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THE 1975 JINR-CERN SCHOOL OF PHYSICS

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Electromagnetic Interactions of Leptons and Hadrons

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Notation

We adopt the following notations and conventions [1,2]

1) The metric tensor $g^{\mu\nu}$ is given by

$$g^{00}=1,\ g^{11}=g^{22}=g^{33}=1;\quad g^{\mu\nu}=0,\ \mu\neq\nu,$$
$$\mu,\nu=0,1,2,3.$$

The scalar product of four-vectors

$$a\cdot b = a^{\mu}b_{\mu} = g^{\mu\nu}a_{\mu}b_{\nu} = a^{0}b^{0}-\vec{a}\cdot\vec{b},$$

where

$$a^{\mu}=(a^{0},\vec{a}),\quad a_{\mu}=(a^{0},-\vec{a}).$$

In particular on the mass shell

$$p^{2}=\varepsilon^{2}-\vec{p}^{\,2}=M^{2},\quad \epsilon=\sqrt{\vec{p}^{\,2}+M^{2}}=p^{0}.$$

2) The anticommutator of γ-matrices

$$\gamma^{\mu}\gamma^{\nu}+\gamma^{\nu}\gamma^{\mu}=2g^{\mu\nu}.$$

The Dirac equation for positive frequency spinor wave functions reads as

$$(\hat{p}-M)\,u(p)=0,$$

where

$$\hat{p}=\gamma\cdot p=\gamma^{0}p^{0}-\vec{\gamma}\cdot\vec{p}.$$

The spinor $u(p)$ is normalized by the condition

$$\bar{u}(p)\, u(p) = 1 , \quad \bar{u} = \overset{*}{u} \gamma^0 .$$

For negative frequency spinors we have

$$(\hat{p} + M)\, v(p) = 1 ,$$

$$\bar{v}(p)\, v(p) = -1 .$$

The matrix

$$\gamma_5 = i\, \gamma_0 \gamma_1 \gamma_2 \gamma_3 .$$

Under the hermitian conjugation

$$\overset{+}{\gamma}_0 = \gamma_0 , \quad \overset{+}{\vec{\gamma}} = -\vec{\gamma} , \quad \overset{+}{\gamma}_5 = \gamma_5 .$$

3) The one-particle state vectors are normalized as

$$\langle p' | p \rangle = \frac{\mathcal{E}(\vec{p})}{M} (2\pi)^3 \delta^3 (\vec{p} - \vec{p}') ,$$

$$\mathcal{E}(\vec{p}) = \sqrt{\vec{p}^{\,2} + M^2} .$$

1. Deep inelastic electron-nucleon scattering

Since the electron to a high accuracy is an elementary point-like particle with the well known electromagnetic properties, electron scattering is an ideal probe of the structure of other more complex objects such as atoms, nuclei, and hadrons. By bombarding a target with the beam of known energy and detecting only the outgoing electrons, one can determine the charge and magnetic moment distributions within the object and hence, gain information on the constituents inside. Thus, we possess a unique opportunity as if to glance into the composite object and in this meaning one could speak about the "leptonic illumination" (Bjorken).

In the case of hadrons like the nucleon we know very little about their basic structure elements, though we believe that we are dealing with composite objects in that sense or another. So much interesting appeared the results of the experiments on the deep inelastic electron-nucleon scattering, which revealed the point-like behaviour of cross sections for these inclusive processes. The idea about such a kind of behaviour of the total cross sections for lepton-hadron reactions time was first pur forward by prof. M.A.Markov more than ten years ago.

The discovered behaviour of the cross section may be interpreted in such a way, as if electron (muon) scatters on quasifree point-like elements (partons) constituting a nucleon.

In addition to a large (point-like) magnitude of the cross section, experiments revealed also a remarkable regularity, namely, the scale invariant (automodel) behaviour of inelastic structure functions, i.e., the absence of any dimensional parameters, characterizing the structure of nucleon constituents (Bjořken, Bogolubov, Matveev, Muradyan, Tavkhelidze).

