ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2-88-254

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NEXT-NEXT-TO-LEADING $O(\bar{\alpha}_s^3)$ QCD CORRECTION TO σ_{tot} $(e^+e^- \to$ HADRONS): ANALYTICAL CALCULATION AND ESTIMATION OF THE PARAMETER $\Lambda_{\overline{MS}}$

Submitted to "Physics Letters B"

1988

1. INTRODUCTION

The e^+e^-- annihilation into hadrons is one of the most informative processes in elementary particle physics. Both experimental [1] and theoretical (see, e.g. [2]) analysis of the behaviour of its basic characteristic $R(s) = \sigma_{tot}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ allows one to obtain important information about the properties of hadrons and their constituents, i.e. quarks and gluons. In the zeroth order of perturbation theory (PT)QCD prediction for R(s) is in qualitative agreement with experiment. However, in order to test quantitatively the QCD predictions it is necessary to take into account the effects of higher PT corrections. Until recently R(s) has been known in QCD up to the next-to-leading $O(\bar{\alpha}_s^2)$ PT correction [3] . In this work we present the result of analytical calculation of the next-next-to-leading $O(\bar{\alpha}_s^3)$ correction to R(s) and obtain the new estimation of the parameter $\Lambda_{\overline{MS}}$ based on the analysis of the combined PETRA and PEP results [4,28] . We show that not only the next-to-leading PT correction but the higher order PT effects are very important for comparing QCD with experiment and determining the value of the QCD parameter $\Lambda_{\overline{MS}}$. We also discuss the problem of comparing asymptotic PT series of QCD with experiment.

2. THE OUTLINE OF CALCULATIONS

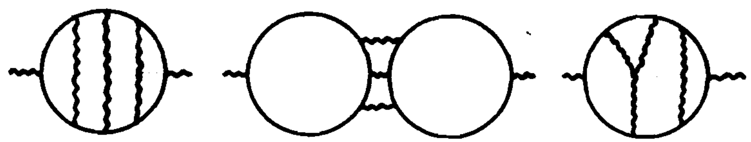
Throughout this work we follow the calculational program outlined in refs. [5,6] and use the notations introduced there. In the course of calculations it is convenient to use the D - function

$$D(Q^2) = -\frac{3}{4}Q^2\frac{d}{dQ^2}\Pi(Q^2) = Q^2\int_0^\infty \frac{R(s)}{(s+Q^2)^2}ds. \qquad (2.1)$$

Here $Q^2 = -q^2$ is the Euclidean transferred momentum and Π is the hadronic vacuum polarization function defined as

$$i\int e^{iqx}\langle 0|T(J_\mu(x)J_\nu(0))|0\rangle d^4x = = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(-q^2)/16\pi^2, \qquad (2.2)$$

where J_μ is the hadronic electromagnetic current. It can be shown that in order to calculate the next-next-to-leading corrections to the D -function, it is necessary to use the two-loop approximation of the bare expansion parameter $a_B = (\alpha_s/\pi)_B$, to calculate the three-loop approximation of the bare expression $\Pi_B(a_B)$ and to find the four-loop approximation of the photon wave function renormalization constant Z_3 . At this level, over 100 diagrams contribute to Z_3 . The typical examples are shown in figs.1. All the calculations have been done within the dimensional regularization in $n = 4-2\varepsilon$ space-time dimensions and the minimal subtraction (MS) scheme.



(a) (b) (c)

Figure 1.

The application of the methods of infrared rearrangement [7,6] and the infrared R^* - operation [8] allowed us to reduce the calculation of the four-loop approximation of Z_3 to the evaluation of the three-loop massless propagator-type integrals up to $O(\varepsilon^0)$ terms. These integrals as well as the three-loop approximation of $\Pi_B(a_B)$ have been calculated with the integration-by-parts algorithm[9] . Some basic scalar integrals used in the calculations have been calculated with the Gegenbauer polynomial x-space technique[6] . All analytical calculations have been done with the help of the SCHOONSCHIP program[10] which implements the integration-by-parts algorithm[9] . The whole running time at the CDC-6500 computer exceeds 200 hours. All the direct calculations have been done at two stages. At the first stage, we have found the counterterms of 58 diagrams which contribute to Z_3 in QED (see, e.g., figs. 1a, 1b). As a result, the four-loop approximations of the QED β - function in the MS-scheme and the QED Ψ - function have been obtained[11]. At the second stage, the final QCD result for the D-function of eq.(2.1) has been obtained. The details and the tests of calculations will be described in a more extended publication.

