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ON SIGMA MODEL FORMULATION OF GREEN - SCHWARZ SUPERSTRING

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(supergroup) G with generators \mathcal{T}^{M} . The generic \mathfrak{S} - model action is as follows

$$A = - \Im \int d^{2} \xi \sqrt{-g} g^{ab} \langle \omega_{a} \omega_{b} \rangle_{\underline{I}} - \frac{1}{2} \beta \int d^{3} \xi \mathcal{E}^{ABC} \langle \omega_{A} [\omega_{B}, \omega_{c}] \rangle_{\underline{I}} .$$
(1)

Here

$$\omega_a \equiv \omega_a^M T^M = U^{-1}(\boldsymbol{s}) \frac{\partial}{\partial \boldsymbol{s}^a} U(\boldsymbol{s}) \tag{2}$$

are left-invariant Cartan's 1-forms, \mathcal{W}^{A} are their extension to a three-dimensional region V with boundary $\partial \mathsf{V}$ as the D = 2 space-time, ϑ and β are coupling constants, $g^{\mathsf{a}\mathsf{b}}$ is a Riemannian metric on $\partial \mathsf{V}$. Two constant matrices $\mathsf{P}_{1,\overline{1}}^{\mathsf{N}\mathsf{M}} \equiv \langle \mathsf{T}^{\mathsf{N}} \mathsf{T}^{\mathsf{M}} \rangle_{\overline{1,\overline{1}}}$ figuring in the definition (1) specify a pattern'of breaking the invariance under the right group multiplications $\mathcal{U}(\boldsymbol{\xi}) \rightarrow \mathcal{U}(\boldsymbol{\xi}) \mathcal{U}_{\mathcal{R}}$. If the averages $\langle \ldots \rangle$ are chosen to coincide with the cyclic operation $\{\mathsf{t}_{\mathsf{T}}\{\ldots\}\}$ for supergroups), then $\mathsf{P}_{\overline{1,\overline{1}}} \sim S^{\mathsf{N}\mathsf{M}}$, the right invariance is unbroken and one is faced with the familiar case of principal chiral field \mathfrak{S} -model. In other cases, e.g. when the target manifold is a homogeneous space of G, matrices $\mathsf{P}^{\mathsf{N}\mathsf{M}}$ do not reduce to the unit ones and the right invariance is necessarily broken. Just this situation occurs for superstrings.

A severe restriction on matrices $P_{\underline{i}\underline{i}}^{\textit{NM}}$ comes from the standard WZW condition

$$S\Omega_3 = d\Omega_2$$
 (3)

$$\Omega_{3} = \langle \omega \wedge d \omega \rangle_{\underline{II}}, \omega = \omega_{A} d \boldsymbol{\xi}^{A}, \quad (4)$$

where Ω_2 is a two-form. For the principal chiral field \mathcal{C} -models the condition (3) is always satisfied. Nontrivial examples of WZW terms for other target manifolds including some homogeneous group spaces have been given in $^{/7/}$. We will demonstrate here that the GS action provides one more important example of this kind.

Recall that the ratio of constants χ , β in eq. (1) is strictly fixed by requiring conformal invariance. This fixing is achieved already at the classical level by imposing extra local symmetries on the action (1): the Kac - Moody symmetry in the case of ordinary WZW \mathfrak{S} - models⁽⁸⁾ and the fermionic gauge one in the superstring case^(1,6).

B61. Spectra and Backward and A

1. Superstrings in the covariant formulation $^{1/}$ (both the Green - Schwarz (GS) and heterotic ones) have a beautiful geometric interpretation as the D = 2 nonlinear \mathcal{C} - models with the D = 10 superspaces (N = 2 or N = 1) as target manifolds $^{2,3/}$. Nontrivial interaction terms entering into the covariant superstring action and needed for its equivalence to the corresponding light-cone gauge one are nothing else than the generalized Wess - Zumino - Witten (WZW) terms $^{2/}$. Thus there reveals a profound affinity between superstrings and ordinary D = 2 WZW \mathcal{C} - models $^{4,5/}$ which opens new horizons for thinking about the geometry governing the superstring dynamics.

Unfortunately, for the GS superstring this exciting analogy is somewhat obscure in what concerns the WZW term. As distinct from the case of ordinary G' - models, this term cannot be directly constructed out of the Cartan 1-form defined on the N = 2 D = 40 Poincare supertranslation algebra which is usually assumed to be the underlying algebra of GS superstring. Its construction is rather tricky, which makes it difficult, e.g., to generalize the GS action to more complicated situations.

