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A CLASSICAL NO-INTERACTION THEOREM FOR BOSONIC STRINGS AND ITS QUANTUM SIGNIFICANCE

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INTRODUCTION

The functional integral approach to first-quantized string theory[1] relies on classically defined quantities. For instance, all random trajectories, over which the functional integration is performed, belong in topological sense to the same surface; put it differently, quantum fluctuations do not produce cuts in the world sheet. This invariance of contour is a characterizing feature of a first-quantized theory.

One is therefore entitled to ask about the classical probability for the existence of topologies for which the world sheet describes interactions among strings. These sheets are surfaces with cuts on the boundaries: these are cuts proper for open strings and holes for closed ones.

That this is not a trivial question is suggested by the case of classical relativistic particles, where topologies with lines crossing each other are infinitely unlikely in space-time dimensions greater than two[2].

We find that a similar result --the vanishing probability for the existence of world sheets describing the crossing of two or more strings (equivalently, world sheets with cuts)-- holds for classical bosonic strings in dimensions D equal or greater than three.

This is a general conclusion: All theories of classical bosonic strings, the interaction of which is based on joining and splitting, are free (in $D \geq 3$). The superstring case introduces no additional features.

At the same classical level, it is interesting to look next into the possibility of other interactions (i.e., interactions <u>not</u> through joining and splitting).

A no-interaction theorem already known to be true for classical particles[3] can then be generalized to strings. The avoidance of such a theorem selects a class of theories --namely those with a covariant gauge fixing-- for which direct interactions between strings can exist. Such a theorem does not generalize --at least in a straightforwardly way-- to superstrings and we do not consider them in the following.

We finally discuss the significance of our results for the first-quantized theory, showing that interacting strings cannot be properly derived by an underlying classical picture. Instead, the interaction must be put in by hand, postulating the desired topologies. Such a result weakens the appeal of the first-quantized approach and advocates the second-quantized theory as the most appropriate one.

JOINING AND SPLITTING OF STRINGS

The classical probability for a collision (between particles or strings) to occur is given by the ratio of volumes of phase space of, respectively, the initial conditions for which the particles/strings do meet and the total volume of available initial conditions.

According to this definition, collisions--but for particular choices of space-time dimensions-- have vanishing probability of taking place because at least one of the canonical variables is constrained to a finite part of phase space.

This result is readily understood considering first two point particles: Their free world lines are straight and "always" meet in space-time $D = 2$. The only possibility for them not to collide is to have exactly the same momenta; the phase space is thus constrained.

Adding an extra dimension ($D = 3$) makes the probability to vanish, the extra dimension providing for each line that meets the other world line an infinity of lines that do not.

Strings are a little subtler. For a finite mutual separation they have a finite probability of meeting in $D = 3$, their interaction defining a world segment. However, the strings never meet for asymptotic initial conditions. Were they infinitely extended, this probability would have been one.

This result is more easily visualized in terms of rigid and non-rotating stricks and rings, their world sheets being straight strips.

In $D = 4$, strings, even for finite separations, have a vanishing probability to collide. Were they infinitely extended, this probability would have been one again. In this case, the locus of their intersection is in general only one point!

The vanishing probability for collisions between two (<u>not</u> infinitely long) strings comes from the constraints in phase space that their initial conditions have to satisfy in order for them to meet. For example, in the rest frame of the first string, for any initial position of the second one, all components of its momentum --except one-- belong to a finite volume of phase space.

Finally, in $D \geq 5$ even infinitely long strings never collide.

Hence, an ideal gas of classical relativistic bosonic strings does not interact in $D \geq 3$. Moreover, an open string never closes and vice versa.

Collision here means joining of strings and, by time reversal, splitting as well. Accordingly, dubious cases like open strings closing by two internal points can be solved by time reversal.

OTHER INTERACTIONS

Within a classical Hamiltonian approach*, once collisions are ruled out, one is left with two possibilities: Either an interaction carried by an infinite number of additional degrees of freedom (a field) or by an action at a distance.

Whereas for particles an interaction by way of fields is a natural choice, the hope for strings to be the only fundamental constituents rests precisely on the non-existence of such a field. This leaves, at least at the classical level, only action-at-a-distance interactions.

Classical string theories based on such direct interactions are severely restricted by imposing the world-sheet condition (WSC). Such a WSC is understood in the following sense.

