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DISCRETE SYMMETRIES IN N=2 SUPERSYMMETRIC THEORIES

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1. <u>Introduction</u>. Recently, several attempts have been made to use N=2 supersymmetry for constructing phenomenological models [1]. In this connection it is important to know what can be said in N=2 supersymmetry concerning violation of discrete symmetries, space reflection (P), charge conjugation (C) and combined conjugation (CP). For example, can space parity be broken before a breakdown of the N=2 supersymmetry? It is well known that N=2 supersymmetric theories are vectorlike (see, e.g., [2]), i.e. they contain an equal number of left and right fermions in each multiplet. However, this does not mean that space parity is conserved. E.g., we shall see below that in a (vectorlike) N=2 theory with two central charges space parity can be broken. At the same time we shall prove that all the N=2 supersymmetric theories do conserve space parity if they have no central charges or if they have only one central charge. At the same time charge parity can be broken in an N=2 theory without central charges.

Present studies of discrete symmetries become possible due to the use of the harmonic superspace approach. Just within this approach one can embrace the most general N=2 supersymmetric interactions and just this is necessary for our analysis.

When discussing discrete symmetries for harmonic superfield we shall start with their standard definitions for usual fields in the Minkowski space-time. We use the following definitions and conventions. Dirac four-component spinors Ψ will be represented by a pair of two-component Weyl spinors ψ and φ,

$$\Psi_\alpha(x) = \begin{pmatrix} \psi_\alpha(x) \\ \bar\varphi^{\dot\alpha}(x) \end{pmatrix}, \quad \alpha, \dot\alpha = 1, 2, \tag{1.1}$$

$$\bar\varphi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\varphi_{\dot\beta} = \varepsilon^{\dot\alpha\dot\beta}\overline{\varphi_\beta}, \quad \varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}, \quad \varepsilon^{\alpha\beta}\varepsilon_{\beta\gamma} = \delta^\alpha_\gamma .$$

For a Majorana fermion these two component spinors coincide, $\varphi_\alpha \equiv \psi_\alpha$.

Under space reflection

$$P: \quad \vec{x}\,' = -\vec{x}, \quad x_0' = x_0 \tag{1.2}$$

spinor fields are transformed as follows

$$P:\ \psi'_\alpha(x') = \eta_P (\sigma_0)_{\alpha\dot\alpha}\bar\varphi^{\dot\alpha}(x) = \eta_P\, \bar\varphi^{\dot\alpha}(x) \tag{1.3}$$
$$\bar\varphi^{\dot\alpha\prime}(x') = \eta_P\, \psi_\alpha(x), \qquad |\eta_P| = 1,$$

while bosonic ones according to

$$P:\quad \Phi'(x') = \eta_{PB}\,\Phi(x),\ \ \eta_{PB} = \pm 1. \tag{1.4}$$

Under the charge conjugation spinors φ and ψ turn one into another and bosons undergo the complex conjugation

$$C:\ \ \psi'_\alpha(x) = \eta_C\, \varphi_\alpha(x),\ \ \varphi'_\alpha(x) = \bar\eta_C\, \psi_\alpha(x),\ |\eta_C| = 1 \tag{1.5}$$
$$\Phi'(x) = \eta_{CB}\,\Phi^*(x),\qquad |\eta_{CB}| = 1.$$

Finally, under the combined conjugation CP the space coordinates change their sign (2). Two-component spinors undergo the complex conjugation

$$CP:\ \psi'_\alpha(x') = \eta_{CP}\,\bar\psi^{\dot\alpha}(x),\ \ \varphi'_\alpha(x') = \bar\eta_{CP}\,\bar\varphi^{\dot\alpha}(x) \tag{1.6}$$

and bosonic fields are transformed according to

$$\Phi'(x') = \eta^B_{CP}\,\Phi^*(x),\qquad |\eta_{CP}| = |\eta^B_{CP}| = 1. \tag{1.7}$$