In the present letter, we demonstrate that the analogy between the GS superstring and the D = 2 WZW G - models can be completely restored provided one starts with the product of two N = 1, D = 10supertranslation groups as the symmetry group. The WZW term is constructed out of the corresponding Cartan 1-forms in entirely the same way as for ordinary chiral fields. N = 2 supersymmetry of the action is guaranteed due to a specific choice of the target maniforld. We deduce a zero-curvature representation for the GS superstring equations of motion which suggests their classical integrability in an arbitrary gauge and irrespective of the topology of a world sheet. The local fermionic supersymmetry inherent to the GS action $^{11,6/}$ proves to be realized as a gauge symmetry of the zero-curvature condition.

2. The two-dimensional WZW ϵ' -models can be described in terms of the matrix field U (Ξ' , Ξ') taking values in some group

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3. Let us show that the GS action^{/1/} corresponds to a WZW \mathfrak{S} - model with $G = G^4 \otimes G^2$, G^4 and G^2 being two mutually commuting N = 1, D = 10 Poincaré supertranslation groups.

An element $U(\mathbf{s}^{\circ}, \mathbf{s}^{i}) \in \mathbf{G}$ and the corresponding left-invariant Cartan 1 - forms are as follows

$$U(\mathbf{s}^{\circ},\mathbf{s}^{4}) = U^{4}(\mathbf{s}^{\circ},\mathbf{s}^{4}) U^{2}(\mathbf{s}^{\circ},\mathbf{s}^{4}), \qquad (5)$$

$$U^{j}(\mathbf{s}^{\circ},\mathbf{s}^{4}) = \exp\{i[\frac{1}{2}x^{j\mu}(\mathbf{s})P_{\mu}^{j} + \Theta^{j\alpha}(\mathbf{s})Q_{\alpha}^{j}]\}, \qquad (6)$$

$$j=1,2 \quad (\text{no summation over } j!)$$

$$U^{-1}(\mathbf{s})\partial_{\alpha}U(\mathbf{s}) \equiv \omega_{\alpha} = \omega_{a}^{4} + \omega_{\alpha}^{2} = \sum_{j=1}^{2}i(\frac{1}{2}\omega_{\alpha}^{j\mu}P_{\mu}^{j} + \omega_{\alpha}^{j\alpha}Q_{\alpha}^{j}) =$$

$$= i\sum_{j=1}^{2}[\frac{1}{2}(\partial_{\alpha}x^{j\mu} + i\partial_{\alpha}\Theta^{j}P^{A}\Theta^{j})P_{\mu}^{j} + \partial_{\alpha}\Theta^{j\alpha}Q_{\alpha}^{j}], \qquad (7)$$
where the generators $(P_{\mu}^{j}, Q_{\alpha}^{j})$ obey the relations*
$$\{Q_{\alpha}^{i}, Q_{\beta}^{j}\} = -\Gamma_{\alpha\beta}^{\mu}(P_{\mu}^{+}S^{ij} + P_{\mu}^{-}G_{3}^{ij}), \qquad (8)$$

$$P_{\mu}^{\pm} = \frac{1}{2}(P_{\mu}^{4} \pm P_{\mu}^{2}), \qquad G_{3}^{ij} = (\frac{1}{2}O_{\mu}^{-}). \qquad (8)$$

The D = 10 Lorentz group is regarded as an outer automorphism group acting on vector μ, ν, λ ... and spinor d, β, γ ... indices of generators. It is worthwhile to point out that the superalgebra (8) possesses neither SO(2) nor SO(1.1) automorphisms.

Left action of G on elements U(6) induces the standard N = 1 supertranslations in two N = 1, D = 10 superspaces $(x^{4/2}, \theta^{4/2})$, $(x^{2/2}, \theta^{2/2})$ $x^{j/2} \mapsto x^{j/2} + i \in i^{\alpha} [\gamma^{\mu}_{\alpha\beta} \theta^{j\beta}, \theta^{j\beta} \mapsto \theta^{j\beta} + \epsilon^{j\beta}$ (9)

For what follows, it will be crucial that the $G^1 \otimes G^2$ transformations (9) look as the N = 2 supersymmetry when applied on the coordinates $(x^{\mu} = x^{4\mu} + x^{2\mu}, \Theta^{j\alpha})$ parametrizing the homogeneous space

