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THE BEHAVIOUR OF THE STRUCTURE FUNCTIONS AND RATIO $R=\sigma_L/\sigma_T$ IN DEEP INELASTIC SCATTERING FOR $x\sim 0$ AND $x\sim 1$ AND THEIR SCHEME-INVARIANT PARAMETRIZATION

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1. Introduction

In recent years, in connection with the progress in Quantum Chromodynamics (QCD) it is necessary to compare the theory and experimental data (ED) for different processes. One of the most intensively investigated processes is the deep inelastic photon scattering (DIS) on a hadron. Theoretically, it is convenient to consider not the DIS structure functinns (SF) but their moments whose coefficient functions (CF) can be calculated in the perturbation theory (PT) and improved with the help of the renormalization group method (RGM)/1/. However it is convenient to compare with experimental data (ED) the SF themselves. They cannot be reproduced analytically from their moments, therefore different parametrizations due, for example, to Feynman and Field /2/, Buras, Gaemers, and others/3/ are used. Following /2/, Lopez and Yndurain (they calculated the leading order (LO) /4/ and the next-to-leading order (NLO)/5/ of PT) have shown that one can find the parametrization parameters by analysing the SF behaviour for $x\sim 0$ and $x\sim 1$ /6-8/. Their parametrisations are simple and exactly satisfy certain sum rules. For $x\sim 0$ and $x\sim 1$ they also satisfy exactly the evolution equations of QCD and for the intermediate values of x the inaccuracy is less than 1% (see /8/). However, the procedure of finding the SF behaviour in the LO (especially in the singlet channel) is extremely labour-consuming and difficult for understanding (see/9,10/). The origin of any coefficients in the parametrizations is not clear because use is made not of the Casimir operators of SU(N) group but of their values for N=3. Moreover, the obtained coefficients are scheme-dependent (SD) (that is, the coefficients depend on the way of removing divergences).

Much attention has been paid to the problem of scheme-dependence in quantum field theory (see, for example, /11-14/). In paper /12-14/ the so-called scheme-invariant perturbation theory (SIPT) was suggested and developed, in which the series in the coupling constants was introduced each of which corresponds to a certain physical process (or quantity).

In short, the essence of the SIPT is the following: a certain physical quantity

$$R = \alpha(1+r_1\alpha+\ldots) \tag{1}$$

can be expanded in the new SI coupling constant

$$R^{SI} = a(Q^2). \tag{2}$$

The expansion parameters $\alpha(Q^2)$ and $a(Q^2)$ in a two-loop approximation satisfy the equations:

$$1/\alpha(Q^2) + \frac{\beta_1}{\beta_0}\ln\alpha(Q^2) = \beta_0\ln(Q^2/\Lambda^2)\,,\quad 1/a(Q^2) + \frac{\beta_1}{\beta_0}\ln a(Q^2) = \beta_0\ln(Q^2/\Lambda_a^2) \tag{3),(4}$$

and the scales Λ_a and Λ are connected as follows

$$\beta_0\ln(\Lambda_a/\Lambda) = r_1. \tag{5}$$

Hereafter β_0 and β_1 are the first two coefficients of the $\alpha(Q^2)$-expansion of β-function: $\beta(\alpha) = -\sum_{m=0}\beta_m[\alpha]^{m+2}$.

In the present paper, we consider the behaviour of the quark and gluon distributions (DI) in the first two orders of PT. The SF and ratio $R=\sigma_L/\sigma_T$ (σ_L and σ_T are the cross sections for the longitudinal and transverse polarized photon scattering on the nucleon) in both PT orders are obtained with the help of the Mellin integration of Wilson coefficients and the DI. This approach is more clear. The SF parametrizations are constructed following papers /4,5/. Moreover, we consider the SI behaviour of the SF and ratio R for $x\sim 0$ and $x\sim 1$ and construct their SI parametrizations. The merits of these parametrizations are described in detail in section 4. Here we note that the SI parametrizations of the nonsinglet SF (see /15/) agree with ED of group /17/ better than the orthodox parametrizations /5,16/.

We draw the graphs of the parametrisations of the ratio $R=\sigma_L/\sigma_T$. The curves well agree with the ED of different groups /18-22/.

In Appendix 1, using the Mellin transformation we reconstruct the Q^2-evolution of the SF in the region $x\sim 1$ (see /6/).

In Appendix 2, we get the α_S-correction to the longitudinal SF using the α_S-corrections to their moments obtained in papers /23,24/.

In Appendix 3 analytical values of the parametrization coefficients in the vicinity of $x=0$ and $x=1$, respectively, are given.

2. The behaviour of the DI, SF and R for $x\to 1$

There are different possibilities of the connection (in the NLO) of the DIS SF and the parton (quark and gluon) DI /25/. We choose the parton DI so that the anomalous dimensions of Wilson operators and β-function are responsible for their evolution, and the connection between the SF and parton DI is determined by the Wilson coefficients /25,26/.

1. Knowing that for a certain value of Q^2 the DI for $x\to 1$ (see, for example /18,20/) behave as follows:

$$\Delta(x,Q_0^2) = \Delta_0(1-x)^{\nu^{NS}(Q_0^2)},\ \Sigma(x,Q_0^2) = \Sigma_0(1-x)^{\nu^S(Q_0^2)},\ G(x,Q_0^2) = G_0(1-x)^{\nu^G(Q_0^2)}$$

and using the Q^2-evolution of the DI moments /25/, we can determine the DI behaviour at different Q^2:

$$\Delta^{(i)}(x,Q^2) = \Delta(x,Q_0^2)\otimes T^{(i)}_{NS}(x,Q^2)$$

$$\Sigma^{(i)}(x,Q^2) = \Sigma(x,Q_0^2)\otimes T^{(i)}_{\psi\psi}(x,Q^2) + G(x,Q_0^2)\otimes T^{(i)}_{\psi G}(x,Q^2) \approx \Sigma(x,Q_0^2)\otimes T^{(i)}_{\psi\psi}(x,Q^2)$$

$$G^{(i)}(x,Q^2) = G(x,Q_0^2)\otimes T^{(i)}_{GG}(x,Q^2) + \Sigma(x,Q_0^2)\otimes T^{(i)}_{G\psi}(x,Q^2) \approx \Sigma(x,Q_0^2)\otimes T^{(i)}_{G\psi}(x,Q^2),$$

where

$$F_1(x)\otimes F_2(x) = \int_x^1 F_1(y)F_2(x/y)\,dy/y\,.$$

The values of the coefficients $T^{(i)}_{NS}$ and $T^{(i)}_{jm}$ $(j,m=\psi,G)$ are given in Appendix 1. We can neglect the contribution of the gluon DI because $2\le\nu_0^G-\nu_0^S\le 7$ (see /18,20,27/).

Hence, we get

to the LO

$$\Delta^{(1)}(x,Q^2) = A_{NS}[\alpha(Q^2)]^{-d_0}\frac{(1-x)^{\nu^{NS}(\alpha)}}{\Gamma(1+\nu^{NS}(\alpha))}\left(1+y^{NS}(x,Q^2)(1-x)\right),\ \Sigma^{(1)}(x,Q^2) = A_S[\alpha]^{-d_0}\frac{(1-x)^{\nu^S(\alpha)}}{\Gamma(1+\nu^S(\alpha))}\left(1+y^S\cdot(1-x)\right) \tag{8}$$

$$G^{(1)}(x,Q^2) = \frac{C_F}{C_A-C_F}A_S[\alpha(Q^2)]^{-d_0}\frac{(1-x)^{\nu^S(\alpha)+1}}{\Gamma(2+\nu^S(\alpha))}\,\frac{\left(1+y^G(x,Q^2)\cdot(1-x)\right)}{\left(\ln\left(\frac{1}{1-x}\right)+\tilde{\gamma}+\psi(2+\nu^S(\alpha))\right)}$$

to the NLO

$$\Delta^{(2)}(x,Q^2) = \Delta^{(1)}(x,Q^2)\left(1+[\alpha(Q^2)\tilde{Z}^{NS}(x,Q^2)-\alpha(Q_0^2)\tilde{Z}^{NS}(x,Q_0^2)]\right)$$

$$\Sigma^{(2)}(x,Q^2) = \Sigma^{(1)}(x,Q^2)\left(1+[\alpha(Q^2)\tilde{Z}^{\psi}(x,Q^2)-\alpha(Q_0^2)\tilde{Z}^{\psi}(x,Q_0^2)]\right),\ G^{(2)}(x,Q^2) = G^{(1)}(x,Q^2)\left(1+[\alpha(Q^2)\tilde{Z}^{G}(x,Q^2)-\alpha(Q_0^2)\tilde{Z}^{G}(x,Q_0^2)]\right), \tag{9}$$

where

$$\nu^i(\alpha) = \nu_0^i - \frac{2^2C_F}{\beta_0}\ln\alpha(Q^2)\,,\quad \tilde{\gamma} = \gamma - \frac{11C_A-9C_F-4T_FC_A/C_F}{12(C_A-C_F)}. \tag{10}$$

$\Gamma(x)$ and $\psi(x)$ are the Euler Γ-function and ψ-function, respectively, and γ is the Euler constant. The quark DI coincides in form (within the notation) with the ones obtained in papers /4,5/. Here $\xi^{NS}=\xi^{\psi}$, where $\xi=(\nu, y, \tilde{Z})$ (see /5,16/ and Appendix 3). The values of the coefficients $\tilde{Z}(x,Q^2)$ and $y^i(x,Q^2)$ $(i=\psi,G)$ are given in Appendix 3.

We note that in the LO and NLO the different coupling constants $\alpha_{LO}(Q^2)$ and $\alpha_{\overline{MS}}(Q^2)$ are used. They satisfy the equations:

$$1/\alpha_{LO}(Q^2) = \beta_0\ln(Q^2/\Lambda^2_{LO})\,,\quad 1/\alpha_{\overline{MS}}(Q^2) + \frac{\beta_1}{\beta_0}\ln\alpha_{\overline{MS}}(Q^2) = \beta_0\ln(Q^2/\Lambda^2_{\overline{MS}}).$$

In the present paper, we use different values of $\Lambda_{\overline{MS}}$ (and the corresponding Λ_{LO}):

$\Lambda^{(1)}_{\overline{MS}} = 105$ MeV ($\Lambda_{LO} = 90$ MeV) , $\Lambda^{(2)}_{\overline{MS}} = 210$ MeV ($\Lambda^{(2)}_{LO} = 180$ MeV) obtained, respectively, by the groups EMC /18/ and BCDMS /20/.

Hereafter

$$C_A = N\,,\ C_F = \frac{N^2-1}{2N} \quad\text{and}\quad T_F = f/2$$

for the SU(N) gauge group and f quarks flavours.

2. The SF are expressed through the Mellin integration of the known Wilson coefficients and the DI:

to the LO [25]:

$$F_2^{(1)NS}(x,Q^2)=\Delta^{(1)}(x,Q^2)\,,\; F_2^{(1)S}(x,Q^2)=\Sigma^{(1)}(x,Q^2)\,,\; F_L^{(1)NS}(x,Q^2)=\alpha(Q^2)B_L^{(1)NS}(x)\otimes\Delta^{(1)}(x,Q^2)$$

$$F_L^{(1)S}(x,Q^2)=\alpha(Q^2)\left[B_L^{(1)\psi}(x)\otimes\Sigma^{(1)}(x,Q^2)+B_L^{(1)G}(x)\otimes G^{(1)}(x,Q^2)\right] \tag{11a}$$

to the NLO [24,25,28]:

$$F_2^{(2)NS}(x,Q^2)=\Delta^{(2)}(x,Q^2)\otimes\left(1+\alpha(Q^2)B_2^{(1)NS}(x)\right),\; F_2^{(2)S}(x,Q^2)=\Sigma^{(2)}(x,Q^2)\otimes\left(1+\alpha(Q^2)B_2^{(1)\psi}(x)\right)+\alpha(Q^2)G^{(2)}(x,Q^2)\otimes B_2^{(1)G}(x)$$

$$F_L^{(2)NS}(x,Q^2)=\alpha(Q^2)B_L^{(1)NS}(x)\left(1+\alpha(Q^2)R_L^{(2)NS}(x)\right)\otimes\Delta^{(2)}(x,Q^2) \tag{11b}$$

$$F_L^{(2)S}(x,Q^2)=\alpha(Q^2)\{\left[B_L^{(1)\psi}(x)\left(1+\alpha(Q^2)R_L^{(2)\psi}(x)\right)\right)\otimes\Sigma^{(2)}(x,Q^2)+B_L^{(1)G}(x)\left(1+\alpha(Q^2)R_L^{(2)G}(x)\right)\otimes G^{(2)}(x,Q^2).$$

Here $B_K^{(1)NS}(x)=B_K^{(1)\psi}(x)$. Hereafter the index K runs over 2 and L. The values of the coefficients $B_K^{(1)j}(x)$ and $R_L^{(2)j}(x)$ $(j=NS,\psi,G)$ are given in Appendix 2.

