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THE ELECTROMAGNETIC α^3 CONTRIBUTIONS TO e^+e^--ANNIHILATION INTO FERMIONS IN THE ELECTROWEAK THEORY. TOTAL CROSS SECTION σ_T AND INTEGRATED ASYMMETRY A_{FB}

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1. Introduction

One of the important processes in high energy physics is the annihilation of point-like fermions,

$$e^+e^- \longrightarrow (\gamma, Z^\circ) \longrightarrow f^+f^-(\gamma). \tag{1.1}$$

This reaction allows the precise determination of the mass M_Z and the width Γ_Z of the weak neutral gauge boson as well as the study of other features of the electroweak theory /1/ at the new colliders SLC and LEP. These new experimental feasibilities represent a great challenge for theorists who want to ensure the analysis of data with sufficient accuracy and reliability /2,3/. In this respect the potentially large QED radiative corrections necessarily require special care. The fruitful competition of different theoretical approaches led to a permanently rising number of complementary results. Analytic formulae allow a deeper understanding of essential features of the process and a fast numerical orientation, whereas Monte-Carlo (MC) studies are the ideal tool for the interpretation of data obtained in experiments. In fact, very effective MC-algorithms have been developed /3-5/.

In this article, we present a systematic and to some extent complete analytic investigation of the α QED radiative corrections to reaction (1) within the electroweak standard theory. We obtain compact expressions for the angular distribution $d\sigma/dc$ ($c = \cos\theta$ with θ the cms scattering angle between e^+ and f^+); the total cross section σ_T,

$$\sigma_T = \sigma(-1, +1) \tag{1.2}$$

and the integrated forward-backward asymmetry A_{FB},

$$A_{FB} = \left[\sigma(0,1) - \sigma(-1,0)\right] / \sigma_T, \tag{1.3}$$

where we use the notation

$$\sigma(c_1, c_2) = \int_{c_1}^{c_2} dc \, \frac{d\sigma}{dc} \, . \qquad (1.4)$$

In the context of the electroweak theory it is technically difficult to obtain analytic results including hard photon bremsstrahlung, much more difficult than in pure QED. To simplify the problem we treat the photon totally inclusive (i.e. no photon observation at all). The only further assumption we made is the ultra-relativistic approximation: $m_e^2, m_f^2 \ll s, M_Z^2, M_Z \cdot \Gamma_Z$. The mass M_Z and the width Γ_Z /6/ of the Z-boson are taken into account without any further approximation at arbitrary beam energy $E = \sqrt{s}/2$.

The QED-corrections arising from diagrams shown in Fig. 1 are model-independent in the sense that they depend only on the mass and width as well as on the vector and axial-vector couplings of the weak neutral boson but are not sensitive to further details of the electroweak theory. Of course, they may be combined with the genuine weak loop corrections/7,8/ to form the complete α EWRC (electroweak radiative corrections). Further, multiphoton initial state radiation/9,10/ has to be added to really compete with the experimentally accessible accuracy.

Generally, attempts to get analytic results on the angular distribution $d\sigma/dc$ /11-15/ and on the integrated C-odd asymmetry A_{FB} /15,16/ are scarce though exist for some other distributions; see, e.g.,/2,4,8/. Concerning the total cross section, there has been done much work on initial (and on the more trivial final) state radiation/4,5,10,11,14/, whereas the remarkably simple analytic expressions for their interference to be presented in this part of the paper are to our knowledge lacking in the literature. Of course, it is not difficult to extend the results presented here to the case of a longitudinally polarized electron beam to get the left-right asymmetry A_{LR} or to the production of longitudinally polarized fermions allowing the study of the asymmetry A_{pol} .

The angular distribution $d\sigma/dc$ is a basic quantity for the theoretical analysis of reaction (1). With a simple one-dimensional integration over the analytic expression to be presented in the second part of this article one may simulate a more realistic (but yet idealized) experimental situation for σ_T and A_{FB} by excluding the beam pipe region or a broader region of low angles for a detector (demanding, e.g., that $|c| < 0.9$). This cut also excludes much of the large amount of hard photon bremsstrahlung. If there is required a

more realistic treatment of hard photons, the analytic formulae of this study may be combined with any MC-program for hard bremsstrahlung simulation. But, in contrast to the usual approach one has to <u>subtract</u> from the analytic expression the <u>observed</u> hard photon events (and not to <u>add</u> the <u>non-observed</u> hard photon events). This seems to be an interesting new ansatz for the calculation of observables, at least in the aspect of an independent check of more common procedures.