3. THE QCD RESULTS

All direct calculations discussed in the previous section have been done in the Euclidean region of momentum transfers. However, to obtain the theoretical expression for $R(s)$ it is necessary to transform the final result into the physical region by means of the following representation

$$R(s)=\frac{1}{2\pi i}\int_{-s-i\varepsilon}^{-s+i\varepsilon} dQ^2\,\frac{D(Q^2)}{Q^2}\,. \tag{3.1}$$

Eq. (3.1) leads to the appearance of the additional scheme independent correction in the next-next-to-leading order we are interested in, which is proportional to π^2

$$R(s)=D(s,a)-3\sum Q_f^2\,\pi^2\,\frac{\beta_0^2}{3}a^3+O(a^4), \tag{3.2}$$

where $a=\alpha_s/\pi$, β_0 is the first coefficient of the QCD function which has been calculated in[12] in the MS-scheme at the three-loop level

$$\frac{1}{\pi}\mu^2\frac{\partial\alpha_s}{\partial\mu^2}=\mu^2\frac{\partial a}{\partial\mu^2}=\beta(a)=-\beta_0a^2-\beta_1a^3-\beta_2a^4=$$
$$=-\beta_0a^2(1+C_1a+C_2a^2)$$
$$\beta_0=(11-\tfrac{2}{3}f)\tfrac{1}{4}\ ,\quad \beta_1=(102-\tfrac{38}{3}f)\tfrac{1}{16}$$
$$\beta_2=\left(\frac{2857}{2}-\frac{5033}{18}f+\frac{325}{54}f^2\right)\frac{1}{64}\,. \tag{3.3}$$

The additional contribution to $R(s)$ in eq.(3.2) appears after taking into account the effects of analytical continuation of the term $\ln^3(Q^2/\mu^2)\rightarrow(\ln(s/\mu^2)\pm i\pi)^3$ which arises in eq.(3.1) after the integration of the $a^3\ln^2(Q^2/\mu^2)$ term in the expression for the D-function. These effects have been discussed earlier in the case of e^+e^- -annihilation[13,14] and $J/\psi(\Upsilon)\rightarrow g\rightarrow$ hadron process[15] . An analogous correction has also appeared in calculations of the next-next-to-leading order corrections to the total hadronic decay width of the neutral Higgs boson of the standard electroweak theory[16] . As well as in that case[16] , taking into account the π^2 term decreases the numerical value of the analysed PT coefficient. Thus, we will not redefine the expansion parameter $\bar{\alpha}_s$ in the space-like region in contrast with the proposals of refs.[13,14] .

Solving the renormalization group (RG) equation, we obtain the following analytical expression for $R(s)$ in QCD in the $\overline{\text{MS}}$ - scheme :

$$
\begin{aligned}
R^{\overline{MS}}(s) = 3\sum Q_f^2 \Big\{ 1 + \bar{a} + \Big[\Big(\frac{730}{3} - 176\zeta(3)\Big) - \Big(\frac{44}{3} - \frac{32}{3}\zeta(3)\Big)f\Big]\frac{\bar{a}^2}{16} \\
+ \Big[\Big(\frac{3503881}{144} - \frac{473744}{27}\zeta(3) + \frac{25120}{9}\zeta(5)\Big) - \\
- \Big(\frac{62776}{27} - \frac{16768}{8}\zeta(3) + \frac{1600}{9}\zeta(5)\Big)f + \\
+ \Big(\frac{4832}{81} - \frac{1216}{27}\zeta(3)\Big)f^2 - \pi^2\Big(11 - \frac{2}{3}f\Big)^2\frac{4}{3}\Big]\frac{\bar{a}^3}{64}\Big\} \\
- \Big(\sum Q_f\Big)^2\Big(\frac{560}{27} - \frac{320}{3}\zeta(3)\Big)\frac{\bar{a}^3}{64},
\end{aligned}
\tag{3.4}
$$