The SF $xF_3(x,Q^2)$ is expressed through $\Delta(x,Q^2)$ and the valence quark DI. Hence its dependence on x and Q^2 is the same as for $F_2^{NS}(x,Q^2)$. Therefore we will not distinguish between the functions $F_2^{NS}(x,Q^2)$ and $xF_3(x,Q^2)$ (the difference in their parametrization coefficients will be given in Appendix 3).

Substituting into equations (11) the DI in the form (8) and (9) respectively, we get for the SF

to the LO
$$F_2^{(1)j}(x,Q^2)=A_j[\alpha(Q^2)]^{-d_0}\frac{(1-x)^{\nu(\alpha)}}{\Gamma(1+\nu(\alpha))}\left(1+y_2(x,Q^2)(1-x)\right) \qquad (j=NS,S)$$

$$F_L^{(1)j}(x,Q^2)=A_j\cdot 4C_F[\alpha(Q^2)]^{1-d_0}\frac{(1-x)^{\nu+1}}{\Gamma(2+\nu(\alpha))}\left(1+y_L(x,Q^2)(1-x)\right),\; \bar{K}^{(1)}(x,Q^2)=4C_F\frac{\alpha(Q^2)(1-x)}{(1+\nu(\alpha))} \tag{12}$$

to the NLO

$$F_K^{(2)j}(x,Q^2)=F_K^{(1)j}\cdot\left(1+\alpha(Q^2)\tilde{C}_K^j(x,Q^2)\right) \tag{13}$$

$$\bar{R}^{(2)}(x,Q^2)=\bar{R}^{(1)}(x,Q^2)\left(1+\alpha(Q^2)\tilde{C}_{\bar{R}}(x,Q^2)\right),$$

where the coefficients $y_K, \tilde{C}_K^j(x,Q^2)$ are given in Appendix 3. Note that the coefficients $\tilde{C}_K^j(x,Q^2)\equiv\tilde{C}_K(x,Q^2)$ are independent of j.

Here
$$\bar{R}=R/(1+R) = F_L(x,Q^2)/F_2(x,Q^2)$$

and

$$F_K(x,Q^2)=\sum_{j=NS,S}\delta_j F_K^j(x,Q^2)\quad,\tilde{C}_{\bar{R}}=\tilde{C}_L-\tilde{C}_2\,.$$

The values of the coefficients δ_j are defined by the quark charges. We can neglect the contribution of the gluon DI because it is $\sim(1-x)^2$ for the any quantity from equations (11) and (12). This constitutes just the difference of the parametrizations of $F_L^j(x,Q^2)$ and $\bar{R}(x,Q^2)$ from those obtained in papers [5,29]. The parametrizations of $F_2^j(x,Q^2)$ coincide (up to the notation) with the ones obtained in papers [4,5].

3. To obtain the SI behaviour of the SF and $\bar{R}$, we have to transform the F_K^j and $\bar{R}$ to new variables $\hat{F}_K^j$ and $\hat{\bar{R}}$ so that

to the LO
$$\hat{F}_K^{(1)j}=\alpha(Q^2),\;\hat{\bar{R}}^{(1)}=\alpha(Q^2),$$

to the NLO
$$\hat{F}_K^{(2)j}=\alpha(Q^2)\left(1+\tilde{\delta}_K\cdot\alpha(Q^2)\right),\;\hat{\bar{R}}^{(2)}=\alpha(Q^2)\left(1+\tilde{\delta}_{\bar{R}}\cdot\alpha(Q^2)\right).$$

Denote this transformation by φ. Then we get to the NLO

$$F_K^{(2)j}=\varphi^{-1}\left(\hat{F}_K^{(2)j}\right)=F_K^{(1)j}\left(1-\tilde{\delta}_K\cdot\alpha(Q^2)\tilde{d}_K(x,Q^2)\right),\;\bar{R}^{(2)}=\varphi^{-1}\left(\hat{\bar{R}}^{(2)}\right)=\bar{R}^{(1)}\left(1-\tilde{\delta}_{\bar{R}}\cdot\alpha(Q^2)\cdot\tilde{d}_{\bar{R}}(x,Q^2)\right).$$

The values of $\tilde{d}_K(x,Q^2)$ and $\tilde{d}_{\bar{R}}(x,Q^2)$ are given in Appendix 3.

Hence we get
$$\tilde{\delta}_K=-\tilde{C}_K(x,Q^2)/\tilde{d}_K(x,Q^2)\quad,\tilde{\delta}_{\bar{R}}=-\tilde{C}_{\bar{R}}(x,Q^2)/\tilde{d}_{\bar{R}}(x,Q^2).$$

Constructing the SIPT for the new quantities $\hat{F}_K^j$ and $\hat{\bar{R}}$ we obtain
$$\hat{F}_K^{SI,j}=a_K(x,Q^2)\;,\;\hat{\bar{R}}^{SI}=a_{\bar{R}}(x,Q^2),$$

where a_K and $a_{\bar{R}}$ are solutions of equations (4) and (5) for $\tau_{1,K}=\tilde{\delta}_K$ and $\tau_{1,\bar{R}}=\tilde{\delta}_{\bar{R}}$, respectively.

Finally, we get the SI analog of equations (13) in the form

$$F_2^{SI,j}(x,Q^2)=A_j[a_2(x,Q^2)]^{-d_0}\frac{(1-x)^{\nu(a_2)}}{\Gamma(1+\nu(a_2))}\quad,F_L^{SI,j}(x,Q^2)=A_j\cdot 4C_F[a_L(x,Q^2)]^{1-d_0}\frac{(1-x)^{\nu(a_L)+1}}{\Gamma(2+\nu(a_L))}$$

$$\bar{K}^{SI}(x,Q^2)=4C_F\,a_{\bar{R}}(x,Q^2)\,(1-x)/(1+\nu(a_{\bar{R}})). \tag{14}$$

In Fig. 1 the graphs are drawn for the ratios $\Lambda_K(x,Q^2)/\Lambda_{\overline{MS}}$ and $\Lambda_{\bar{R}}(x,Q^2)/\Lambda_{\overline{MS}}$ versus the variable x. The dependence is weak on the quark flavours and the variable Q^2 and thus it is not considered (the curves are built for $f=4$). The values of both the ratios increase with increasing x (they tend to infinity for $x\to 1$) since $\tilde{C}_K(x,Q^2)/\tilde{d}_K(x,Q^2)\underset{x\to 1}{\sim}\ln(1-x)$, $\tilde{C}_{\bar{R}}(x,Q^2)/\tilde{d}_{\bar{R}}(x,Q^2)\underset{x\to 1}{\sim}\ln(1-x)$ (see Appendix 3).

3. The Behaviour of DI, SF and R for $x\sim 0$

1. Considering the DI behaviour for $x\to 0$ to be Regge-like [30] and accepting the dominant role of the gluon distribution in this region: $G(x,Q_0^2)/\Sigma(x,Q_0^2)\underset{x\to 0}{\approx}3.7$ (see [1,8]) we get the DI behaviour in the form [4,5,25] (hereafter $Q_0^2=5\,GeV^2$):

$$\Delta^{(1)}(x,Q^2)=\Delta_0[\alpha(Q^2)]^{-d_\lambda^{NS}}x^{1-\lambda}\left(1+\Delta_1\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{(d_\lambda^{NS}-d_{\lambda_1}^{NS})}\cdot x^{(\lambda-\lambda_1)}\right),$$

$$\Sigma^{(1)}(x,Q^2)=\Sigma_0[\alpha(Q^2)]^{-d_\delta^+}x^{1-\delta}\left(1+\Sigma_1\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{(d_\delta^+-d_{\delta_1}^+)}\cdot x^{(\delta-\delta_1)}\right), \tag{15}$$

$$G^{(1)}(x,Q^2)=\xi(\delta)\,\Sigma_0\left[\alpha(Q^2)\right]^{-d^+_\delta}x^{1-\delta}\left(1+G_1\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{(d^+_\delta-d^+_{\delta_1})}x^{(\delta-\delta_1)}\right),$$

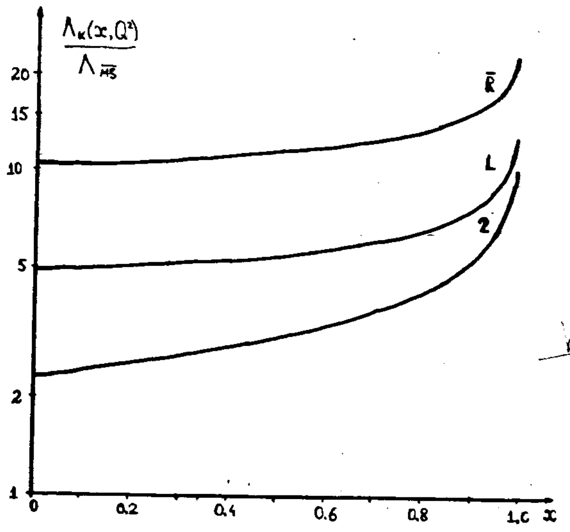


Fig. 1. The ratios $\Lambda_K(x,Q^2)/\Lambda_{\overline{MS}}$ and $\Lambda_{\bar{R}}(x,Q^2)/\Lambda_{\overline{MS}}$ versus variable x. The indices 2,L and R correspond to the curves of the transverse and longitudinal SF and ratio R, respectively.

where $\xi(\delta)=\left(\gamma_+^{(0)\delta}-\gamma_{\psi\psi}^{(0)\delta}\right)/\gamma_{\psi G}^{(0)\delta}$

to the NLO

$$\Delta^{(2)}(x,Q^2)=\Delta^{(1)}(x,Q^2)\left(1+(\alpha(Q^2)-\alpha(Q_0^2))\,Z^{NS}_\lambda\right),\ \Sigma^{(2)}(x,Q^2)=\Sigma^{(1)}(x,Q^2)\left(1+(\alpha(Q^2)-\alpha(Q_0^2))\,Z^+_\delta\right)$$

$$G^{(2)}(x,Q^2)=G^{(1)}(x,Q^2)\left(1+(\alpha(Q^2)-\alpha(Q_0^2))\,Z^+_\delta\right).$$

All the notation is given in Appendix 3.