This article is organized as follows. In Chapter 2, we introduce the definitions and notation. Chapter 3 contains the C-even QED-corrections to the total cross-section σ_T and Chapter 4 the C-odd contributions to the integrated forward-backward asymmetry A_{FB} . Both the results are numerically compared with an MC-calculation. In the Appendix, we comment on the method used to carry out the analytic integration of hard bremsstrahlung.

2. Definitions

The differential cross section corresponding to the diagrams of Fig. 1 may be parametrized as follows:

$$\begin{aligned}
\frac{d\sigma}{dc} = \frac{\pi\alpha^2}{2s} \Big\{ & Q_f^2 \Big[1 + c^2 + \frac{\alpha}{\pi} \left(F_0 + Q_f F_1 + Q_f^2 F_2 \right) \Big] + \\
& + 2|Q_f| v_e v_f \, \mathrm{Re} \Big[\chi (1 + c^2) + \frac{\alpha}{\pi} \chi \left(G_0 + Q_f G_1 + Q_f^2 G_2 \right) \Big] + \\
& + 2|Q_f| a_e a_f \, \mathrm{Re} \Big[\chi \, 2c + \frac{\alpha}{\pi} \chi \left(G_3 + C_f G_4 + Q_f^2 G_5 \right) \Big] + \\
& + (v_e^2 + a_e^2)(v_f^2 + a_f^2) |\chi|^2 \Big[1 + c^2 + \frac{\alpha}{\pi} \mathrm{Re} \left(H_0 + Q_f H_1 + Q_f^2 H_2 \right) \Big] + \\
& + 4 v_e a_e v_f a_f |\chi|^2 \Big[2c + \frac{\alpha}{\pi} \mathrm{Re} \left(H_3 + Q_f H_4 + Q_f^2 H_5 \right) \Big] \Big\} .
\end{aligned} \qquad (2.1)$$

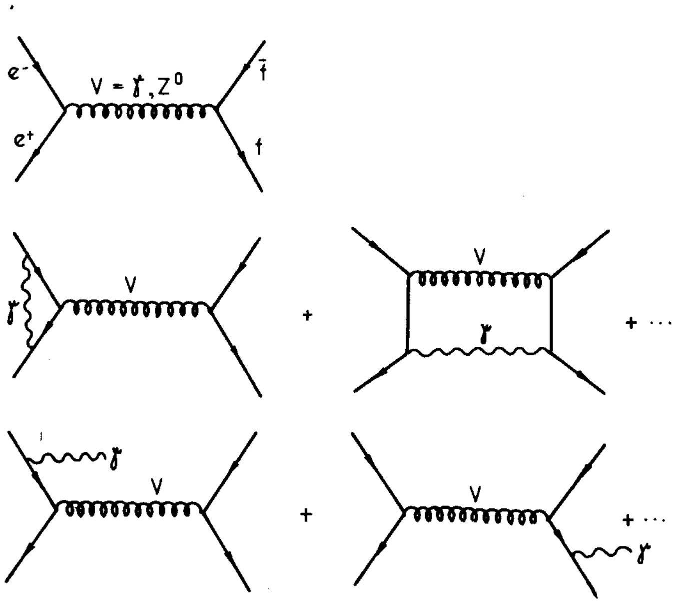

Fig. 1. The QED α^3 radiative contributions to the e^+e^--annihilation into a fermion pair considered in this article.