The paper is planned as follows. We start with a brief account of harmonic space elements needed in what follows (for details see /3,4/) (section 2). Section 3 contains the definition of space reflection in N=2 supersymmetric theories without central charges and the theorem that such theories conserve always space parity if they do not involve higher derivatives. The introduction of central charges within the harmonic superspace formalism is considered in section 4. In the next section it is proved that the theorem on parity conservation in N=2 theories remains valid also when there is one central charge. However, it can be violated in the presence of two central charges. A corresponding counter example is given in section 6. The combined parity CP is also violated in this example. In subsequent sections 7 and 8 we are discussing the charge conjugation and combined (CP) conjugation within the harmonic superspace approach and here we give an example of charge parity breaking. Unfortunately, we have not succeeded in finding a natural example of N=2 theory that conserves the combined CP parity while violating space parity and charge parity each taken separately. A generalization of G-parity within the harmonic superspace is given in section 9.

2. Basic notions of the harmonic N=2 superspace. This superspace is coordinatised as

$$\{x^m,\ \theta^+_\alpha,\ \bar\theta^+_{\dot\alpha},\ \theta^-_\alpha,\ \bar\theta^-_{\dot\alpha},\ u^\pm_i\}, \tag{2.1}$$

where $\theta, \bar\theta$ are Grassmann coordinates. Isospinor harmonics $u^\pm_i$ parametrize a two-dimensional sphere $S^2 = SU(2)/U(1)$, where the SU(2)-group corresponds to the N=2 supersymmetry automorphisms. By definition, the harmonics obey the relation*)

$$u^{+i} u^-_i = 1,\quad i = 1,2. \tag{2.2}$$

They are considered up to U(1) transformations

$$(u^\pm)' = \exp(\pm i\beta)\, u^\pm. \tag{2.3}$$

Indices $\pm$ mean charges of this U(1) group. The connection with usual Grassmann coordinates $\theta_{\alpha i}, \bar\theta^i_{\dot\alpha}$ is given by relations

$$\theta^\pm_\alpha = \theta^i_\alpha\, u^\pm_i,\ \ \bar\theta^\pm_{\dot\alpha} = \bar\theta^i_{\dot\alpha}\, u^\pm_i,\ \ \overline{(\theta_{\alpha i})} = \bar\theta^i_{\dot\alpha}. \tag{2.4}$$

The rigid N=2 supersymmetry is realized on the coordinates (2.1) as follows

$$\delta x^m = -2i(\varepsilon^i\sigma^m\bar\theta^+ + \theta^+\sigma^m\bar\varepsilon^i)u^-_i,$$
$$\delta\theta^\pm_\alpha = \varepsilon^i_\alpha u^\pm_i,\ \ \delta\bar\theta^\pm_{\dot\alpha} = \bar\varepsilon^i_{\dot\alpha} u^\pm_i,\ \ \delta u^\pm_i = 0. \tag{2.5}$$

Harmonic superspace (2.1) contains an analytic subspace with coordinates

$$\{\zeta^M = (x^m, \theta^+_\alpha, \bar\theta^+_{\dot\alpha}),\ u^\pm_i\} \tag{2.6}$$

This analytic subspace is clearly closed under N=2 supersymmetry transformations (2.5). It is real under an appropriate complex conjugation $\sim$:

$$\widetilde{x^m} = x^m,\ \ \widetilde{\theta^+_\alpha} = \bar\theta^+_{\dot\alpha},\ \ \widetilde{\bar\theta^+_{\dot\alpha}} = -\theta^+_\alpha, \tag{2.7}$$
$$\widetilde{u^\pm_i} = u^{\pm i},\ \ \widetilde{u^{\pm i}} = -u^\pm_i.$$

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*) Isoindices $i, j, \ldots$ are raised and lowered as follows

$$A^i = \varepsilon^{ij}A_j,\ \ A_j = \varepsilon_{jk}A^k,\ \ \varepsilon^{ij} = -\varepsilon^{ji},\ \ \varepsilon^{ij}\varepsilon_{jk} = \delta^i_k.$$

All N=2 supermultiplets are described by the analytic superfields, i.e. by superfields defined in the analytic superspace (2.6). These superfields carry some U(1) charges; e.g., the N=2 matter multiplet (hypermultiplet) is represented by an analytic superfield of change + 1:

$$q^+(\zeta,u)=f^+(x,u)+\theta^{+\alpha}\psi_\alpha(x,u)+\bar\theta^+_{\dot\alpha}\bar\varphi^{\dot\alpha}(x,u)+\dots \tag{2.8}$$

