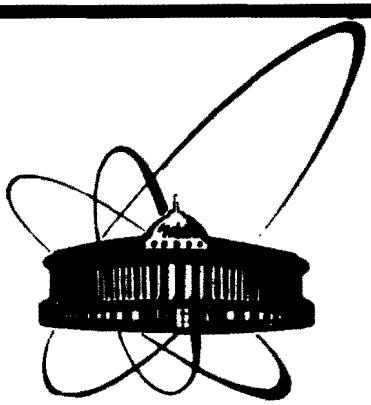

объединенный
институт
ядерных
исследований
дубна

E2-88-421

V.K.Mitrjushkin, A.M.Zadorozhny,
G.M.Zinovjev*

ON THERMODYNAMIC PROPERTIES OF CHROMOPLASMA

Submitted to "Physics Letters B"

*Institute of Theoretical Physics, Kiev, USSR

1988

At present it is well established that there is a transition to the deconfinement phase at high enough temperatures in gauge theories . This transition, predicted in refs. /1,2/, has then been confirmed by the numerical calculations in the framework of the lattice approach (see, for instance, refs. /3-9/). According to these calculations, at high enough temperatures chromoplasma behaves as a gas of almost free gluons and quarks. At the same time at temperatures just above the phase transition temperature θ_c plasma is a rather complicated and nontrivial object. Note that the temperature region close to θ_c is of great interest for experimentalists as just this temperature interval is most probable for obtaining chromoplasma in nucleus-nucleus collisions. At present, we have no unequivocal idea about the excitation modes of chromoplasma. It is not improbable (see review /10/ and refs. therein) that at temperatures close to θ_c an important role is played by collective excitation modes of chromoplasma which give rise to electric and magnetic masses. These collective excitations can be both phonon /11/ and massive /12/. As a result, a qualitative picture may be thought to be the following /12-14/: In the large momentum region the dominating part is played by almost noninteracting gluons whereas at momenta of an order of m_D and m_M (the Debye and magnetic masses, respectively) the main contribution comes from collective excitations both massive and massless. These collective excitations are assumed to be singlet with respect to colour (dynamical confinement hypothesis) and electric and magnetic massive modes give similar contributions /14/.

The role and nature of collective excitation modes of chromoplasma can be elucidated by investigating thermodynamic functions within the lattice approach. The standard Wilson action is

$$S = \beta \sum_{Pl_s} \left(1 - \frac{1}{2} Sp\, U_{Pl_s} \right) + \beta \sum_{Pl_t} \left(1 - \frac{1}{2} Sp\, U_{Pl_t} \right), \qquad (1)$$

where U_{Pl_s} and U_{Pl_t} are plaquette variables corresponding to spacelike and timelike plaquettes, respectively, and $\beta = 4/g^2_{bare}$.

The calculations were made on the lattice $4*12^3$ with periodic boundary conditions in the range of β values : $2.2 \le \beta \le 2.8$.

The Monte-Carlo method was used to calculate the contribution of electric and magnetic modes to the internal energy E, pressure P and the so-called interaction measure $\delta = E-3P$ in the SU(2)-gauge theory. Temperature θ was determined by the known two-loop expression

$$\theta/\Lambda_L = \frac{1}{N_t a \Lambda_L} =$$

$$= \frac{1}{N_t}\left(\frac{11}{6\pi^2\beta}\right)^{51/121} \exp\left\{\frac{3\pi^2\beta}{11}\right\}\ (\ 1+ O(g^2)\ ), \qquad (2)$$

where a is the lattice spacing and $N_t=4$ is the number of sites along the time axis.