In the other language, employing the duality ideas, one can say that a substantial part of the scaling behaviour emerges as a result of averaging over a sum of large number of nucleon resonances, exited by an incoming lepton (Bloom and Gilman).

While preparing this section we widely used lectures and reviews [3-6].

1.1. Kinematics of the process

Consider the inclusive reaction of inelastic electron-nucleon scattering

$$e + N \longrightarrow e' + \underbrace{\sum_n \text{hadrons}}_{\text{unobserved}} \qquad (1.1)$$

where only the final electron is detected. This process in one-photon (e^2) approximation is shown graphically in Fig.1.

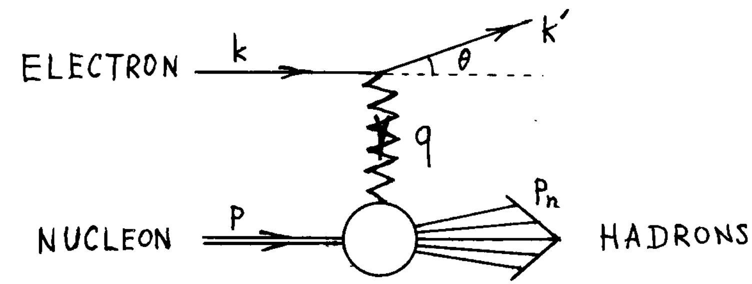


Fig.1

where k and k' are the initial and final four-momenta of an electron of mass m , $q = k - k'$ is the four-momentum transfer carried by the virtual photon, and p is the target nucleon's four-momentum with $p^2 = M^2$.
Hadrons in the final state $|n\rangle$ owing to the conservation law have the total four-momentum

$$P_n = p + q$$

and the effective invariant mass squared

$$P_n^2 = s = (p+q)^2 = M^2 + q^2 + 2(p\cdot q), \qquad (1.2)$$

It is also helpful to introduce the invariant variable (M is the nucleon mass)

$$\nu = \frac{(p\cdot q)}{M} \qquad (1.3)$$

which in laboratory (lab.) frame of reference (initial nucleon at rest, $\vec{p} = 0$) is equal to the virtual photon's energy (or the electron energy transfer)

$$\nu = q^0_{lab} = E - E'$$

where

$$E = (p\cdot k)/M \quad , \quad E' = (p\cdot k')/M$$

are the energies of the initial and final electrons in the lab.frame. The invariant momentum transfer squared

$$q^2 = (k-k')^2 = 2m^2 - 2(k\cdot k') \; , \quad k^2 = k'^2 = m^2$$

in the lab.frame takes the form

$$q^2 = -2EE'(1-\cos\theta) = \\ = -4EE'\sin^2\frac{\theta}{2} < 0 \, , \tag{1.4}$$

where θ is the scattering angle and the electron mass has been neglected compared to its energy. We adhere this approximation throughouti in what follows. Sometimes we shall also use positive-definite variable

$$Q^2 = -q^2 \geqslant 0 \tag{1.5}$$

Knowing ν and q^2 from measuring the incident and scattered electron, one can easily determine from eq. (1.2) the effective mass squared of the final hadrons

$$s = M^2 + q^2 + 2M\nu . \tag{1.6}$$

Using the selfevident inequality

$$s \geqslant M^2$$

one immediately obtains the boundary of the physical region of inelastic electroproduction

$$q^2 + 2M\nu \geqslant 0 \quad , \quad Q^2 \leqslant 2M\nu . \tag{I.7}$$

In the following we shall often use the dimension variable

$$\omega = -\frac{2M\nu}{q^2} = \frac{2M\nu}{Q^2} = \frac{s-M^2}{Q^2} + 1 \tag{I.8}$$

in terms of which inequality (I.7) takes the form

$$\omega \geqslant 1 . \tag{I.7a}$$

The inequality $q^2 \leqslant 0$ ($\omega < \infty$) serves as another boundary of the physical region, which is shown graphically in Fig.2.