Manifest covariance naturally realizes the Poincare invariance of the Hamiltonian mechanics of a bosonic string. However, manifest

*For a non-Hamiltonian point of view, see ref./4/ and /2/ for particles, and ref./5/ for strings.

covariance is not necessary and Poincare invariance does not coincide with it: Poincare invariance only states the equivalence between different observers; manifest covariance requires that all quantities be four-tensors (namely, linear representations of the Poincare group).

What is particularly desirable in manifest covariance is that, without any further assumptions, at the same time Poincare algebra is represented in phase space by Poisson brackets[6] and each point of the trajectory swept out by the string transforms like a space-time event: the world sheet.

The existence of such a world sheet nicely agrees with our physical intuition.

Whenever covariance is not manifest, while Poincare algebra can still be represented in phase space, the existence of a world sheet does not follow any longer and it must be enforced. The dynamical variables span in this case a nonlinear representation of the Poincare group.

This additional requirement --the WSC-- led for relativistic particles in the form of a world line condition(WLC)to a rather surprising no-interaction theorem, about the mutual incompatibility among canonical formalism, interactions and WLC[9].

This result is easily generalized to bosonic strings. At the same time, the experience gained in the case of particles[7] can be used in finding appropriate conditions apt to circumvent it.

WSC

We begin by defining the WSC for a bosonic string in a gauge belonging to the orthonormal family. Such a condition is here enforced rather than deduced because we are interested in non-manifestly covariant gauges (e.g.: light-cone).

Reparametrization invariance of the string gives us two constraints:

$$K_1 = \pi^2 + x'^2, \qquad (1)$$

$$K_2 = \pi \cdot x', \qquad (2)$$

where $x^\mu(\tau,\sigma)$ and $\pi^\mu(\tau,\sigma)$ are the canonical variables and $x' \equiv \partial_\sigma x$.

Therefore, the Hamiltonian is a linear combination of K_1 and K_2 with arbitrary coefficients[9]. To determine these two coefficients, one must add two more constraints (gauge fixing):

$$\Omega_1 = h \cdot \pi - \gamma, \qquad (3)$$

$$\Omega_2 = h \cdot x - \gamma \tau, \qquad (4)$$

where h is a fixed vector and γ = const. in an orthonormal gauge.

The time independence of these gauge fixings determines the two unknown coefficients in the Hamiltonian. Note that only Ω_2 is explicitly time dependent. The WSC is then:

$$\frac{\partial\Omega_2/\partial\tau}{\{\Omega_2, K_1\}}\{x^\mu, K_1\}\,\delta\tau - \{x^\mu, K_2\}\,\delta\sigma = \{x^\mu, K_a\}\,C^{-1}_{ab}\{\Omega_b, G\}, \qquad (5)$$

where C^{-1}_{ab} (a, b = 1,..., 4) is the inverse matrix of the Poisson brackets among constraints and gauge fixings: $C^{-1}_{ab} = (\{\Omega_a, K_b\})^{-1}$, G generates an infinitesimal Poincare transformation.

The left-hand side of Eq.(5) is the reparametrization of the world sheet necessary for the phase-space Poincare transformations to coincide with the usual geometrical ones. The WSC is the requirement for $\delta\tau$ and $\delta\sigma$ to exist. For the free case, they can always be found, namely the WSC is always satisfied. In a manifestely covariant formalism this is also true ($\delta\tau = \delta\sigma = 0$); only a non-manifestly covariant gauge fixing requires an additional reparametrization in order for the gauge fixing to be preserved under a Poincare transformation.

In the interacting case the WSC leads to a no-interaction theorem.

THE THEOREM

We now prove such a theorem. We consider only two strings interacting by means of a potential depending on the relative position. The WSC is now:

$$\begin{aligned} &\gamma\, C^{-1}_{ab}\{x^\mu_{(\alpha)}, K_a\}\,\delta\tau_{(\alpha)} + \{x^\mu_{(\alpha)}, K_{(\alpha)}\}\,\delta\sigma_{(\alpha)} = \\ &= -\{G, \Omega_a\}\,C^{-1}_{ab}\{x^\mu_{(\alpha)}, K_b\}, \end{aligned} \qquad (6)$$

where α = 1,2 identifies the string and an index between parentheses is not summed;

$$K_1 = \pi_{(1)} \cdot x'_{(1)} \quad ; \quad K_3 = \pi^2_{(1)} + x'^2_{(1)} + V$$
$$K_2 = \pi_{(2)} \cdot x'_{(2)} \quad ; \quad K_4 = \pi^2_{(2)} + x'^2_{(2)} + V \qquad (7)$$

and

$$\Omega_1 = \frac{1}{2} h \cdot (x_{(1)} - x_{(2)}) \quad ; \quad \Omega_3 = \frac{1}{2} h \cdot (\pi_{(1)} - \pi_{(2)})$$
$$\Omega_2 = \frac{1}{2} h \cdot (x_{(1)} + x_{(2)}) - \gamma\tau \quad ; \quad \Omega_4 = \frac{1}{2} h \cdot (\pi_{(1)} + \pi_{(2)}) + \gamma \qquad (8)$$

are the constraints generalizing the one-string case, V being the interaction potential.