2. The SF is connected with the DI by equations (11), where the Mellin integration is replaced by the ordinary product. Hence, we get to the LO

$$F_2^{(1)NS}(x,Q^2)=\mathcal{D}_{NS}\left[\alpha(Q^2)\right]^{-d^{NS}_\lambda}x^{1-\lambda},\quad F_2^{(1)S}(x,Q^2)=\mathcal{D}_S\left[\alpha(Q^2)\right]^{-d^+_\delta}x^{1-\delta}$$

$$F_L^{(1)NS}(x,Q^2)=\mathcal{D}_{NS}\,B^{(1)NS}_{L,\lambda}\left[\alpha(Q^2)\right]^{1-d^{NS}_\lambda}x^{1-\lambda},\quad F_L^{(1)S}(x,Q^2)=\mathcal{D}_S\,B^{(1)+}_{L,\delta}\left[\alpha(Q^2)\right]^{1-d^+_\delta}x^{1-\delta}\qquad(16a)$$

$$\bar{R}^{(1)}(x,Q^2)=B^{(1)+}_{L,\delta}\,\alpha(Q^2),$$

where

$$B^{(1)+}_{L,\delta}=B^{(1)\psi}_{L,\delta}+\xi(\delta)\,B^{(1)G}_{L,\delta}$$

to the NLO

$$F_K^{(2)NS}(x,Q^2)=F_K^{(1)NS}(x,Q^2)\left(1+\alpha(Q^2)C^{NS}_{K,\lambda}\right),\quad F_K^{(2)S}(x,Q^2)=F_K^{(1)S}(x,Q^2)\left(1+\alpha(Q^2)C^+_{K,\delta}\right)$$

$$\bar{R}^{(2)}(x,Q^2)=\bar{R}^{(1)}(x,Q^2)\left(1+\alpha(Q^2)\left[C^+_{L,\delta}-C^+_{2,\delta}\right]\right).\qquad(16b)$$

In the expression for $\bar{R}$ we neglect the contribution of the nonsinglet part giving the corrections $\sim x^{\delta-\lambda}\approx x^1$.

To obtain the SI analog of equations (16), we transform F_K^j $(j=NS,S)$ and $\bar{R}$ to new variables $\check{F}_K^j$ and $\check{\bar{R}}$ having an expansion in the form (1). This operation can be done in different ways (see [14]), which leads to different SIPT. We will use one of them (see [12,14]):

$$F_K^{NS}\to\check{F}_K^{NS}=\left(F_K^{NS}\right)^{-1/\bar{d}^{NS}_{\lambda,K}}\Big/\mathcal{D}_{NS}f_K^{NS}x^{1-\lambda}=\alpha(Q^2)\left(1-\alpha(Q^2)C^{NS}_{K,\lambda}/\bar{d}^{NS}_{\lambda,K}\right)$$

$$F_K^{S}\to\check{F}_K^{S}=\left(F_K^{S}\right)^{-1/\bar{d}^{+}_{\delta,K}}\Big/\mathcal{D}_{S}f_K^{S}x^{1-\delta}=\alpha(Q^2)\left(1-\alpha(Q^2)C^{+}_{K,\delta}/\bar{d}^{+}_{\delta,K}\right),$$

where $\bar{R}\to\check{\bar{R}}=\bar{R}\big/B^{(1)+}_{L,\delta}=\alpha(Q^2)\left(1+\alpha(Q^2)\left(C^+_{L,\delta}-C^+_{2,\delta}\right)\right)$

$$f_K^j=\left\{\begin{matrix}1 & K=2 & d^j_\lambda\\ B^{(1)j}_{L,\lambda} & K=L & d^j_{\lambda}+1\end{matrix}\right\}\bar{d}^j_{\lambda,K}\qquad(j=NS,+).$$

Constructing the SIPT for the new quantities $\check{F}_K^j$ $(j=NS,S)$ and $\check{\bar{R}}$ we get

$$\check{F}_K^{NS}=a_K^\lambda(Q^2),\ \check{F}_K^S=a_K^\delta(Q^2),\ \check{\bar{R}}=a_{\bar{R}}(Q^2),$$

where a_K^λ, a_K^δ and $a_{\bar{R}}$ satisfy equations (1) and (2) for $z^K_{1,\lambda}=-C^{NS}_{K,\lambda}/\bar{d}^{NS}_{\lambda,K}$, $z^K_{1,\delta}=-C^+_{K,\delta}/\bar{d}^+_{\delta,K}$ and $z_{1,\bar{R}}=C^+_{L,\delta}-C^+_{2,\delta}$, respectively.

Finally, we get the SI analog of equation (16b) in the form

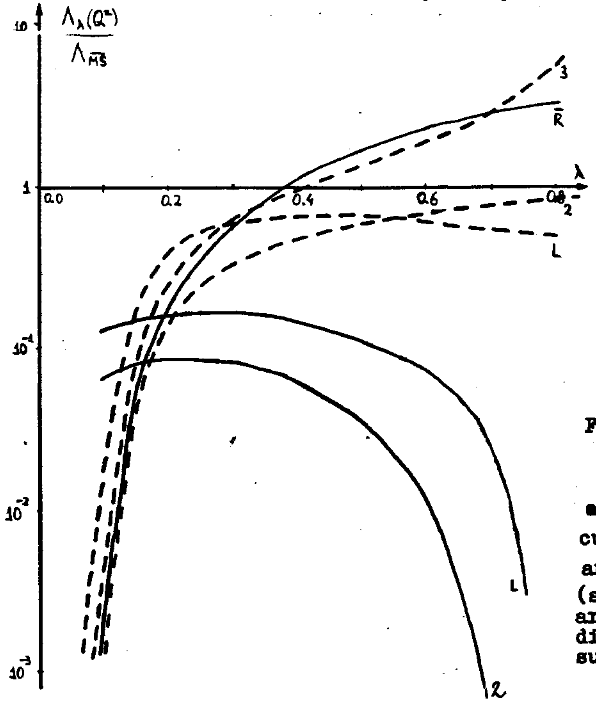


Fig. 2. The ratios $\Lambda_K^{NS}(\lambda)/\Lambda_{\overline{MS}}$ and $\Lambda_3(\lambda)/\Lambda_{\overline{MS}}$ (dashed curves), $\Lambda_K^+(\lambda+1)/\Lambda_{\overline{MS}}$ and $\Lambda_{\bar{R}}(\lambda+1)/\Lambda_{\overline{MS}}$ (solid curves), which are denoted by the indices 2,L,3 and R, versus variable λ.

$$F_2^{SI,NS}(x,Q^2) = \mathcal{D}_{NS}[a_2^\lambda(Q^2)]^{-d_\lambda^{NS}} x^{1-\lambda}, \quad F_2^{SI,S}(x,Q^2) = \mathcal{D}_S[a_2^\delta(Q^2)]^{-d_\delta^+} x^{1-\delta}$$

$$F_L^{SI,NS}(x,Q^2) = \mathcal{D}_{NS} B_{L,\lambda}^{(1)NS} [a_L^\lambda(Q^2)]^{1-d_\lambda^{NS}} x^{1-\lambda}, \quad F_L^{SI,S}(x,Q^2) = \mathcal{D}_S B_{L,\delta}^{(1)+} [a_L^\delta(Q^2)]^{1-d_\delta^+} x^{1-\delta} \quad (16c)$$

$$\bar{R}^{SI}(x,Q^2) = B_{L,\delta}^{(1)+} a_{\bar{R}}(Q^2)$$

In Fig. 2 we plot the ratios $\Lambda_K^{NS}(\lambda)/\Lambda_{\overline{MS}}$, $\Lambda_3(\lambda)/\Lambda_{\overline{MS}}$, $\Lambda_K^+(\lambda+1)/\Lambda_{\overline{MS}}$ and $\Lambda_K^+(\lambda+1)/\Lambda_{\overline{MS}}$ versus the variable λ. As is seen, all ratios, with the exception of $\Lambda_K^+(\lambda+1)/\Lambda_{\overline{MS}}$, decrease (increase) for decreasing (increasing) λ. The ratios $\Lambda_K^+(\lambda+1)/\Lambda_{\overline{MS}}$ have the singularity for $\lambda=0.8$ as $d_{1.8}^+ - d_{1.8}^- = 1$ (see (A9) in Appendix 3). All ratios slightly depend on the quark flavours used in calculation (the curves are built for $f=4$).

4. The SI Parametrization of the SF and R

Analysing the SF and R for $x \to 0$ and $x \to 1$, we can construct their parametrizations following papers /4,5/:

to the LO

$$F_2^{(1)NS}(x,Q^2) = \Big\{ \mathcal{D}_{NS}[\alpha(Q^2)]^{-d_\lambda^{NS}} (x^{1-\lambda} - x^{\mu_{NS}(\alpha)}) + A_{NS}[\alpha(Q^2)]^{-d_0} \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(\alpha))} x^{\mu_{NS}(\alpha)} \Big\} (1-x)^{\nu(\alpha)}$$

$$F_2^{(1)S}(x,Q^2) = \Big\{ \mathcal{D}_S[\alpha(Q^2)]^{-d_\delta^+} (x^{1-\delta} - x^{\mu_S(\alpha)}) + A_S[\alpha(Q^2)]^{-d_0} \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(\alpha))} x^{\mu_S(\alpha)} \Big\} (1-x)^{\nu(\alpha)} \quad (17a)$$

$$F_L^{(1)NS}(x,Q^2) = \Big\{ \mathcal{D}_{NS} B_{L,\lambda}^{(1)NS} [\alpha(Q^2)]^{1-d_\lambda^{NS}} (x^{1-\lambda} - x^{\mu_{NS}(\alpha)}) + A_{NS} B_L^{(1)NS}(x) [\alpha(Q^2)]^{1-d_0} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(\alpha))} x^{\mu_{NS}(\alpha)} \Big\} (1-x)^{\nu(\alpha)+1}$$

$$F_L^{(1)S}(x,Q^2) = \Big\{ \mathcal{D}_S B_{L,\delta}^{(1)+} [\alpha(Q^2)]^{1-d_\delta^+} (x^{1-\delta} - x^{\mu_S(\alpha)}) + A_S B_L^{(1)\Psi}(x) [\alpha(Q^2)]^{1-d_0} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(\alpha))} x^{\mu_S(\alpha)} \Big\} (1-x)^{\nu(\alpha)+1}$$

$$\bar{R}^{(1)}(x,Q^2) = \Big\{ B_{L,\delta}^{(1)+}(1-x) + B_L^{(1)\Psi}(x)/(1+\nu(\alpha)) \Big\} \alpha(Q^2)(1-x)$$

to the NLO

$$F_2^{(2)NS}(x,Q^2) = \Big\{ \mathcal{D}_{NS}[\alpha(Q^2)]^{-d_\lambda^{NS}} (1+\alpha(Q^2) C_{2,\lambda}^{NS})(x^{1-\lambda} - x^{\mu_{NS}(\alpha)}) + A_{NS}[\alpha(Q^2)]^{-d_0} x^{\mu_{NS}(\alpha)} \cdot \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(\alpha))} \Big\} (1-x)^{\nu(\alpha)} (1+\alpha(Q^2)\tilde{C}_2^\Psi(x,Q^2)) \quad (17b)$$

$$F_2^{(2)S}(x,Q^2) = \Big\{ \mathcal{D}_S[\alpha(Q^2)]^{-d_\delta^+} (1+\alpha(Q^2) C_{2,\delta}^+)(x^{1-\delta} - x^{\mu_S(\alpha)}) + A_S[\alpha(Q^2)]^{-d_0} x^{\mu_S(\alpha)} \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(\alpha))} \Big\} \cdot (1-x)^{\nu(\alpha)} (1+\alpha(Q^2)\tilde{C}_2^\Psi(x,Q^2))$$

$$F_L^{(2)NS}(x,Q^2) = \Big\{ \mathcal{D}_S B_{L,\lambda}^{(1)NS} [\alpha(Q^2)]^{1-d_\lambda^{NS}} (1+\alpha(Q^2) C_{L,\lambda}^{NS})(x^{1-\lambda} - x^{\mu_{NS}(\alpha)}) + A_{NS} B_L^{(1)\Psi}(x) \cdot [\alpha(Q^2)]^{1-d_0} x^{\mu_{NS}(\alpha)} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(\alpha))} \Big\} (1-x)^{\nu(\alpha)+1} \cdot (1+\alpha(Q^2)\tilde{C}_L^\Psi(x,Q^2))$$

$$F_L^{(2)S}(x,Q^2) = \Big\{ \mathcal{D}_S B_{L,\delta}^{(1)+} [\alpha(Q^2)]^{1-d_\delta^+} (1+\alpha(Q^2) C_{L,\delta}^+)(x^{1-\delta} - x^{\mu_S(\alpha)}) + A_S B_L^{(1)\Psi}(x) \cdot [\alpha(Q^2)]^{1-d_0} x^{\mu_S(\alpha)} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(\alpha))} \Big\} (1-x)^{\nu(\alpha)+1} (1+\alpha(Q^2)\tilde{C}_L^\Psi(x,Q^2))$$

$$\bar{R}^{(2)}(x,Q^2) = \Big\{ B_{L,\delta}^{(1)+}(1-x)(1+\alpha(Q^2)(C_{L,\delta}^+ - C_{2,\delta}^+)) + B_L^{(1)\Psi}(x) \frac{1+\alpha(Q^2)\tilde{C}_{\bar{R}}(x)}{1+\nu(\alpha)} \Big\} \alpha(Q^2)(1-x).$$

The quantities $\mu_{NS}(\alpha)$ and $\mu_S(\alpha)$ take account of the Regge-trajectories with the exception of the leading trajectories that define λ and δ. To a good accuracy, $\mu_{NS}(\alpha)=1$ and $\mu_S(\alpha)=0$ (see /4,5/).