where $\zeta(3) = 1.20205\ldots$, $\zeta(5) = 1.03692$ are the Riemann zeta functions. Notice the cancellations of $\zeta(4)$-terms in the obtained result. We have no explanation of this fact. In the numerical form eq.(3.4) reads

$$
\begin{aligned}
R^{\overline{MS}}(s) = 3\sum Q_f^2\{1 + \bar{a} + (1.986 - 0.115f)\bar{a}^2 + \\
+ (70.985 - 1.200f - 0.005f^2)\bar{a}^3\} - \Big(\sum Q_f\Big)^2 1.679\,\bar{a}^3 .
\end{aligned}
\tag{3.5}
$$

The term containing the factor $(\sum Q_f)^2$, which does not appear in the previous orders of PT, results from the QCD analogs of the QED light-by-light diagrams (see fig.1b)with SU(3)-group factors proportional to $d^{abc}d^{abc}$. This term is scheme independent. However, other coefficients do depend on the subtraction scheme used. The question of the scheme dependence of the results obtained will be considered elsewhere.

4. DISCUSSIONS OF THE RESULTS AND DETERMINATION OF THE PARAMETER $\Lambda_{\overline{MS}}$

We have obtained that in the $\overline{MS}$-scheme the coefficient of the next-next-to-leading correction to $R(s)$ is large. Thus, the question arises: how to involve $O(\bar{\alpha}_s^3)$ terms in procedures

of comparing theory with experiment and determining the correct values of the parameter $\Lambda_{\overline{MS}}$ in various regions of energies? Indeed, it is known (see, e.g. [17]) that PT series of quantum field theory are the asymptotic ones and, as distinct from $g\varphi^4$ and QED, in QCD they have sign constant character. It is commonly considered that the error of the sum of asymptotic series is estimated by the value of a first thrown away term[18]. Hence, in order to minimize the error, it is necessary to find a minimal perturbative term, throw it away and take into account all the preceding ones, So it is important to find the minimal term among PT terms with asymptotically increasing coefficients. However, in the model $g\varphi^4$ the asymptotic $n!$ growth predicted by the asymptotic estimates [17] has not been observed even at the five-loop level[19] . That is why we will first consider the case when the minimal term in the PT series for $R(s)$ is numbered among the unknown higher order terms and include the calculated correction in the analysis of the experimental data.

We will use the data obtained at the PETRA and PEP colliders far above the thresholds of the production of the b-quarks in the process $e^+e^- \to \gamma, Z^0 \to$ hadrons. The recent analysis of these data with taking into account both the value of the $O(\alpha_s^2)$ corrections in the $\overline{MS}$ - scheme and the known mass effects for the world average value of $\sin^2\theta_W = 0.23$ gives the results

$$\bar{\alpha}_s(34^2\ \mathrm{GeV}^2) = 0.169 \pm 0.025,\quad \bar{a}_{nl} = \alpha_s/\pi = 0.054 \pm 0.008\ [4],\ (4.1)$$

where index $\underline{nl}$ means that the next-to-leading correction has been taken into account*1.

*1. For the consideration of the more recent results[28] see Sect. 7.

Let us now take into account the calculated next-next-to-leading correction and find the corresponding value of the parameter $\Lambda_{\overline{MS}}$. The analysis will be made in two different ways: (a) the direct analysis in the $\overline{MS}$-scheme and (b) the analysis in the framework of the approach[20] known in the literature as the fastest appearant convergence (FAC) criterion (this approach has been also discussed in[21]). We will call it "the effective scheme approach". Substituting $f = 5$ into eq.(5) and introducing the index nnl to indicate the next-next-to-leading PT order and the index eff for the effective scheme results, we present the expression for R(s) in this region of energies in the following form

$$R(s) = R_0\left[1 + \bar{a}_{nnl} + r_1 \bar{a}_{nnl}^2 + r_2 \bar{a}_{nnl}^3 + \cdots\right] = R_0\left[1 + \bar{a}_{eff}^{(3)}\right]. \qquad (4.2)$$