On the mass shell the hypermultiplet describes four physical real scalar fields (two complex scalars $f^i(x)$) and Dirac spinor field $\psi_\alpha(x)$, $\bar\varphi^{\dot\alpha}(x)$. Off the mass shell the hypermultiplet contains an infinite number of auxiliary component fields in its decomposition in harmonics

$$f^+(x,u)=f^i(x)u^+_i+f^{(ijk)}(x)u^+_i u^+_j u^-_k+\cdots \tag{2.9}$$

and analogously for fermions. Each term of the harmonic decomposition has the same U(1) charge (+1 in (2.9)). This charge is carried by the harmonics, while the field components do not carry it. The physical fields are described in (2.8) by the first terms of the decompositions of $f^+(x,u)$, $\psi_\alpha(x,u)$ and $\bar\varphi^{\dot\alpha}(x,u)$. All the higher terms represent auxiliary fields that have to be eliminated by means of the equations of motion.

The hypermultiplet free action has the form

$$S=\frac{1}{2}\int d\zeta^{-4}du\;\tilde q^+\overleftrightarrow{\mathcal{D}}^{++}q^+, \tag{2.10}$$

where $d\zeta^{-4}du=d^4x\,d^2\theta^+d^2\bar\theta^+du$ is the analytic superspace volume element, having U(1) charge − 4, and $\mathcal{D}^{++}$ is a harmonic derivative, consistent with (2.2)

$$\mathcal{D}^{++}=u^{+i}\frac{\partial}{\partial u^{-i}}-2i\,\theta^+\sigma^m\bar\theta^+\partial_m. \tag{2.11}$$

Integration over harmonics is defined by the following rule

$$\int du\cdot 1=1\,,\quad \int du\; u^+_{(i}u^+_j\dots u^-_{k)}=0 \tag{2.12}$$

The free equation of motion that follows from (2.10)

$$\mathcal{D}^{++}q^+=0 \tag{2.13}$$

cuts the infinite tail of auxiliary fields; on the mass shell we have

$$q^+(\zeta,u)=f^i(x)u^+_i+\theta^{+\alpha}\psi_\alpha(x)+\bar\theta^+_{\dot\alpha}\bar\varphi^{\dot\alpha}(x)+ +2i\,\theta^+\sigma^m\bar\theta^+\partial_m f^i(x)u^-_i, \tag{2.14}$$

where physical fields $f^i(x)$, $\psi_\alpha(x)$ and $\bar\varphi^{\dot\alpha}(x)$ obey the Klein-Gordon and Dirac equation, respectively.

The second important example is the Maxwell N=2 supermultiplet. It is described by the real analytic superfield $V^{++}(\zeta,u)$ having U(1) charge +2, entering as a connection into the harmonic derivative $\mathcal{D}^{++}$. It has the gauge transformation law

$$\mathcal{D}=D^{++}+V^{++}\,,\quad V^{++\prime}=V^{++}+D^{++}\lambda\,, \tag{2.15}$$

where $\lambda=\lambda(\zeta,u)$ is a real parameter of the gauge group.

The general self-coupling for hypermultiplets without higher derivatives has the form /4/

$$S=\int d\zeta^{-4}du\left[\frac{1}{2}\tilde q^+\overleftrightarrow{\mathcal{D}}^{++}q^++\mathcal{L}^{(+4)}(q,\tilde q,u)\right], \tag{2.16}$$

where the hyper-Kähler prepotential $\mathcal{L}^{+4}(q,\tilde q,u)$ is an arbitrary function of its arguments having U(1) charge +4. (There are no restrictions on the number of hypermultiplets). After elimination of the auxiliary fields eq. (2.16) describes a nonlinear sigma-model without potential terms.

