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## **MODELLING OF NARROW AIR SHOWERS**

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## 1. INTRODUCTION

Experimental results have been published<sup>/1/</sup> recently on the structure of an extensive air shower (EAS) within a narrow central region about its axis. In this connection the evaluation of three-dimensional nuclear - electromagnetic showers with account of the characteristics of an experimental installation becomes of interest for the analysis of the experimental data. At present numerous models are available for the evaluation of nuclear and electromagnetic air showers<sup>/2/</sup>; these models provide satisfactory descriptions and correspondingly interpretations of the experimental data on the hadron spectrum, the muon spectrum, and so on.

In this work a Monte Carlo program<sup>/3/</sup> for three-dimensional simulation of nuclear - electromagnetic air showers is applied for evaluating the characteristics of electromagnetic showers; the model is based on the extrapolation of accelerator data to the region of super-high energies.

Calculations are made for the distributions of central densities (of electromagnetic particles) by integrating over radii  $R \approx 0.4$  m and  $R \approx 0.8$  m, and for the spectrum over the total number of particles in a shower, as well as for the distributions of over the fractions of the full number of particles in an (maximum) detector setting the thresholds to be  $N_0 \geq 170$  and  $N_0 \geq 625$  particles. Comparison with the experimental data is performed.

## 2. THE MAIN FEATURES OF THE MODEL

A primary proton enters the standard atmosphere<sup>/4/</sup> where it undergoes interaction in accordance with the usual exponential law. A logarithmic dependence was assumed for the mean charged particle multiplicity on energy<sup>/5/</sup>, while the Koba - Nielsen formula<sup>/6/</sup> was taken as the multiplicity distribution. Each interaction act was required to satisfy energy and momentum conservation. Particle production was considered in two regions: of fragmentation and of pionization. The mean multiplicities in these regions were taken from refs.<sup>/5/</sup> and<sup>/7/</sup>. The energy spectrum of secondary particles was cho-

sen in accordance with ref.<sup>/8/</sup>, and distribution of transverse momenta was considered to be exponential for  $P_{\perp} \leq 1$  GeV/c, while for  $P_{\perp} > 1$  GeV/c the enhancement of the transverse momentum was taken into account<sup>/9/</sup>. The integral energy spectrum of the primary protons was taken to be of the form

$$I(E_p) = 0.856 \cdot E_p^{-1.7} \text{ (cm}^{-2} \text{ s}^{-1} \text{ str}^{-1} \text{)}. \quad (1)$$

In our calculations the difference between the interaction of a proton passing through the atmosphere with the nucleus of an atom making up the air and its interaction with a nucleon was not taken into account, since the main part of the pions initiating electromagnetic cascades lies within the region of  $x \approx 0.1 \div 0.4$ , owing to the slope of the primary proton spectrum (1) being steep. In this region of  $x$  the ratio<sup>/5/</sup>

$$\frac{1}{\sigma_{air}^{in}} \frac{d\sigma_{air}(x)}{dx} / \frac{1}{\sigma_p^{in}} \frac{d\sigma_p(x)}{dx},$$

differs little from unity.

## 3. THE PROCEDURE FOR SHOWER SIMULATION

A proton interacting with the nucleus of an atom that presents in the air produces pions (only the production of pions is assumed). In accordance with isotopic invariance a third part of the pions is considered to be  $\pi^0$ s, which initiate electromagnetic cascades. The spatial distribution of particles in these cascades is described, in the Greisen approximation, by the Nishimura-Kamata function (NKG)<sup>/10/</sup>:

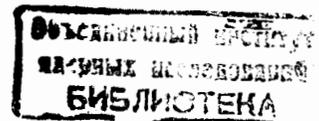
$$f(R, R_0, s) = c(s) \left(\frac{R}{R_0}\right)^{s-2} \left(1 + \frac{R}{R_0}\right)^{s-4.5}, \quad (2)$$

