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NEUTRINO OSCILLATIONS AND THE PRIMORDIAL NUCLEOSYNTHESIS

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1. INTRODUCTION

The problem of neutrino oscillations in the early Universe and their effect on the primordial nucleosynthesis has been considered by several authors/1-4,7/. Mainly the possibility for creation of neutrino-antineutrino asymmetry by CP-violating resonant neutrino oscillations has been studied/2-4/.

In this note we establish that vacuum neutrino oscillations may influence the number density and the energy distribution of electron neutrinos, and thus directly influence the kinetics of the neutron-to-proton transitions, determining ^4He production.

The kinetics of oscillating neutrinos in the primeval plasma was studied in ref./1/. Here we follow this line of work.

The kinetic equations for the oscillating neutrinos in the expanding universe are written for the density matrix of neutrinos. The evolution of the neutrino density matrix $\rho(t)$ is described for the period just before the premordial nucleosynthesis $t \sim 1$ sec.

The influence of neutrino oscillations between nonthermalized sterile neutrinos and active ones on the weak $n \cdot p$-transitions and on the subsequent synthesis of ^4He is studied in more detail for the case when neutrino oscillations become considerable after the decoupling of the electron neutrino from the plasma.

The evolution of n/p-density ratio is numerically calculated for different values of the neutrino mass differences and mixing angles: $\delta m^2 = |m_1^2 - m_2^2| \le 10^{-7} eV^2$, $\theta \in [0, \frac{\pi}{4}]$.

A considerable increase in the ^4He production can be observed for a certain range of the model parameters. Consequently, the observational data on primordical ^4He abundance put limits on the possible oscillation parameters, i.e. new cosmological restrictions on oscillation parameters of neutrinos are established.

2. THE MODEL

In the standard cosmological model only left-handed neutrinos exist. They maintain their thermal equilibrium with the primeval plasma by the rapid weak processes with electrons, positrons e^-, e^+:

$$\begin{aligned} \nu_e + e^- &\to \nu_e + e^- \\ \nu_e + e^+ &\to \nu_e + e^+ \\ \bar{\nu}_e + e^- &\to \bar{\nu}_e + e^- \\ \bar{\nu}_e + e^+ &\to \bar{\nu}_e + e^+ \\ \nu_e + \bar{\nu}_e &\to e^+ + e^- . \end{aligned} \qquad (1)$$

These reactions keep the neutrino equilibrium distribution down to $T_F \sim 3$ MeV, when the rate of the processes (1) becomes comparable with the rate of the Universe expansion. After the freezing of these processes, $T < T_F$, neutrinos still preserve their equilibrium distribution because the expansion does not disturb the equilibrium distribution of massless particles.

So, in the standard Big Bang nucleosynthesis scenario/6/, at the nucleosynthesis epoch (1-0.1 MeV), neutrino number densities and neutrino energy distribution are assumed the equilibrium ones:

$$n_\nu(E_\nu) = \exp(-E_\nu/T)/(1+\exp(-E_\nu/T)).$$

Here E_ν is the neutrino's energy and T is the temperature. Electron neutrinos participate in the reactions with nucleons:

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^- \\ p + \bar{\nu}_e &\leftrightarrow n + e^+ . \end{aligned} \qquad (2)$$

The weak processes (2) and the diluting effect of the expansion determine the evolution of the neutron and proton number densities. When the rate of reactions (2) equals the expansion rate, the neutron-to-proton ratio (n/p) freezes:

$$\Gamma_w = \Gamma_H \quad , \quad \Gamma_w \sim G_F^2 E^2 n, \quad \Gamma_H \sim 1.66 g^{1/2} M_{Pl}^{-1} T^2,$$

E is the mean energy of the interacting particles, n denotes the particle number density, M_{Pl} is the Planck mass and g is the number of the relativistic degrees of freedom. After freezing

the n/p ratio slightly decreases due to the slow neutron decay till $T \sim 0.1$ MeV, when nucleosynthesis processes leading to ^4He synthesis begin.

The premordially producted mass fraction of ^4He can be expressed as $Y_p(^4He) \sim 2(\frac{n}{p})_F/(1+\frac{n}{p})_F$. Hence, it essentially depends both on the expansion rate and on the rate of the weak reactions (2).

In our model of oscillating neutrinos both Γ_w and Γ_H are shifted from their standard nucleosynthesis values and for a certain range of values of $\delta m^2, \theta$ the effect on the premordial nucleosynthesis is considerable.

We assume the presence of right-handed neutrinos, ν_s (SU(2) x U(1) singlets). Oscillations between the sterile neutrinos and the active ones are possible, i.e. we study the Majorana and Dirac mixing scheme/9/.