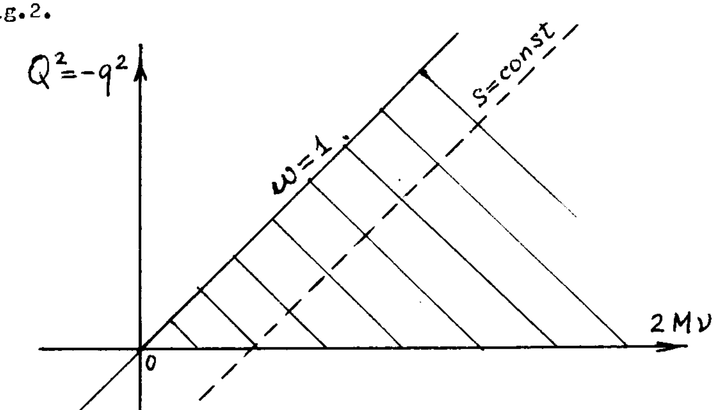


Fig.2

According to the rules of quantum electrodynamics (QED) [1,2] the amplitude of inelastic electro-production can be written in terms of the S_matrix element

$$S_{fi} \equiv \langle f | S | i \rangle = \langle f | i \rangle + i(2\pi)^4 \delta^4(p_n + k' - p - k) T_{fi}$$

$$T_{fi} \equiv \langle f | T | i \rangle =$$

$$= 4\pi\alpha \frac{g^{\mu\nu}}{q^2} \bar{u}(k') \gamma_\mu u(k) \langle p_n | \mathcal{J}_\nu(0) | p \rangle , \quad (1.9)$$

where $\mathcal{J}_\nu(x)$ is the hadronic electromagnetic current operator single-particle states and the Dirac spinors are normalized by the condition

$$\langle \vec{p} | \vec{p}' \rangle = \frac{\varepsilon(\vec{p})}{M} (2\pi)^3 \delta(\vec{p} - \vec{p}')$$

$$\bar{u}(p) u(p) = 1 \quad , \quad \varepsilon(p) = \sqrt{\vec{p}^2 + M^2}$$

and $\alpha = e^2/4\pi = 1/137$ the fine structure constant.

We are interested in the differential cross section for the process (1.1), where only the final electron is detected and various (unobserved) hadron states are produced. According to the usual rules [2] the invariant differential cross section for this process has the form

$$d\sigma = \frac{mM}{\sqrt{(p\cdot k)^2 - m^2 M^2}} \sum_n |T_{fi}|^2 (2\pi)^4 \delta^4(p + q - p_n) \frac{m\, d^3k'}{(2\pi)^3 \varepsilon'_m} \approx \quad (1.10)$$

$$\approx \frac{m^2}{E} \sum_n |T_{fi}|^2 (2\pi)^4 \delta^4(p + q - p_n) \frac{d^3k'}{(2\pi)^3 \varepsilon'_m} , \quad \varepsilon'_m = \sqrt{\vec{k}'^2 + m^2} ,$$

where the final states $|n\rangle$ are summed over. On account of eq. (1.7) a phase space element in the lab.frame looks like

$$\frac{d^3k'}{\varepsilon'_m} = |\vec{k}'| dE' d\Omega = \pi \frac{dE'}{E} dq^2 \quad (1.10a)$$

In the general case of a reaction with both polarized initial electrons and nucleons the double differential cross section summed over the final electrons polarization in the lab.frame can be represented as

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{4\alpha^2 E'}{q^4 E} L^{\mu\nu}(\sigma) W_{\mu\nu}(s) \quad (1.11)$$

where the leptonic tensor

$$L_{\mu\nu}(\sigma) = m^2 \sum_{\sigma'} \bar{u}^\sigma(k) \gamma_\mu u^{\sigma'}(k') \bar{u}^{\sigma'}(k') \gamma_\nu u^\sigma(k) = \quad (1.12)$$

$$= \frac{1}{4} \mathrm{Tr} \left[\frac{1}{2}(1 + \gamma_5 \hat{\sigma})(\hat{k} + m) \gamma_\mu (\hat{k}' + m) \gamma_\nu \right] ,$$

and the hadronic structure tensor

$$W_{\mu\nu}(s) = (2\pi)^3 \sum_n \langle p, s | \mathcal{J}_\mu(0) | p_n \rangle \langle p_n | \mathcal{J}_\nu(0) | p, s \rangle \delta^4(p + q - p_n) . \quad (1.13)$$