where $r_1 = 1.411$ and $r_2 = 64.860$. The expression for R_0 can be found,e.g. in ref.[4] . It contains the information not only about γ - exchange, but about Z^0 - exchange and their interference also. We neglect in (4.2) the $(\Sigma Q_f)^2$ - term of eq. (3.5) since within dimensional regularization its generalization to the case of the Z^0 axial vector coupling needs additional careful consideration. It should be noted, however, that for $f = 5$ this scheme independent term is suppressed by both the small coefficient (see eq.(3.5)) and the factor $(\Sigma Q_f)^2/3\Sigma Q_f^2 = 1/33$. From the experimental result for $\bar{a}_{nl}$ we have that $\bar{a}_{eff}^{(2)} = \bar{a}_{nl}(1 + r_1 \bar{a}_{nl}) = 0.058 \pm 0.009$. Solving now numerically the equation $\bar{a}_{eff}^{(3)} = \bar{a}_{eff}^{(2)}$, we obtain the corrected value of

$\bar{\alpha}_s(34^2\ \mathrm{GeV}^2)$ in the $\overline{MS}$-scheme: $\bar{a}_{nnl} = 0.048^{+0.005}_{-0.006}$,
$\bar{\alpha}_s(34^2\ \mathrm{GeV}^2) = 0.151^{+0.016}_{-0.019}$.

It should be reminded that the introduced constants $\bar{a}_{nl}$, $\bar{a}_{nnl}$, $\bar{a}_{eff}^{(2)}$ and $\bar{a}_{eff}^{(3)}$ obey different RG equations of the types (3.3). They have the following forms:

$$\frac{\partial \bar{a}_{nl}}{\partial \ln|s|} = -\beta_0 \bar{a}_{nl}^2 (1 + c_1 \bar{a}_{nl}) \qquad (4.3)$$

$$\frac{\partial \bar{a}_{nnl}}{\partial \ln|s|} = -\beta_0 \bar{a}_{nnl}^2 (1 + c_1 \bar{a}_{nnl} + c_2 \bar{a}_{nnl}^2) \qquad (4.4)$$

$$\frac{\partial \bar{a}_{eff}^{(2)}}{\partial \ln|s|} = -\beta_0 \left(\bar{a}_{eff}^{(2)}\right)^2 (1 + c_1 \bar{a}_{eff}^{(2)}) \qquad (4.5)$$

$$\frac{\partial \bar{a}_{eff}^{(3)}}{\partial \ln|s|} = -\beta_0 \left(\bar{a}_{eff}^{(3)}\right)^2 \left(1 + c_1 \bar{a}_{eff}^{(3)} + \tilde{c}_2 \left(\bar{a}_{eff}^{(3)}\right)^2\right). \qquad (4.6)$$

For $f = 5$ from eq.(3.3) we have $\beta_0 = 1.917$, $c_1 = 1.261$, $c_2 = 1.475$ and $\tilde{c}_2$ can be found from the property that the quantity $\rho_2 = c_2 + r_2 - c_1 r_1 - r_1^2$ is scheme invariant[22-24] . In the effective scheme $r_1 = r_2 = 0$ we then have $\tilde{c}_2 = \rho_2 = 62.565$.

There are several methods of extracting the values of the parameter $\Lambda_{\overline{MS}}$. In the framework of the first of them one should exactly solve the RG equations (4.3)-(4.6). Let us introduce the following designations:

$$\Psi_{nl}(a) = \frac{1}{\beta_0 a} + \frac{c_1}{\beta_0} \ln \frac{c_1 a}{1 + c_1 a} \qquad (4.7)$$

$$\Psi_{nnl}(a, c_2) = \Psi_{nl}(a) + \frac{c_1}{2\beta_0} \ln \frac{(1 + c_1 a)^2}{1 + c_1 a + c_2 a^2} + \frac{2c_2 - c_1^2}{\sqrt{\Delta}\,\beta_0}\left[\mathrm{arctg}\,\frac{c_1 + 2c_2 a}{\sqrt{\Delta}} - \mathrm{arctg}\,\frac{c_1}{\sqrt{\Delta}}\right], \qquad (4.8)$$

where $\Delta = 4c_2 - c_1^2$. In the next-to-leading order the solutions of eq. (4.3), (4.5) in the $\overline{MS}$- and effective scheames read

$$\ln \frac{s}{\Lambda^2_{\overline{MS}}}\Big|_{\sqrt{s}=34\,GeV} = \Psi_{n\ell}(\bar{a}_{n\ell}) \qquad (4.9a)$$

$$\ln \frac{s}{\tilde{\Lambda}^2_{eff}}\Big|_{\sqrt{s}=34\,GeV} = \Psi_{n\ell}(\bar{a}^{(2)}_{eff}). \qquad (4.9b)$$

