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# EVOLUTION OF THE PARTICLE SPECTRUM SPONTANEOUSLY CREATED IN THE EXPANDING UNIVERSE

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## 1. Introduction

The evolution laws of quantum fields in classical background fields have been elaborated during the last two decades (for some reviews see Birrell and Davies 1982, Parker 1977, Grib et al. 1980). A natural framework where this special branch of quantum field theory applies is given by the cosmological evolution.

As stated firstly by Sexel and Urbantke (1967) and Parker (1968, 1969) the dynamics of the gravitational background field causes a nonconservation of the particle number observable. Thus, in an expanding universe, particles with finite rest mass which are conform-invariantly coupled to the gravitation field, are created spontaneously. The evolution equations for the spectra of the spontaneously created particles have been derived, e.g. for Bosons by Grib and Mamaev (1969, 1971) and Grib et al. (1971) and for Fermions by Mamaev et al. (1976). The particle spectra depend on the actual dynamics of the classical gravitation field, for which one needs some model.

According to the cosmological standard model, the early phases of the universe were quickly evolving accompanied by mostly adiabatically cooling background radiation. This basically simple and smooth picture is brought off equilibrium in eras where, according to our present understanding of elementary particle physics, symmetry breaking processes occured, such as the GUT symmetry breaking, the electroweak symmetry breaking and the confinement transition. These phases become the inflationary ones, during which the space is anomalously fastly expanding driven by the vacuum energy density (Guth and Weinberg 1981).

The available estimates of the cosmological particle production (cf. Birrell and Davies 1982) up to now neglect such non-smooth evolution scenarios and are constrained to rather simplified model universes, e.g. with power law expansion (cf. Audretsch and Schäfer 1978, Grib et al. 1980).

The present work is aimed to report evaluations of the spectra of spontaneously created particles on distinguished mass scales in a different evolution scenario. Namely we consider a universe being initially radiation dominated; at a certain time the vacuum energy is supposed to dominate, thus causing an inflationary expansion during the supercooling. After terminating the inflationary interlude, the universe evolves again as radiation dominated one.

In exploiting the method of Hamiltonian diagonalisation we follow the history of both the very heavy particles ranging on the GUT mass scale and the lighter particles ranging from the electroweak scale down to the conjectured finite neutrino mass. Particularly we address our present work (i) to compare with previous results (Grib et al. 1980, Audretsch and Schäfer 1978) when beginning with a finite aged universe, (ii) to investigate the influence of inflationary phases and (iii) to check how sharp the spontaneous particle creation does end up at the Compton time of the respective particle type. Especially we are interested in the question whether the density of spontaneously created particles can become comparable with the radiation background, thus offering a new channel for huge entropy production useful for solving, e.g. the monopole problem (Kämpfer et al. 1987).

The paper is organised as follows. In section 2 we briefly recall the Hamiltonian diagonalisation method. In section 3 we describe the scenario which we consider, i.e. the inflationary interlude in an otherwise radiation dominated universe. The results obtained in solving the evolution equations for the spontaneously created particle spectra are reported in section 4. The physical implications and the summary of our findings are given in section 5.

## 2. Hamiltonian diagonalisation method

Hamiltonian diagonalisation is a suitable means for describing particle production in an external field. The formalism is decribed in detail, e.g. by Grib et al. 1980, thus we will restrict here ourselves to a short outline.