Only left-handed currents are allowed. The sterile neutrinos do not participate in the ordinary weak interactions, so, in the case without oscillations they decouple from the plasma much earlier than the active neutrinos do ($T_s^{dec} >> T_\nu^{dec} \sim 3$ MeV). Then sterile neutrinos are not heated by the numerous annihilations of particles that have taken place in the process of plasma's cooling from T_s^{dec} to 3 MeV. As a result, the temperature of the sterile neutrinos at the nucleosynthesis period will be considerably lower than that of the active particles: $T_s << T_\nu$ for $T_\nu \leq 3$ MeV.

When transitions between active and sterile neutrinos are allowed, sterile neutrinos may not decouple from the plasma much earlier than the active ones, or may regain their thermal equilibrium if already decoupled/7/. The reactions of active neutrinos with the plasma are the source of thermalization for the sterile neutrinos, because when oscillating into active neutrinos they have ability to react with e^- and e^+ of the plasma and to thermalize. If neutrino oscillations become effective after the decoupling of active neutrinos $T < 3$ MeV, it looks realistic that sterile particles would not be thermalized. We discuss this case. So, the essential point in our research is the element of nonequilibrium introduced by the nonthermalized sterile neutrinos, after the temperature falls below 3 MeV and oscillations become effective*.

*The case of considerable oscillations at $T \gtrsim T_\nu^{dec} \sim 3$ MeV, has been considered elsewhere/2-4/, where the CPV-effect of resonant neutrino oscillations due to different interactions of ν_e and ν_s (or ν_e and ν_μ) with the plasma has been treated/5/.

We estimate the pure effect of $\mu + \mathcal{D}$ vacuum oscillations of nonthermalized neutrinos on nucleosynthesis of ^4He .

Oscillations are effective when the oscillation rate $\Gamma_{osc} \sim \delta m^2/4\pi\bar{E}$ becomes comparable with or greater than the expansion rate: $\Gamma_{osc} \gtrsim \Gamma_H$.

At $T \sim 3$ MeV, the mean energy of the sterile particles is less than that of the ordinary neutrinos. $\Gamma^S_{osc} \sim \frac{\delta m^2}{4\pi E_s} > \Gamma_{osc} \sim \frac{\delta m^2}{4\pi \bar{E}}$, $\bar{E}_s = \varepsilon \bar{E}$, $\varepsilon < 1$. Consequently, the oscillations of the sterile neutrinos become effective earlier than those of the active ones. Then the requirement $\Gamma_{osc} \lessgtr \Gamma_H$ at $T \lesssim 3$ MeV sets an upper limit on the mass differences $\delta m^2 \lesssim 10^{-7}$ eV^2 for $\varepsilon \sim 1/10$.

When oscillations become effective, the total particle density of the active neutrinos $n_\nu \sim T_\nu^3$ is much greater than that of the sterile ones $n_s \sim \varepsilon^3 T^3$. The oscillations will tend to reestablish the statistical equilibrium between different oscillating neutrinos. Roughly speaking, as a result of transitions to sterile ones, the number density of the active neutrinos will decrease in comparison to their standard equilibrium value. Moreover, oscillations also change the energy distribution of active neutrinos. If these changes are great enough just before the nucleosynthesis epoch $T \sim 1$ MeV, they may influence the kinetics of reactions with nucleons (2). Both the change of the total number of electron neutrinos and the distortion of their equilibrium energy distribution lead to changes in the rate of the weak processes (2). ($\Gamma_W \sim n_\nu(E)$). The addition of new particles to the model increases the expansion rate in comparison with the standard one. $\Gamma_H \sim g^{1/6}$. The increase in g as well as the decrease of the total number of neutrinos cause an increase of the freezing temperature of the nucleons, and consequently an increase of $(n/p)_F$ and the produced ^4He . The effect of the distortion of the energy distribution of neutrinos has two aspects. An average decrease of the energy of active neutrinos should lead to a decrease of the weak reactions rate $\Gamma_W \sim \bar{E}^2$ and subsequently to an increase in the freezing temperature T_F . On the other hand, there exists an energy threshold for the reaction $\bar{\nu}_e + p \to n + e^+$. And in case when, due to oscillations, the energy of the relatively greater part of neutrinos becomes smaller than that threshold one should expect a decrease of the produced neutrons, and consequently, of the $(n/p)_F$ /8/ . However, the detailed calculations show that this effect is less noticeable compared with the previous ones. So, the total result of neutrino oscillations is a considerable increase of the $(n/p)_F$ and consequently of ^4He in comparison to the standard values. The relative increase in (^4He) may reach 10-20% which is large enough to eliminate the possibility of such oscillations. In what fallows we reveal more precisely the kinetics of oscillating neutrinos in the early universe and the effect of the nonequilibrium oscillations on the production of ^4He .