Interaction with a gauge field is obtained by lengthening (2.15) of the derivative $\mathcal{D}^{++}$ in the action (2.16). The gravitational interaction is constructed also by a corresponding lengthening of the harmonic derivative. Of course, the kinetic terms for the gauge and gravity superfield are needed, etc. We shall not reproduce here the complete expression for the action principle; it can be found in /3-5/. We would like to stress, however, its remarcable peculiarity: all the N=2 supersymmetric interactions without central charges do not explicitly contain superspace variables x^m, θ^+_α, $\bar\theta^+_{\dot\alpha}$ and derivatives with respect to them. This property will be crucial in the subsequent considerations. Note that in theories with central charges the Grassmann variables θ^+, $\bar\theta^+$ enter explicitly into the Lagrangian (see below).

3. Space reflection and parity conservation in N=2 supersymmetry without central charges. The symmetries in superspace are realized both on coordinates and superfields. We shall start with coordinates. Under space reflections we have (1.2)

$$P:\quad \vec{x}^{\,\prime}=-\vec{x}\,,\quad x_0^{\prime}=x_0 \tag{3.1}$$

and spinor coordinates are transformed as

$$P:\ (\theta^+_\alpha)' = \eta_P\,\bar\theta^{+\dot\alpha},\quad (\bar\theta^+_{\dot\alpha})' = -\bar\eta_P\,\theta^{+\alpha},\quad |\eta_P| = 1 \tag{3.2}$$

so that the analytic superspace (2.6) is closed under P. We shall postulate also that harmonics do not change *)

$$\mathbb{P}:\ u^{\pm\,\prime}_i = u^{\pm}_i . \tag{3.3}$$

The volume element and the harmonic derivative in (2.10), (2.16), etc., turn out to be invariants

$$(d\zeta^{-4}du)' = d\zeta^{-4}du\,,\quad D^{++\,\prime} = D^{++}. \tag{3.4}$$

The analytic superfields (without external indices) are postulated to be scalars. For example,

$$q^{+\,\prime}(\zeta',u') = q^+(\zeta,u);\quad V^{++\,\prime}(\zeta',u') = V^{++}(\zeta,u). \tag{3.5}$$

It is easy to see that (3.5), (3.1)-(3.3) are compatible (see (2.8), (2.9)) with the transformation laws for the component field under space reflection (1.3), (1.4), (1.1) with the corresponding correlations in phases for bosons and fermions. It is easy now to prove the general theorem.

<u>The space parity cannot be violated in any N=2 supersymmetric theory without central charges and higher derivatives</u>.

Indeed, the Lagrangian for self-couplings of hypermultiplets (as well as for gauge and gravitational couplings) contains only matter superfields (and gauge and gravitational ones) but it does not contain x^m, θ^+_α and $\bar\theta^+_{\dot\alpha}$ in the absence of central charges and higher derivatives. Only harmonic derivatives enter and also thermonics could enter. However, they are sterile under space reflections. Therefore the statement of the theorem is evident because of scalarity of superfields and of invariance of the supervolume (2.10). So, it remains to analyse central-charge modificatinns of N=2 theories.

<u>Digression</u>. The rule (3.5) for the hypermultiplet space reflection says that all four physical bosons are scalars. At the same time if one represents the on-mass-shell hypermultiplet by two N=1 chiral superfields, then two of the bosons will be scalars while the remaining two will be pseudoscalars. An explanation is as follows. The definition (3.5) is possible within N=2 supersymmetry only (and, of course, for component fields) and it does not admit a reduction to N=1 supersymmetry. Indeed, the laws (3.2), (3.3) imply that

$$\theta^{+\,\prime}_\alpha = \eta_P\,\bar\theta^{+\dot\alpha} \Rightarrow \theta^{1\,\prime}_\alpha = \eta_P\,\bar\theta^{\dot\alpha 1} = \eta\,\bar\theta^{\dot\alpha}_2 . \tag{3.6}$$