Using the definitions (1.2), (1.3) we derive:

$$\sigma_T = \sigma_0 \Big\{ Q_f^2 \Big[1 + \frac{\alpha}{\pi} \left(F_0^T + Q_f^2 F_2^T \right) \Big] + 2 |Q_f| v_e v_f \, \mathrm{Re} \Big[\chi + \frac{\alpha}{\pi} \chi \left(G_0^T + Q_f^2 G_2^T \right) \Big] + 2 |Q_f| a_e a_f \frac{\alpha}{\pi} Q_f \, \mathrm{Re}\left(\chi G_4^T \right) + (v_e^2 + a_e^2)(v_f^2 + a_f^2) |\chi|^2 \Big[1 + \frac{\alpha}{\pi} \mathrm{Re} \left(H_0^T + Q_f^2 H_2^T \right) \Big] + 4 v_e a_e v_f a_f |\chi|^2 \frac{\alpha}{\pi} Q_f \, \mathrm{Re} \left(H_4^T \right) \Big\}, \tag{2.2}$$

$$A_{FB} = \frac{\sigma_0}{\sigma_T} \Big\{ \frac{\alpha}{\pi} Q_f^3 F_1^T + 2 |Q_f| v_e v_f \frac{\alpha}{\pi} Q_f \, \mathrm{Re} \left(\chi G_1^T \right) + 2 |Q_f| a_e a_f \, \mathrm{Re} \Big[\frac{3}{4} \chi + \frac{\alpha}{\pi} \chi \left(G_3^T + Q_f^2 G_5^T \right) \Big] + (v_e^2 + a_e^2)(v_f^2 + a_f^2) |\chi|^2 \frac{\alpha}{\pi} Q_f \, \mathrm{Re} \left(H_1^T \right) + 4 v_e a_e v_f a_f |\chi|^2 \Big[\frac{3}{4} + \frac{\alpha}{\pi} \mathrm{Re} \left(H_3^T + Q_f^2 H_5^T \right) \Big] \Big\}. \tag{2.3}$$

The QED radiative corrections are contained in the functions F, G, H:

$$\{ F_i^T, G_i^T, H_i^T \} = \frac{3}{4} \int_0^1 dc \, \{ F_i, G_i, H_i \}, \quad i = 0, 1, \ldots, 5. \tag{2.4}$$

These functions depend only on particle masses, the Z^0-width and on the beam energy. Strictly speaking, one should add yet the fermionic vacuum polarization (see, e.g.[4]) to get the complete α^3 QED contribution. The reader should have in mind that this has not been done here.

In (2.1)-(2.3) Q_f is the charge of the produced fermion, $Q_\mu = -1$. The v and a are vector and axial-vector couplings to the massive neutral vector boson. In the standard electroweak theory they become

$$a_f = 1, \quad v_f = 1 - 4 s_w^2 |Q_f|. \tag{2.5}$$

Following the recommendations of the study group of electroweak radiative corrections at LEP[3], we use the following definition of χ:

$$\chi = k \varkappa \, (1 - \delta r)^{-1}. \tag{2.6}$$

Here real constant k measures the relative strengths of the photon and weak neutral boson couplings, in the standard theory:

$$k = \frac{g^2}{16 c_w^2 e^2}, \tag{2.7}$$

where we use the on-mass-shell renormalization scheme: $c_w^2 = 1 - s_w^2 = M_W^2 / M_Z^2$, $g = e / s_w$. The complex kinematic variable $\varkappa$ relates the corresponding propagators:

$$\varkappa = \frac{s}{s - M^2} \tag{2.8}$$

with

$$M^2 = M_Z^2 - i M_Z \Gamma_Z . \tag{2.9}$$

In eq. (2.6) the quantity δr is the radiative correction to the muon decay constant G_μ . So we can rewrite (2.6) as follows

$$\chi = \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{8\pi\alpha} \frac{s}{s - M^2} = 0.38894 \left(\frac{M_Z}{93}\right)^2 \frac{s}{s - M^2} . \tag{2.10}$$

The point-like QED-cross section σ_0 is

$$\sigma_0 = \frac{4\pi\alpha^2}{3s} . \tag{2.11}$$

A longitudinal polarization λ of the electron beam may be taken into account by the following modification of electron couplings:

$$\begin{aligned} v_e v_f &\longrightarrow (v_e - \lambda a_e) v_f , \\ a_e a_f &\longrightarrow (a_e - \lambda v_e) a_f , \\ (v_e^2 + a_e^2)(v_f^2 + a_f^2) &\longrightarrow (v_e^2 + a_e^2 - 2\lambda v_e a_e)(v_f^2 + a_f^2) , \\ 4 v_e a_e v_f a_f &\longrightarrow 2\left[2 v_e a_e - \lambda (v_e^2 + a_e^2)\right] v_f a_f . \end{aligned} \tag{2.12}$$

Alternatively, the creation of a fermion with a definite helicity state ($S_f = \pm 1$) may be described by a corresponding change in the fermion couplings. This simple procedure of inclusion of a longitudinal polarization is no longer true in the presence of genuine weak loop corrections because these destroy the factorisation property of the couplings valid here. But even for weak corrections one may obtain some adequate substitutions which are only slightly more complicated than (2.12) as has been demonstrated in [17].