$R_0 = 95$  m for the observation level of 840 g/cm<sup>2</sup> (altitude of 1700 m),

$$f(R, R_0, s) R dR = 1.$$

The shower age  $s$ , is determined by the formula<sup>/10/</sup>:

$$s = \frac{3t}{t + 2 \ln(E/\beta)}, \quad (3)$$



where  $t = T/37.1$  and  $\beta = 0.081$  GeV. The number of particles in an electromagnetic shower is given by the expression:

$$N = \frac{0.31}{\ln(E/\beta)} \exp(t(1 - 3/2 \ln s)), \quad (4)$$

$$F(N, R, R_0, s) = Nf(R, R_0, s).$$

The transition effect due to the concrete roof over the experimental installation and the walls of the vessel, containing the scintillator, having a certain thickness<sup>/11/</sup>, was taken into account:

$0.4 \leq s \leq 1.35$	$R(m)$	$s < 0.4, s > 1.35$
$F(R) = 5.1 - 1.4R$	$0 < R \leq 1.5$	$0 \leq R \leq 4.5$
$F(R) = 3.9 - 0.6R$	$1.5 < R \leq 2.5$	$f(R, s) = \frac{5.12R(0.178s - 0.51)}{4.18s}$
$F(R) = 3.525 - 0.45R$	$2.5 < R \leq 3.5$	
$F(R) = 3.55 - 0.4R$	$3.5 < R \leq 4.5$	
$F(R) = 2.45 - 0.2R$	$4.5 < R \leq 5.5$	by $R > 4.5$
$F(R) = 2.25 - 0.15R$	$5.5 < R \leq 8.5$	$f(R, s) = F(R).$
$F(R) = 0.95$		

(5)

The experimental installation represented a continuous "carpet" of 400 detectors,  $(0.7 \times 0.7) m^2$  each<sup>/12/</sup>. The readout of each detector is generated in the form of integer numbers  $N$  from 0 to 36 (above this value saturation occurs), which are then transformed into the numbers of particles by the formula:

$$M = 9 \cdot (1.25)^{N-1}. \quad (6)$$

For the dispersion of each individual detector an expression was applied that was obtained in studies of real showers<sup>/13/</sup>:

$$\left(\frac{\sigma}{M}\right)^2 = \frac{1}{M} + 0.0169 + (N-1)^2 (4 \cdot 10^{-3})^2 + \frac{1}{3} \left[ \frac{(2-s)0.35}{R} \right]^2, \quad (7)$$

where the first term takes into account the Poisson density fluctuations, and the other summands are due to dispersion of the thresholds and of the slopes of the characteristics of the logarithmic transformator for each individual detector, as well as to the steps of the transformer being discrete and to the dimensions of the detector itself.

$$M' = M + \sigma A_1,$$

where  $A_1$  is a random number generated by the Gaussian distribution.

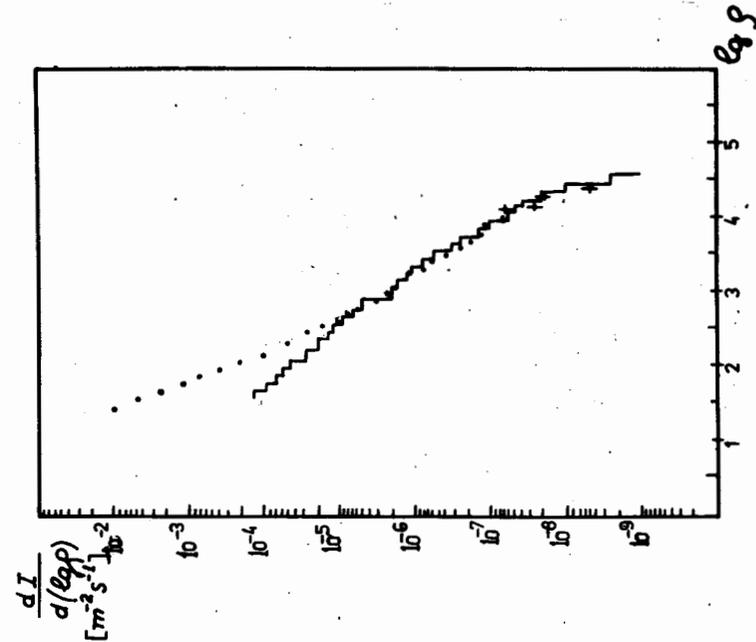


Fig. 1. Differential spectrum of central densities  $N_1$  ( $R = 0.4$  m). Points - data from ref./11/, histogram - our calculation.