The lepton and nucleon polarizations are characterized by the spin four-vectors σ^{μ} and S^{μ} satisfying the conditions

$$S^2 = \sigma^2 = -1 \ , \quad \sigma\cdot k = S\cdot p = 0$$

so that in the rest frame, e.g., $\vec{p}=0$

$$p = (M, \vec{0}) \ , \ S = (0, \vec{S}) \ , \quad \vec{S}^2 = 1 \ ,$$

where $\vec{S}$ is the usual spin pseudovector.

From the hermiticity of the electromagnetic current operator

$$\overset{+}{\mathcal{J}}_{\mu}(x) = \mathcal{J}_{\mu}(x)$$

the hermiticity properties of the structure tensor $W_{\mu\nu}$ follow

$$\overset{*}{W}_{\mu\nu} = W_{\nu\mu} \ . \qquad \text{(1.13a)}$$

Exploiting the translation invariance property

$$\langle p_n | \mathcal{J}_{\mu}(x) | p\rangle = e^{i(p_n - p)\cdot x} \langle p_n | \mathcal{J}_{\mu}(0) | p\rangle ,$$

it is straightforward to verify that the hadronic tensor describing the nucleon structure can be written as the Fourier transform of a matrix element of the current commutator:

$$W_{\mu\nu}(s) = \frac{1}{2\pi}\int d^4x \, e^{iq\cdot x} \langle p,s | [\mathcal{J}_{\mu}(x), \mathcal{J}_{\nu}(0)] | p,s\rangle , \qquad \text{(I.14)}$$

where the second term vanishes due to energy conservation for $\nu > 0$.

Expression (I.14) shows that tensor $W_{\mu\nu}$ up to a constant factor is the "imaginary" (absorptive) part of the forward off-shell Compton amplitude for virtual photons of mass squared equal to q^2. In fact such an amplitude up to some real finite polynomial in q^0 (irrelevant to demonstration of the above statement) can be represented as

$$C_{\mu\nu}(q) = ie^2 \int d^4x \, e^{iq\cdot x} \langle p | T \mathcal{J}_{\mu}(x) \mathcal{J}_{\nu}(0) | p\rangle , \qquad \text{(I.15)}$$

where the time-ordered product of currents

$$T\mathcal{J}_{\mu}(x)\mathcal{J}_{\nu}(0) = \theta(x^0)\mathcal{J}_{\mu}(x)\mathcal{J}_{\nu}(0) + \theta(-x^0)\mathcal{J}_{\nu}(0)\mathcal{J}_{\mu}(x).$$

Making use of the T-product definition and integral representation of the θ- function, we finally obtain a dispersion-like relation

$$C_{\mu\nu}(q) = e^2 \int_0^{\infty} dq_0' \left[\frac{W_{\mu\nu}(q_0', \vec{q})}{q_0' - q_0 - i0} - \frac{W_{\mu\nu}(-q_0', \vec{q})}{q_0' + q_0 - i0} \right] \qquad \text{(I.16)}$$

Hence, employing a symbolic formula

$$\frac{1}{x - i0} = \mathcal{P}\frac{1}{x} + i\pi\delta(x)$$

we immediately find

$$\operatorname{Im} C_{\mu\nu}(q) = 4\pi^2\alpha\, W_{\mu\nu}(q) . \tag{1.17}$$

Graphically eq. (1.17) is presented in Fig. 3

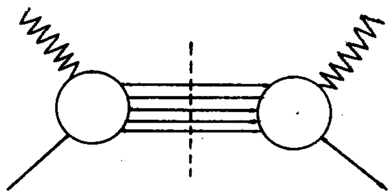


Fig.3

More precious meaning of eq. (1.17) consists in that the hadronic tensor $W_{\mu\nu}$ is given by the discontinuity of the forward off-shell Compton amplitude across the S-cut (in ν-variable):