*We use the standard D = 10 conventions $\Gamma_{\alpha\beta}^{\mu} = (\chi^{\mu})_{\alpha}^{S} C_{\beta\beta}^{\prime} = \chi^{\mu\nu}\Gamma_{\alpha\beta}, \nu$, where χ^{μ} are the D = 10 Dirac matrices, $C_{\beta\beta} = -C_{\beta\beta}$ is the charge conjugation matrix and $\chi^{\mu\nu} = \text{diag}(1, -1), \quad \Theta^{j\alpha}$ are 32 - component Majorana - Weyl spinors. $G^4 \otimes G^2/G^-$ where G^- is an abelian subgroup with the generator P_{μ}^- . Indeed,

$$x^{\mu} \rightarrow x^{\mu} + i \sum_{j=1}^{2} \epsilon^{j\alpha} \Gamma^{\mu}_{\alpha\beta} \Theta^{j\beta} , \Theta^{j\beta} \rightarrow \Theta^{j\beta} + \epsilon^{j\beta}$$
(10)

while P_{μ}^{+} generates ordinary D = 10 translations. The generator P_{μ}^{-} is zero on the set $(x^{\mu}, \Theta^{j^{\alpha}})$ so the (N = 1) \oplus (N = 1) superalgebra (8) is reduced to the N = 2 supertranslations algebra as far as this superspace is concerned. Thus the latter can be identified with N = 2 D = 10 superspace. An important consequence of these observations is that the G¹ \otimes G² left-invariant G - model action will be automatically N = 2 supersymmetric if the target manifold is chosen to be G¹ \otimes G²/G⁻. The latter option amounts to requiring the action to be invariant under the right gauge G⁻ - transformations

$$U^{4}(\mathfrak{s}) \mapsto U^{4}(\mathfrak{s}) \exp\left[\frac{i}{2}a^{\mu}(\mathfrak{s})P^{4}_{\mu}\right], U^{2}(\mathfrak{s}) \mapsto U^{2}(\mathfrak{s}) \exp\left[-\frac{i}{2}a^{\mu}(\mathfrak{s})P^{2}_{\mu}\right], (11)$$

$$\omega_{a}^{j} \mapsto \omega_{a}^{j} + \frac{i}{2} \partial_{a} Q^{\mu}(\boldsymbol{\varepsilon}) \sum_{k=1}^{2} (\boldsymbol{\varepsilon}_{3})^{jk} P_{\mu}^{k} . \qquad (12)$$

Now we are prepared to formulate the principles which unambiguously lead to the GS covariant superstring action as that of the WZW \mathfrak{S} - model.

i. The \mathbf{c}' - model action is constructed on the supergroup $G = G^1 \otimes G^2$ by the generic formula (1).

ii. The target manifold is taken to be $G^1 \otimes G^2/G^-$, i.e. the action respects invariance under transformations (11), (12).

iii. The three-form entering as a density in the WZW term obeys the standard condition (3).

iv. The action gives rise to the correct kinetic terms, that is of the second order in the time derivative for bosons and of the . first order for fermions.

These natural requirements identify the \mathcal{C} -model action with the GS superstring one modulo the gauge fermionic symmetry. The proof goes as follows. One starts from the action (1) with $\mathcal{W}a$ given by eq. (7) assuming for a moment that the matrices $P_{\underline{I},\underline{\overline{\mu}}}^{\mathcal{N}\mathcal{M}}$ have a most general form allowed by Lorentz invariance and Grassmann parity

$$\langle \mathsf{P}^{i}_{\mu} \mathsf{P}^{j}_{\nu} \rangle_{\underline{I},\underline{\Pi}} = \mathcal{E}_{\mu\nu} \mathsf{P}^{ij}_{\underline{I},\underline{\Pi}} , \langle \mathsf{P}^{i}_{\mu} Q^{j}_{\alpha} \rangle = 0, \langle Q^{i}_{\alpha} Q^{j}_{\beta} \rangle_{\underline{I},\underline{\Pi}} = \mathcal{E}_{\alpha\beta} \mathsf{P}^{ij}_{\underline{I},\underline{\Pi}}$$
(13)

(the coupling constants ∂ , β are assumed to be included into the definition of these matrices). The condition iv demands $q^{ij} = 0$ as q^{ij} may appear only in the bilinear part of the action (1) and would result in kinetic terms containing two time derivatives of fermion fields. Then the conditions ii), iii) yield

$$P_{\underline{I}}^{ij} = 4\ell_{\underline{I}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, P_{\underline{II}}^{ij} = 4\ell_{\underline{II}}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}, \quad (14)$$
$$\ell_{\underline{I},\underline{II}} = \frac{1}{4}P_{\underline{I},\underline{II}}^{22} .$$