$\bar\theta^{\dot\alpha}_2$ is complex conjugated to $\theta^{\alpha 2}$ and not to $\theta^{\alpha 1}$, i.e. one Grassmann variable θ^1_α turns into the complex conjugate of another, θ^2_α. It is possible only if there are two <u>different</u> Grassmann coordinates. It is impossible in the framework of N=1 superfields where we have only one Grassmann variable in our disposition. For N=2 parity definition admitting a reduction to the N=1 case one has to change the postulated law for harmonics (3.3) by

$$\mathbb{P}:\quad u^{\pm\,\prime}_i = u^{\pm i} . \tag{3.3'}$$

4. <u>Introduction of central charges</u>. As is well known, the N=2 supersymmetry algebra admits inclusion of two central charges Z_1, Z_2 commuting with all generators. Then the commutator of spinor generators takes the form

$$\{Q^i_\alpha, Q^j_\beta\} = \varepsilon^{ij}\varepsilon_{\alpha\beta}\,(Z_1 + iZ_2) . \tag{4.1}$$

In the presence of central charges the harmonic derivative explicitly acquires Grassmann variables /3/ (see also /6/)

$$D^{++} \to D^{++}_{cc} = D^{++} + (\theta^{+\alpha}\theta^+_\alpha - \bar\theta^+_{\dot\alpha}\bar\theta^{+\dot\alpha})Z_1 + i(\theta^{+\alpha}\theta^+_\alpha + \bar\theta^+_{\dot\alpha}\bar\theta^{+\dot\alpha})Z_2 \tag{4.2}$$

Now there appears a possibility of a space parity violation.

Let us recall, in the harmonic superspace language, the introduction of central charges. To this aim the presence of U(1) isometries is necessary. These isometries have to commute with supersymmetry. So in the case of two central charges the theory has to have two such mutually commuting U(1) isometries /7/. These isometries are represented by some transformations (τ are parameters)

$$\delta q^+ = \lambda^+(q^+,u)\cdot\tau \tag{4.3}$$

that leave the action invariant (e.g., (2.16) in the case of the sigma-model). An effect of the central charge operator on q^+ is defi-

*) In principle, they could also undergo a discrete transformation acting on the index i and reducing to the phase after being used twice.

ned by the Killing vector $\lambda^+(q^+,u)$

$$Z q^+ = i m \lambda^+(q^+,u), \qquad (4.4)$$

where m is a parameter of the dimensionality of mass. So the action of the N=2 sigma-model involving several hypermultiplets q_a, $a=1,..n$ and two central charges has the form

$$S=\int d\zeta^{-4} du \left\{ \frac{1}{2} \tilde{q}_a^+ \overleftrightarrow{\mathcal{D}}^{++}_{c.c.} q_a^+ + \mathcal{L}^{+4}(q,u) \right\} =$$

$$= \int d\zeta^{-4} du \left\{ \frac{1}{2} \tilde{q}_a^+ \overleftrightarrow{D}^{++} q_a^+ + \mathcal{L}^{+}(q,u) + \right. \qquad (4.5)$$

$$+\frac{1}{2} i m_1 (\theta^+\theta^+ - \bar\theta^+\bar\theta^+) \tilde{q}_a^+ \lambda^+_{a,1}(q,u) - \frac{m_2}{2} (\theta^+\theta^+ + \bar\theta^+\bar\theta^+) \tilde{q}_a^+ \lambda^+_{a,2}(q,u)$$

$$+ \text{Herm. Conj.}$$

where λ^+_{a1}, λ^+_{a2} are Killing vectors for two U(1) isometries and m_1, m_2 are parameters of mass. In fact, m_1, m_2 define both the masses of hypermultiplets and potentials /7/. Without central charges (4.5) reduces to (2.16), i.e., to the pure sigma-model without potential terms. We start with

5. The case of one central charge, when, say, $m_2=0$. Then Grassmann variables appear in (4.5) only in the form

$$\frac{1}{2} i m_1 (\theta^+\theta^+ - \bar\theta^+\bar\theta^+) \tilde{q}_a^+ \lambda^+(q^+,u) + \text{Herm. Conj.} \qquad (5.1)$$

However in transformation law (3.2) there is a free parameter, the phase η_p. Choosing $\eta_p = i$ we achieve scalarity of the θ -factor

$$(\theta^+\theta^+ - \bar\theta^+\bar\theta^+)' = \theta^+\theta^+ - \bar\theta^+\bar\theta^+. \qquad (5.2)$$

It becomes clear now that all the arguments in the proof of the above theorem (for the central chargeless case) remain also valid for the case of one central charge. So, the more general theorem is proved:

Space-parity is conserved in any N=2 supersymmetric theory without central charges and with one central charge (in the absence of higher derivatives).