$$4\pi^2\alpha\, W_{\mu\nu}(\nu, q^2) = \operatorname{disc}_S C_{\mu\nu}(s, q^2) =$$

$$= \{ C_{\mu\nu}(s+i0, q^2) - C_{\mu\nu}(s-i0, q^2) \} . \tag{1.17a}$$

Decompose leptonic tensor $L_{\mu\nu}(\sigma)$ into pieces symmetric $[S]$ and antisymmetric $[A]$ under permutation $\mu \leftrightarrow \nu$

$$L_{\mu\nu} = L_{\mu\nu}^{[S]} + i\, L_{\mu\nu}^{[A]} , \tag{1.18}$$

where neglecting the lepton mass

$$L_{\mu\nu}^{[S]} = \frac{1}{2}\left[k_\mu k'_\nu + k_\nu k'_\mu + g_{\mu\nu}\frac{q^2}{2} \right] = L_{\nu\mu}^{[S]} \tag{1.19a}$$

$$L_{\mu\nu}^{[A]} = \frac{1}{2} m\, \varepsilon_{\mu\nu\lambda\tau}\, q^\lambda \sigma^\tau = - L_{\nu\mu}^{[A]} . \tag{1.19b}$$

Actually decomposition (1.18) follows from the hermiticity property of $L_{\mu\nu}$

$$\overset{*}{L}_{\mu\nu} = L_{\nu\mu} .$$

The symmetric piece as one should expect, coincides with the result of averaging over the polatizations of the initial lepton

$$L_{\mu\nu}^{[S]} = \frac{1}{2} \sum_{\sigma} L_{\mu\nu}(\sigma) . \tag{1.20}$$

Now owing to the hermiticity property (1.13a), the hadronic tensor $W_{\mu\nu}$ can be decomposed in a similar way into symmetric and antisymmetric pieces

$$W_{\mu\nu} = W_{\mu\nu}^{[S]} + i\, W_{\mu\nu}^{[A]} , \tag{1.21}$$

where , as in the previous case, the symmetric part corresponds to averaging over the initial nucleon polarizations. Utilizing the relativistic and gauge invariance of the theory we can put the spin idenepdent part $W_{\mu\nu}^{[S]}$ into the form

$$W_{\mu\nu}^{[S]} = \frac{1}{2}\sum_{s} W_{\mu\nu}(s) = W_{\nu\mu}^{[S]} =$$

$$= \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] W_1(\nu, Q^2) + \frac{1}{M^2}\tilde{P}_\mu \tilde{P}_\nu W_2(\nu, Q^2), \tag{1.22}$$

$$\tilde{P}_\mu = p_\mu - q_\mu \frac{(p\cdot q)}{q^2}, \quad (\tilde{P}\cdot q) = 0,$$

which insures the implementation of the requirements of current conservation

$$q^\mu W_{\mu\nu}^{[S]} = q^\nu W_{\mu\nu}^{[S]},$$

and of PT-invariance $W_{\mu\nu}^{[S]} = W_{\nu\mu}^{[S]}$.

In order to find an explicit gauge and Lorentz invariant decomposition into structure functions of the spin dependent piece $W_{\mu\nu}^{[A]}$ we notice that it must be linear in the spin vector S. Indeed, this follows from the expression for the spin 1/2 density matrix

$$u^s(p)\,\bar{u}^s(p) = \frac{\hat{p}+M}{2M}\,\frac{1}{2}\left(1+\gamma_5 \hat{s}\right) \tag{1.23}$$

and from the constraints imposed by PT-invariance:

$$W_{\mu\nu}^{[A]}(s) = -W_{\mu\nu}^{[A]}(-s). \tag{1.23a}$$

Thus the spin dependent tensor $W_{\mu\nu}(s)$ can be represented as

$$iW_{\mu\nu}^{[A]}(s) = \frac{1}{4M}\mathrm{Tr}\left[(\hat{p}+M)\gamma_5 \hat{s}\, G_{\mu\nu}\right], \tag{1.24}$$