Thus the eight original parameters $P_{I,\underline{u}}^{\iota,\underline{j}}$ have been reduced to the two independent ones $\ell_{\overline{I}}$, $\ell_{\overline{u}}$. The action is written eventually

$$A = \ell_{\underline{I}} \int d^{3} \mathfrak{s} \sqrt{-g} \mathfrak{g}^{\alpha \delta} (\omega_{a}^{\mu} + \omega_{a}^{\nu}) (\omega_{\boldsymbol{\delta}\mu}^{\mu} + \omega_{\boldsymbol{\delta}\mu}^{\mu}) + \ell_{\underline{II}} \int d^{3} \mathfrak{s} \mathcal{E}^{ABC} (\omega_{A}^{\mu} + \omega_{A}^{\nu\mu}) \partial_{B} (\omega_{C\mu}^{\mu} - \omega_{C\mu}^{\nu}).$$

$$(15)$$

Now one has to pass in the WZW term to integration over ∂V using the property that locally $\Omega_3 = d \tilde{\Omega}_2$ (here important is the identity for $D = 10 \quad \emptyset$ - matrices $\prod_{\mu}^{M} \prod_{\gamma S, \mu} + \text{cyclic} (\alpha, \beta, \delta) =$ = 0). As a result, one arrives at the GS superstring covariant action, up to a freedom in choosing the value of $\ell_{\overline{H}} / \ell_{\overline{1}}$

$$A = l_{\bar{1}} \int d^{2} \underline{s} \left\{ \sqrt{-g} g^{ab} (\partial_{a} x^{\mu} + i \sum_{j=1}^{\bar{z}} \partial_{a} \theta^{j} \Gamma^{\mu} \theta^{j}) (\partial_{b} x_{\mu} + i \sum_{j=1}^{\bar{z}} \partial_{b} \theta^{j} \Gamma_{\mu} \theta^{j}) - \frac{i l_{\bar{1}}}{2} \mathcal{E}^{ab} (\partial_{a} x^{\mu} + \frac{i}{2} \sum_{j=1}^{\bar{z}} \partial_{a} \theta^{j} \Gamma^{\mu} \theta^{j}) (\sum_{\kappa,j=1}^{\bar{z}} \partial_{b} \theta^{\kappa} \mathcal{E}^{\kappa j}_{3} \Gamma_{\mu} \theta^{j}) \right\}.$$
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The genuine GS action possessing local fermionic symmetry arises at $l_{\bar{l}} / l_{\bar{l}} = 2^{/1/}$.

Thus we see that the correct \mathcal{C} - model interpretation of GS superstring should be based on the supergroup $G^1 \otimes G^2$ and, in accord with the condition ii), target space of \mathcal{C} -model coincides with N = 2, D = 10 superspace $(\mathcal{X}^{\mathcal{H}}(\mathcal{E}), \mathcal{O}^{\mathcal{I}}(\mathcal{E}))$. There remains a trace of original $G^1 \otimes G^2$ group structure in the WZW term. As is seen from eq. (15), this term essentially involves the 1- form $\omega^{\mathcal{I}}_{\mu} - (\omega^2_{\mu}$ associated with generator \mathcal{P}^{-}_{μ} . We wish to point out once more that, starting with the Cartan form ω defined on the N = 2, D = 10 supertranslation algebra, it is impossible to write the GS superstring WZW term according to the generic \mathcal{C} - model formula(1). Note that the obvious reason why the internal SO(2) symmetry inherent to N = 2 superalgebra is broken in the GS action $^{(1-3)}$ is the absence of such a symmetry in the underlying superalgebra (8). 4. It is of interest to rewrite the GS superstring equations of motion and transformations of fermionic gauge symmetry in manifestly geometric terms of Cartan's forms.