Using the SI approximations of the SF for $x \to 0$ and $x \to 1$ obtained in the previous sections we construct the SI parametrizations in the form:

$$F_2^{SI,NS}(x,Q^2) = \Big\{ \tilde{\mathcal{D}}_{NS}[a_2^\lambda(Q^2)]^{-d_\lambda^{NS}} (x^{1-\lambda} - x^{\mu_{NS}(a_2^\lambda)}) + \tilde{A}_{NS}[a_2(x,Q^2)]^{-d_0} x^{\mu_{NS}(a_2^\lambda)} \cdot \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(a_2(x)))} \Big\} (1-x) \cdot (1-x)^{\nu(a_2(x))}$$

$$F_2^{SI,S}(x,Q^2) = \Big\{ \tilde{\mathcal{D}}_S[a_2^\delta(Q^2)]^{-d_\delta^+} (x^{1-\delta} - x^{\mu_S(a_2^\delta)}) + \tilde{A}_S[a_2(x,Q^2)]^{-d_0} x^{\mu_S(a_2^\delta)} \frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(a_2(x)))} \Big\} (1-x)^{\nu(a_2(x))}$$

$$F_L^{SI,NS}(x,Q^2) = \Big\{ \tilde{\mathcal{D}}_{NS} B_{L,\lambda}^{(1)NS} [a_L^\lambda(Q^2)]^{1-d_\lambda^{NS}} (x^{1-\lambda} - x^{\mu_{NS}(a_L^\lambda)}) + \tilde{A}_{NS} B_L^{(1)NS}(x) [a_L(x,Q^2)]^{1-d_0} x^{\mu_{NS}(a_L^\lambda)} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(a_L(x)))} \Big\} (1-x)^{\nu(a_L(x))+1} \quad (17c)$$

$$F_L^{SI,S}(x,Q^2) = \Big\{ \tilde{\mathcal{D}}_S B_{L,\delta}^{(1)+} [a_L^\delta(Q^2)]^{1-d_\delta^+} (x^{1-\delta} - x^{\mu_S(a_L^\delta)}) + \tilde{A}_S B_L^{(1)\Psi}(x) [a_L(x,Q^2)]^{1-d_0} \cdot x^{\mu_S(a_L^\delta)} \frac{\Gamma(1+\nu_0)}{\Gamma(2+\nu(a_L(x)))} \Big\} (1-x)^{\nu(a_L(x))+1}$$

$$\bar{R}^{SI}(x,Q^2) = \Big\{ B_{L,\delta}^{(1)+}(1-x) a_{\bar{R}}^\delta(Q^2) + B_L^{(1)\Psi}(x) a_{\bar{R}}(x,Q^2)/(1+\nu(a_{\bar{R}}(x))) \Big\} (1-x),$$

where

$$\mu_{NS}(a_K^\lambda) \approx \mu_{NS}(\alpha) \approx 1 \quad ; \quad \mu_S(a_K^\delta) \approx \mu_S(\alpha) \approx 0.$$

Generally speaking, the coefficients $\tilde{A}, \tilde{B}$ and A, B are different as the corrections $\alpha(Q^2)\tilde{C}_K(x,Q^2)$ are large and the result changes essentially if they are written or not in the braces (see the parametrization in the NLO in paper /5,16/).

In papers /23,24/ the moments of the SF were treated in an SI way. Thus, for the SF there was an infinite set of the coupling constants (for each moment its own coupling constant). The SF were reconstructed numerically by the Yndurain method /31/. In paper /24/ the first nine moments ($n \leq 9$) are involved in this procedure. In equation (17c) the parametrizations depend on two coupling constants, in which the SI processing of the SF moments is effectively-taken into account in two kinematical regions of the variable x : $x \sim 0$ and $x \sim 1$. The coupling constants are not independent, rather they are connected with each other (see (4) and (5)).

In the considered region of Q^2 (for $x \gtrsim 0.3$) besides the logarithmic $(\sim \alpha_s)$ correction, it is essential also the power corrections of the form m^2/Q^2. Here we take into account the corrections to the SF due to the nucleon (target) mass $(m = m_N)$ and the parton transverse moment in the nucleon. We get the so-called ξ -scalling [32]:

$$F^j_{K,m}(x,Q^2) = \frac{x^2/\xi^2}{(1+4x^2m^2/Q^2)} F^j_K(\xi,Q^2) + \frac{6m^2}{Q^2} \cdot \frac{x^3}{(1+4x^2m^2/Q^2)} \int_\xi^1 \frac{d\xi'}{(\xi')^2} F^j_K(\xi',Q^2), \quad (18)$$

where

$$\xi = 2x\Big/\left(1+(1+4m^2x^2/Q^2)^{1/2}\right). \quad (19)$$

To an accuracy of m^2/Q^2 the expression (16) for the new SF $F_m(x,Q^2)$ is given in the form

$$F^j_{K,m}(x,Q^2) = F^j_K(x,Q^2) + \frac{m^2x^2}{Q^2}\left\{6x\int_x^1 \frac{dy}{y^2} F^j_K(y,Q^2) - x\frac{\partial}{\partial x}F^j_K(x,Q^2) - 4F^j_K(x,Q^2)\right\} \quad \begin{matrix}(20)\\ (j=NS,S)\end{matrix}.$$

Analysing equation (20) for $x \to 0$ and $x \to 1$ we obtain that in our case the contribution of the target mass is factorized as follows [5]:

$$F^{NS}_{2,m}(x,Q^2) = F^{NS}_2(x,Q^2)\left(1+\left[6/\lambda - 5 + \lambda + \frac{x\nu(\alpha)}{1-x}\right]\frac{x^2m^2}{Q^2}\right)$$

$$F^S_{2,m}(x,Q^2) = F^S_2(x,Q^2)\left(1+\left[6/\delta - 5 + \delta + \frac{x\nu(\alpha)}{1-x}\right]\frac{x^2m^2}{Q^2}\right) \quad (21a)$$

$$F^{NS}_{L,m}(x,Q^2) = F^{NS}_L(x,Q^2)\left(1+\left[6/\lambda - 5 + \lambda + \frac{x(\nu(\alpha)+1)}{1-x}\right]\frac{x^2m^2}{Q^2}\right)$$

$$F^S_{L,m}(x,Q^2) = F^S_L(x,Q^2)\left(1+\left[6/\delta - 5 + \delta + \frac{x(\nu(\alpha)+1)}{1-x}\right]\frac{m^2x^2}{Q^2}\right).$$

Hence, we have for $\bar{R}_m(x,Q^2) \approx F^S_{L,m}(x,Q^2)/F^S_{2,m}(x,Q^2)$ (see [33])

$$\bar{R}_m(x,Q^2) = \bar{R}(x,Q^2)\left(1+\frac{m^2x^3}{Q^2(1-x)}\right). \quad (21b)$$

As seen the parametrizations (21) become absurd for $x \to 1$ because the expansion parameter (19) is $m^2x^3/Q^2(1-x)$ in this region. For a correct analysis of the parametrizations (21) as $x \to 1$, we present the ξ-parameter (19) in the form:

$$\xi\big|_{x=1} = 1 - m^2/Q^2 + \frac{3}{2}\left(m^2/Q^2\right)^2 + \ldots$$

The values of parametrizations (21) for $x = 1$ to LO in m^2/Q^2 are as follows

$$F^j_{2,m}\big|_{x=1} = A_{NS}[\alpha(Q^2)]^{-d_0}\frac{\Gamma(1+\nu_0)}{\Gamma(1+\nu(\alpha))}\left(\frac{m^2}{Q^2}\right)^{\nu(\alpha)} \qquad (j=NS,S)$$

$$F^j_{L,m}\big|_{x=1} = \frac{4C_F}{1+\nu(\alpha)} \cdot \frac{m^2}{Q^2} \cdot \alpha(Q^2)\, F^j_{2,m}\big|_{x=1}, \quad \bar{R}_m\big|_{x=1} = \frac{4C_F}{1+\nu(\alpha)}\frac{m^2}{Q^2}\alpha(Q^2)$$

Hence, $F^j_{K,m}(x,Q^2)$ and $\bar{R}_m(x,Q^2)$ are not equal to zero (but they do not increase unlimitedly) and tend to zero as powers of m^2/Q^2 when $Q^2 \to \infty$.

The earlier obtained infinities resulted from the fact that we took the first terms of the expansion in $\frac{m^2x^3}{Q^2(1-x)}$ and obtained the absurd expressions for $x \to 1$. The above analysis of the parametrizations (21) for $x \to 1$ indicates that the whole series is not an absurd expression, that is, the following situation takes place (ν is non-integer):

$$\left(1-x+\frac{m^2x^3}{Q^2}\right)^\nu = (1-x)^\nu\left(1+\frac{m^2x^3}{Q^2(1-x)}\right)^\nu = (1-x)^\nu \sum_{k=0}^{\infty} C^k_\nu \left(\frac{m^2x^3}{Q^2(1-x)}\right)^k.$$

The contribution due to the nonzero parton transverse moment in the nucleon has the form [33,34]

$$\bar{R}^{prim} = \frac{m^2}{Q^2}\,\frac{4x^3}{F_2(x,Q^2)}\int_x^1 \frac{dx'}{(x')^2} F_2(x',Q^2). \quad (22)$$

Analysing equation (22) for $x \to 0$ and $x \to 1$ we have

$$\bar{R}^{prim} \approx \frac{4m^2}{Q^2}(x^2)^{\frac{\nu+4}{\nu+3}}\,\frac{(1-x)}{\nu+1}.$$

This contribution is not large in the region $Q^2 \geq 3$ GeV^2 (see Fig. 3) and it coincides with the values obtained in the papers [34,35].

In Fig. 3 we present the graphs of different parametrizations of $R(x,Q^2) = \sigma_L/\sigma_T$. We use the mean values obtained by the ED fit (23): $\lambda = 0.5$ and $\delta = 1.5$. In Fig. 3b the theoretical curves are built for $\Lambda^{(1)}_{\overline{MS}} = 105$ MeV [18] and in others the curves are built for $\Lambda^{(2)}_{\overline{MS}} = 210$ MeV [20]. All parametrizations well agree with ED of SLAC group [21] and BCDMS group [20]. ED of EMC group [18] place down of the parametrization on the whole. However the negative mean value of R obtained by EMC group [18] and the presence of the large uncertainty of the ED doesn't allow us to infer about the bad conformity of the QCD parametrization and the experiment. The correction due to the target mass is very small and it is not plotted in Fig.3. The correction due to the parton transverse moment in the nucleon is not large (see also [34,35]) even in the small Q^2 region.