The parameter $\Lambda_{\overline{MS}}$ is connected with $\tilde{\Lambda}_{\overline{MS}}$ and $\tilde{\Lambda}_{eff}$ in the following ways:

$$\Lambda^2_{\overline{MS}} = \tilde{\Lambda}^2_{\overline{MS}}\left(\frac{c_1}{\beta_0}\right)^{c_1/\beta_0}$$

$$\Lambda^2_{\overline{MS}} = \Lambda^2_{eff}\exp(-r_1/\beta_0) = \tilde{\Lambda}^2_{eff}\exp(-r_1/\beta_0)\left(\frac{c_1}{\beta_0}\right)^{c_1/\beta_0}. \qquad (4.10)$$

Taking into account the numerical values for $\bar{a}_{n\ell}$, $\bar{a}^{(2)}_{eff}$, c_1, β_0 and r_1 we obtain from eqs. (4.9) and (4.10) the corresponding estimates in the framework of the $\overline{MS}$-scheme and effective scheme approaches:

$$(\Lambda_{\overline{MS}})_{n\ell} = 585^{+481}_{-320}\ \mathrm{MeV} \qquad (4.11a)$$

$$(\Lambda_{\overline{MS}})_{n\ell} = 553^{+415}_{-298}\ \mathrm{MeV}. \qquad (4.11b)$$

In order to take into account the next-next-to-leading PT corrections both to $R(s)$ and the β-functions, one should solve the following equations:

$$\ln \frac{s}{\Lambda^2_{\overline{MS}}}\Big|_{\sqrt{s}=34\,GeV} = \Psi_{nn\ell}(\bar{a}_{nn\ell}, c_2), \qquad (4.12a)$$

$$\ln \frac{s}{\tilde{\Lambda}^2_{eff}}\Big|_{\sqrt{s}=34\,GeV} = \Psi_{nn\ell}(\bar{a}^{(3)}_{eff}, \tilde{c}_2). \qquad (4.12b)$$

As a result, we find two corrected numerical values of the parameter $\Lambda_{\overline{MS}}$ in the framework of the $\overline{MS}$-scheme and effective scheme approaches

$$(\Lambda_{\overline{MS}})_{nn\ell} = 326^{+201}_{-169}\ \mathrm{MeV} \qquad (4.13a)$$

$$(\Lambda_{\overline{MS}})_{nn\ell} = 241^{+139}_{-117}\ \mathrm{MeV}. \qquad (4.13b)$$

Let us now find the values of $\Lambda_{\overline{MS}}$ in the framework of the second method which presupposes expansion of the solutions of eqs. (4.9), (4.12) in powers of $1/\ln(s/\Lambda^2)$. The corresponding representations of the running coupling constants can be expressed in terms of the following functions:

$$\varphi_{n\ell}(\Lambda) = \frac{1}{\beta_0 \ln(s/\Lambda^2)} - \frac{c_1}{\beta_0^2}\frac{\ln\ln(s/\Lambda^2)}{\ln^2(s/\Lambda^2)} \qquad (4.14)$$

$$\varphi_{nn\ell}(\Lambda, c_2) = \varphi_{n\ell}(\Lambda) + \frac{1}{\beta_0^3 \ln^3(s/\Lambda^2)}\left(c_1^2 \ln^2\ln(s/\Lambda^2) - c_1^2\ln\ln(s/\Lambda^2) + c_2 - c_1^2\right).$$

In the next-to-leading PT order we have

$$\bar{a}_{n\ell} = \varphi_{n\ell}(\Lambda_{\overline{MS}})\Big|_{\sqrt{s}=34\ GeV} \qquad (4.15a)$$

$$\bar{a}^{(2)}_{eff} = \varphi_{n\ell}(\Lambda_{eff})\Big|_{\sqrt{s}=34\,GeV}. \qquad (4.15b)$$

From eqs. (4.15) we obtain the corresponding estimates in the framework of the $\overline{MS}$-scheme and effective scheme approaches.