3. THE KINETICS OF NEUTRINO OSCILLATIONS

For processes with oscillating neutrinos the kinetic equations must be written for the density matrix of neutrinos ρ /1/.

The equilibrium form of ρ is $\rho_{ij} = \delta_{ij} \exp \frac{\mu - E}{T}$ in the case without oscillations. So, in the standard model, where neutrinos are in equilibrium, one can work with particle densities instead of ρ .

In an equilibrium background, the introduction of oscillations slightly shifts ρ from its diagonal form, due to the extreme smallness of the ν mass in comparison with the characteristic temperatures ($T \sim 1$ MeV) and to the fact that equilibrium distributions of massless particles is not changed by the expansion*. So, it is again preferable to work simply in terms of particle densities.

However, in the case of oscillations between nonthermalized sterile neutrinos and the active neutrinos $\nu_s \leftrightarrow \nu_e$, one has to work in terms of the ρ- matrix, because in this case ρ essentially differs from its equilibrium value.

The transitions between different neutrino flavours are proved to have a negligible effect, because $T_\ell \sim T_{\ell'}$ (ℓ is the flavour index)/1,3/. So, for simplicity in what fallows we will accept a simple $\mathcal{D}+\mathcal{M}$. mixing scheme. Mixing is present only in the electron sector:

$$\nu_e = c\nu_1 - s\nu_2$$
$$\bar{\nu}_s = s\nu_1 + c\nu_2 .$$

ν_s denotes the sterile electron antineutrino, $c \equiv \cos\theta$, $s \equiv \sin\theta$ and θ is the mixing angle in the electron sector. All other mixing angles are put equal to zero. ν_1 and ν_2 are Majorana particles with masses m_1 and m_2 , respectively.

*For this reason the effect of $\nu_e \leftrightarrow \nu_\mu$ oscillations on particle densities of ν_e and on nucleosynthesis is proved to be small/1/.

Using the approach of refs.[1,10] we write the kinetic equation for the density matrix of the oscillating neutrinos in the primeval plasma of e^-, e^+ of the expanding Universe:

$$\frac{\partial \varrho}{\partial t} = H p_\nu \frac{\partial \varrho}{\partial p_\nu} + i\left[\mathcal{H}_o, \varrho\right] +$$
$$+ \int d\Omega(\bar{\nu}, e^+, e)\left[n_e n_{\bar{e}} \mathcal{A}\mathcal{A}^+ - \tfrac{1}{2}\{\varrho, \mathcal{A}^+ \bar{\varrho} \mathcal{A}\}_+\right] +$$
$$+ \int d\Omega(e, \nu', e')\left[n'_e \mathcal{B}\varrho'\mathcal{B}^+ - \tfrac{1}{2}\{\mathcal{B}^+\mathcal{B}, \varrho\}_+ n_e\right] + \qquad (3)$$
$$+ \int d\Omega(e^+, \nu', e^{+\prime})\left[n'_{\bar{e}} C\varrho' C^+ - \tfrac{1}{2}\{C^+ C, \varrho\}_+ n_{\bar{e}}\right].$$

where $\int d\Omega(a,b,c) = \frac{(2\pi)^4}{2E_\nu}\frac{d^3p_a}{(2\pi)^3 2E_a}\cdot\frac{d^3p_b}{(2\pi)^3 2E_b}\cdot\frac{d^3p_c}{(2\pi)^3 2E_c}\,\delta^4(p_a + p_b - p_c - p_\nu)$,

p_ν, E_ν are respectively the momentum and the energy of neutrino, ϱ_{ij} is the ij - element in the density matrix of the massive Majorana neutrinos. n stands for the density number of the interacting particles. The bar denotes the corresponding antiparticle. $\mathcal{H}_o$ is the free neutrino hamiltonian:

$$\mathcal{H}_o = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}.$$

$\mathcal{A}$ is the amplitude of the process $e^+e^- \to \nu_i \nu_j$, $\mathcal{B}$ is the amplitude of the process $e^-\nu_i \to e^-\nu_j$, C is that of the process $\nu_i e^+ \to \nu_j e^+$. They can be expressed through the known amplitudes $\mathcal{A}_e$ $(e^-e^+ \to \nu_e\bar{\nu}_e)$, $\mathcal{B}_e$ $(e^-\nu_e \to e^-\nu_e)$ and C_e $(e^+\nu_e \to e^+\nu_e)$:

$$\mathcal{A} = \alpha \mathcal{A}_e, \quad \mathcal{B} = \alpha \mathcal{B}_e, \quad C = \alpha C_e$$
$$\alpha_{ij} = \mathcal{U}^*_{ie}\,\mathcal{U}_{je}.$$

where $\mathcal{U}$ is the unitary matrix in the expression: $\nu_i = \mathcal{U}_{il}\nu_l$, $l = e, s$.