We consider a massive, real, scalar field, being conformally coupled to the classical gravitational field, with the Lagrangian (cf. Chernikov and Tagirov 1968 for a foundation)

$$L = {}^1/_2\ (-g)^{1/2}\ \{g^{\mu\nu}\,\phi_{,\mu}\,\phi_{,\nu} - (m^2 + {}^1/_6\ R)\phi^2\} \qquad (1)$$

in a spatially flat Friedmann Robertson Walker metric

$$ds^2 = dt^2 - R(t)^2\ (dx^2 + dy^2 + dz^2). \qquad (2)$$

The corresponding Hamiltonian reads

$$H(t) = R^4 \int d^3x\ \{-{}^1/_4\ \dot\phi + {}^1/_2\,\dot\phi^2 + {}^1/_4\ \dot R\,\phi\,\dot\phi\ /\ R - {}^1/_6\ R^{-2}\,\eta^{ab}\,\phi_{,a}\,\phi_{,b} - {}^1/_{12}\ R^{-2}\,\eta^{ab}\,\phi_{,a,b}\,\phi + {}^1/_4\ m^2\phi^2 + {}^1/_4\ \phi^2\ (\dot R^2\ R^{-2} - \ddot R\,/\ R)\}, \qquad (3)$$



where $\eta_{ab}$ denotes the spatial metric and a,b = 1,2,3. After inserting the mode expansion

$$\phi(x,t) = (2\pi^3)^{-1/2} \int d^3k\ R^{-1}\ (2\ k)^{-1/2}\ \{\phi(k)\ H(k,t)\ e^{ik\,x} + h.\ c.\} \qquad (4)$$

one recovers diagonal as well as off-diagonal terms of the Hamiltonian. Using the Bogoliubov transformation

$$\phi(k) = \alpha^*(k,t)\ \Phi(k,t) - \beta^*(k,t)\ \Phi^+(-k,t) \qquad (5)$$

the Hamiltonian becomes diagonal if the coefficients obey the equations

$$\dot\alpha = {}^1/_2\,\dot\omega\,\omega^{-1}\ \beta\ \exp\{2\,i\int dt'\,\omega(t')\ /\ R(t')\},$$
$$\dot\beta = {}^1/_2\,\dot\omega\,\omega^{-1}\ \alpha\ \exp\{-2\,i\int dt'\,\omega(t')\ /\ R(t')\}, \qquad (6)$$

where

$$\omega = (k^2 + m^2\ R^2)^{1/2}, \qquad (7)$$

and a dot means the derivative with respect to t. Instead of the two complex Bogoliubov coefficients $\alpha$ and $\beta$ which fulfil the subsidary condition $\alpha^*\,\alpha - \beta^*\,\beta = 1$, it is more convenient to introduce the three real functions

$$s(k,t) = \beta^*(k,t)\ \beta(k,t), \qquad (8)$$
$$u(k,t) = 2\ \mathrm{Re}\{\alpha\ (k,t)\beta^*\ (k,t)\ \exp[-2i\int d\tau\,\omega\ (\tau)/R(\tau)]\},$$
$$v(k,t) = -2\ \mathrm{Im}\{\alpha\ (k,t)\beta^*\ (k,t)\ \exp[-2i\int d\tau\,\omega\ (\tau)/R(\tau)]\}.$$

Exploiting eq.(6) one obtains the following set of coupled differential equations

$$\dot s(k,t) = {}^1/_2\,\dot\omega\,\omega^{-1}\ u(k,t),$$
$$\dot u(k,t) = \dot\omega\,\omega^{-1}\ (2\ s(k,t) + 1) - 2\,\omega\ v(k,t)\ /\ R(t),$$
$$\dot v(k,t) = 2\ \omega\ u(k,t)\ /\ R(t). \qquad (9)$$

The initial conditions

$$s(k,t_0) = u(k,t_0) = v(k,t_0) = 0 \qquad (10)$$

reflect the absence of the respective particles at $t_0$.

The particles density can be calculated from the formula (we use here units with $h/2\pi = c = 1$)

$$dn(k,t) = [(2\ \pi^2\ R^3(t)]^{-1}\ s(k,t)\ k^2\ dk \qquad (11)$$

indicating that s(k,t) represents the time-dependent distribution function.

It should be noticed that the Hamiltonian diagonalisation method must be applied with some care (cf. Birrell and Ford 1979) with respect to the underlying particle concept in fastly varying external fields. Otherwise, it has been proven that the results recover the findings obtained with the in-out formalism (Grib et al. 1980). The latter fact enforces us to apply the Hamiltonian diagonalisation method as suitable approach for investigating the time dependence of the spontaneously created particles.