The energy density E, pressure P and interaction measure δ were determined by the expressions /5/:

$$E/\theta^4 = 12\, N_t^4 \left[g^{-2} \left(\langle Pl_s \rangle - \langle Pl_t \rangle \right) + \right.$$

$$\left. + C_s \left(\langle Pl_{sym} \rangle - \langle Pl_s \rangle \right) + C_t \left(\langle Pl_{sym} \rangle - \langle Pl_t \rangle \right) \right] =$$

$$= E_E/\theta^4 + E_M/\theta^4 \qquad (3a)$$

$$P/\theta^4 = 4\, N_t^4 \left[g^{-2} \left(\langle Pl_s \rangle - \langle Pl_t \rangle \right) + \right.$$

$$\left. + C_s \left(\langle Pl_{sym} \rangle - \langle Pl_s \rangle \right) + C_t \left(\langle Pl_{sym} \rangle - \langle Pl_t \rangle \right) - \right.$$

$$\left. - a \frac{dg^{-2}}{da} \left(\langle Pl_s \rangle + \langle Pl_t \rangle - 2 \langle Pl_{sym} \rangle \right) \right] =$$

$$= P_E/\theta^4 + P_M/\theta^4 \qquad (3b)$$

$$\delta/\theta^4 = (E - 3 P)/\theta^4 =$$

$$= 12\, N_t^4\, a \frac{dg^{-2}}{da} \left(\langle Pl_s \rangle + \langle Pl_t \rangle - 2 \langle Pl_{sym} \rangle \right) =$$

$$= \delta_E/\theta^4 + \delta_M/\theta^4, \qquad (3c)$$

where $\langle Pl_s \rangle$ and $\langle Pl_t \rangle$ are the spacelike and timelike plaquettes, respectively, and $\langle Pl_{sym} \rangle$ is the plaquette average calculated on the hypercubic lattice. We have used values of coefficients C_s and C_t calculated within one-loop aproximation /15/: $C_s = .1140$, $C_t = -.06758$. The electric and magnetic contributions to the thermodynamic functions are determined in a standard way through a lattice generalisation of the strength tensor $F_{\mu\nu}$. Correspondingly, in formulae (3a) - (3c) electric (magnetic) components of the thermodynamic functions are composed of spacelike (timelike) plaquettes and $\langle Pl_{sym} \rangle$. It is worth noting that all thermodynamic functions listed above are normalised in such a way that their values at zero temperature are equal to zero by definition.

Our values for $E(\theta)$, $P(\theta)$ and $\delta(\theta)$ and their electric and magnetic components are shown in figs.1-3[1]. The dashed lines in these figures denote the behaviour of the thermodynamic quantities E/θ^4, P/θ^4 and δ/θ^4 calculated by the perturbation theory within the one-loop approximation /16/. It is seen from these figures that at large enough temperatures the behaviour of energy E and pressure P is in the agreement with the Stefan-Boltzman law (taking into account one-loop corrections). At temperatures close to the critical one in the deconfinement phase the values of P/θ^4 are strongly suppressed in comparison with the values of E/θ^4. This is quite natural since for the second order phase transition in the vicinity of the critical temperature the thermodynamic functions should have the following behaviour:

$E \simeq \text{const} \mid \theta - \theta_c \mid^{1-\alpha}$; $P \simeq \text{const} \mid \theta - \theta_c \mid^{2-\alpha}$,

where α is the relevant critical exponent for specific heat (the notion of universality classes /17,18/ suggested us that $\alpha \simeq 0.12$ as in the 3D-Ising model).

It is possible to reach some understanding of the behaviour of the chromoelectric part of energy using the Hamiltonian approach to SU(2)-gluodynamics in the strong coupling limit. In this regime the partition function is of the form

---

[1]We are indebted to prof. H. Satz for providing us with the values of $\langle Pl_{sym} \rangle$ on the lattice 12^4.

$$Z = \int \prod_i d\gamma_i \prod_{links} D(\gamma_k, \gamma_j) \qquad (4a)$$

$$D(\gamma_i, \gamma_j) = 1 + \sum_k \exp\left(- \frac{g^2}{2\, a\, \theta}\, k(k+1) \right) X_k(\gamma_i)\, X_k(\gamma_j), \qquad (4b)$$

where γ_i is the SU(2) group determined in site i, θ is the temperature, $a = a_s$ is the lattice spacing in spatial directions and $X_k(\gamma)$ is the character for kth representation of SU(2) group ($k = 1/2, 1, 3/2 \ldots$). At temperatures close to the critical one the expression for the partition function can be exposed in the form /1,2/

$$Z = \int \prod_i d\gamma_i \exp(-S_{eff}) \qquad (5)$$

$$S_{eff}(\gamma) = \varkappa \sum_{links} X_{1/2}(\gamma_i)\, X_{1/2}(\gamma_j); \qquad \varkappa = \exp\left(- \frac{g^2}{8\, a\, \theta} \right),$$