One can ask whether this theorem remains true in the presence

of two central charges. In this case one also deals with vectorlike representations, and if just vectorlikeness is important, then one should expect here parity conservation. However, we shall give now a

6. Counterexample of parity nonconservation in the presence of two central charges. Let there be two hypermultiplets, $a=1,2$, and the self-coupling has the following simple form

$$\mathcal{L}^{+4} = \lambda \tilde{q}_1^+ q_1^+ \tilde{q}_2^+ q_2^+. \qquad (6.1)$$

The U(1) isometries and the corresponding Killing vectors are

$$\delta q_1^+ = i \nu_1 q_1^+, \quad \delta q_2^+ = 0 \quad \text{and Herm. Conj.} \qquad (6.2)$$

$$\lambda_1^+(q,u) = i q_1^+ \quad \text{and Herm. Conj.} \qquad (6.3)$$

$$\delta q_1^+ = 0, \quad \delta q_2^+ = i \nu_2 q_2^+ \quad \text{and Herm. Conj.} \qquad (6.4)$$

$$\lambda_2^+(q,u) = i q_2^+ \quad \text{and Herm. Conj.} \qquad (6.5)$$

Correspondingly, the action will be written as

$$S = \int d\zeta^{-4} du \left\{ \frac{1}{2} \left(\tilde{q}_1^+ \overleftrightarrow{D}^{++} q_1^+ + \tilde{q}_2^+ \overleftrightarrow{D}^{++} q_2^+ \right) \right. \qquad (6.6)$$

$$+ \lambda \tilde{q}_1^+ q_1^+ \tilde{q}_2^+ q_2^+$$

$$- m_1 (\theta^+\theta^+ - \bar\theta^+\bar\theta^+) \tilde{q}_1^+ q_1^+ + i m_2 (\theta^+\theta^+ + \bar\theta^+\bar\theta^+) \tilde{q}_2^+ q_2^+.$$

This model will violate the space parity for any choice of the phase in the law (3.2). Indeed, at $\eta_p = \pm i$ $\tilde{q}_1^+ q_1^+$ would be scalar, while $\tilde{q}_2^+ q_2^+$ would change its sign. In this case one cannot use (3.5) and conventional parity definition for q_2^+. Instead, q_2^+ would undergo the CP-transformation. Even in this case $\mathcal{L}^{+4}$(6.1) would change its sign so the action (6.6) would not be invariant. At $\eta_p = \pm 1$ would change its sign and again there would be no invariance of the action (6.6). An analysis of the general case $\eta_p = e^{i\varphi}$ is given in the Appendix. The conclusion is that the space-parity is violated in the sigma-model (6.6). This violation is possible due the presence of two central charges.

7. Charge conjugation and CP-conjugation for analytic superfield have to be defined in accordance with the corresponding conjugations for their component fields (1.5), (1.6). This condition lead us to the following definition of the complex conjugation C.

$$\mathbb{C}: \quad x^{m\prime} = x^m \tag{7.1}$$

$$\theta_\alpha^{+\prime} = \eta_{\theta c}\theta_\alpha^+ , \quad \bar\theta_{\dot\alpha}^{+\prime} = \bar\eta_{\theta c}\bar\theta_{\dot\alpha}^+ \tag{7.2}$$

$$u_i^{+\prime} = u_i^+ , \quad u_i^{-\prime} = u_i^- \tag{7.3}$$

$$D^{++\prime} = D^{++} \tag{7.4}$$

$$\left(d\zeta^{-4} du\right)' = d\zeta^{-4} du \tag{7.5}$$

$$q^{+\prime}(\zeta', u') = \eta_{qc}\, q^+(\zeta, u); \quad \tilde q^{+\prime}(\zeta', u') = \bar\eta_{qc}\, q^+ \tag{7.6}$$

$$V^{++\prime}(\zeta', u') = - V^{++}(\zeta, u) \tag{7.7}$$

(we shall not here give rules for charge conjugation for gravitational superfields). Choice (7.7) for the transformation of the gauge superfield V^{++} is dictated by the charge conjugation law of D^{++} (7.4) and, at the same time, by the charge conjugation rule