So, the first term in eq.(3) describes the effect of the expansion the second one is responsible for oscillations of ν_e and ν_s and the last terms describe the weak reactions $e^-e^+ \to \nu_i\bar{\nu}_j$, $e^+\nu'_j \to e^+\nu_i$ and $e^-\nu'_j \to e^-\nu_i$ respectively. With the use of identities:

$$\left[\mathcal{H}_o, \varrho\right] = \left(\sqrt{p^2+m_1^2} - \sqrt{p^2+m_2^2}\right)\begin{pmatrix} 0 & \varrho_{12} \\ -\varrho_{21} & 0 \end{pmatrix}$$

$$\mathcal{B}\varrho'\mathcal{B}^+ = \begin{pmatrix} c^2 & -cs \\ -cs & s^2 \end{pmatrix}\left(c^2\varrho'_{11} - 2cs\,\mathrm{Re}\,\varrho'_{12} + s^2\varrho'_{22}\right)|\mathcal{B}_e|^2$$

$$\{\mathcal{B}^+\mathcal{B}, \varrho\}_+ = \begin{pmatrix} 2c^2\varrho_{11} - 2cs\,\mathrm{Re}\,\varrho_{12} & \varrho_{12} - cs(\varrho_{11}+\varrho_{22}) \\ \varrho_{21} - cs(\varrho_{11}+\varrho_{22}) & 2s^2\varrho_{22} - 2cs\,\mathrm{Re}\,\varrho_{12} \end{pmatrix}|\mathcal{B}_e|^2$$

$$\{\varrho, \mathcal{A}^+\bar{\varrho}\mathcal{A}\} = |\mathcal{A}_e|^2\begin{pmatrix} 2(\varrho_{11}c^2 - cs\,\mathrm{Re}\,\varrho_{12}) & \varrho_{12} - cs(\varrho_{11}+\varrho_{22}) \\ \varrho_{21} - cs(\varrho_{11}+\varrho_{22}) & 2s^2\varrho_{22} - 2cs\,\mathrm{Re}\,\varrho_{12} \end{pmatrix}\left(c^2\bar{\varrho}_{11} - 2cs\,\mathrm{Re}\,\bar{\varrho}_{12} + s^2\bar{\varrho}_{22}\right)$$

equation (3) can be written in the more explicit form:

$$\frac{\partial \varrho}{\partial t} = H p_\nu \frac{\partial \varrho}{\partial p_\nu} + i\begin{pmatrix} 0 & \varrho_{12} \\ -\varrho_{21} & 0 \end{pmatrix}\left(\sqrt{p_\nu^2+m_1^2} - \sqrt{p_\nu^2+m_2^2}\right) +$$
$$+ \mathbf{T}\int d\Omega(e, e^+, \bar{\nu})\, n_e n_{\bar{e}} |\mathcal{A}_e|^2 - \tfrac{1}{2}\mathbb{R}\int d\Omega(e, e^+, \bar{\nu})|\mathcal{A}_e|^2\left(c^2\bar{\varrho}_{11} - 2cs\,\mathrm{Re}\,\bar{\varrho}_{12} + s^2\bar{\varrho}_{22}\right)$$
$$+ \mathbf{T}\int d\Omega(e, \nu', e')\, n'_e\left(c^2\varrho'_{11} - 2cs\,\mathrm{Re}\,\varrho'_{12} + s^2\varrho'_{22}\right)\left(|\mathcal{B}_e|^2 + |C_e|^2\right) -$$
$$- \tfrac{1}{2}\mathbb{R}\int d\Omega(e, \nu', e')\, n_e\left(|\mathcal{B}_e|^2 + |C_e|^2\right)$$

$$\mathbf{T} \equiv \begin{pmatrix} c^2 & -cs \\ -cs & s^2 \end{pmatrix}, \quad \mathbb{R} \equiv \begin{pmatrix} 2c^2\varrho_{11} - 2cs\,\mathrm{Re}\,\varrho_{12} & \varrho_{12} - cs(\varrho_{11}+\varrho_{22}) \\ \varrho_{21} - cs(\varrho_{11}+\varrho_{22}) & 2s^2\varrho_{22} - 2cs\,\mathrm{Re}\,\varrho_{12} \end{pmatrix},$$

In numerous works on neutrino oscillations, neutrino density numbers have been calculated in the approximation when the oscillations are only taken into account. Next, these density numbers have been used in the reactions discussed as in the case without oscillations. This approximation is valid only when neutrino oscillations proceed much faster than the other processes, i.e. when $\Gamma_{osc} \gg \Gamma_{reactions}$. Otherwise, one has to work with the kinetic equation for the density matrix (3).