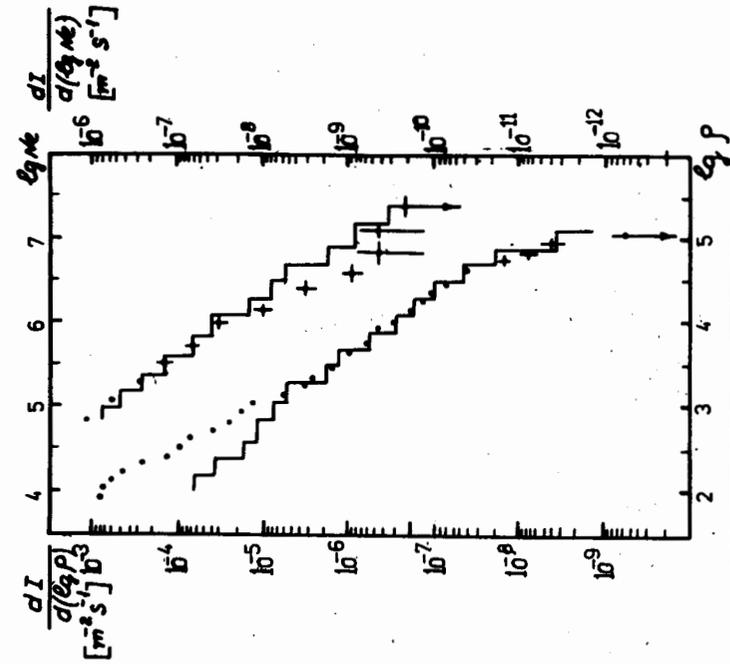


Fig. 2. Differential spectrum of central densities  $N_4$  ( $R = 0.8$  m) (left scale) and differential spectrum of showers over number of particles (right scale). Points - experimental data from ref./11/ histogram - our calculation.

#### 4. RESULTS OF OUR SIMULATION

Simulation of nuclear-electromagnetic cascades in the atmosphere was performed applying the procedure described in Section 3. The distribution was determined of central densities for different integration radii, corresponding to adding up densities for different numbers of detectors. In Figs. 1 and 2 presented are the density distributions  $N_1$  (a single detector, integration radius  $R \approx 0.4$  m),  $N_4$  (the sum of densities in four maximum detectors,  $R \approx 0.8$  m) and  $N_c$  (spectrum for the total number of particles in the shower). Showers were selected on the basis of a large release of energy within the area with  $R \approx 0.4$  m (this corresponds to the selection of a single detector with a high energy deposit). The thresholds for the energy release were chosen to be the following:

$$\begin{aligned} 6 \leq N < 13 \\ 13 \leq N < 17 \\ 17 \leq N < 20 \\ 20 \leq N < 24 \\ 24 \leq N < 29 \\ 29 \leq N < 36 \\ N \geq 36 \end{aligned}$$

where  $N$  is determined by formula (6).

The points in Figs. 1 and 2 are experimental ones from ref. [1] (multiplied by the normalizing factor  $1/\lg 1.25$ , included in transition from the distribution over  $\rho$  to the distribution over  $\lg \rho$ ), the histogram is calculated. The results of calculation are in agreement with the experimental data, with the exception of the region of small  $\lg \rho$ . This discrepancy is due to the NKG-function (2), made use of in the present study, at low energies; for this reason a more precise evaluation of electromagnetic showers is required in this region.