$$(\Lambda_{\overline{MS}})_{n\ell} = 600^{+466}_{-330}\ \mathrm{MeV} \qquad (4.16a)$$

$$(\Lambda_{\overline{MS}})_{n\ell} = 560^{+423}_{-297}\ \mathrm{MeV}. \qquad (4.16b)$$

After using the information about the next-next-to-leading order corrections instead of eqs. (4.25), we obtain

$$\bar{a}_{nn\ell} = \varphi_{nn\ell}(\Lambda_{\overline{MS}}, c_2)\Big|_{\sqrt{s}=34\,GeV} \qquad (4.17a)$$

$$\bar{a}^{(3)}_{eff} = \varphi_{nn\ell}(\Lambda_{eff}, \tilde{c}_2)\Big|_{\sqrt{s}=34\,GeV}. \qquad (4.17b)$$

Solving them numerically and taking into account the relation between $\Lambda_{\overline{MS}}$ and Λ_{eff} we find results which follow from the $\overline{MS}$-scheme and the effective scheme approaches

$$(\Lambda_{\overline{MS}})_{nnl} = 325^{+200}_{-175}\ \text{MeV} \tag{4.18a}$$

$$(\Lambda_{\overline{MS}})_{nnl} = 211^{+135}_{-104}\ \text{MeV}\,. \tag{4.18b}$$

Comparing the results (4.18) with (4.13) we observe that the values of $\Lambda_{\overline{MS}}$ depend on both the form of representing the solutions of the RG equations (compare eqs. (4.12) and (4.17)) and the ways of extracting the numerical values of the $\Lambda_{\overline{MS}}$ - parameter (either the $\overline{MS}$-scheme or the effective scheme approaches). This difference is due to different ways of taking into account the amounts of information about totally unknown and uncontrollable terms of order $O(\ell_n^{-4}(s/\Lambda^2))$. Notice, however, that the results $(\Lambda_{\overline{MS}})_{nnl} = 325^{+200}_{-175}$ MeV, obtained by analysing in the $\overline{MS}$-scheme, are almost insensitive to the methods of extracting its value (compare eqs. (4.13a) and (4.18a)). In any case, we arrive at the definite conclusion that taking into account the calculated $O(\alpha_s^3)$ next-next-to-leading corrections decreases twice the values of $\Lambda_{\overline{MS}}$. Thus, in addition to the observation made in ref.[25], we claim that in order to analyse self-consistently the QCD prediction for physical quantities it is important to use not only next-to-leading corrections but higher order PT effects as well. Indeed, the next-to-leading order corrections allow one to fix the renormalization scheme[25]. Using higher order corrections one can make more rigorous statements about the real numerical values of the parameter $\Lambda_{\overline{MS}}$ and the theoretical errors of the QCD PT series.

To minimise the theoretical uncertainties of the obtained

estimates one should find the region of intersection of the numerical intervals (4.13) and (4.18). The final results

$$(\Lambda_{\overline{MS}})_{nnl} = 157 - 346\ \text{MeV} \tag{4.19}$$

are in better agreement with the values of $\Lambda_{\overline{MS}}$, extracted from other processes[26] with taking into account the next-to-leading PT effects only, then the results (4.11), (416) obtained from the alalysis of $R(s)$ in the next-to-leading PT order. Of course the comparison should be made with care. Indeed, the values of $\Lambda_{\overline{MS}}$ also depend on the numbers of flavours taken into account in the analysis of experimental data (see,e.g.[24]). But this theoretical dependence does not affect the obtained conclusions since it is even smaller than the magnitudes of the experimental error bars of the numerical values of the parameter $\Lambda_{\overline{MS}}$ [24]. The large value of the calculated correction may indicate that for the presently available energies of PEP and TRISTAN and even for the future energies of SLC and LEP the correction is experimentally detectable and should be involved in procedures of analysing the e^+e^- - data.