The well agreement of the QCD parametrizations with the ED for $R = \sigma_L/\sigma_T$ is the test for the quantum chromodynamics concerning the constant of the quarks and gluons coupling. This test together with the before obtained test on the three gluon vertex [28], suggested in the paper [36] are the rather good check of the QCD predictions.

In paper [16] the global fit of the ED has been carried out and all independent constants of the parametrization have been obtained. They have the following form to the NLO:

$$41 \text{ MeV} \leq \Lambda_{\overline{MS}} \leq 216 \text{ MeV} \qquad 0.27 \leq \nu_0 \leq 0.70,$$

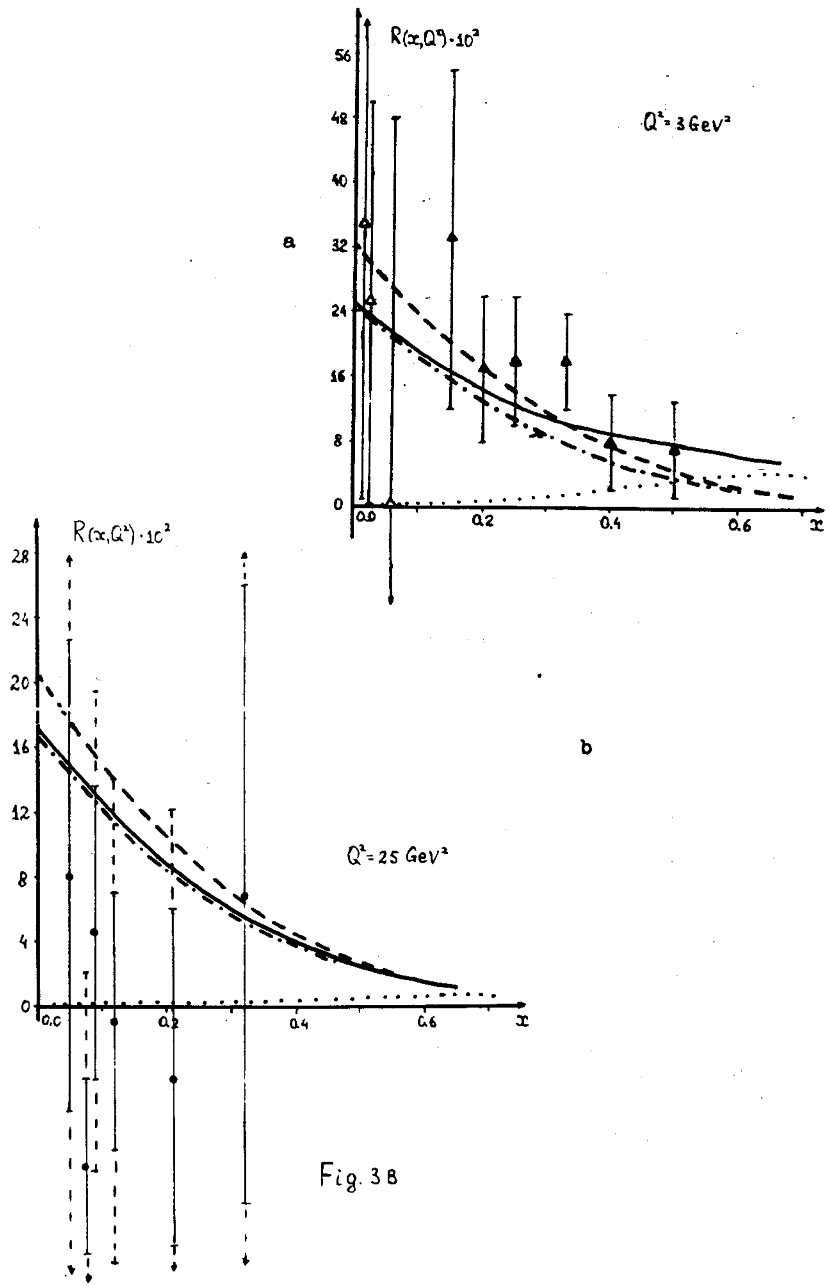
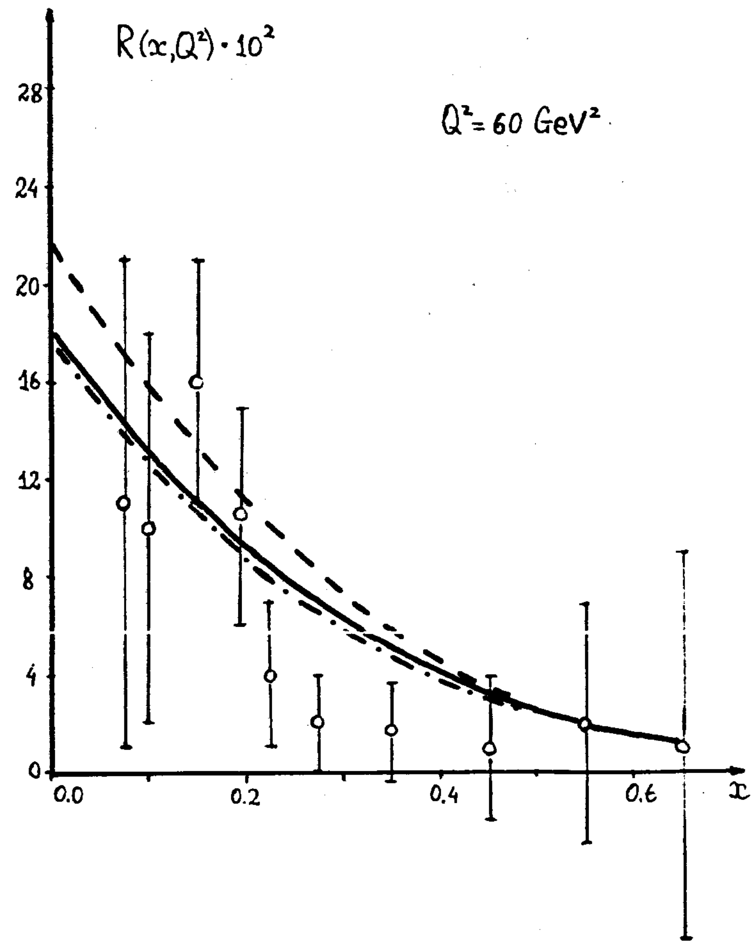


Fig. 3B



c

<u>Fig.3.</u> The graphs of the parametrizations $R^{(1)}(x,Q^2)$, $R^{(2)}(x,Q^2)$ and $R^{ST}(x,Q^2)$ are shown by the dashed, dotted-dashed and solid curves, respectively. The symbols ●, △, ▲ and ○ are the data of the groups EMC [18] , EMC [19] , SLAC [21] and BCDMS [20], respectively. The dotted curve corresponds to the contribution due to the parton transverse moment.

$$0.49 \le \lambda \le 0.52 \qquad 1.3 \le \delta \le 1.7 \qquad\qquad (23)$$
$$4.10 \le A_{NS} \le 5.10 \qquad 1.24 \le A_S \le 1.43 .$$

Finally, we point out some reasons emphasizing the importance of the SI parametrization of the SF:

1. The coefficients $C^{\delta}_{k,\lambda}$ and $\tilde{C}^{i}_{k}(x,Q^2)$ $(i = NS, +)$ in the parametrizations (17a) depend on the scheme of removal of divergences, hence each scheme, generally speaking, requires a new fit of the data. Thus, the parameters (23) of equations (17a) obtained in the fit are DS. This is certainly inconvenient.

2. The SI parametrization takes account of the highest orders of PT, that is one may say that the contribution of a subsequent perturbation order should not change the result essentially.

3. The SI expansion in the coupling constant is in a sense more successful than the ordinary one (in analogy with the RGM). For example, the coefficients $\tilde{C}_k(x,Q^2)$ contain terms $\sim \ln(1-x)$ and $\sim \ln^2(1-x)$, and increase strongly for $x \to 1$. The SI coupling constants $a_k(x,Q^2)$ contain only the term $\sim \ln(1-x)$.

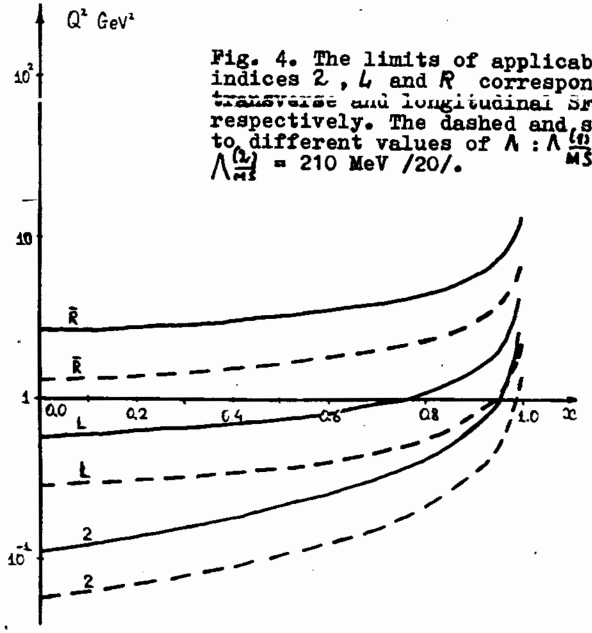


Fig. 4. The limits of applicability of the SIPT. The indices 2, L and R correspond to the curves of the transverse and longitudinal SF and ratio $R = \sigma_L/\sigma_T$, respectively. The dashed and solid curves correspond to different values of Λ: $\Lambda^{(4)}_{\overline{MS}}$ = 105 MeV /18/ and $\Lambda^{(4)}_{\overline{MS}}$ = 210 MeV /20/.

Moreover, we can easily establish the limits of applicability of the SIPT (hence, the PT as well) using the following receipt: whether there exists a solution of equations of type (4) and (5) or not

(see [15]). Really, the left-hand side of equation (4) has a minimum at $\alpha = \beta_1/\beta_0$. Hence, equation (4) has a solution for

$$Q^2 \geq \Lambda_a^2 \exp\left[\left(1-\ln(\beta_1/\beta_0)\right)\beta_1/\beta_0^2\right],$$

where

$$a = \{x, \psi; \lambda, NS; \delta, +\}.$$

The curves corresponding to the boundary of the kinematic-variable (x, Q^2) region, where the PT is applicable are drawn in Fig. 4. The PT is applicable in the (x, Q^2) region above the curves. Analogous graphs can be constructed for the variables (λ, Q^2) and (δ, Q^2).

4. In paper [15] the graphs for both the parametrizations (17a) and (17c) for the nonsinglet SF are constructed with values (23) obtained in the data fit for the parametrization (17a). The curves corresponding to the SI parametrization are in the best agreement with experiment. Hence, the SI parametrization is more successful. Strictly speaking, the SI parametrization (and the target mass contribution) requires its own data fit. For the coefficients obtained from this fit the agreement of the theory and experiment should be better. This procedure will be realized in the future.

5. Conclusion

In paper [27,28] the SF moments to the NLO have been treated ~~with the help of the SIPT, and then the SF have been numerically re-~~ constructed by the Yndurain method [30]. In the present paper, the inverse operation is performed: the SF and $R(x,Q^2)$ are exactly reconstructed in the vicinity of points $x=0$ and $x=1$, then to the NLO they are processed with the help of the SIPT. The obtained equations are used for the parametrizations of the SF and $R(x,Q^2)$. In this paper, mass corrections due to the target mass and the parton transverse moment are also taken into account which is essential in the region $x>0.25$ for small Q^2 (see [15]). In this region, also essential are the heavy-quark corrections (see [5]). For the correct inclusion of the heavy quark corrections it is essential to consider the massive SIPT (see [13]) that will be done in a subsequent paper.