The influence of the interactions has been estimated to range on the per cent scale (cf.Lotze 1985).

## 3. Setting the scenario with an inflationary interlude

Hitherto it has been usual to solve eqs.(9) by considering limiting cases, e.g. the very beginning of the particle production or the times after the Compton time (Grib et al. 1976, Mamaev et al. 1976). Other authors exploit the in-out formalism (Parker 1968, Audretsch and Schäfer 1978, and further refs. in Birrell and Davies 1982) and are restricted to rather simple evolution scenarios; e.g. Audretsch and Schäfer (1978) consider the pure radiation universe matched in addition to a mirror universe.

Here we wish to model an inflationary epoch as intermezzo in an otherwise radiation dominated universe. To be explicit, we consider the scale factor (De Grand and Kajantie 1984, Kämpfer et al. 1987, Kämpfer 1988)

$$R(t) = R_0 \, \mathrm{sh}^{1/2} \, (2 \, C \, B^{1/2} \, t) \tag{12}$$

with $C = (8 \pi / 3)^{1/2} / M_p$, $M_p = 1.22 \cdot 10^{19}$ GeV being the Planck mass. The quantity B denotes the vacuum energy density which is estimated by $B \sim T_c^4$, where $T_c$ stands for the critical temperature for a symmetry breaking phase transition. The latter one can be the GUT transition ($T_c \sim 10^{15}$ GeV), the electroweak transition ($T_c \sim 10^2$ GeV) or the confinement transition ($T_c \sim 10^{-1}$ GeV). For early times, $t \ll (C B^{1/2})^{-1}$, eq.(12) describes the adiabatically cooling radiation dominated universe with $R \sim t^{1/2}$. The vacuum energy dominates at $t > t_{inf}$,

$$t_{inf} = (2 \, C \, B^{1/2})^{-1}, \tag{13}$$

and the space approaches quickly the inflationary expansion $R \sim \exp \{ 2 \, C \, B^{1/2} \, t \}$. We assume a supercooling up to $t = q \; t_{inf}$ with q as yet unspecified parameter to fix the end of the inflation era. In the approximation of a sudden phase transition we describe the subsequent era, where all vacuum energy is transformed into thermal (radiation) energy, by

$$R = R_1 \, t^{1/2} \quad \text{for} \quad t > q \, t_{inf} \tag{14}$$

with $R_1 = R_0 \, \mathrm{sh}^{1/2} \, (2 \, C \, B^{1/2} \, t_{inf} \, q)$.



The given scenario should be considered as semirealistic model for a symmetry breaking phase transition (DeGrand and Kajantie 1984, Kämpfer 1987, 1988). It reflects the main features needed for exploring the effect of inflationary growth of the space onto the spontaneous particle production. Other, more involved scenarios are considered, e.g. by Guth and Weinberg (1981), Albrecht and Steinhardt (1982), Linde (1984) and Kämpfer et al.(1987). Having a definite evolution scenario at disposal we can try to solve the eqs.(9). In doing this we find it convenient to introduce the new time variable $\tau$ via

$$\tau = \log \{ t \, M_p \} \tag{15}$$

and a dimensionless momentum variable suggested by the form of eq.(11)

$$p(k,\tau) = k \, / \, m \, R \tag{16}$$

being the ratio of the physical momentum $k / R$ to the respective particle mass. Now, eqs.(9) take the form

$$\begin{aligned} s' &= {}^1/2 \, (R' / R) \, u \, \Omega^{-2} \\ u' &= (R' / R) \, (2 \, s + 1) \, \Omega^{-2} - 2 \, m \, e^{\tau} \, \Omega \, v \, / \, M_p \\ v' &= 2 \, m \, e^{\tau} \, \Omega \, u \, / \, M_p \end{aligned} \tag{17}$$

with $\Omega = (1 + p^2)^{1/2}$ and s, u, v and p as funtions of k and $\tau$: a dash means the derivative with respect to $\tau$.