3. The integrated C-even corrections

The analytic calculations done with SCHOONSCHIP[18] will not be described in this article. Some reamrks on the definitions and the strategy used together with some references of more technical orien-

tation may be found in the Appendix. Here we only remember that the QED-corrections considered are a gauge-invariant sum of vertex (or box diagram) corrections and of initial- or final-state radiation (or their interference). The infrared finiteness is ensured due to the inclusion of both the loop diagrams and the soft photon radiation. Since we integrate over the complete photon phase space, the result doesn't contain any cut-off parameter. Instead, it is Lorentz-invariant.

The initial-state corrections are

$$F_0^T = d + t \left(L_f - \frac{7}{6}\right), \tag{3.1}$$

$$G_0^T = d + t \left[R + \frac{1}{2} + (1 + R^2) L_R\right], \tag{3.2}$$

$$H_0^T = d + t \left[2R + \frac{1}{2} - |R|^2 + \frac{2R}{R - R^*} (1 - R)(1 + R^2) L_R\right], \tag{3.3}$$

with

$$d = \frac{\pi^2}{3} - \frac{1}{2} , \qquad t = L_e - 1 . \tag{3.4}$$

The pure QED function F_0^T is known from[11]. The other two initial-state corrections depend on one additional, complex parameter R ,

$$R = \frac{M^2}{s} \tag{3.5}$$

with M^2 as defined in (2.9). The R^* is its complex conjugate. Further,

$$L_a = \frac{s}{m_a^2} , \quad a = e, f , \tag{3.6}$$

$$L_R = \ln (1 - 1/R) \tag{3.7}$$

The use of complex variables in final expressions allows a very compact notation compared to a more conservative, real-variable approach. This is especially evident for the C-odd functions but may be also realised comparing (3.2)-(3.3) with the real expressions for G_0^T, H_0^T which may be found in[5].

All initial-state-radiation functions show the well-known QED mass-singularity term t due to the emission of a photon from an electron line. The other mass singularity in F_0^T arising there from the photon propagator kinematics is regularized by the final fermion mass in L_f . In G_0^T, H_0^T this singularity has been naturally repla-

Table 1. Individual contributions to σ_T as defined in (2.2) in units of σ_0 (2.10) as functions of $\sqrt{s}=2E$. The A, Z and I are the corresponding Born values due to photon exchange (A), Z-boson exchange (Z) and their interference (I). The parameters are M_Z=93 GeV, Γ_Z =2.5 GeV, s_W^2=0.23. (see also footnote to Table 2).

s(GeV)	60	82	92.5	93.0	93.5	100
σ_0(nB)	0.02413	0.01292	0.01015	0.01004	0.00994	0.00869
A	1	1	1	1	1	1
I	-0.00354	-0.01714	-0.06305	0	0.06467	0.03579
Z	0.07772	1.84241	179.00599	212.01602	186.59627	8.15261
F_0	0.60431	0.65436	0.67415	0.67504	0.67593	0.68710
F_2	0.00174	0.00174	0.00174	0.00174	0.00174	0.00174
G_0	0.00011	0.00188	0.00039	-0.02916	-0.05094	-0.00530
G_2	-0.00001	-0.00003	-0.00011	0	0.00011	0.00006
G_4	-0.00120	-0.01161	-0.04879	0.00271	0.05554	0.03214
H_0	-0.00824	-0.36007	-63.06491	-66.01508	-41.76859	10.45730
H_2	0.00014	0.00321	0.31185	0.36936	0.32507	0.01420
H_4	-0.00001	-0.00008	-0.00035	-0.00001	0.00034	0.00021
σ_T	1.671	3.115	117.817	148.021	146.900	20.376
σ_T'	1.664	3.100	117.792	147.993	146.869	20.357
MC: σ_T'	1.665	3.102	117.899	148.139	147.006	20.360
$\Delta\sigma_T'$	±0.001	±0.003	±0.096	±0.121	±0.120	±0.017
N= 750000						

tions is quite good. But far from the resonance σ_T differs essentially from σ_T' (several sigmas), which is quite obvious, because the resonance box approximation is valid only near the resonance.