The equations of motion following from the action (15) can be written as

$$\partial_{a} \left[P_{+}^{ab} \left(\omega_{b\mu}^{1} + \omega_{b\mu}^{2} \right) - \frac{l_{\bar{I}}}{l_{\bar{I}}} \varepsilon^{ab} \omega_{b\mu}^{1} \right] = 0, \qquad (17a)$$

$$P^{ab}_{+}(\omega^{1}_{b\mu}+\omega^{2}_{b\mu})\Gamma^{\mu}_{a\beta}\omega^{1\beta}_{a}=0, \qquad (17b)$$

$$\mathcal{P}_{a}\left[P_{-}^{ab}\left(\omega_{b\mu}^{1}+\omega_{b\mu}^{2}\right)+\frac{\ell_{II}}{\ell_{I}}\varepsilon^{ab}\omega_{b\mu}^{2}\right]=0, \qquad (18a)$$

$$P_{-}^{ab}(\omega_{b\mu}^{1}+\omega_{b\mu}^{2})\Gamma_{\alpha\beta}^{\mu}\omega_{a}^{2\beta}=0, \qquad (18b)$$

$$T_{ab} = (\omega_{a}^{4\mu} + \omega_{a}^{2\mu})(\omega_{b\mu}^{4} + \omega_{b\mu}^{2}) - \frac{1}{2}g_{ab}g^{cd}(\omega_{c}^{4\mu} + \omega_{c}^{2\mu})(\omega_{d\mu}^{4} + \omega_{d\mu}^{2}) = 0$$
(19)
where

$$P_{\pm}^{ab} = \sqrt{-g} g^{ab} \pm \frac{1}{2} \left(\frac{\ell_{\parallel}}{\ell_{\perp}} \right) \mathcal{E}^{ab} . \qquad (20)$$

One should also add to the system (17)-(19) the Maurer - Cartan equations

$$\partial_{a}\omega_{g}^{j\mu} - \partial_{e}\omega_{a}^{j\mu} = 2i\omega_{e}^{j\alpha}\Gamma_{\alpha\beta}^{\mu}\omega_{a}^{j\beta}, \qquad (21)$$
$$\partial_{a}\omega_{g}^{j\alpha} - \partial_{e}\omega_{a}^{j\alpha} = 0.$$

The equations (17)-(19), (21) put together are equivalent to the original equations $GS^{/1/}$ (with $l_{\underline{y}}/l_{\underline{i}}$ = 2) and can be regarded as a generalization of the Cartan form representation of the equations of ordinary WZW nonlinear G' - models⁽⁹⁾.

The transformations of local fermionic symmetry $^{1,6/}$ leaving invariant the action and the equations of motion at $\ell_{\bar{l}l}/\ell_{\bar{l}} = 2$ are represented through the Cartan forms as

$$\widetilde{\omega}^{1\alpha} = P_{+}^{ab} \left(\omega_{b\mu}^{1} + \omega_{b\mu}^{2} \right) C^{\alpha\beta} \Gamma_{\beta\delta}^{\mu} \mathscr{X}_{a}^{1\delta} (\mathfrak{z}^{\circ}, \mathfrak{z}^{1}), \qquad (22)$$

$$\widetilde{\omega}^{2a} = P_{-}^{ab} \left(\omega_{b\mu}^{1} + \omega_{b\mu}^{2} \right) C^{\alpha\beta} \Gamma_{\beta\delta}^{\prime \mu} \mathscr{Z}_{a}^{2b} (\mathfrak{Z}^{\circ}, \mathfrak{Z}^{1}) ;$$

where $\mathscr{D}_{a}^{\mathsf{M}}(\mathbf{\xi})$ are transformation parameters and we have introduced the left-invariant variations

$$(\mathcal{U}^{j})^{-1} S(\mathcal{U}^{j}) \equiv \frac{i}{2} \widetilde{\omega}^{j\mu} \mathcal{P}^{j}_{\mu} + i \widetilde{\omega}^{j\mu} \mathcal{Q}^{j}_{\mu}$$
(23)

and assumed eq. (19) to be fulfilled (it is easy as well to write the transformation of the D = 2 metric g^{ab}).

5. Surprisingly, the GS superstring equations written in the form (17), (18) admit a zero-curvature representation at $\ell_{\bar{I}}/\ell_{\bar{I}}$ = 2. These amount to the integrability conditions

$$\begin{bmatrix} L_a^1, L_b^1 \end{bmatrix} = \begin{bmatrix} L_a^2, L_b^2 \end{bmatrix} = 0, \qquad (24)$$

where

$$L_{a}^{4} = \partial_{a} - i\lambda^{2} \mathcal{E}_{a\beta} P_{+}^{\beta c} (\omega_{c}^{4\mu} + \omega_{c}^{2\mu}) R_{\mu}^{4} - 2\lambda \omega_{a}^{1\alpha} S_{a}^{\prime 1},$$

$$L_{a}^{2} = \partial_{a} + i\lambda^{2} \mathcal{E}_{a\beta} P_{-}^{\beta c} (\omega_{c}^{4\mu} + \omega_{c}^{2\mu}) R_{\mu}^{2} - 2\lambda \omega_{a}^{2\alpha} S_{a}^{\prime 2},$$

$$(25)$$

$$\lambda_{a}^{2} = \delta_{a}^{2} + i\lambda^{2} \mathcal{E}_{a\beta} P_{-}^{\beta c} (\omega_{c}^{4\mu} + \omega_{c}^{2\mu}) R_{\mu}^{2} - 2\lambda \omega_{a}^{2\alpha} S_{a}^{\prime 2},$$