We will study the evolution of the neutrino density matrix only for the case of electron neutrino oscillations.

According to our assumptions oscillations become noticeable after the decoupling of electron neutrinos. So, the neutrino kinetics down to 3 MeV does not differ from the standard situation, i.e. down to $T \sim 3$ MeV electron neutrinos maintain their equilibrium distribution, while the sterile neutrinos should have decoupled at the earlier epoch as far as they participate in superweak interactions : $T_s < T_\nu$, $n_s << n_\nu$ $(n_s \sim T_s^3 \quad n_\nu \sim T^3)$.

Kinetic equation (3) for the case $T < 3$ MeV after the decoupling of electron neutrinos reduces to :

$$\frac{\partial \rho}{\partial t} = Hp_\nu \frac{\partial \rho}{\partial p_\nu} + i\,[\mathcal{H}_o, \rho]. \qquad (4)$$

The initial condition reads:

$$\rho^o = \begin{pmatrix} c^2 & -cs \\ -cs & s^2 \end{pmatrix} \exp\left(-\frac{E_\nu}{T_o}\right) \qquad T = T_o \sim 3 \text{ MeV}.$$

The differential equation (4) is easily solved by using the substitution $\mu = \frac{m}{T}$, $\lambda = \frac{p_\nu}{T}$. Then, after some simple calculations we obtain:

$$\rho = \exp\left(-\frac{E_\nu}{T}\right) \begin{pmatrix} c^2 & -cs\exp\left(\frac{iBT}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_o^3}\right)\right) \\ -cs\exp\left(-\frac{iBT}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_o^3}\right)\right) & s^2 \end{pmatrix},$$

where $B = 0.1\, M_{PL}\, g^{-\frac{1}{2}} |m_1^2 - m_2^2|$ and $T = 1.56 g^{\frac{1}{4}} t^{-\frac{1}{2}}$.

The density number of the electron neutrinos $n_\nu \equiv \rho_{LL}$ will then read:

$$\rho_{LL} = \exp\left(-\frac{E_\nu}{T}\right)\left[1 - 2c^2s^2 + 2c^2s^2 \cos\frac{BT}{E_\nu}\left(\frac{1}{T^3} - \frac{1}{T_o^3}\right)\right]. \qquad (5)$$

So, in the case of ν- oscillations proceeding in the way described above, the evolution of active neutrinos with the gradual cooling of the Universe $\rho_{LL}(T(t))$ is explicitly described by (5). It differs from the equilibrium distribution of active neutrinos in the standard model $\rho_{LL}^{eq} \sim \exp\left(-\frac{E_\nu}{T}\right)$ by the multiple $\left[1 - 2c^2s^2 + 2c^2s^2\cos\frac{BT}{E_\nu}\left(\frac{1}{T^3} - \frac{1}{T_o^3}\right)\right]$. As was expected, this distribution reduces to the standard one if the mixing angles or δm^2 are put equal to zero. At great B , i.e. $\delta m^2 \gtrsim 10^{-7}$ eV^2, when $\cos\left[\frac{BT}{E_\nu}\left(\frac{1}{T^3} - \frac{1}{T_o^3}\right)\right]$ is frequently oscillating, it can be averaged and then $\rho_{LL} = \exp\left(-\frac{E}{T}\right).(1-2c^2s^2)$. From (5) it is evident that the energy distribution of active neutrinos is also changed by the oscillations.

4. THE KINETICS OF NUCLEONS

The kinetic equation describing the evolution of the neutron number density n_n for the case of oscillating neutrinos $\nu_e \leftrightarrow \nu_s$ reads:

$$\frac{\partial}{\partial t} n_n = H \frac{\partial}{\partial p_n} n_n + \int d\Omega(e,p,\nu)\, |A(ep \to \nu_e n)|^2 (n_e n_p - n_n \rho_{LL}) -$$
$$- \int d\Omega(e^+,p,\bar\nu)\, |A(e^+ n \to p\bar\nu)|^2 (n_{e^+} n_n - n_p \bar\rho_{LL}). \qquad (6)$$

Instead of a similar equation for n_p the identity $N_n + N_p = const$ is used. $\left(N = \frac{1}{(2\pi)^3}\int d^3p\, n(p)\right)$.

The first term describes the expansion and the second one describes the processes $e^- + p \leftrightarrow n + \nu_e$ and $p + \bar\nu_e \leftrightarrow e^+ + n$ directly influencing the nucleon density.