4. The integrated C-odd corrections

The initial-state QED corrections to the integrated forward--backward asymmetry (2.3) are:

$$\frac{1}{3}G_3^T=-\frac{1}{8}+\frac{1-R}{1+R}(1-\ln 2)+(1-R)\ln^2 2+\frac{1}{4}(1+2R)Li_2(1)+ \tag{4.1}$$
$$+\frac{1}{2}\frac{1-R}{1+R}\left[-\frac{1+3R}{1+R}\ln 2+\frac{7-R}{4(1-R)}\right]t+R\frac{1+R^2}{(1+R)^2}D_3,$$

$$\frac{1}{3}H_3^T=-\frac{2R}{|1+R|^2}-\left|\frac{1-R}{1+R}\right|^2\ln 2+|1-R|^2\ln^2 2+\frac{1}{4}\left(1+4R-2|R|^2\right)Li_2(1)+$$
$$+\left(\frac{7}{8}-\frac{2R}{|1+R|^2}\right)(t+1)+\left(-\frac{5}{2}+2\frac{6R-1}{|1+R|^2}+4\frac{1-R^2}{|1+R|^4}\right)t\ln 2+ \tag{4.2}$$
$$+2R^2\frac{1+R^2}{(1+R)^2}\frac{1-R^*}{R-R^*}D_3.$$

The following abbreviations are used:

$$D_3=D_1+D_2+\ln^2 2+(t-2\ln 2)L_R, \tag{4.3}$$

$$D_2=D_0+Li_2\left(-\frac{1}{R}\right)-Li_2(1)+\ln\left(\frac{R+1}{R-1}\right)\ln\left(-\frac{1}{R}\right), \tag{4.4}$$

$$D_{0,1}=Li_2\left(\mp\frac{1+R}{1-R}\right)+\frac{1}{2}L_R^2, \tag{4.5}$$

$$Li_2(z)=-\int_0^1\frac{dt}{t}\ln(1-zt). \tag{4.6}$$

These corrections have no analogue in pure QED. Both G_3^T and H_3^T contain the electron mass-singularity term t (3.4) and H_3^T develops a radiative tail beyond the resonance in the same way as H_0^T. Additionally, the Euler dilogarithm with a complex argument and a dependence on $\ln 2$ arise here. Both dependences are closely connected with the behaviour of C-odd functions on the integration boundary c=0 (2.4). This fact will become more evident from the corresponding spectra in $\cos\theta$ to be presented in the second part of this article. As a result, the C-odd functions are of considerably more complexity than the corresponding C-even functions.

The QED bremsstrahlung and the interference of photon box diagrams with the QED Born-graph yield the following pure QED contributions to A_{FB}:

$$F_{1br}^T = \rho + \frac{3}{4}\left[1 - 16\ln 2 - Li_2(1)\right], \tag{4.7}$$

$$F_{1box}^T = -\rho + \frac{3}{4}\left(1 + 6\ln 2 + \ln^2 2\right) - \frac{i\pi}{2}\left(2 - 5\ln 2\right),$$
$$\rho = (1 + 8\ln 2)\,P_{IR}. \tag{4.8}$$

The function F_1^T in (2.3) may be composed as follows[15]:

$$F_1^T = Re\,(f_1^T), \tag{4.9}$$

$$f_1^T = F_{1br}^T + F_{1box}^T, \tag{4.10}$$

$$F_1^T = -\frac{3}{4}Li_2(1) + \frac{3}{4}\ln^2 2 - \frac{15}{2}\ln 2 + \frac{3}{2} = -4.572. \tag{4.11}$$

The pure QED bremsstrahlung F_{1br}^T is a real function for the same reasons as $F_{0,2}^T$. Due to the vanishing two-photon production threshold, the photon box diagrams have an energy-independent imaginary part. This imaginary part interferes with the complex (due to the M^2-dependence of (2.7)) Z-boson Born-graph and contributes to the photon-Z-boson interference function G_1^T.