where in the most general from

$$G_{\mu\nu} = \frac{1}{2M}\left\{[\gamma_\mu,\gamma_\nu](p\cdot q) - [\gamma_\mu,\hat{q}]p_\nu - [\hat{q},\gamma_\nu]p_\mu\right\} G_1(\nu,Q^2) +$$

$$+\frac{1}{2}\left\{[\gamma_\mu,\gamma_\nu]q^2 - [\gamma_\mu,\hat{q}]q_\nu - [\hat{q},\gamma_\nu]q_\mu\right\} G_2(\nu,Q^2). \tag{1.25}$$

The invariant structure functions $G_{1,2}$ are defined according to Bjorken [7]. The trace calculations lead to a simpler expression

$$W_{\mu\nu}^{[A]} = \varepsilon_{\mu\nu\rho\sigma}\, q^\rho s^\sigma\, G_1(\nu,Q^2) +$$

$$+\frac{1}{M}\varepsilon_{\mu\nu\rho\sigma}\, q^\rho\left[(p\cdot q)s^\sigma - (s\cdot q)p^\sigma\right] G_2(\nu,Q^2). \tag{1.26}$$

The requirements of current conservation on account of anti-symmetry of the tensor $\varepsilon_{\mu\nu\rho\sigma}$ are identically fulfilled

$$q^\mu W_{\mu\nu}^{[A]} = q^\nu W_{\mu\nu}^{[A]} = 0.$$

The hermiticity properties (1.13a) of the hadronic tensor $W_{\mu\nu}$ provide the conditions

$$\overset{*}{W}{}_{\mu\nu}^{[S,A]} = W_{\mu\nu}^{[S,A]} , \tag{1.13b}$$

which result in the reality of the structure functions $W_{1,2}$ and $G_{1,2}$.

The product of the hadronic and leptonic tensors entering expression (1.11) for the differential cross section with the help of decomposition into symmetric and antisymmetric pieces can be rewritten as

$$L^{\mu\nu} W_{\mu\nu} = L^{\mu\nu}_{[S]} W^{[S]}_{\mu\nu} - L^{\mu\nu}_{[A]} W^{[A]}_{\mu\nu} . \tag{1.27}$$

According to eqs. (1.19), (1.20) and (1.22) the first term in eq. (1.27) corresponds to averaging over the spins of the initial lepton and nucleon, so that the spin independent piece of the differential cross section (1.11) has the form

$$\frac{d^2\sigma_{av}}{d\Omega' dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left[2W_1(\nu,Q^2)\sin^2\frac{\theta}{2} + W_2(\nu,Q^2)\cos^2\frac{\theta}{2}\right]. \tag{1.28}$$

Exactly this quantity was measured till now in the experiments on the deep inelastic electron-nucleon scattering in which the remarkable phenomenon of scaling was discovered.

The second term in eq. (1.27) describes the spin dependent effects and, as is obvious from its expression, in order to observe them one needs to scatter polarized electrons (muons) on polarized protons. Such experiments are planned at a number of laboratories (SLAC, FNAL). Under the assumption of time reversal (T) invariance the spin components normal to the scattering plane gives no effect. It is therefore sufficient to restrict oneself to the two independent configurations for the proton spin parallel and transverse to the beam direction in the electron scattering plane.

Hence, it follows that one must measure two asymmetries in the lab.frame:

1) $\overset{\sigma \rightarrow}{k \longrightarrow} \qquad \overset{\bullet\; p}{\rightleftarrows s}$

$$\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega' dE'} - \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega' dE'} = \frac{4\alpha^2 E'}{Q^2 E}\left[(E+E'\cos\theta)\, G_1(\nu,Q^2) - Q^2 G_2(\nu,Q^2)\right] , \tag{1.29a}$$

2) $\overset{\sigma \rightarrow}{k \longrightarrow} \qquad \overset{\circ\; p}{\uparrow\downarrow s}$

$$\frac{d^2\sigma^{\uparrow\leftarrow}}{d\Omega' dE'} - \frac{d^2\sigma^{\uparrow\rightarrow}}{d\Omega' dE'} = \frac{4\alpha^2 E'}{Q^2 E} E'\sin\theta\left[G_1(\nu,Q^2) + 2E\, G_2(\nu,Q^2)\right] . \tag{1.29b}$$

Thus is principle it is possible to separate G_1 and G_2.