## 4.Results

Now we report the results in solving eqs.(17). In figure 1 the spectra as function of p are displayed for different times in case of $m/M_p = 10^{-4}$. The respective particles range on the GUT mass scale and might represent the Higgs boson $M_x$. Up to the Compton time $\tau_c \sim 9$ the spectrum remains fairly constant indicating massive particle production. Afterwards the particle production becomes ineffective, and the already produced particles become red shifted. The quadratic increase of the lhs of the spectrum peak suggest a similarity with a thermal distribution - but we find that thermal distributions are much narrower. The above mentioned

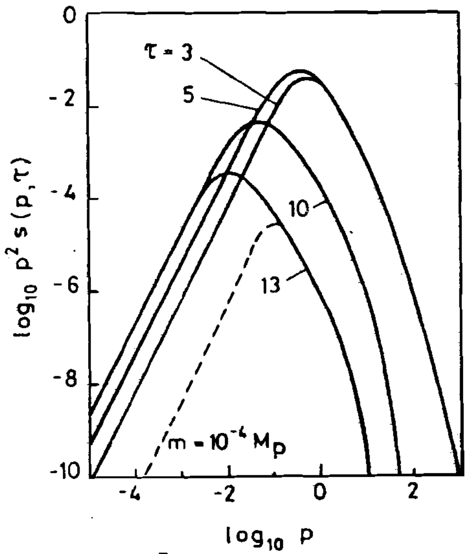


Figure 1: The spectra $p^2 * s(p,t)$ of spontaneously created particles with mass $m = 10^{-4}\ M_p$ at different times.
heavy lines: particle creation starts at Planck time or earlier.
dashed line: particle creation starts at t = 10.

inflation era apoear much later, so that for such heavy particles the spontaneous production process ceases before inflation. We mention that the spectra approach very fastly to their "saturation" form. i.e. there is no difference whether the particle production starts at Planck time $\sim 1/M_p$ or somewhat before. Otherwise, if the particle production starts shortly before the respective Compton time $\sim 1/m$, the spectral density remains below the "saturation" values as displayed in fig.1. This holds for all lighter particles too.

Let us now consider particles the Compton time of them being after the inflation epoch. In figure 2 the spectra are displayed for $m/M_p = 10^{-17}$. The respective particles range on the electroweak mass scale and might represent, e.g. the Higgs bosons $W^{\mp}$, Z. Qualitatively our numerical findings apply to all lighter particles too. The inflationary epoch is determined by our choice $T_c = 10^{15}$ GeV, being representative for the GUT transition. Interestingly, the spectra $p^2 * s(p,\tau)$ do not reflect any



particularity when compared with a pure radiation universe or when varying the duration of the inflation (determined by the value of q). At the very beginning ($\tau < 10$), however there appears the linear increase of the spectra for not too small values of p indicating clearly a non-thermal distribution. The spectrum remains constant up the Compton time $\tau_c \sim 39$. When approaching to Compton time, the rhs of the spectrum becomes steeper and steeper. After the Compton time the red shift becomes operative. The peak of the spectrum decreases and is red shifted too. Eventually, after long times, the peak is so far red shifted, that the lhs side shows a quadratic increase, and the spectrum can be