This equation differs from the standard scenario one only in the substitution of ρ_{LL} from eq.(5) instead of $n_\nu^{eq} \sim \exp\left(-\frac{E_\nu}{T}\right)$. The number density of electrons is assumed to be the equilibrium one. This assumption is justified by the fact that the rate of reactions of electrons with the photons is enormous, and as a result, the deviation of n_e from its equilibrium value is negligible.

$\bar\rho_{LL} = \rho_{LL}$ because of CP-invariance of all the processes discussed $n_{e^+} = n_e$ as in the standard model.

The initial condition for eq.(6) is the equilibrium distribution of neutrons and protons at $T \sim 3$ MeV.

Number densities per unit volume can be expressed through $n(p)$ as $N = (2\pi)^{-3}\int d^3p\; n(p)$. Performing integration in (6) one can obtain the following equation for the time evolution of the neutron number density:

$$\frac{\partial N_n}{\partial t} = -3H N_n + G_F^2 \frac{(1+3\alpha^2)}{\pi^3} T^3 \Big\{ -N_n \left[(1-2c^2s^2) I_1 + 2c^2s^2 J_1 \right] + \tag{7}$$

$$+ e^{-\frac{\Delta m}{T}} N_p I_1 + e^{-\frac{\Delta m}{T}} N_p \left[(1-2c^2s^2) I_2 + 2c^2s^2 J_2 \right] - N_n I_2 \Big\}$$

where

$$I_1 = \sum_{k,n=0}^{\infty} (-1)^{k+n} e^{-ny} \int_0^{\infty} dx\, \mathcal{X}(x,y)$$

$$\mathcal{X}(x,y) = e^{-(n+k+1)x} x^2 (y+x)^2 \left(1 - \frac{\mu^2 y^2}{2(x+y)^2}\right)$$

$$J_1 = \sum_{k,n=0}^{\infty} (-1)^{k+n} e^{-ny} \int_0^{\infty} dx\, \mathcal{X}(x,y) \cos\frac{B}{x}$$

$$I_2 = \sum_{k,n=0}^{\infty} (-1)^{k+n} e^{-ny} (\mu y)^3 \int_0^{\infty} d\xi\, Z(\xi,y)$$

$$J_2 = \sum_{k,n=0}^{\infty} (-1)^{k+n} e^{-ny} (\mu y)^3 \int_0^{\infty} d\xi\, Z(\xi,y) \cos\left(\frac{B}{y(1+\sqrt{1+\xi^2}\,\mu)}\right)$$

$$Z(\xi,y) = \xi^2 y^2 \left(1+\mu\sqrt{1+\xi^2}\right)^2 \exp\left[-\mu y \sqrt{1+\xi^2}(1+k+n)\right]$$

and

$$\mu = m_e/\Delta m\,, \quad y = \Delta m/T\,, \quad \Delta m = m_n - m_p = 1.293\ \text{MeV}.$$

Equations (7) have been numerically integrated for the temperature range of interest $0.3 \leq T \leq 3$ MeV, at different δm^2, $\delta m^2 \leq 10^{-7} eV^2$ and different values of the mixing angle $\theta \in (0, \frac{\pi}{4})$. The effect of oscillations on nucleonsynthesis becomes negligible for $\theta < \frac{\pi}{15}$ and $\delta m^2 \leq 10^{-11}$ eV^2, $g = \frac{50}{4}$.

The results of the numerical integration of eq.(7) for $\delta m^2 \in (10^{-7} - 10^{-11})$ eV^2 and $\theta = \pi/4$ are plotted in Figs.1 and 2.