1.2. Unitarity and positivity conditions

As is shown above, hadronic tensor $W_{\mu\nu}$ is proportional to the imaginary part of the off-shell Compton amplitude for the forward scattering of virtual photons on nucleons. The optical theorem, which is the consequence of the S-matrix unitarity, relates this forward amplitude to the total cross section for the absorption of virtual photons. Unlike real ones virtual photons in addition to the two transverse (T) polarization states possess a longitudinal (L) one as well. Thus, it is possible to express the invariant structure functions $W_{1,2}$, $G_{1,2}$ in terms of the photoabsorption cross sections for various polarization states. It is convenient to utilize the formalism of the helicity amplitudes:

$$\langle \lambda', s' | T | \lambda, s \rangle = \overset{*}{\varepsilon}{}^{\mu}_{\lambda'}\, C_{\mu\nu}(s', s)\, \varepsilon^{\nu}_{\lambda} \tag{1.30}$$

where the virtual photon's helicity wave functions in the lab. frame have the form

$$\varepsilon^{T}_{\pm 1} = \frac{-1}{\sqrt{2}}\left(0, \pm 1, i, 0\right),$$

$$\varepsilon^{L}_{0} = \frac{1}{\sqrt{-q^2}}\left(|\vec{q}|, 0, 0, q^0\right),$$

$$q = (q^0, 0, 0, |\vec{q}|),\ q^0 = \nu,\ |\vec{q}| = \sqrt{\nu^2 - q^2},$$

and satisfy the conditions

$$\overset{*}{\varepsilon}{}^{T}_{\mu}\,\varepsilon^{\mu}_{T} = -1,\quad \overset{*}{\varepsilon}{}^{L}_{\mu}\,\varepsilon^{\mu}_{L} = +1,\quad q_{\mu}\,\varepsilon^{\mu}_{T,L} = 0.$$

For the s - channel forward helicity amplitudes for virtual Compton scattering we introduce the abbreviated notations:

$$\begin{aligned} \langle 1, 1/2 | T | 1, 1/2 \rangle &= T_{1/2} \\ \langle 1, -1/2 | T | 1, -1/2 \rangle &= T_{3/2} \\ \langle 0, 1/2 | T | 0, 1/2 \rangle &= T_{L} \\ \langle \pm 1, \pm 1/2 | T | 0, \pm 1/2 \rangle &= T_{1/2\, L}. \end{aligned} \tag{1.31}$$

The tensor Compton amplitude $C_{\mu\nu}$ can be also splitted into the symmetric and antisymmetric pieces

$$C_{\mu\nu} = C^{[S]}_{\mu\nu} + i\, C^{[A]}_{\mu\nu},$$

which have exactly the same invariant decomposition as their "imaginary" parts $W^{[S,A]}_{\mu\nu}$ displayed by the expressions (1.22) and (1.26). We shall denote the invariant structure functions entering $C^{[S]}_{\mu\nu}$ as $C_{1,2}(\nu, Q^2)$ and the ones entering $C^{[A]}_{\mu\nu}$ as $H_{1,2}(\nu, Q^2)$. Thus on the basis of eq. (1.17) we have the relations

$$\begin{aligned} \mathrm{Im}\, C_{1,2} &= 4\pi^2\alpha\, W_{1,2} \\ \mathrm{Im}\, H_{1,2} &= 4\pi^2\alpha\, G_{1,2}, \end{aligned} \tag{1.32}$$

where as before the "imaginary" part means the discontinuity across the s - cut (in ν - variable).