The SF parametrization to the LO has been applied [37] to the ratio $R^{A'}_{k,A} = AF^{A'}_k(x,Q^2)/A'F^{A}_k(x,Q^2)$ in the rescaling model [38]. Here $F^A_k(x,Q^2)\cdot\frac{1}{A}$ is the SF of the nucleus A. In the future the result obtained here will be used to analyse the ED of $F_2(x,Q^2)$ for $0.75 \le x \le 0.95$ recently obtained by the BCDMS group and also for the ratio $R^{A'}_{k,A}$ in other EMC-effect models (see [39] and references therein).

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Appendix 1

Consider the first two expansion terms of the anomalous dimensions γ^n_{NS} and γ^n_{ij} $(i,j=\Psi,G)$ in the coupling constant $\alpha(Q^2)$:

$$\gamma^n_{NS}=\sum_{m=0}\gamma^{(m)n}_{NS}\left[\alpha(Q^2)\right]^{m+1}, \quad \gamma^n_{ij}=\sum_{m=0}\gamma^{(m)n}_{ij}\left[\alpha(Q^2)\right]^{m+1}. \tag{A1}$$

Their values are given in papers /5,10/.

1. For the determination of the DI behaviour for $x\to 1$ we expand the coefficients (A1) in series of large n /9,10/. The expansions of the standard functions of the anomalous dimensions are given in paper /15/. Hence we have

to the LO with accuracy $O(1/n)$

$$\gamma^{(0)n}_{NS}=\gamma^{(0)n}_{\Psi\Psi}\underset{n\to\infty}{=\!=}2^3C_F\left(\ln n+\gamma-3/4+1/2n\right), \quad \gamma^{(0)n}_{\Psi G}\underset{n\to\infty}{=\!=}-\frac{2^3T_F}{n} \tag{A2a}$$

$$\gamma^{(0)n}_{G\Psi}\underset{n\to\infty}{=\!=}-\frac{2^2C_F}{n}, \quad \gamma^{(0)n}_{GG}=2^3C_A\left(\ln n+\gamma-\frac{11}{12}+T_F/3C_A+\frac{1}{2n}\right),$$

to the NLO with accuracy $O(1)$

$$\gamma^{(1)n}_{NS}\underset{n\to\infty}{=\!=}\gamma^{(1)n}_{\Psi\Psi}\underset{n\to\infty}{=\!=}8C_F\left(a^\Psi_1(\ln n+\gamma)+a^\Psi_2\right) \tag{A2b}$$

$$\gamma^{(1)n}_{\Psi G}\underset{n\to\infty}{=\!=}\gamma^{(1)n}_{G\Psi}\underset{n\to\infty}{=\!=}0, \quad \gamma^{(1)n}_{GG}=8C_A\left(a^G_1(\ln n+\gamma)+a^G_2\right).$$

where

$$a^\Psi_1=a^G_1=C_A\left(67/9-2\zeta(2)\right)-20T_F/9\;;\quad a^G_2=\frac{4}{3}T_F+\frac{C_FT_F}{C_A}-C_A\left(3\zeta(3)+\frac{8}{3}\right)$$

$$a^\Psi_2=\frac{3}{2}(2C_F-C_A)\left(\zeta(2)-2\zeta(3)-\frac{1}{8}\right)-C_A\left(13\zeta(2)+\frac{43}{48}\right)+\frac{4}{3}T_F\left(\zeta(2)+\frac{1}{8}\right).$$

Thus, for the anomalous dimensions of the multiplicative renormalized operators we have (see /25/):

to the LO

$$\gamma^{(0)n}_{-}=\gamma^{(0)n}_{\Psi\Psi}, \quad \gamma^{(0)n}_{+}=\gamma^{(0)n}_{GG}, \tag{A3a}$$

to the NLO

$$\gamma^{(1)n}_{--}=\gamma^{(1)n}_{\Psi\Psi}, \quad \gamma^{(1)n}_{++}=\gamma^{(1)n}_{GG}, \quad \gamma^{(1)n}_{-+}=0 \tag{A3b}$$

$$\gamma^{(1)n}_{+-}=\gamma^{(1)n}_{GG}-\gamma^{(1)n}_{\Psi\Psi}-\frac{2(C_A-C_F)}{C_F}\,n(\ln n+\tilde{\gamma})\,\gamma^{(1)n}_{\Psi G}.$$

2. To determine the Mellin coefficients of the integrations (6), we consider following /6/ the integrals. (Here we use the notation: $a=\frac{4C_F}{\beta_0}$, $t=\frac{\alpha(Q_0^2)}{\alpha(Q^2)}$, $w=1/x$ and $p=a\ln t$):

a) $$T_1=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}dn\,t^{-a\ln n}\cdot w^{n-1}=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{dn}{n^p}\,w^{n-1}=\frac{\ln^{p-1}w}{w\Gamma(p)}$$

b) $$T_2=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}dn\,t^{-a\ln n}\cdot\ln n\cdot w^{n-1}=\frac{d}{da}\left(\frac{\ln^{p-1-a}w}{w\Gamma(p-a)}\right)\Big|_{a=0}=\frac{\ln^{p-1}w}{w\Gamma(p)}\left(\Psi(p)-\ln\ln w\right)$$

c) $$T_3=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}dn\,t^{-a\ln n}\,w^{n-1}/\ln(n/n_0)=-n_0^p\int^{\delta}d\delta'\,\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}dn\,\frac{w^{n-1}}{(n/n_0)^{p+\delta'}}\Big|_{\delta'=0}= \tag{A4}$$

$$=\int^{\delta}d\delta'\,n_0^{\delta'}\,\frac{\ln^{p-1+\delta'}w}{w\Gamma(p+\delta')}\Big|_{\delta=0}=I(n_0\ln w,p)\,\frac{\ln^{p-1}w}{w\Gamma(p)},$$

where

$$I(x,p)=-\int^{\delta}d\delta'\,x^{\delta'}\,\frac{\Gamma(p)}{\Gamma(p+\delta')}\Big|_{\delta=0}$$

and

$$\ln(1/n_0)\equiv\tilde{\gamma}. \tag{A5}$$

For the integral $I(x,p)$ we find a more simple expression. Expanding $\Gamma(p+\delta')$ in δ' we have

$$(-)\,I(x,p)=\int^{\delta}d\delta'\,x^{\delta'}\left\{1-\Psi(p)\delta'+\left(\Psi^2(p)+\Psi^{(1)}(p)\right)\frac{(\delta')^2}{2}-\dots\right\}\Big|_{\delta=0}= \tag{A6}$$

$$=\left(1+\frac{\Psi(p)}{\ln x}+\left(\Psi^2(p)+\Psi^{(1)}(p)\right)/\ln^2x+\dots\right)/\ln x-\frac{\Gamma(p)}{\ln x}\sum_n\left[\left(\frac{d}{d\delta}\right)^n\Gamma^{-1}(p-\delta)\right]/(\ln x)^n\Big|_{\delta=0}.$$

Summing up the latter series we get

$$I(x,p)=\Gamma(p)\left(1/\left(\frac{d}{d\delta}-\ln x\right)\right)\Gamma^{-1}(p-\delta)\Big|_{\delta=0}. \tag{A7}$$

The expression (A7) with good accuracy can be represented as

$$I(x,p)=\frac{1}{\Psi(p)-\ln x}$$

which corresponds to elimination of $\Psi^{(m)}(p)$ $(m\geq 1)$ in the expansion of the Γ-function (see (A.6)).

3. Using equations (A2)-(A7) we obtain the required Mellin coefficients in the form

to the LO $$T^{(1)}_{NS}(w)=T^{(1)}_{\Psi\Psi}(w)=\exp\left[(3/4-\gamma)p\right]\frac{\ln^{p-1}w}{w\Gamma(p)}\left(1-\frac{\ln w}{2}\right)$$

$$T^{(1)}_{G\Psi}(w)=\frac{C_F}{C_A-C_F}\exp\left[(3/4-\gamma)p\right]\frac{\ln^{p}w}{w\Gamma(p+1)}\left(I(n_0\ln w,p+1)-I(n_0\ln w,p+2)\frac{\ln w}{2}\right),$$

to the NLO

$$T^{(2)}_{NS}(w)=T^{(2)}_{\Psi\Psi}(w)=T^{(1)}_{\Psi\Psi}(w)\left(1+\left(\alpha(Q^2)-\alpha(Q_0^2)\right)\left[a^\Psi_1\left(\Psi(p)+\gamma-\ln\ln w\right)+a^\Psi_2\right]\right)$$

$$T^{(2)}_{G\Psi}(w)=T^{(1)}_{G\Psi}(w)\left(1+\left(\alpha(Q^2)-\alpha(Q_0^2)\right)\left[a^\Psi_1 I^{-1}(n_0\ln w,p+1)+a^\Psi_2+a^\Psi_1(\tilde{\gamma}-\gamma)\right]\right). \tag{A8}$$

Appendix 2

The coefficients $B_k^{(1)j}(x)$ $(j=NS,\psi,G)$ are given in paper /37/. They have the form:

$$B_2^{(1)NS}(x)=B_2^{(1)\psi}(x)=2C_F\Big[(1+x^2)\Big(\frac{\ln(1-x)}{1-x}\Big)_+-\frac{3}{2}\Big(\frac{1}{1-x}\Big)_+-\frac{1+x^2}{1-x}\ln x+3-2x-$$
$$-\Big(\frac{9}{2}+2\zeta(2)\Big)\delta(1-x)\Big]$$
$$B_2^{(1)G}(x)=2T_F\Big[(x^2+(1-x^2))\ln\Big(\frac{1-x}{x}\Big)+6(1-x)x\Big] \qquad \text{(A8)}$$
$$B_L^{(1)NS}(x)=B_L^{(1)\psi}(x)=4C_Fx\,,\quad B_L^{(1)G}(x)=16\,T_F\,x(1-x)\,,$$

where

$$\int_0^1 dx\,\varphi(x)\big(f(x)\big)_+\equiv\int_0^1 dx\,f(x)\big(\varphi(x)-\varphi(1)\big).$$

The expressions obtained by expanding equation (A8) in the moments coincide with the ones given in papers /24,38/.