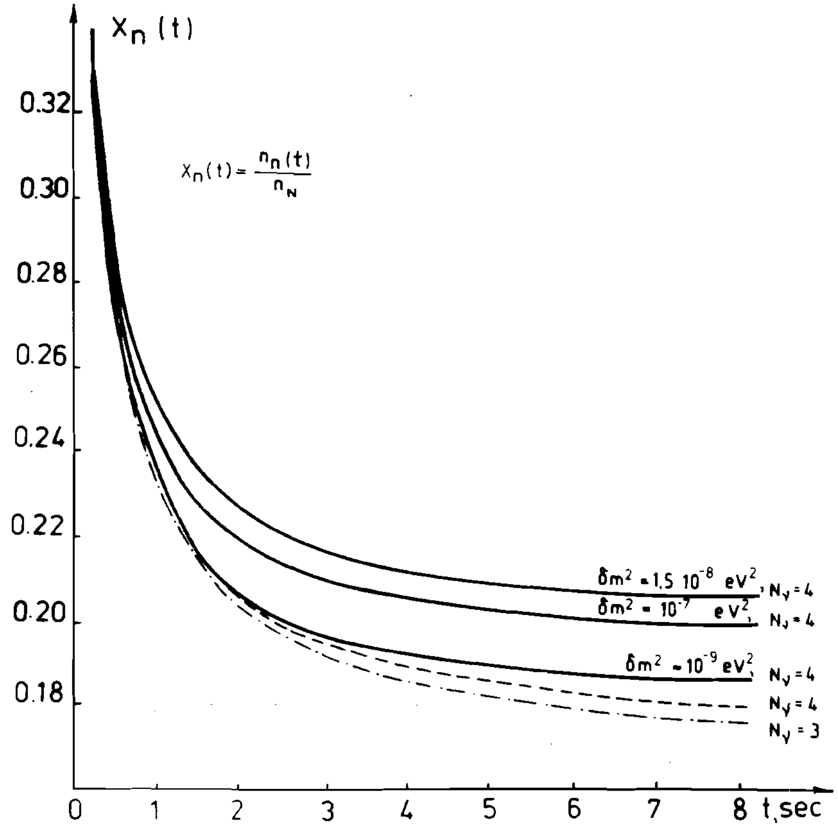

Fig. 1. Solid lines give the evolution of the number density of neutrons relative to nucleons $X_n(t) = n_n(t)/n_N$ for the models with oscillations with $\delta m^2 = 10^{-7}$ eV^2, 10^{-8} eV^2, 10^{-9} eV^2, and $g = 50/4$ $(N_\nu = 4)$, i.e. mixing only in the electron section. The dashed line gives the evolution of the relative neutron density for the standard models of primordial nucleosynthesis.

When independent mixings in the μ and τ sector are also allowed, the main effect is the possible increase of the relativistic degrees of freedom g at $T \sim 1$ MeV. Then the overproduction of ^4He would be even greater and the constraints on the δm^2_{ij} and θ_i would be more strict.

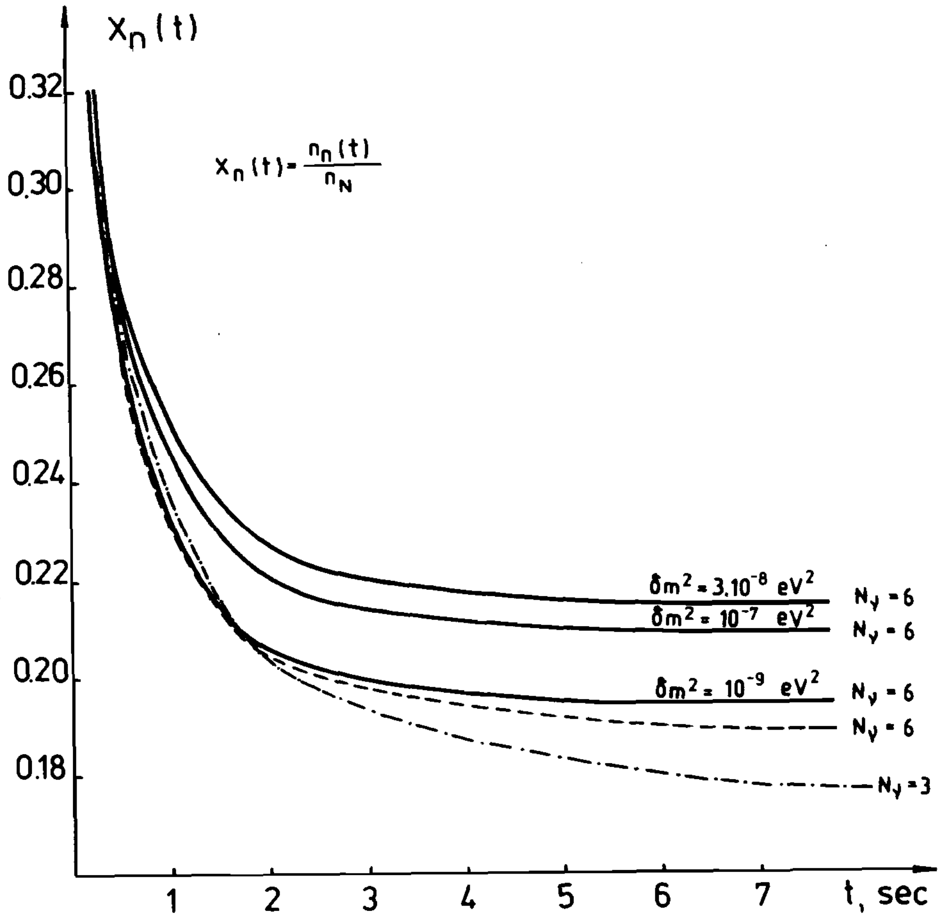

Fig. 2. Solid lines give the evolution of the number density of neutrons relative to nucleons $X_n(t) = n_n(t)/n_N$ for the models with oscillations with $\delta m^2 = 10^{-7}$, 10^{-8}, 10^{-9} eV^2, $\theta = \frac{\pi}{4}$ and $g = 16$, $N_\nu = 6$, i.e. independent mixing in all sectors is allowed. The dashed line gives the evolution of the relative neutron density for the standard models.

