}{k_0\gamma_0 - k_i\gamma_i - m_a + i\varepsilon}\\ &= \left(\frac{\Lambda_+^{(a)}(\vec{k})}{k_0 - E_a(\vec{k}) + i\varepsilon} + \frac{\Lambda_-^{(a)}(\vec{k})}{k_0 + E_a(\vec{k}) - i\varepsilon}\right)\gamma_0 \qquad (A\ 2)\\ &= \gamma_0\left(\frac{\Lambda_+^{\prime(a)}(\vec{k})}{k_0 - E_a(\vec{k}) + i\varepsilon} + \frac{\Lambda_-^{\prime(a)}(\vec{k})}{k_0 + E_a(\vec{k}) - i\varepsilon}\right).\end{aligned}$$

After inserting (A 2) into (7) and performing the integral over k_0 with the help of the formulae

$$\int_{-\infty}^{+\infty} d\varepsilon \frac{1}{[a-\varepsilon \pm i\delta][b+\varepsilon \pm i\delta]} = \pm \frac{2\pi i}{a+b\pm i\delta} ,$$

$$\int_{-\infty}^{+\infty} d\varepsilon \frac{1}{[a-\varepsilon \pm i\delta][b+\varepsilon \mp i\delta]} = 0$$

one obtains (for $\mathcal{P}_H = (M_H, \vec{0})$)

$$\mu^2 \Gamma^H(M_H) = \frac{1}{(2\pi)^3} \int d^3k \left(\frac{\Lambda_+^{\prime(1)}(\vec{k})\Gamma^H(M_H)\Lambda_-^{(2)}(\vec{k})}{E_T(\vec{k}) - M_H - i\varepsilon} + \frac{\Lambda_-^{\prime(1)}(\vec{k})\Gamma^H(M_H)\Lambda_+^{(2)}(\vec{k})}{E_T(\vec{k}) + M_H - i\varepsilon} \right), \qquad \text{(A 3)}$$

where

$$E_T(\vec{k}) = E_1(\vec{k}) + E_2(\vec{k}) .$$

Using the expressions for the projection operators this equation can be rewritten in the form

$$\mu^2 \Gamma^H(M_H) = \int \frac{d^3k}{(2\pi)^3} \{ E_T [S_1^{-2}\gamma_0 \Gamma^H(M_H)\gamma_0 S_2^{-2} - \Gamma^H(M_H)] + 2M_H [\Gamma^H(M_H)\gamma_0 S_2^{-2} - S_1^{-2}\gamma_0 \Gamma^H(M_H)]\} \cdot (M_H^2 - E_T^2 + i\varepsilon)^{-1} .$$

Now by inserting into this representation for the Bethe - Salpeter wave function the decomposition (8) one receives after some algebra the system (9).

2. Normalization of the Bethe - Salpeter wave function

The 16-component Bethe - Salpeter wave function is normalized as follows:

$$\sum_H \int \frac{d^4\mathcal{P}}{(2\pi)^4} (\Gamma^H(\mathcal{P}))^+ \Gamma^H(\mathcal{P}) (\mathcal{P}_0 - M_H) M_H = 1 . \qquad \text{(A4)}$$

Here $\mathcal{P}_0$ denotes the bound state energy. As one can see from (A 4) a normalization is possible only beyond $\vec{\mathcal{P}} = 0$. To derive a normalization condition for the lower-component Bethe - Salpeter wave functions we consider the action (3) and make an expansion around small values ($\mathcal{P}_0 - M_H$). Let us concentrate upon the term

$$S_{free\,2} = -i \frac{N_c}{2} \mathrm{tr} \int \frac{d^4\mathcal{P}}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \gamma_\nu^H \underset{\sim}{G}_{m_1}(\tfrac{\mathcal{P}}{2}+k) \underset{\sim}{\Phi}(\mathcal{P}) \underset{\sim}{G}_{m_2}(\tfrac{\mathcal{P}}{2}-k) \gamma^{H\nu} \underset{\sim}{\Phi}(\mathcal{P}),$$

which is obtained from the second part of the action (3) by means of the Fourier transform (6). Using (A 2) it can be rewritten analogously to (A 3) in the following manner:

$$S_{free\,2} = -\frac{N_c}{2} \mathrm{tr} \int \frac{d^4\mathcal{P}}{(2\pi)^4} \int \frac{d^3k}{(2\pi)^3} \left[\frac{\Lambda_+(\vec{k})\Gamma^H(\mathcal{P})\Lambda'_-(\vec{k})}{E_T(\vec{k}) - \sqrt{\mathcal{P}^2} - i\varepsilon} + \frac{\Lambda_-(\vec{k})\Gamma^H(\mathcal{P})\Lambda'_+(\vec{k})}{E_T(\vec{k}) + \sqrt{\mathcal{P}^2} - i\varepsilon} \right] \Gamma(\mathcal{P}). \qquad \text{(A 5)}$$