The coefficients $R_L^{(2)j}(x)$ $(j=NS,\psi,G)$ are obtained by the relations between characteristic functions and their moments (see /39/). Here we also use the connection between the functions in the anomalous dimensions of the Wilson operators and their moments (see /40/). The coefficients $R_L^{(2)j}(x)$ have the form:

$$R_L^{(2)NS}(x)=(C_A-2C_F)\Big[8S_{1,2}(-x)+4Li_3(-x)+4Li_3(x)+4Li_2(-x)\big(3\ln x-4\ln(1+x)\big)-4\zeta(3)-$$
$$-2\ln^2x\ln\frac{1-x}{1+x}-4\ln^2(1+x)\ln x+\frac{2}{3}\ln^3x+\frac{4(2+10x^2-5x^3+3x^5)}{5x^3}Li_2(-x)+\frac{4(2+5x^2-5x^3+3x^5)}{5x^3}\cdot$$
$$\cdot\ln(1+x)\ln x+\frac{2(5-3x)}{5x}\ln^2x-\Big(\ln(1-x)+\ln(1+x)-2\ln x+\frac{2+5x^2+3x^5}{5x^3}\Big)4\zeta(2)-\frac{23}{3}\ln(1-x)+$$
$$+4\ln x\cdot\frac{6+21x+77x^2-9x^3}{15x^2}-\frac{144-3078x-289x^2+216x^3}{90x^2}\Big]+2C_F\Big[Li_2(x)+\ln^2(1-x)+\ln^2x$$
$$-\frac{3-22x}{3}\ln x-3\zeta(2)-\frac{6-25x}{3x}\ln(1-x)-\frac{3-22x}{3}\ln x-\frac{78-355x}{36x}-\frac{4}{3}T_F\Big[\ln\frac{x^2}{1-x}-\frac{6-25x}{6x}\Big]$$

$$R_L^{(2)\psi}(x)=R_L^{(2)NS}(x)+2^3T_F\Big[Li_2(x)+\ln^2x-\zeta(2)+\frac{2+x-4x^2}{2x}\ln x-\frac{1-3x+2x^3}{3x^2}\ln(1-x)$$
$$-\frac{7-48x+57x^2-16x^3}{36x^2}\Big]$$

$$R_L^{(2)G}(x)=2C_A\Big[\ln^2(1-x)+2\ln x\ln\frac{1+x}{1-x}+2Li_3(-x)+\frac{3}{1-x}\ln^2x+\frac{2(3-x)}{1-x}\big(Li_2(-x)-\zeta(2)\big)$$
$$+\frac{1-3x-27x^2+29x^3}{3x^2(1-x)}\ln(1-x)+\frac{24+204x-317x^3}{24x(1-x)}\ln x+\frac{10+24x+483x^2-517x^3}{72x^2(1-x)}\Big]-$$
$$-\frac{2C_F}{1-x}\Big[Li_2(x)+\frac{2(5-3x)}{15}\ln^2x+\frac{2+10x^3-12x^5}{15x^3}\big(Li_2(-x)+\ln x\cdot\ln(1+x)\big)+\frac{1+3x-4x^2}{2x}\ln(1-x)$$
$$+\frac{2-5x^3-12x^5}{15x^3}\cdot\zeta(2)+\frac{13+82x-36x^2}{30x}\ln x+\frac{1+206x-243x^2+36x^3}{30x}\Big],$$

where $$S_{n,p}(x)=\frac{(-1)^{n+p+1}}{(n-1)!\,p!}\int_0^1\frac{dt}{t}\ln^{n-1}(t)\ln^p(1-xt)\,;\quad Li_n(x)\equiv S_{n-1,1}(x)\,,\ n\geq 2.$$

The coefficients $R_L^{(2)j}(x)$ $(j=NS,\psi,G)$ are rather unwieldy, however, they are much simpler than the ones (for $R_L^{(2)NS}(x)$) obtained in paper /39/.

Appendix 3.

1. The region $x\sim 1$. The coefficients $\tilde{Z}_k(x,Q^2)$; $\tilde{C}_2=\tilde{B}_2^{(1)}+\tilde{Z}_2$, $\tilde{C}_L(x,Q^2)=\tilde{R}_L^{(2)}(x,Q^2)+\tilde{Z}_L(x,Q^2)$, $\tilde{d}_k$, y_k and α_0 of the parametrizations (8), (9) for the DI and (12), (13) for SF are determined by the integrations (6) and (10), respectively. They have the form:

$$\beta_0=\frac{11}{3}C_A-\frac{4}{3}T_F\,,\ \beta_1=\frac{34}{3}C_A^2-\frac{20}{3}C_AT_F-4C_FT_F\,;\ \alpha_0=\frac{4C_F}{\beta_0}\Big(\frac{3}{4}-\gamma\Big)\,,\ \tilde{d}(x,Q^2)=\frac{4C_F}{\beta_0}\Big(\Psi_x(1+\nu(\alpha))+$$
$$+\frac{3}{4}\Big)\equiv\tilde{d}_2(x,Q^2)\,,\ \tilde{d}_L(x,Q^2)=\tilde{d}_2(x,Q^2)+1\,,\ \tilde{d}_{\bar{R}}(x,Q^2)=1+\frac{4C_F}{\beta_0}\Big/(1+\nu(\alpha))$$
$$\tilde{Z}_2(x,Q^2)=\frac{4C_F}{\beta_0}\Big(C_1^{Z\psi}\cdot\Psi_x(1+\nu(\alpha))+C_2^{Z\psi}\Big)\,,\ \tilde{Z}_L(x,Q^2)=\frac{4C_F}{\beta_0}\Big(C_1^{Z\psi}\cdot\Psi_x(2+\nu(\alpha))+C_2^{Z\psi}\Big)$$
$$\tilde{B}_2^{(1)}(x,Q^2)=2C_F\Big(\Psi_x^2(1+\nu(\alpha))+2C_1^{B\psi}\cdot\Psi_x(1+\nu(\alpha))+C_2^{B\psi}\Big)$$
$$\tilde{R}_L^{(2)}(x,Q^2)=2C_F\Big(\Psi_x^2(2+\nu(\alpha))+2C_1^{R\psi}\cdot\Psi_x(2+\nu(\alpha))+C_2^{R\psi}\Big)$$
$$y^{NS}(x,Q^2)=y^{\psi}(x,Q^2)=\frac{4C_F}{\beta_0}\ln\Big(\alpha(Q_0^2)\big/\alpha(Q^2)\Big)\cdot\frac{\nu-3/2}{\nu+1}\equiv y_2(x,Q^2)\,,\ y_L(x,Q^2)=\frac{\nu+1}{\nu+2}(y_2+3)-2$$
$$y^G(x,Q^2)=\frac{4C_F}{\beta_0(\nu+2)}\ln\Big(\alpha(Q_0^2)\big/\alpha(Q^2)\Big)\Big[\frac{1}{\tilde{\Psi}_x(\nu+3)}-\frac{3}{2}(\nu+2)-\Big(\frac{\tilde{\Psi}_x(\nu+2)+\tilde{\Psi}_x(\nu+3)}{(\nu+2)\cdot\tilde{\Psi}_x(\nu+3)}-$$
$$-\frac{3}{2}(\nu+2)\Big)\Big/\tilde{\Psi}_x(\nu+2)\Big],$$

where

$$C_1^{B\psi}=\frac{3}{2}\,,\ C_2^{B\psi}=\Psi^{(1)}(1+\nu)-\zeta(2)-\frac{9}{2}\,;\ C_1^{R\psi}=2\zeta(2)-\frac{9}{4}-\frac{C_A}{C_F}\Big(\zeta(2)-\frac{23}{12}\Big)-\frac{T_F}{3C_F}$$
$$C_2^{R\psi}=\Psi^{(1)}(2+\nu)+6\zeta(3)-5\zeta(2)-\frac{53}{36}-\frac{C_A}{C_F}\Big(3\zeta(3)-2\zeta(2)-\frac{215}{36}\Big)-\frac{19}{9}\frac{T_F}{C_F}$$
$$C_1^{Z\psi}=a_1^{\psi}-\beta_1/\beta_0\,,\ C_2^{Z\psi}=a_2^{\psi}+\frac{3}{4}\beta_1/\beta_0\,,$$

and $\zeta(m)$ are the Riemannian ζ-functions, and $\Psi_x(1+\nu)=\ln\frac{1}{1-x}+\gamma+\Psi(1+\nu)$, $\tilde{\Psi}_x(1+\nu)=\Psi_x'(1+\nu)+\tilde{\gamma}-\gamma$.

The coefficients a_1^{ψ} and a_2^{ψ} are given in Appendix 1.

In Fig. 5 we present the graphs of $\hat{y}_k=\frac{1-x}{\alpha(Q^2)}y_k$ and $\tilde{C}_k(x,Q^2)$. One can see that the coefficients $\hat{y}_k(x,Q^2)$ are much smaller than the values of $\tilde{C}_k(x,Q^2)$ in the considered region. Hence, the coefficients $y_k(x,Q^2)$ can be neglected in the parametrisations (1[illegible]), (1[illegible]) and (17).

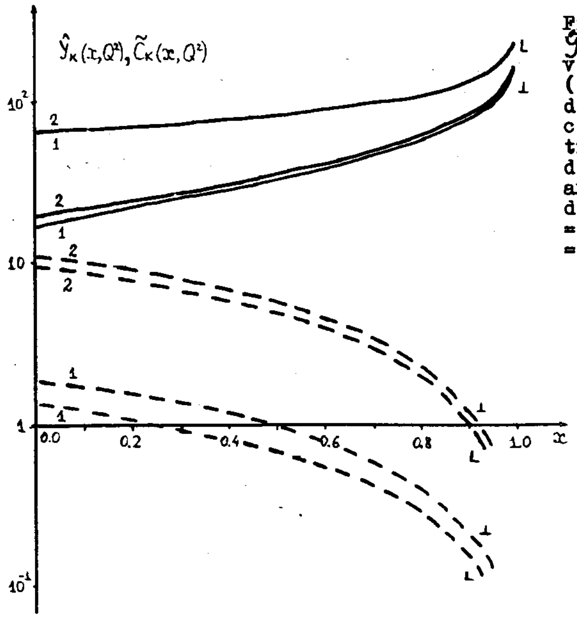

Fig. 5. The graphs of $\hat{g}_K(x,Q^2)$ (dashed curves) and $\tilde{C}_K(x,Q^2)$ (solid curves). The indices $\perp$ and L mark the corrections of the transverse and longitudinal SF. The indices 1 and 2 correspond of the different values Q^2 = = 2 GeV^2 and Q^2 = = 200 GeV^2.

2. The region $x\sim 0$. In the parametrizations (15) and (16) the coefficients d_λ^{NS} and d_δ^+, C_λ^{NS} and C_δ^+ have the form:

$$d_\lambda^{NS} = -\gamma_{NS}^{(0)\lambda}/2\beta_0\,, C_{K,\lambda}^{NS} = \bar{B}_{K,\lambda}^{NS} + \gamma_{NS}^{(1)\lambda}/2\beta_0 - \beta_1\gamma_{NS}^{(0)\lambda}/2\beta_0^2 \tag{A9}$$

$$d_\delta^+ = -\gamma_+^{(0)\delta}/2\beta_0\,, C_{K,\delta}^+ = \bar{B}_{K,\delta}^+ + \gamma_{++}^{(1)\delta}/2\beta_0 - \beta_1\gamma_+^{(0)\delta}/2\beta_0^2 - \gamma_{+-}^{(1)\delta}/(2\beta_0 + \gamma_+^{(0)\delta} - \gamma_-^{(0)\delta}),$$

where

$$\bar{B}_{K,\lambda}^{j} = \begin{cases} B_{2,\lambda}^{(1)j} & \text{for} \quad K=2 \\ R_{L,\lambda}^{(2)j} & \text{for} \quad K=L \end{cases}$$

The coefficients $\bar{B}_{K,\delta}^{(1)+}$; $R_{2,\delta}^{(2)+}$; $\gamma_{++}^{(1)\delta}$; $\gamma_{+-}^{(1)\delta}$; $\gamma_+^{(0)\delta}$; $\gamma_-^{(0)\delta}$ are expressed through the coefficients $\bar{B}_{K,\delta}^{(m)i}$ and $\gamma_{ij}^{(m)\delta}$ $(m=1,2)$ $(i,j=\psi,G)$ (see/25/).

The analytical form of quantities $\gamma_{ij}^{(m)\delta}$ and $\gamma_{NS}^{(m)\lambda}$, $B_{K,\lambda}^{(m)NS}$ and $B_{K,\delta}^{(m)i}$, $R_{L,\lambda}^{(2)NS}$ and $R_{L,\delta}^{(2)i}$ in the right-hand side of equations (A9) is given in papers /5,10/,/37,38/ and /23,24,28/, respectively, however, it is cumbersome. The coefficients can be simplified for λ =0.5 and δ =1.5.