In the Table the relative increase of ^4He in comparison with the standard one at $T \sim 0.3$ MeV, $\theta = \frac{\pi}{4}$ and for $N_\nu = 4$ and $N_\nu = 6$ is reported. In the first column the kinetic effect in case $N_\nu = 4$ is reflected in comparison to the total effect, including also the increase of relativistic degrees of freedom, shown in the second column. It is obvious that the kinetic effect of oscillations plays a considerable role in overproduction of ^4He.

δm^2 eV^2	R, % $g=43/4$	$g=50/4$	$g=16$
10^{-11}	0	2.7	7.0
10^{-10}	0.1	2.8	7.1
10^{-09}	3.6	5.9	9.6
10^{-08}	13.2	16.4	20.1
10^{-07}	10.3	13.1	17.8

Tabl. Relative increase of ^4He in comparison with the standard ^4He value:

$$R = \frac{Y_p^{osc}(^4He) - Y_p^{s}(^4He)}{Y_p^{s}(^4He)}$$

for different δm^2 and $N_\nu = 3, 4, 6$.

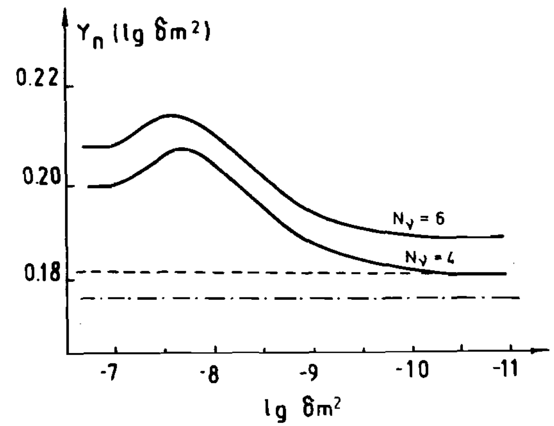

Fig. 3. The number density of neutrons relative to nucleons as a function of the neutrino mass differences. $\theta = \pi/4$, $N_\nu = 4, 6$.

On Fig. 3 the dependence of $n/(n+p)_F$ on δm^2 is shown. As could be predicted from the eq.(5), for great $\delta m^2 \sim 10^{-7}$ eV^2 when the oscillations of neutrino densities are fast, the resultant $n/(n+p)$-ratio tends to a constant value, corresponding to

$\rho_{LL} \sim 1-2c^2s^2$. The overproduction of ^4He $\geq 5\%$ for $\theta \geq \frac{\pi}{15}$, $\delta m^2 \geq 10^{-9}$ and $N_\nu = 4$ $(g_* = \frac{20}{4})$ is considerable.

Consequently, the discussed quite realistic model of nucleosynthesis with nonequilibrium neutrino oscillations $\nu_e \leftrightarrow \nu_s$ is an example of the case when working in terms of ρ -matrix is inevitable.

Future investigations must include both the kinetic influence of neutrino oscillations (discussed above) and the CP-violating effects of the plasma.

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Нейтринные осцилляции и первичный нуклеосинтез

Изучена модификация стандартной модели первичного нуклеосинтеза при наличии нейтринных осцилляций. Рассмотрена кинетика осциллирующих нейтрино в первичной плазме (T ~ ~ 3 МэВ) расширяющейся Вселенной. Исследована конкретная модель осцилляции между активными и нетермализованными стерильными нейтрино. Показано, что для определенной области осцилляционных параметров осцилляции сильно влияют на нейтрон-протонные переходы и на последующий синтез гелия (^{4}He). Противоречие с имеющимися экспериментальными данными для первично образованного гелия позволяет наложить ограничения на осцилляционные параметры: $\delta m^2 < 10^{-9}$ эв^2, $\theta \geq \pi/15$.

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Neutrino Oscillations and the Primordial Nucleosynthesis

A possible modification of the standard Big Bang nucleosynthesis scenario, allowing neutrino oscillations, is studied. A concrete model of oscillations between active and nonthermalized sterile neutrinos is investigated. The effect on the production of ^{4}He has been proved considerable for a certain range of oscillation parameters. The contradiction with the observational data on primordial abundance of helium in the Universe sets bounds on the oscillation parameters: $\delta m^2 < 10^{-9}$ eV^2 for $\theta \geq \pi/15$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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