The initial-final-state interference functions due to Z-boson exchange are:

$$\begin{aligned} H_{1br}^T = {} & \rho - \frac{1}{4}(5R-3) - \frac{3}{4}\left(1+R-2R^2\right)Li_2(1) + \\ & + \frac{1}{1+R}\left(-12-3R+8R^2+5R^3\right)\ln 2 + \frac{1}{2}(1-R)\left(5-R+5R^2\right)Li_2\left(\frac{1}{R}\right) - \\ & - \frac{R}{1+R}\left(1-4R+R^2\right)L_R + \frac{1}{2}\left(5-3R+6R^2\right)D_1 + \frac{R}{2}\left(6-3R+5R^2\right)D_2, \end{aligned} \tag{4.12}$$

$$\begin{aligned} H_{1box}^T = {} & -\rho + \frac{3}{2} - R + \left(9-4R-4R^2\right)\ln 2 + 2\ln^2 2 + \frac{1}{2}(-5+4R)L_Z + \\ & + \frac{1}{2}\left[4 - 9R + 3R^2 + 2\left(-5+3R-6R^2\right)\ln 2\right]L_R + \\ & + \left(1-3R+6R^2-8R^3\right)\left[Li_2\left(1-\frac{1}{2R}\right) - Li_2\left(1-\frac{1}{R}\right)\right] + 4R^3\left[Li_2(1) - Li_2\left(1-\frac{1}{R}\right)\right]. \end{aligned} \tag{4.13}$$

The asymmetry A_{FB} contains the sum of (4.12)-(4.13):

$$H_1^T = H_{1br}^T + H_{1box}^T. \tag{4.14}$$

The initial-final-state and photon-Z-boson-exchange interference functions G_1^T are the average of the corresponding functions f_1^T and H_1^T:

$$G_1^T = \frac{1}{2}\left(f_1^T + H_1^T\right). \tag{4.15}$$

At first sight this identity, which we derived by direct calculation, looks mysterious, but a more deep analysis shows that relations of that kind hold already on the level of traces and are valid for any initial-final interference. A similar relation holds even for G_4 and H_4 though this is not so obvious due to the absence of a pure QED-function F_4. Evidently, the initial-state photon-Z-boson interference cannot be a simple sum of photon and Z-boson exchange terms because of their very different mass signularities and radiative tail properties.

The final-state contribution to the integrated asymmetry vanishes identically after summing up the vertex correction and bremsstrahlung:

$$G_5^T = H_5^T = 0. \tag{4.16}$$

This behaviour is present already for the angular distribution (2.1) and may be understood as a consequence of the following fact. In the limit of small fermion masses, it is not possible to define a non-vanishing axial-vector-type self-energy of the photon or Z-boson due to fermions because there is only one independent momentum for the composition of tensor structures. The imaginary part of the self--energy is proportional to the final-state radiation correction. This connection allows a simple derivation of (4.16) and also explains (3.18).

The resonance contributions of the box diagrams as introduced in Sect. 3 are (see the comment after eq.(3.20)):

$$g_{1box}^T = \frac{1}{2}\left(F_{1box}^T + h_{1box}^T\right), \tag{4.17}$$

$$\begin{aligned} h_{1box}^T = {} & -\rho - (1+8\ln 2)\left[\ln(R-1) + \frac{1}{2}L_Z\right] + \frac{1}{2} + \\ & + (1+4\ln 2)\ln 2 + 2Li_2(1). \end{aligned} \tag{4.18}$$

The numerical contributions to A_{FB} from the integrated C-odd functions are shown in Table 2. In the energy interval considered,

Table 2. Individual contributions to A_{FB} as defined in (2.3). Specifications are those of Table 1.