To mimic the proof given in ref.[8] for particles, we separate Eq. (6) in two parts:

$$\{x^\mu_{(\alpha)}, K_{(\alpha)}\}\left[\gamma C^{-1}_{a(\alpha)} \delta\tau_{(\alpha)} + \delta\sigma_{(\alpha)}\right] = -\{G, \Omega_a\} C^{-1}_{a(\alpha)} \{x^\mu_{(\alpha)}, K_{(\alpha)}\} \qquad (9)$$

and

$$\{x^\mu_{(\alpha)}, K_\beta\} \gamma C^{-1}_{2\beta} \delta\tau_{(\alpha)} = -\{G, \Omega_a\} C^{-1}_{a\beta} \{x^\mu_{(\alpha)}, K_\beta\} \cdot \qquad (10)$$

where β = 3,4 and we use the orthogonality between $\{x^\mu_{(\alpha)}, K\}$ and $\{x^\mu_{(\alpha)}, K_\beta\}$ to isolate the σ-reparametrization in Eq. (9).

Eq. (10) is analogous to the WLC for two particles and it is only satisfied either when the interaction V is equal to zero or when Ω_a --the gauge fixing-- are Poincare invariant.

The proof from this point does not differ from the two-particle case, to which we refer[7].

INTERACTIONS CONSISTENT WITH WSC

The previous theorem shows that only for the free strings the WSC can be preserved independently of manifest covariance. In the interacting case, this is only possible if Eq. (8), where the evolution parameter is singled out, is manifestly covariant (i.e., the vector h must be identified with $P = \int d\sigma\,[\pi_{(1)} + \pi_{(2)}]$).

For the free bosonic string this was already suggested by Rohrlich[9], providing a consistent quantum theory in any dimension. In this case, however, the Hamiltonian is not one of the ten generators of the Poincare group[7].

A more general gauge fixing, that also allows interactions, is the proper-time one[10]. This is not a canonical gauge fixing because it is defined using the lagrangian multipliers.

CONCLUSION: QUANTUM SIGNIFICANCE

The vanishing probability for a collision between two strings shows that string theory cannot be interpreted as the first quantization of a model describing joining and splitting of classical strings.

In the interacting case, the absence of an underlying classical mechanics implies that the topologies of the world sheet over which the integration is performed must be postulated. Such a definition singles out topologies with a zero probability to exist at the classical level.

A similar problem would arise if one wanted to rewrite ϕ^3 field theory as the first-quantized version of particles interacting by 3-body collisions.

Whether such a result is a real drawback or just a semantic problem it will depend on the interpretation one wishes to attach to the functional integral: Quantum extension of a classical picture or technical device to produce the correct amplitudes.

A more natural way to preserve the joining-splitting picture would be to interpret strings as quantum excitations of a second-quantized field. In this approach, strings meet because they are fields; fields, permeating the entire space, always interact through "collisions". There are no world sheets, only Feynman-like diagrams.

Still, it would be wrong to depict the classical limit as the mechanics of joining-splitting strings. We know that the classical limit is a theory of particles and fields in mutual interaction. A way toward properly defining such a limit may be in terms of an action at a distance between strings, in one of the manifestly covariant gauges previously discussed.

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Теорема о невзаимодействии классических бозонных струн и ее квантовое значение

Показано, что в пространствах размерностей три и выше классические бозонные струны не объединяются и не расщепляются. Это доказанное нами утверждение составляет часть теоремы о невзаимодействии. Рассмотрен также квантовый случай.

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Delamotte B., Fabbrichesi M.E. E2-88-410

A Classical No-Interaction Theorem for Bosonic Strings and Its Quantum Significance

We show that, in dimensions equal or greater than three, classical strings do not join or split. This is part of a no-interaction theorem that is also proved. The quantum theory is considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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