$$\mathbb{C}: \quad \left(q^+ \tilde q^+\right)' = - q^+ \tilde q^+ \tag{7.8}$$

that follows from (7.6). That definition ensures that the hypermultiplet kinetic term, its gauge and gravitational interactions will be invariant under charge conjugation. For example,

$$\int d\zeta^{-4} du \left(\tfrac{1}{2}\tilde q^+ \overleftrightarrow{D}^{++} q^+ + V^{++}\tilde q^+ q^+\right) \rightarrow \tag{7.9}$$

$$\rightarrow \int (d\zeta^{-4} du)' \left(-\tfrac{1}{2} q^+ \overleftrightarrow{D}^{++\prime} \tilde q^+ - V^{++\prime} q^+ \tilde q^+\right) =$$

$$= \int d\zeta^{-4} du \left(\tfrac{1}{2}\tilde q^+ \overleftrightarrow{D}^{++} q^+ + V^{++} q^+ \tilde q^+\right).$$

To achieve invariance of the central charge terms containing $\tilde q^+ q^+$, it is enough to fix the phase of spinor coordinates choosing

$$\eta_{\theta c} = \pm i . \tag{7.10}$$

All the above examples including the P-parity counterexample (6.6) will be invariant under charge conjugation with the phase (7.10), We conclude that in the example (6.6) the space parity is violated in parallel with the combined parity CP, but the charge parity does conserve.

Naturally, we would like to construct the models violating the charge parity C (in the ideal case, such ones, in which CP is conserved, while P and C are violated). Let us show that in N=2 supersymmetric models one can violate the charge parity.

8. An example of charge parity violation in the N=2 sigma-model. Let us choose $\mathcal{L}^{+4}$ in the form

$$\mathcal{L}_1^{+4} = \lambda (q^+)^4 + \bar\lambda (\tilde q^+)^4 . \tag{8.1}$$

It would conserve charge parity C iff the phase η_{qc} (7.6) for the hypermultiplet is correlated with the coupling constant

$$\lambda (\eta_{qc})^4 = \bar\lambda (\bar\eta_{qc})^4 . \tag{8.2}$$

So, η_{qc} is fixed. Now we introduce one more self-coupling

$$\mathcal{L}_2^{+4} = g (q^+)^3 u_1^+ + \bar g (\tilde q^+)^3 u^{+1} . \tag{8.3}$$

The requirement of its charge invariance leads to another fixing of the phase

$$g (\eta_{qc})^3 = \bar g (\bar\eta_{qc})^3 . \tag{8.4}$$

Generally, fixings (8.2) and (8.4) are incompatible and the charge parity is violated in a sigma-model with the self-interaction

$$\mathcal{L}^{+4}(q, u) = \mathcal{L}_1^{+4}(q, u) + \mathcal{L}_2^{+4}(q, u) . \tag{8.5}$$

9. Generalized internal G-parity. The harmonic superspace contains additional coordinates, viz. harmonics. Correspondingly, one can define in it new kinds of reflections. In conclusion of this paper we shall discuss one of them corresponding to G-parity. Scalar fields entering into the hypermultiplet form a doublet with respect to the SU(2)-group of automorphisms of the N=2 supersymmetry algebra. Using it one can extend the charge conjugation operation. To this end one retains valid all the rules (7.1-7) except the rule (7.3). This rule is changed by adding to the transformation law a SU(2)-matrix, the square of which is some purely phase transformation, e.g.,

$$u_i^{\pm\prime} = \Lambda_i^{\ j} u_j^{\pm}; \quad \Lambda_i^{\ k}\Lambda_k^{\ j} = -\delta_i^{\ j}. \tag{9.1}$$

Then bosonic fields will transform as (taking into account (7.6))

$$G: \qquad \Phi^{i\prime}(x) = \eta_G \Lambda^i_j \bar{\Phi}^j(x). \tag{9.2}$$

For $\Lambda = i\tau_1$ one identifies (9.2) with the known G-conjugation (see, e.g.[8]). It is evident that this G-parity can be rather easily violated. For instance, self-couplings $\mathcal{L}^+(q,u)$ preserve it only for rather special cases of the dependence of $\mathcal{L}^+(q,u)$ on u. Therefore, the requirement of G-parity conservation will select a definite class of interactions. We shall not here discuss this question which, may be, is of a certain interest.