$\sqrt{s}$ (GeV)	60	82	92.5	93.0	93.5	100
$\sigma_0(nB)$	0.02413	0.01292	0.01015	0.01004	0.00994	0.00869
A	0	0	0	0	0	0
I	-24.84033	-64.48485	-6.27147		5.15924	20.58333
Z	0.08817	1.12134	2.88018	2.71523	2.40791	0.75847
F_1	0.63553	0.34096	0.00901	0.00717	0.00723	0.05212
G_1	-0.00100	-0.00323	-0.00066	-0.00035	-0.00008	0.00084
G_3	1.07973	7.41178	0.05346	-2.29662	-4.05141	-2.94114
H_1	-0.00676	0.02753	0.03843	0.02742	0.01686	0.00774
H_3	-0.00959	-0.22009	-1.01485	-0.84558	-0.53917	0.96717
A_{FB}	-23.054	-55.807	-4.306	-0.393	3.001	19.429
A'_{FB}	-23.050	-55.489	-4.105	-0.287	3.022	19.281
MC: A'_{FB}	-22.973	-55.399	-3.822	-0.156	2.774	19.408
$\Delta A'_{FB}$*	±0.100	±0.100	±0.100	±0.100	±0.100	±0.100
N= =750000						

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* To the MC-results in Tables 1,2 we have assigned the statistical errors as follows $\Delta\sigma_T'/\sigma_T' = (1/N_{ev} - 1/N_c)^{1/2}$, $\Delta A_{FB} = (1/N_{ev} - 1/(2N_c))^{1/2}$, where N_{ev} and N_c are the numbers of useful generated and crude events in the MC program, respectively (for all $\sqrt{s}$ $N_c \simeq 2N_{ev}$). Expression for ΔA_{FB} was obtained for $|A_{FB}| \ll 1$. The errors for σ_T are somewhat greater than those assigned by the authors of MUSTRAAL. The discussion of the statistical and systematic errors in the MUSTRAAL and detailed comparison of analytic and MC calculations as well as a recipe how to update the MC-program to reproduce analytic results will be presented in a subsequent publication.

the Born interference term is largest with the exception of the Z-boson pole where it vanishes due to the zero of χ (see (2.9)). Generally, the influence of the different corrections depends on two components, on the relative weight, they get from the kinematical factor of order either 1 or χ or $|\chi|^2$, and on their own magnitude for a given energy. With the exception of G_1^T any of the functions has a region of sensible influence. Here again we confronted the analytic results with those obtained numerically with MUSTRAAL.

We remark that in the neighbourhood of the resonance the agreement of two calculations is not so good as for σ_T'. Note also that the nonleading imaginary parts (e.g. those due to powers of R) are noticeable here, hence they have to be taken into account carefully. Near the pole, the interplay of several contributions reduces the A_{FB}, which complicates the phenomenological analysis of this observable /2/. Here we do not study details of this behaviour because in the second part of this article the more realistic differential cross section $d\sigma/dc$ (2.1) may be analysed.

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Appendix

This Appendix contains some remarks on the technique used to study the process

$$e^-(k_1, m_e) + e^+(k_2, m_e) \to f^-(p_1, m_f) + f^+(p_2, m_f) + \gamma(p). \qquad \text{(A.1)}$$

The problem to be solved here is the integration of hard bremsstrahlung. The Lorentz-invariant integration phase space is parametrized as follows:

$$\int d\Gamma = \frac{\pi^2}{4s} \int_{-1}^{+1} d\cos\theta \int_0^1 x\,dx \, \frac{1-x}{1-x+m_f^2/s} \, \frac{1}{4\pi} \int_{-1}^{+1} d\cos\theta_R \int_0^{2\pi} d\varphi_R . \qquad \text{(A.2)}$$

The θ_R and φ_R are angles of the photon in the rest system of fermion f^- plus photon:

$$\vec{p}_1 + \vec{p} = 0. \qquad \text{(A.3)}$$

The twofold integration over these angles is carried out first. Then, integration over the normalized energy x of the fermion f^+ in the centre-of-mass system,