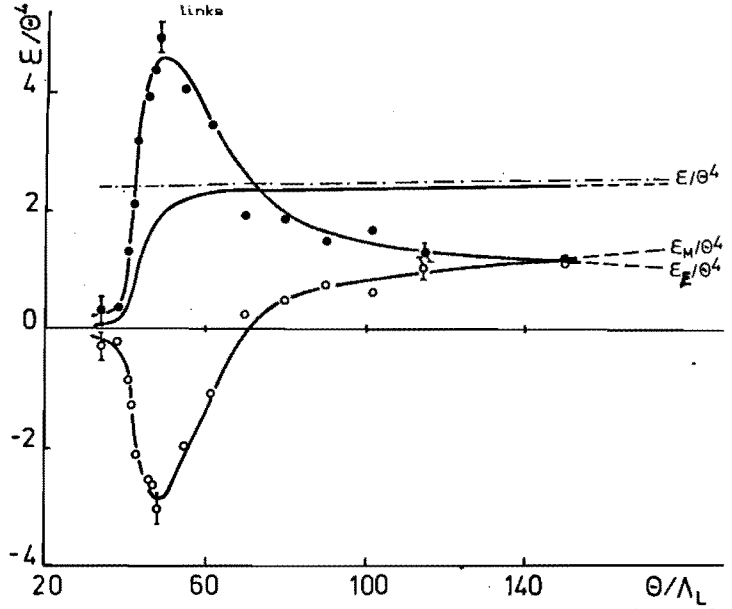

Fig.1. The solid line without points represents qualitatively the behaviour of E/θ^4, full circles and open circles correspond to E_E/θ^4 and E_M/θ^4, respectively. The dash-dotted line denotes the perturbative contribution. Some points represent a typical statistical error.

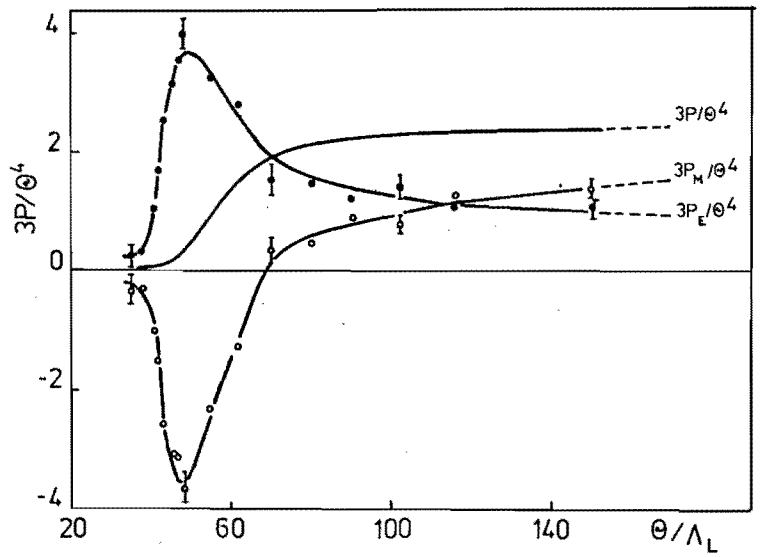

Fig.2. The solid line without points represents qualitatively the behaviour of $3P/\theta^4$, full circles and open circles correspond to $3P_E/\theta^4$ and $3P_M/\theta^4$, respectively. Some points represent a typical statistical error.