We are formulating this important conclusion with a bit of caution since for the obtained values of the running coupling constant α_s in the $\overline{MS}$-scheme $\alpha_s(34^2\ \text{GeV}^2) = 0.151$ the PT series for $R(s)$ obtained from eq. (4.2) has the form

$$R(s) = R_0(1 + 0.048 + 0.003 + 0.007 + \ldots)\,. \tag{4.20}$$

The inclusion of the next-next-to-leading correction in the fit of the experiment presumes that, firstly, the $O(\alpha_s^2)$ - correction in eq. (4.20) is accidentally small, and secondly, the unknown $O(\alpha_s^4)$ corrections to $R(s)$ are not large.

Let us now discuss the possibility that the asymptotic nature of the QCD PT series manifests itself already at the level of the next-next-to-leading correction, that is to say the series (4.20) asymptotically explodes at this level. Further, following the ideology of asymptotic expansions[18] , one must truncate the series (4.20) on the minimal third term and take into account only the tree and leading order terms. In this case it is impossible to determine self-consistently the value of the parameter $\Lambda_{\overline{MS}}$ since all the information about the scheme-dependence is absorbed in the truncated term. Thus, in order to compare self-consistently the predictions of QCD PT series with experiment, it becomes very important to develop methods of summations of QCD sign-constant series.

Note that even if the series (4.20) does not blow up for the considered energies $\sqrt{s} \simeq 34$ GeV, we face the problems discussed above in the low-energy region. Indeed, the calculated by us $O(\bar{\alpha}_s^3)$ correction in eq.(3.5) becomes comparable with the leading $O(\bar{\alpha}_s)$ term for $\bar{\alpha}_s \sim 0.380$. For these values of $\bar{\alpha}_s$ the PT approximation for R(s) obviously blows up. It seems, that in the region of larger values of $\bar{\alpha}_s$ the quantity R(s) should be approximated by only two terms of the PT series (3.5) with all that this implies.

To conclude this section we want to note that from the point of view of studying the region of applicability of asymptotic QCD PT predictions, it is highly desirable (i) to estimate the contributions of the $O(\bar{\alpha}_s^4)$ corrections to $R(s)$ which is an unrealistic problem, and (ii) to calculate the effects of the next-next-to-leading order corrections to characteristics of other physical processes, say, deep inelastic lepton-hadron scattering. This is practicable due to the existence of the methods of calculating the necessary corrections in the MS-scheme of both anomalous dimensions of composite operators[9] and coefficient functions of operator product expansions[27] .

6. CONCLUSION

We have calculated the next-next-to-leading $O(\bar{\alpha}_s^3)$ QCD correction to $R(s) = \sigma_{tot}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. The obtained correction is large, e.g., in the $\overline{MS}$-scheme at $\sqrt{s} = 34$ GeV it is over 2.5 times larger than the previous next-to-leading correction. Of course, it cannot be ruled out that some contributions of the lower PT corrections are accidentally small. That is why we include the $O(\bar{\alpha}_s^3)$ correction in the procedure of analysing PEP and PETRA data. As a result, taking into account this correction drastically (twice) decreases the value of $\Lambda_{\overline{MS}}$. To obtain final estimates we have considered two forms of representing the running coupling constant in terms of the Λ -parameter ((I) explicit solutions of RG equations, (II) reexpansions of these solutions in terms of $1/\ln(s/\Lambda^2)$) and two ways of comparing QCD predictions with experiment based on application of (a) the $\overline{MS}$-scheme and (b) the effective scheme approaches. We have got the following four results: (Ia) $(\Lambda_{\overline{MS}})_{nnl} = 326^{+201}_{-169}$ MeV; (Ib) $(\Lambda_{\overline{MS}})_{nnl} = 241^{+139}_{-117}$ MeV (IIa) $(\Lambda_{\overline{MS}})_{nnl} = 325^{+200}_{-175}$ MeV and (IIb) $(\Lambda_{\overline{MS}})_{nnl} = 211^{+135}_{-104}$ MeV. They intersect in the region $(\Lambda_{\overline{MS}})_{nnl} = 157\text{-}346$ MeV. We also consider the possibility that the asymptotic PT series explodes already at the level of the next-next-to-leading corrections, discuss the regions of applicability of the obtained results and comment upon the problems of involving the calculated corrections into the procedure of analysing the e^+e^- data in various regions of energies.