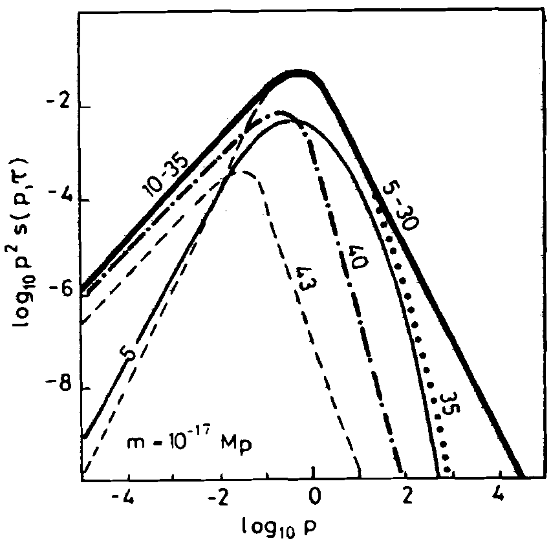


Figure 2: The spectra $p^2 * s(p,t)$ of spantaneously created particles with mass $m = 10^{-17}\ M_p$ at different times (heavy, dotted, dashed and dot-dashed lines). The inclusion of an inflationary supercooling period at $t \sim 20$, parametrised by $T_c = 10^{15}$ GeV and $q < 1.2$ in eq.(14) does not alter the spectra. The numbers denote time or time intervalls t. The thin line depicts a spectrum at t = 35 for the case of spontaneous particle creation starting at $t_0 = 30$; it indicates that the particle modes did not reach their saturation.

represented by a thermal distribution (Notice that huge oscillations of the Bogoliubov coefficients after Compton time hamper the numerical treatment by standard methods).

The difference between radiation universe and the appearence of an inflationary epoch becomes evident when considering the distribution function $s(\varkappa = k/m, \tau)$ as function of $\tau$ for several values of k as displayed in figure 3. In figure 3a (pure radiation

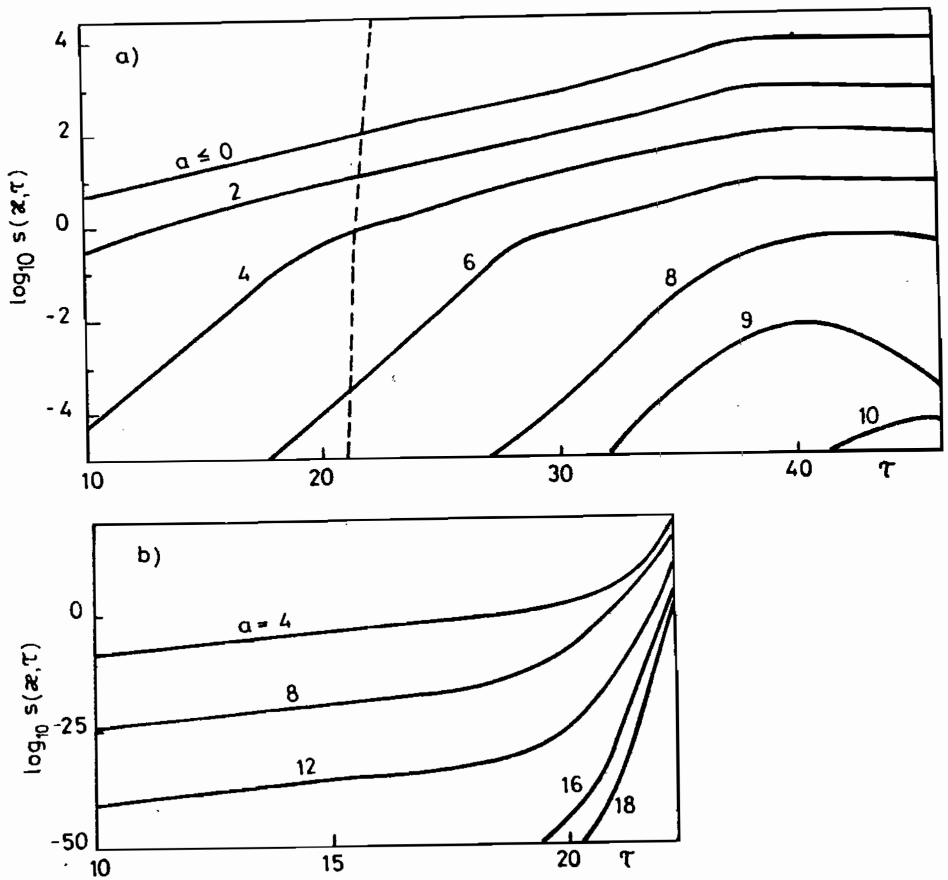


Figure 3: Time evolution of the distribution function s(k,t) for different modes $\varkappa = k/m = 10^{a}$ for $m = 10^{-17}\ M_p$.