The coefficients of the parametrizations contain the characteristic function (see/15/) obtained by analytical continuation from the integer values of the argument n (as for the moments) to the region of noninteger values of λ and δ. Following paper /15/ we get

$$\gamma_{NS}^{(0)\lambda=1/2} = 16C_F(7/24 - \ln 2),\ \gamma_{\psi\psi}^{(0)\delta=3/2} = 16C_F\left(\frac{107}{120} - \ln 2\right),\ \gamma_{\psi G}^{(0)\delta=3/2} = -8T_F\cdot\frac{46}{105}$$

$$\gamma_{G\psi}^{(0)\delta=3/2} = -4C_F\cdot\frac{46}{15}\,,\quad \gamma_{GG}^{(0)\delta=3/2} = 16C_A\left(\frac{213}{580} - \ln 2\right) + \frac{8}{3}T_F$$

$$\gamma_{NS}^{(1)\lambda=1/2} = 16C_F\Big\{(C_A - 2C_F)\Big[\tilde{F}(1/2) - 11\zeta(3) + \left(8\ln 2 - \frac{13}{2}\right)\zeta(2) - \frac{32}{3}G - \frac{40}{9}\ln 2 + \frac{25009}{864}\Big]$$
$$- C_A\Big[\left(4\ln 2 - \frac{29}{6}\right)\zeta(2) + 3\ln 2 + \frac{3619}{864}\Big] - \frac{4}{3}T_F\Big[\zeta(2) - \frac{5}{3}\ln 2 - \frac{35}{48}\Big]\Big\}$$

$$\gamma_{\psi\psi}^{(1)\delta=3/2} = 16C_F\Big\{(2C_F - C_A)\Big[\tilde{F}(1/2) - \frac{304}{15}G + \left(\frac{103}{30} - 4\ln 2\right)\zeta(2) + \frac{1936}{225}\ln 2 - \frac{54469}{36000}\Big] +$$
$$+ C_A\Big[\left(\frac{217}{30} - 4\ln 2\right)\zeta(2) + \frac{216}{225}\ln 2 - \frac{778583}{108000}\Big] - \frac{4}{3}T_F\Big[\zeta(2) - \frac{5}{3}\ln 2 + \frac{817723}{882000}\Big]\Big\}$$

$$\gamma_{\psi G}^{(1)\delta=3/2} = 4\gamma_{\psi G}^{(0)\delta=3/2}\Big\{C_A\Big[\frac{3}{2}\zeta(2) - 2\ln^2 2 - 4G + \frac{11152}{2415}\ln 2 + \frac{3167}{4830}\Big] + 2C_F\Big[\ln^2 2 + \zeta(2)/2 -$$
$$- \frac{38}{23}\ln 2 + \frac{9839}{41400}\Big]\Big\}$$

$$\gamma_{G\psi}^{(1)\delta=3/2} = 4\gamma_{G\psi}^{(1)\delta=3/2}\Big\{2C_F\Big[\zeta(2)/2 - \ln^2 2 + \frac{79}{690}\ln 2 - \frac{18218}{5175}\Big] + C_A\Big[2\ln^2 2 + \frac{3}{2}\zeta(2) -$$
$$- 4G + \frac{343}{49}\ln 2 - \frac{372637}{169050}\Big] - \frac{4}{3}T_F\Big[\ln 2 - \frac{3}{115}\Big]\Big\}$$

$$\gamma_{GG}^{(1)\delta=3/2} = 16C_A\Big\{C_A\Big[\tilde{F}(1/2) + \left(\frac{24}{7} - 8\ln 2\right)\cdot\zeta(2) - \frac{1024}{105}G - \frac{46331}{11025}\ln 2 + \frac{16506409}{1157625}\Big] +$$
$$+ T_F\Big[\frac{20}{9}\ln 2 + \frac{1702}{1572}\Big] + \frac{24449}{47250}\frac{C_F T_F}{C_A}\Big\}$$

$$B_{L,\lambda=1/2}^{(1)NS} = \frac{8}{3}C_F\,,\ B_{L,\delta=3/2}^{(1)\psi} = \frac{8}{5}C_F\,,\ B_{L,\delta=3/2}^{(1)G} = \frac{64}{25}T_F$$

$$B_{2,\lambda=1/2}^{(1)NS} = 8C_F\Big[\ln^2 2 + \zeta(2)/2 - \frac{25}{12}\ln 2 + \frac{25}{24}\Big],\ B_{3,\lambda=1/2}^{(1)} = B_{2,\lambda=1/2}^{(1)NS} - \frac{16}{3}C_F$$

$$B_{2,\delta=3/2}^{(1)\psi} = 8C_F\Big[\ln^2 2 + \zeta(2)/2 - \frac{197}{60}\ln 2 + \frac{37}{40}\Big],\ B_{2,\delta=3/2}^{(1)G} = \frac{368}{105}T_F\left(\ln 2 - \frac{7}{23}\right)$$

$$R_{L,\lambda=1/2}^{(2)NS} = 4\Big\{(2C_F - C_A)\Big[14\zeta(3) - \tilde{F}(1/2) + \left(\frac{89}{21} - 4\ln 2\right)\zeta(2) + \frac{40}{21}G + \frac{23}{6}\ln 2 - \frac{3887}{168}\Big] + 2C_F\Big[\ln^2 2 +$$
$$+ \zeta(2)/2 - \frac{5}{4}\ln 2 + \frac{443}{144}\Big] - \frac{2}{3}T_F\Big[\frac{5}{4} - \ln 2\Big]\Big\}$$

$$R_{L,\delta=3/2}^{(2)\psi} = 4\Big\{(2C_F - C_A)\Big[3\zeta(3) + \tilde{F}(1/2) - \frac{328}{15}G + \zeta(2)/15 + \frac{23}{6}\ln 2 + \frac{13447}{1080}\Big] + 2C_F\Big[\ln^2 2 +$$
$$+ \zeta(2)/2 - \frac{223}{60}\ln 2 + \frac{12781}{3600}\Big] - \frac{2}{3}T_F\Big[\frac{136939}{14700} - \frac{219}{35}\ln 2\Big]\Big\}$$

$$R_{L,\delta=3/2}^{(2)G} = 4C_A\Big[2\ln^2 2 + \frac{3}{2}\zeta(2) - 4G - \frac{76}{15}\ln 2 + \frac{598447}{88200}\Big] + 2C_F\Big[\frac{7}{9}(\zeta(2) - 8G) + \frac{53}{15}\ln 2 - \frac{7214}{2025}\Big]$$

where

$$\tilde{F}(1/2) = \frac{13}{2}\zeta(3) - 2\ln 2\cdot\zeta(2) + \pi^3/4 - 2G_2$$

and $G \equiv \beta^1(1/2) = \left(\psi^{(1)}(3/4) - \psi^{(1)}(1/4)\right)/2 = 0.9159656$

$$G_2 = \sum_{\ell,k=1}^{\infty} (-1)^{k+\ell} \Big/ \left((\ell - 1/2)(\ell + k - 1/2)^2\right) = 0.614977 .$$

Only the coefficients $B_\lambda^{(1)}$ for the functions $F_2^{NS}(x,Q^2)$ and $xF_3(x,Q^2)$ semewhat differ from each other, all the other parameters are equivalent.

The coefficients $f_1 = f_{\lambda_1}(Q_0^2) / f_\lambda(Q_0^2)$ $(f = \Delta, \Sigma, G)$ in the parametrization (15) are the ratios of the moments of the corresponding DI with $n = \lambda_1$ and $n = \lambda$ (for the singlet channel $\lambda \equiv \delta - 1$). The values λ and δ correspond to the LO and λ_1, δ_1 are the NLO of the Regge trajectory. Moreover, $\lambda_1 \simeq 1$ and $\delta_1 \approx 0$ (see /5/). Using the SF parametrizations obtained in paper /18/ we get

$$\Delta_1 \approx 0.61 \ , \Sigma_1 \approx 0.64 \ , G_1 \approx 0.48 .$$

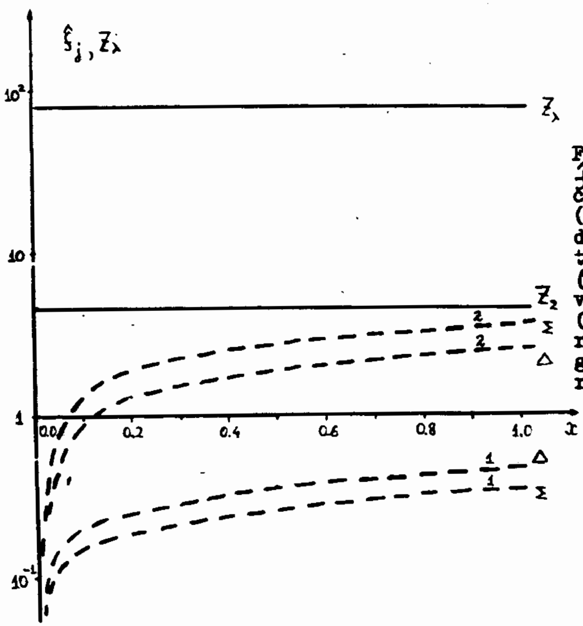

Fig. 6. The graphs of $\hat{f}_j$ $(f = \Delta\, \Sigma)$ (dashed curves) and Z_λ^j $(j = NS, \Psi)$ (solid curves). The indices 1 and 2 correspond to the $Q^2 = 2$ GeV^2 and $Q^2 = 200$ GeV^2, respectively. The indices Ψ and G are marked the corrections of the nonsinglet and singlet DI, respectively.

In Fig. 6 we present the graphs of the values of $\hat{f}_j = \left| f_j \left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_\lambda^j - d_{\lambda_1}^j - 1} \cdot x^{\lambda - \lambda_1} \right|$ $(f = \Delta, \Sigma)$ and Z_λ^j $(j = NS, \Psi)$. One can see that the coefficients $\hat{f}_j$ are much smaller than Z_λ^j in the considered region. Hence, the coefficients f_j can be neglected in the parametrizations (15), (16) and (17).

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Поведение структурных функций и отношения $R = \sigma_L/\sigma_T$ в глубоконеупругом рассеянии при $x \sim 0$ и $x \sim 1$ и их схемноинвариантная параметризация

Рассмотрено поведение кварковых и глюонных распределений в нуклоне, структурных функций глубоконеупругого рассеяния, а также отношения $R = \sigma_L/\sigma_T$, где σ_L и σ_T - сечения рассеяния продольно- и поперечно-поляризованного фотона на нуклоне при $x \sim 0$ и $x \sim 1$ в первых двух порядках теории возмущений. Во втором порядке теории возмущений мы рассматриваем также схемно-инвариантное поведение структурных функций и отношение R в этой области x. По аналогии с работой/5/ мы строим простые параметризации для написанных выше величин. Использование схемно-инвариантной теории возмущений дает простой критерий ее применимости. Мы приводим область перемещенных x, Q^2 /для каждой структурной функции, она, вообще говоря, своя/, где разложение по схемно-инвариантной теории возмущений является корректной операцией. Кроме того, мы даем аналитическое продолжение коэффициентов в разложении Вильсона на полуцелые значения n, что может иметь самостоятельный интерес. В Приложении 2 приведена α_s-поправка к продольной структурной функции, найденная по значению α_s-поправки к ее моментам.

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Kotikov A.V. E2-88-422
The Behaviour of the Structure Functions and Ratio $R = \sigma_L/\sigma_T$ in Deep Inelastic Scattering for $x \sim 0$ and $x \sim 1$ and Their Scheme-Invariant Parametrization

We consider the behaviour of the quark and gluon distributions on a nulceon, the structute functions of deep inelastic scattering and the ratio $R = \sigma_L/\sigma_T$, where σ_L and σ_T are the cross sections for the longitudinal and transverse polarized photon scattering on the nucleon, for $x \sim 0$ and $x \sim 1$ to the first two orders of perturbation theory. In the two orders of perturbation theory we consider also the scheme-invariant behaviour of the structure function and ratio R in this region of x. Following/5/ we construct the parametrization for the above-written functions. In the framework of the scheme-invariant approach there exists a simple criterion of applicability of the perturbation theory. We show the region of variables x and Q^2 (generally speaking, the region is different for the different functions), where the expansion of the scheme-invariant perturbation theory is a correct operation. Moreover, we also give the analytical continuation of the Wilson coefficients to the half-integer n, which may be of independent interest. In Appendix 2 we give α_s-correction to the longitudinal structure function obtained with the help of α_s-correction to its moments.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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