Table 1 ( $N_1$ )

$\gamma$	$\lg \rho$	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5
1.7	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	1.0	6.0	5.1	3.9	1.7	1.0	4.4	2.0
		$10^{-7}$	$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-9}$
2.2	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	8.6	5.3	3.6	1.2	7.0	3.3	3.3	3.0
		$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-9}$

Table 1 Continuation

$\gamma$	$\lg \rho$	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3
1.7	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	2.4	2.6	1.4	1.6	9.0	5.0	3.3	1.6
		$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-10}$	$10^{-10}$	$10^{-10}$	$10^{-10}$
2.2	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	2.3	1.2	6.9	6.6	2.9	1.5	1.5	7.3
		$10^{-9}$	$10^{-9}$	$10^{-10}$	$10^{-10}$	$10^{-10}$	$10^{-10}$	$10^{-10}$	$10^{-11}$

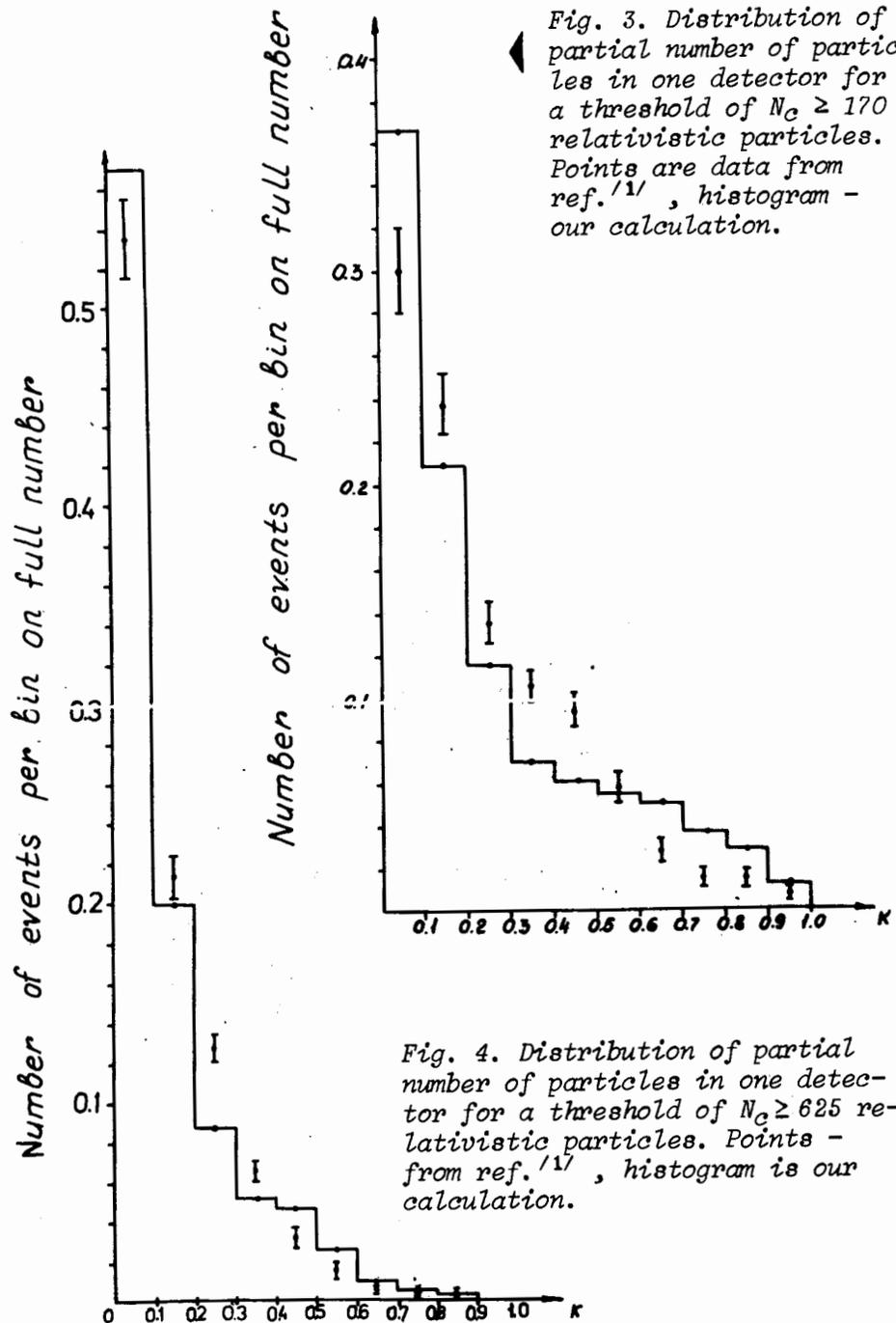
Table 2 ( $N_4$ )