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 $(\mathcal{R}_{\mathcal{P}}, \mathcal{S}_{\alpha})$ constitute two isomorphic mutually (anti)commuting superalgebras

$$[R^{1}_{\mu}, R^{1}_{\nu}] = R^{1}_{[\mu\nu]}, [R^{1}_{\mu}, S^{1}_{\alpha}] = (\Gamma_{\mu})^{\beta}_{\alpha} S^{1}_{\beta}, \{S^{1}_{\alpha}, S^{1}_{\beta}\} = -\Gamma^{\mu}_{\alpha\beta} R^{1}_{\mu} (26)$$

(the same relations hold for R^2_{μ} , S^2_{α}). Note that the relations (26) are not closed, the generator $R^4_{L\mu\nu}$ cannot be set equal to zero and it produces new generators when commuted with R^4_{μ} and S^4_{α} , so the complete zero-curvature representation superalgebra is likely infinite-dimensional. However, only the second and third relations in (26) actually appear in the commutators (24) and just these ones are crucial for proving integrability. It is worth mentioning that the original superalgebra (8) can be consistently regarded as a contraction of (26).

A striking fact about the representation (24), (25) is that the latter exists at the same value of ℓ_{II}/ℓ_{I} = 2 which is selected by fermionic gauge invariance. One may verify that this invariance can be realized as a kind of gauge transformations preserving (24)

$$L_a^j \mapsto G^j(s) L_a^j G^{j-1}(s), G^j(s) = \exp(i \psi^{j\alpha}(s) \beta_{\alpha}^j).$$
 (27)

To check this in the infinitesimal case, we have to put $\psi_{J}^{\mathcal{A}}(\boldsymbol{z})$ proportional to $\boldsymbol{\lambda} \widetilde{\boldsymbol{\omega}}^{\boldsymbol{j}\boldsymbol{\alpha}'}$ given by eqs.(22) and to take into account the equations of motion (17b), *(18b). An interpretation of ordinary strings as integrable systems was discussed in $^{/10/}$.

6. Finally, we would like to point out that the algorithmic construction proposed in this paper applies not only to the case of GS superstring. In principle, one may choose G^1 , G^2 to be more complicated supergroups, e.g., with the nonabelian even parts, and set up superstring on curved supergroup manifolds analogously to strings on group manifolds /11/. The interesting problems ahead are to extend the above consideration to the GS superstring in an arbitrary curved D = 10 supergravity background $\frac{12}{12}$ and to understand in a full generality the implications of integrability property (24). (25). In particular, one may wonder about the existence of an infinite set of conserved currents associated with the zero-curvature representation (24). Furthermore, in view of an intimate relation between this representation and fermionic gauge invariance, one may expect an analogous property of classical integrability to hold for all the other super D-branes respecting such an invariance, e.g. for the D = 11 supermembrane⁽¹⁾.

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Сигма-модельная интерпретация суперструны Грина – Шварца

Ковариантное действие для суперструны Грина - Шварца последовательно выводится как действие С-модели Весса -Зумино - Виттена, ассоциированное с прямым произведением двух N=1,D=10 групп супертрансляций Пуанкаре. Показано, что N=2 суперсимметрия действительно связана со специфическим выбором многообразия отображения. Найдено представление нулевой кривизны для полевых уравнений суперструны Грина - Шварца, которое служит указанием на их полную интегрируемость независимо от топологии мировой поверхности струны.

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Isaev A.P., Ivanov E.A. E2-88-474 On Sigma Model Formulation of Green - Schwarz Superstring

The Green - Schwarz covariant superstring action is consistently deduced as the action of the Wess - Zumino -Witten σ -model defined on the direct product of two N=1, D=10 Poincaré supertranslation groups. N=2 supersymmetry of the action is shown to be related to a specific choice of the target manifold. We propose a zero curvative representation for the GS superstring field equations and interpret the local fermionic supersymmetry of the GS action as a gauge symmetry preserving this representation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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