$$x = 2p_2^0/\sqrt{s} \qquad \text{(A.4)}$$

yields the angular distribution $d\sigma/d\cos\theta$ (2.1). For the calculation of the total cross section σ_T and the integrated forward-backward asymmetry A_{FB} after integrating over the photon angles θ_k, φ_k, we integrate over the scattering angle $\cos\theta$ and then over the fermion energy. So, in fact we did not analytically integrate over the differential in $\cos\theta$ cross section as is suggested by (1.2)-(1.4). These definitions, however, have been used for a numerical integration in order to check the analytic results. Further, at the Z-boson pole, $s=M_Z^2$, and in the small and large s limits we proved analytically that (1.2)-(1.4) are fulfilled. At small (large) s, an additional control is given by the necessary cancellation of powers (inverse powers) of $R=M_Z^2/s$. The analytic calculations have been done with SCHOONSCHIP[18] and consist of several thousand statements, approximately half of them being control calculations. The strategy to be followed is quite different from that of the MC-approach where one is interested in very compact expressions. Here, at each of the subsequent integration steps one reduces by algebraic manipulations the squared matrix element to a minimal, but sufficiently simple set of integrable functions. This set is called the canonical form and can be integrated by means of prepared sets of bremsstrahlung standard integrals [20,21]; see also[22,23]. Before the first integration, the squared matrix element is a complicated sum of ratios of polynomials in the integration variables $\cos\theta_k$, $\cos\varphi_k$. The next step deals already with rational functions and logarithms, and it is here where the Euler dilogarithms (4.6) arise. The whole procedure has a common feature with the chess: the rules are not too complicated but nevertheless the game is quite sophisticated.

The bremsstrahlung integral (A.2) has been divided into soft and hard photon contributions. Here we follow the method developed for deep inelastic ep-scattering in [19], see also[15]. Both the references are concerned with pure QED. But since soft bremsstrahlung factorizes from the Born cross section (as do the QED vertex insertions), the inclusion of Z-boson exchange gives no further complication. The same is not true, of course, for the hard bremsstrahlung integrals[21]. Without going into details, we only remark that due to the relation

$$\frac{1}{|s'-M^2|^2} = \frac{i}{2M_Z\Gamma_Z}\left(\frac{1}{s'-M^2} - \frac{1}{s'-M^{*2}}\right) \tag{A.5}$$

the Z-boson part of the calculation is not considerably more complicated than the photon-Z-boson interference which is linear in the Z-boson propagator. Further, (A.5) shows the mathematical origin of tail effects in initial radiation corrections. The s' is the invariant

energy squared of the Z-boson which has to be integrated over in the case of initial-state radiation (for final-state radiation, $s' = s$). Once again it is obvious here that one has to carry out the complete calculation in the complex plane. The box diagram contributions to the angular distribution are well-known[24]. The integration over the scattering angle[21] yields the compact expressions (3.12)-(3.13), (4.13), (4.15) which seem to be obtained here for the first time.

We would like to conclude the Appendix with a comment on possible further applications of the technique developed. The analytic investigation of the QED corrections in a theory with several heavy neutral bosons[8] is only slightly more complicated. This is due to the fact that the algebra is essentially linear in the Z-boson propagators as is indicated in (A.5). The treatment of a narrow resonance, $M_Z \cdot \Gamma_Z \lesssim m_f^2$, seems also possible with slight modifications. However, an analytic description of the process $e^+e^- \to e^+e^-(\gamma)$ is much more difficult due to the additional t-channel diagrams and their interference with the s-channel diagrams which are studied here.

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Бардин Д.Ю. и др. E2-88-324

Электромагнитные поправки порядка α Кe^+e^--аннигиляции в пару фермионов в теории электрослабого взаимодействия. Полное сечение σ_T и интегральная симметрия A_{FB}

Получены аналитические выражения для полностью проинтегрированных КЭД-вкладов порядка α^3 в полное сечение σ_T и интегральную асимметрию вперед-назад A_{FB} для процесса $e^+e^- \to f^+f^-(\gamma)$. Предполагается, ято фотоны ненаблюдаемы. Расчет выполнен в ультрарелятивистском приближении по массам фермионов, при этом не делается никаких дополнительных приближений по массе M_Z и ширине Γ_Z нейтрального векторного бозона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Bardin D.Yu. et al. E2-88-324

The Electromagnetic α^3 Contributions to e^+e^--Annihilation into Fermions in the Electroweak Theory. Total Cross Section σ_T and Integrated Asymmetry A_{FB}

Analytic expressions are obtained for the integrated α^3 QED contributions to the total cross section σ_T and the forward-backward asymmetry A_{FB} in the process $e^+e^- \to f^+f^-\gamma$. Photons from soft and hard bremsstrahlung are assumed not to be observed. The calculations are performed in the ultrarelativistic approximation in fermion masses, $m_f^2 \ll s, M_Z^2, M_Z\Gamma_Z$, but the mass M_Z and width Γ_Z of the neutral weak gauge boson Z are treated without any further approximations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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