$\gamma$	$\lg \rho$	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8
1.7	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	7.9	4.3	2.3	5.5	2.7	2.6	1.0	2.4
		$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-10}$
2.2	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	4.6	2.0	4.4	1.4	7.2	6.1	2.5	1.5
		$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-10}$	$10^{-10}$	$10^{-10}$	$10^{-10}$

Table 3 ( $N_c$ )

$\gamma$	$\lg \rho$	5.9	6.1	6.3	6.5	6.7	6.9	7.1	7.3
1.7	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	7.5	4.4	2.3	7.3	5.2	1.7	1.4	4.2
		$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-10}$
2.2	$\frac{dI}{d(\lg \rho)} [m^{-2}s^{-1}]$	7.6	3.8	1.5	6.1	2.2	1.4	7.8	1.1
		$10^{-8}$	$10^{-8}$	$10^{-8}$	$10^{-9}$	$10^{-9}$	$10^{-9}$	$10^{-10}$	$10^{-10}$

In Tables 1, 2 and 3 calculations are presented for large  $\rho$  for  $N_1$ ,  $N_4$ , and  $N_c$  with account of the bend in the primary spectrum ( $\gamma = 1.7$ ) at energies  $E_0 \geq 10^8$  GeV<sup>1/4</sup> ( $\gamma = 2.2$ ). From these Tables it can be seen that taking account of the bend in the primary spectrum leads to a significant change in the slope of the density distribution and in the total number



of particles (if the bend is taken to be in the region of  $E_0 \geq 5 \cdot 10^8$  GeV, then calculations for  $\gamma = 1.7$  and for  $\gamma = 2.2$  coincide within the errors for the given intervals of  $\lg \rho$ ).

In integration over the solid angle in expression (1) the effective solid angle, at which observation was performed with the experimental installation, was taken to be 1.7 str.

In Figs. 3 and 4 presented are the distributions (normalized to unity) of the part of the total number of particles detected by one detector ( $k = \rho_{\max} / \rho_{\text{tot}}$ ) for the thresholds  $N_c \geq 170$  and  $N_c \geq 625$  (the points are experimental data from ref.<sup>1/</sup> the errors are statistical, the histogram is calculated; the statistics used for calculations and the experimental statistics are about the same). As one can see from these figures the calculation is in qualitative agreement with the experimental data. For thresholds  $N_c \geq 170, 625$  the mean values of shower characteristics (age, height expressed in radiation units, energy) are the following:

	$\bar{s}$	$\bar{H}$	$\bar{E}$ (GeV)
$N_c \geq 170$	0.6	4.9	$1.7 \cdot 10^8$
$N_c \geq 625$	0.8	10.1	$4.3 \cdot 10^8$

The complete picture obtained at the experimental installation represents a superposition of a large number of showers and therefore the image corresponding to the mean shower differs quite significantly from superpositional picture.

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Архестов Г.Х., Бештоев Х.М. E2-88-343  
Моделирование узких атмосферных ливней

Произведены расчеты распределений центральных плотностей /электромагнитных частиц/ при интегрировании по радиусам  $R \approx 0,4$  м и  $R \approx 0,8$  м, спектра по полному числу частиц, а также распределения доли полного числа частиц в одном /максимальном/ детекторе для порогов  $N_c \geq 170$  и  $N_c \leq 625$  частиц. Проводится сравнение с экспериментальными данными.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Arkhestov G.Kh., Beshtoev Kh.M. E2-88-343  
Modelling of Narrow Air Showers

Calculations are made for the distributions of central densities (of electromagnetic particles) by integrating over radii  $R \approx 0.4$  m and  $R \approx 0.8$  m, and for the spectrum over the total number of particles in a shower, as well as for the distributions of over the fractions of the full number of particles in a (maximum) detector setting the thresholds to be  $N_c \geq 170$  and  $N_c \geq 625$  particles. Comparison with the experimental data is performed.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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