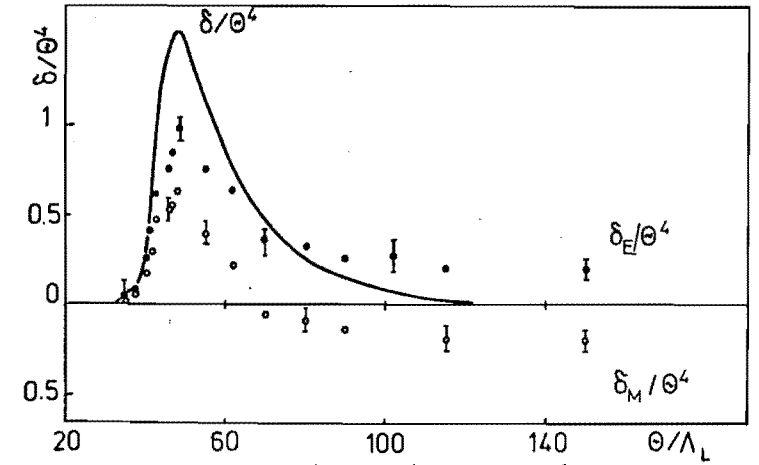

Fig.3. The quantities δ/θ^4, δ_E/θ^4 and δ_M/θ^4 with the same notation as in figs.1,2.

i.e., we have developed the three-dimensional "spin" model of the Ising type , with the difference that the integration in (5) is performed over the Haar measures $d\gamma_i$. The role of spins is played by representations $\frac{1}{2} X_{1/2}(\gamma_i) = s_i$. (A consistent procedure of reducing (4) to partition function of the "spin" system is demonstrated in ref./19/). A model of this type is known to predict a second order phase transition. For a qualitative investigation of the behaviour of thermodynamic functions we use the mean-field approximation. Using the standard technique of the mean-field approximation (see, for instance, ref./20/) we get the equation for the mean "magnetization" $M = \langle s_i \rangle$:

$$M = I_2(24 \varkappa M)/I_1(24 \varkappa M), \qquad (6)$$

where I_2 and I_1 are the modified Bessel functions. This equation at $\varkappa \le \varkappa_c = 1/6$ has only zero solutions for mean magnetization M whereas at $\varkappa \ge \varkappa_c$ the magnetization changes continuously from zero at $\varkappa = \varkappa_c$, i.e. at $\varkappa = \varkappa_c$ the second order phase transition occurs.

The effective action $W(M) = -\ln Z$ in this approximation is

$$\frac{1}{N_s} W(M) = 12 \varkappa M^2 - \ln\left[I_1(24 \varkappa M)/12 \varkappa M \right], \qquad (7)$$

where N_S is the number of sites in the three-dimensional space. Using (7) we get for the internal energy

$$E/\theta^4 = \frac{1}{V\theta^2} \frac{\partial W}{\partial \theta} = \frac{1}{(a\theta)^4} \frac{9g^2}{2} \varkappa M^2 , \qquad (8)$$

where $V = N_s a^3$ is the three-dimensional volume.

With relations (6)-(8) one can easily obtain the behaviour for the internal energy which coincides qualitatively with the one depicted in fig.1 for E_E/θ^4. This fact is hardly surprising in the light of the universality hypothesis mentioned above /17,18/. (Similar calculations for P and δ are more ambiguous because of the presence of derivative dg^{-2}/da which is unknown in the strong-coupling limit).

The problem of interpretation like this for the chromomagnetic contributions remains open. At temperatures just above the critical one the magnetic components E_M and P_M (both of them

being the difference between the corresponding nonzero temperature and zero temperature values) take negative values (fig.1,2) /21/[2]. This means that the contribution of the chromomagnetic excitation modes differs drastically from that of the chromoelectric excitation modes (at least for SU(2) group), and extra excitation mechanisms of chromoplasma are to be searched for. We assume that the temperature dependence of magnetic components of energy and pressure presented here is maybe an indication of an important role played by unstable modes /23,24/ in the QCD vacuum.