10. Conclusion. So space purity is conserved in any N=2 supersymmetric theory without central charges. The same concerns the case of one central charge. Only the introduction of two central charges admits construction of N=2 supersymmetric models with the space parity breakdown (theory remains vectorlike). However, then the combined CP-parity will also be broken, and we cannot conserve it in the examples we have considered. At the same time it is rather easy to find sigma models with broken charge conjugation parity (though they look rather artificial). It will be of interest to find a natural in some sense model where CP will be conserved while C and P will be broken separately. Perhaps, this can be achieved in the models with spontaneous breakdown of P-parity or N=2 supersymmetry. In principle, the harmonic superspace techniques permit one to study this problem.

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Appendix. Proof of the parity violation in the model (6.6).

Let $\eta_p = e^{i\varphi}$. Then the sum of mass terms in the third line of (6.6) will be invariant under P iff

$$(\tilde{q}_1^+ q_1^+)' = \cos 2\varphi\, \tilde{q}_1^+ q_1^+ - \frac{m_2}{m_1}\sin 2\varphi\, \tilde{q}_2^+ q_2^+$$
$$(\tilde{q}_2^+ q_2^+)' = \cos 2\varphi\, \tilde{q}_2^+ q_2^+ + \frac{m_1}{m_2}\sin 2\varphi\, \tilde{q}_1^+ q_1^+. \tag{A.1}$$

Consider linear transformations

$$q_1^{+\prime} = A q_1^+ + B\tilde{q}_1^+ + C q_2^+ + D\tilde{q}_2^+ ,$$
$$q_2^{+\prime} = E q_1^+ + F\tilde{q}_1^+ + G q_2^+ + H\tilde{q}_2^+ . \tag{A.2}$$

In the r.h.s. of (A.1) the terms $q_1^+ q_2^+$, $\tilde{q}_1^+ q_2^+$ and their conjugated must not appear after change (A.2). This requirement leads to the conclusion that in the r.h.s. of (A.2) only one term can be different from zero, i.e. there are the following possibilities

$$q_1^{+\prime} = \eta_1 q_1^+ , \quad q_2^{+\prime} = \eta_2 \tilde{q}_2^+ \quad \text{and Herm. Conj.}$$

or (A.3)

$$q_1^{+\prime} = \eta_1 \tilde{q}_1^+ , \quad q_2^{+} = \eta_2 q_2^+ \quad \text{and Herm. Conj.}$$

They are already considered in the main text. There are no alternatives, e.g.,

$$(q_1^+)' = \sqrt{\frac{m_2}{m_1}}\, q_2^+ , \quad (q_2^+)' = -\sqrt{\frac{m_1}{m_2}}\, \tilde{q}_1^+ \tag{A.4}$$

at $m_1 \neq m_2$ contradict the invariance of kinetic terms, etc.

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Дискретные симметрии в N = 2 суперсимметричных теориях

Методом гармонического суперпространства исследуются дискретные симметрии в N=2 суперсимметричных теориях. Доказана теорема о строгом сохранении пространственной четности в этих теориях без центральных зарядов и с одним центральным зарядом. При двух центральных зарядах теорема уже не верна и дан соответствующий пример N=2 теории с нарушением пространственной и, одновременно, комбинированной /CP/ четности. Возможности нарушения зарядовой четности в N=2 теориях шире и не требуют двух центральных зарядов. Обсуждается также внутренняя G-четность.

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Discrete Symmetries in N=2 Supersymmetric Theories

Discrete symmetries of N=2 supersymmetric theories are studied by means of the harmonic superspace techniques. A theorem is proved that the space parity is strictly conserved in these theory without central charges and with one central charge. However, for the case of two central charges that theorem is not valid and the corresponding example of N=2 theory is given where there is a breakdown of space parity together with combined (CP) parity. Charge parity can be broken in a much wider class of N=2 theories even without two central charges. Internal G-parity is also discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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