Now we expand the dominators in (A 5) around ($\mathcal{P}_0 - M_H$). Then, by taking into account

$$\sqrt{\mathcal{P}^2} - M_H = (\mathcal{P}_0 - M_H) + \dots$$

and introducing the notation $\mathcal{P}_H \equiv \sqrt{\mathcal{P}_H^2}$ the equation (A 5) turns into

$$S_{free\,2} \approx -\frac{N_c}{2} \mathrm{tr} \int \frac{d^4\mathcal{P}}{(2\pi)^4} \left[1 - (\mathcal{P}_0 - M_H) \frac{\partial}{\partial \mathcal{P}_H} \right] \cdot \int \frac{d^3k}{(2\pi)^3} \left[\frac{\Lambda_+(\vec{k})\Gamma^H(\mathcal{P})\Lambda'_-(\vec{k})}{E_T(\vec{k}) - \mathcal{P}_H - i\varepsilon} + \frac{\Lambda_-(\vec{k})\Gamma^H(\mathcal{P})\Lambda'_+(\vec{k})}{E_T(\vec{k}) + \mathcal{P}_H - i\varepsilon} \right] \Bigg|_{\mathcal{P}_H = M_H} \cdot \Gamma(\mathcal{P}) .$$

Comparing this expression with (A 4) one recieves as normalization condition for the lower-component Bethe - Salpeter wave functions

$$\frac{N_c}{2}\frac{1}{M_H}\frac{\partial}{\partial \mathcal{P}_H} tr\int \frac{d^3k}{(2\pi)^3}\left(\frac{\Lambda_+(\vec k)\Gamma^H(\mathcal{P})\Lambda_-'(\vec k)}{E_T(\vec k)-\mathcal{P}_H-i\varepsilon}+\frac{\Lambda_-(\vec k)\Gamma^H(\mathcal{P})\Lambda_+'(\vec k)}{E_T(\vec k)+\mathcal{P}_H-i\varepsilon}\right)\Bigg|_{\mathcal{P}_H=M_H}\Gamma(\mathcal{P})=1. \qquad (A\ 6)$$

The integral in (A 6) is the same as in the Bethe – Salpeter equation (A 3), so that one can use the results (9) on rewriting the Bethe – Salpeter equation to obtain the normalization condition for the quantities L_ℓ^I in (8) explicitly. Finally, one gets

$$\frac{2N_c}{M_H}\frac{\partial}{\partial \mathcal{P}_H} tr_{fl}\left[D^I(\mathcal{P}_H)(L_2^I)^2 + C^I(\mathcal{P}_H)(L_1^I)^2 + 2B^I(\mathcal{P}_H)L_1^I L_2^I\right]\Big|_{\mathcal{P}_H=M_H} = 1. \qquad (A\ 7)$$

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Низкоэнергетический предел билокального мезонного лагранжиана из КХД

Рассматривается низкоэнергетический предел уравнения Бете-Солпитера для кварк-антикваркового связанного состояния с релятивистским ядром. Показано, что этот предел эквивалентен локализации потенциала, т.е. модели Намбу-Йона-Лазино. Без разложения по энергии вычисляется спектр масс низколежащих мезонов с точным учетом S-V, P-A и V-T смешиваний. Полученные результаты сравниваются с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Kalinovsky Yu.L., Kaschluhn L., Pervushin V.N.

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The Low-Energy Limit of a Bilocal Meson Lagrangian from QCD

The low-energy limit of the Bethe-Salpeter equation for quark-antiquark bound states with a relativistic kernel is considered. It is shown that this limit is equivalent to a localization of the potential which becomes a special 4-quark Nambu-Jona-Lasinio one. The mass spectrum of the low-lying mesons is calculated without expanding in energy. Thereby the S-V, P-A and V-T mixings are taken into account exactly. Furthermore the Pion decay constant is determined. We received good agreement with the experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988

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