References

1. Polyakov A.M. Phys.Lett., 1978, 72B, p.477
2. Susskind L. Phys.Rev., 1979, D20, p.2610
3. McLerran L.D., Svetitsky B. Phys.Lett., 1981, 98B, p.195; Phys.Rev., 1981, D24, p.450
4. Kuti J., Polonyi J., Szlachanyi K. Phys.Lett., 1981, 98B, p.199
5. Engels J., Karsch F., Montvay J., Satz H. Phys.Lett., 1981, 102B, p.332; Nucl.Phys., 1982, B205, p. 545
6. Gerdt V.P., Mitrjushkin V.K. Pis'ma Zh. Eksp. Teor. Fiz., 1983, 37, p.400.
7. Kanaya K., Satz H. Phys.Rev., 1986, D34, p.3193
8. Mitrjushkin V.K., Zadorozhny A.M. Phys.Lett., 1987, B185, p.377
9. Engels J., Jersak J., Kanaya K., Laermann E., Lang C.B., Neuhaus T., Satz H. Nucl.Phys., 1987, B280, p.577
10. Karsh F. CERN-TH.4851/87, 1987
11. Carruthers P. Phys.Rev. Lett., 1983, 50, p.1179
12. DeTar C.E. Phys.Rev., 1985, D32, p.276
13. DeGrand T.A., DeTar C.E. Phys.Rev., 1986, D34, p.2469

---

[2]It is worth noting that nonperturbative corrections /22/ changing appreciably the values of coefficients C_s and C_t outside the region of asymptotic scaling do not influence this conclusion.

14. DeGrand T.A., DeTar C.E. Phys.Rev., 1987, D35, p.742
15. Karsh F. Nucl.Phys., 1982 , B205, p.239
16. Heller U., Karsh F. Nucl.Phys., 1985 , B251, p.254
17. Svetitsky B., Yaffe L.G. Nucl.Phys., 1982, B210, p.423
18. Polonyi J., Szlachanyi K., Phys.Lett., 1982, B110, p.395
19. Borisenko O.A., Petrov V.K., Zinovjev G.M. preprint ITP-87-154E, Kiev, 1987
20. Drouffe J.-M., Zuber J.-B. Phys.Rep., 1983, 102, p.1
21. Mitrjushkin V.K., Zadorozhny A.M., Zinovjev G.M. 19th Spring Symposium on High Energy Physics, Cordobang, DDR, April 18-22, 1988
22. Burgers G., Karsch F., Nakamura A., Stamatescu I.O. CERN-TH.4843/87, 1987
23. Nielsen N.K., Olesen P. Nucl.Phys., 1978, B144, p.376
24. Ambjorn J., Nielsen N.K., Olesen P. Nucl.Phys., 1979, B152, p.75

Received by Publishing Department
on June 14, 1988.

Митрюшкин В.К., Задорожный А.М., Зиновьев Г.М. E2-88-421

О термодинамических свойствах хромоплазмы

Вычислялись вклады электрических и магнитных мод во внутреннюю энергию в SU(2)-калибровочной теории на решетке $4 \cdot 12^3$. Показано, что поведение хромоэлектрической части энергии E_E вполне объясняется в рамках модели изинговского типа, в полном соответствии с гипотезой универсальности. В то же время, поведение магнитной части внутренней энергии E_M кардинально отличается от поведения E_E. Характер температурной зависимости E_M указывает на существенно несимметричный характер электрических и магнитных мод возбуждения хромоплазмы, и, быть может, проливает свет на роль нестабильных мод в калибровочных теориях.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Mitrjushkin V.K., Zadorozhny A.M., Zinovjev G.M. E2-88-421

On Thermodynamic Properties of Chromoplasma

The contributions of electric and magnetic modes to some thermodynamic functions in SU(2)-gauge theory on the lattice $4 \cdot 12^3$ were calculated. It is shown that the behaviour of the chromoelectric part of energy E_E can be interpreted within the Ising-type model in agreement with the universality hypothesis. At the same time the behaviour of the magnetic parts of the internal energy and pressure (i.e. E_M and P_M) differs drastically from that of E_E and P_E. The character of the temperature dependence of E_M and P_M exhibited here testifies to the presence of a highly nonidentical properties of electric and magnetic modes of chromoplasma and maybe sheds light on the role of unstable modes in gauge theories.

The investigation has been performed at the Laboratory of Theroetical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988