We are grateful to V.A. Matveev, D.V.Shirkov and A.N.Tavkhelidze for interest in the work, constant support and useful discussions. It is a pleasure to thank K.G.Chetyrkin, D.I.Kazakov,

N.V.Krasnikov, S.A.Kulagin, O.V.Tarasov, F.V.Tkachov, M.E.Shaposhnikov, A.A.Vladimirov and other researches of the theoretical divisions of both INR and JINR for useful discussions at different stages of the work.

7. NOTE ADDED

After this work has been written, we became aware that a more refined analysis of the combined PETRA and PEP data with an improved treatment of the QED radiative corrections gives

$\bar{\alpha}_s(34^2\ \mathrm{GeV}^2) = 0.145 \pm 0.020$

for $\sin^2\Theta_W = 0.23$ [28]*2. Revising our analysis with respect to this new information instead of estimates of eqs. (4.11), (4.16), obtained in the next-to-leading order in the framework of the discussed above approaches, we have the following estimates: (Ia) $(\Lambda_{\overline{MS}})_{nl} = 265^{+226}_{-147}$ MeV; (Ib) $(\Lambda_{\overline{MS}})_{nl} = 255^{+221}_{-145}$ MeV; (IIa) $(\Lambda_{\overline{MS}})_{nl} = 280^{+230}_{-165}$ MeV; (IIb) $(\Lambda_{\overline{MS}})_{nl} = 270^{+228}_{-153}$ MeV. Including the calculated $O(\bar{\alpha}_s^3)$ correction into the analysis, we obtain the corrected value of the running coupling constant in the $\overline{MS}$-scheme $\bar{\alpha}_s(34^2\ \mathrm{GeV}^2) = 0.132^{+0.013}_{-0.016}$ and the new extimates of the parameter $\Lambda_{\overline{MS}}$: (Ia) $(\Lambda_{\overline{MS}})_{nnl} = 157^{+104}_{-86}$ MeV; (Ib) $(\Lambda_{\overline{MS}})_{nnl} = 124^{+88}_{-65}$ MeV; (IIa) $(\Lambda_{\overline{MS}})_{nnl} = 155^{+125}_{-85}$ MeV; (IIb) $(\Lambda_{\overline{MS}})_{nnl} = 107^{+80}_{-55}$ MeV, which should be compared with the previous estimates of eqs. (4.13), (4.18). These intervals intersect in the region $(\Lambda_{\overline{MS}})_{nnl} = 71\text{-}187$ MeV, which is the new estimate of this parameter obtained from PETRA and PEP data provided the considered PT series is not yet in the asymptotic regime for these energies.

*2 Note, however, that in ref. [29] this result has been attributed to $\sqrt{s} = 43$ GeV.

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Received by Publishing Department
on April 18, 1988.

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Следующая за нелидирующей поправка КХД порядка $O(\bar{a}_s^3)$ к σ_{tot} $(e^+e^- \to$ адроны): аналитическое вычисление и оценка параметра $\Lambda_{\overline{MS}}$

Вычислена следующая за нелидирующей поправка КХД порядка $O(\bar{a}_s^3)$ к σ_{tot} $(e^+e^- \to$ адрона). Найденная поправка оказалась большой. Например, при $\sqrt{s}=34$ ГэВ в схеме $\overline{MS}$ она приблизительно в 2,5 раза больше, чем предыдущий член порядка $O(\bar{a}_s^2)$. Учет найденной поправки в процедуре фита комбинированных данных PETRA и PEP уменьшает в два раза значение параметра $\Lambda_{\overline{MS}}$. Обсуждается статус пертурбативных предсказаний КХД.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

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Next-Next-to-Leading $O(\bar{a}_s^3)$ QCD Correction to σ_{tot} $(e^+e^- \to$ Hadrons): Analytical Calculation and Estimation of the Parameter $\Lambda_{\overline{MS}}$

The next-next-to-leading $O(\bar{a}_s^3)$ QCD correction to σ_{tot} $(e^+e^- \to$ hadrons) is calculated. The obtained correction is large. For example at $\sqrt{s} \simeq 34$ GeV in the $\overline{MS}$ - scheme it is about 2.5 times larger than the previous $O(\bar{a}_s^2)$ term. Taking into account this correction in the fit of the combined PETRA and PEP data decreases twice the value of $\Lambda_{\overline{MS}}$. The status of perturbative QCD is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988