a: pure radiation universe. For the sake of comparison the dashed line depicts the mode a = 8 in case of inflation as in b.

b: universe evolving into an inflationary period as described in figure 2.



universe) one observes (i) two different growth modes, $\sim e^{\tau}$ and $\sim e^{\tau/2}$ before Compton time and (ii) the constancy of the low-energy part of the spectrum after the Compton time while the high-energy part is slightly decreasing, thus causing the steepening of the rhs of the spectra. In figure 3b there is displayed, on another scale, the inflationary particle production, showing the very steep increase of the occupation numbers of different modes when the inflation starts.

## 5. Physical implications and summary

From figures 1 and 2 one reads off that the particle density defined in eq.(11) remains fairly constant up the Compton time. An inflationary interlude does not alter this picture. The present analysis suggests that the spectra $p^2 * s(p,\tau)$ are independent of the actual form of $R(\tau)$.

The constant particle density, up to Compton time, suggests the existence of a constant equivalent "temperature" scale in the otherwise adiabatically cooling radiation universe. Such a "temperature" $T_{eq}$ can be defined, even for non-thermal distribution functions, via the entropy density $\sigma$ and $T_{eq} \sim \sigma^{1/3}$. This is one interesting outcome of our investigations. A second point concerns the relation to the density of the radiation background particles. The latter ones undergo the adiabatically cooling with temperature $T_{bg} = T_p\ R_p\ /\ R$, where the index p denotes the Planck time. Estimating the density of the spontaneously produced particles by $n \sim m^3$ (cf. figure 2) we have

$$n / n_{bg} \sim (m / M_p)^3\ (R / R_p)^3, \qquad (18)$$

where $n_{bg} \sim T_{bg}^{\ 3}$ is the background density and $R_p$ is choosen so that $T_{bg}(\tau_p) = T_p$. After inserting eq.(12) for R one observes that $n / n_{bg} \sim 1$ at $\tau = 21.85$. That means, if the adiabatic cooling during inflation (Guth and Weinberg 1981) proceeds sufficiently long (in the given example with $T_c = 10^{15}$ GeV, $m = 10^{2}$ GeV it means $q = 1.15$) the spontaneously created particle density exceeds the radiation background density. This is the second important point to be noticed. Of course, in such a situation, the interactions and the dynamical feed-back onto the evolution of $R(\tau)$ can not be longer neglected.

Finally we mention a point to be investigated in a subsequent paper. Namely, we assumed that the particles' rest mass remains constant during the whole evolution. Current unifying theories, however, suggest the mass generation via spontaneous symmetry breaking, thus particle masses have their own dynamics. To build in this idea, the consistent coupling of the mass generating Higgs field to the gravitational background field must be included. We conjecture that terms $\sim m^{\cdot}$ will enter in eq.(9) and will drive in addition the spontaneous particle production.

In summary, we consider the time evolution of the spectra of spontaneously produced massive particles being conform-invariantly coupled to the classical gravitational background field in an expanding universe. We find that, up to Compton time, these mentioned particles define a constant "temperature" scale in the otherwise expanding and cooling universe. A short inflationary supercooling period, when before Compton time, does not alter the spectra of the spontaneously produced particles. If the supercooling epoch is sufficiently long, than the density of spontaneously created particles can exceed the density of quanta of the radiation background field, thus, e.g. offering the possibility of strong entropy production being important for solving problems of the cosmic standard model (Kämpfer et al. 1987). After Compton time the particle production sharply stops and the spectra become red shifted.

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