It is quite evident that the number of independent helicity amplitudes must be equal to the number of invariant structure functions, i.e., in the present case to four. Thus they can be linearly expressed in terms of each other:

$$\begin{aligned} T_{1/2} &= C_1 + \left[\nu H_1 + q^2 H_2\right], \\ T_{3/2} &= C_1 - \left[\nu H_1 + q^2 H_2\right], \\ T_L &= \left(1 - \frac{\nu^2}{q^2}\right) C_2 - C_1, \\ T_{1/2\,L} &= \sqrt{-2q^2}\left[H_1 + \nu H_2\right]. \end{aligned} \tag{1.33}$$

As is well known the optical theorem in the case of real photons states

$$\operatorname{Im} T(s) = \frac{s - M^2}{2M}\,\sigma^{tot}, \tag{1.34}$$

where s is the centre-of-mass energy squared equal to the effective mass squared of a produced hadron system.

At present it is common to adopt Hand's convention concerning the kinematic flux factor for virtual photons. Namely, the invariant flux factor is taken to be the same as in the case of a real photon with energy ν producing a final hadron state of mass $\sqrt{s}$. Thus the "equivalent virtual photon's energy"

$$K = \frac{s - M^2}{2M} = \nu + \frac{q^2}{2M} = \nu - \frac{Q^2}{2M}. \tag{1.35}$$

Using eqs. (1.32), (1.33) and (1.34) we find the helpful relations

$$\begin{aligned} \sigma_T &= \frac{1}{2}(\sigma_{1/2} + \sigma_{3/2}) = \frac{1}{2K}\operatorname{Im}\left[T_{1/2} + T_{3/2}\right] = \frac{4\pi^2\alpha}{K} W_1, \\ \sigma_L &= \frac{1}{K}\operatorname{Im} T_L = \frac{4\pi^2\alpha}{K}\left[\left(1 + \frac{\nu^2}{Q^2}\right) W_2 - W_1\right], \\ \frac{1}{2}(\sigma_{1/2} - \sigma_{3/2}) &= \frac{1}{2K}\operatorname{Im}\left[T_{1/2} - T_{3/2}\right] = \frac{4\pi^2\alpha}{K}\left[\nu G_1 - Q^2 G_2\right], \\ \sigma_{1/2\,L} &= \frac{1}{K}\operatorname{Im} T_{1/2\,L} = \frac{4\pi^2\alpha}{K}\sqrt{2Q^2}\left[G_1 + \nu G_2\right]. \end{aligned} \tag{1.36}$$

In addition the asymmetry

$$A(\nu, Q^2) = \frac{\sigma_{1/2} - \sigma_{3/2}}{2\sigma_T} = \frac{\nu G_1 - Q^2 G_2}{W_1} \tag{1.37a}$$

and the ratio of the cross sections for longitudinal and transverse "photons"

$$R = \frac{\sigma_L}{\sigma_T} = \left(1 + \frac{\nu^2}{Q^2}\right)\frac{W_2}{W_1} - 1 \tag{1.37b}$$

are often employed.

It is easy to write down the inverse relations

$$\begin{aligned} W_1 &= \frac{K}{4\pi^2\alpha}\,\sigma_T \\ W_2 &= \frac{K}{4\pi^2\alpha}\left(\sigma_T + \sigma_L\right)\frac{Q^2}{Q^2 + \nu^2} \end{aligned} \tag{1.38a}$$

$$G_1 = \frac{K}{4\pi^2\alpha}\left[\frac{Q^2}{\nu}\frac{\sigma_{1/2\,L}}{\sqrt{2Q^2}} + A\sigma_T\right]\frac{\nu}{\nu^2+Q^2}$$

$$G_2 = \frac{K}{4\pi^2\alpha}\left[\nu\,\frac{\sigma_{1/2\,L}}{\sqrt{2Q^2}} - A\sigma_T\right]\frac{1}{\nu^2+Q^2}\,. \qquad (1.38b)$$

In terms of the above introduced total photoabsorption cross sections the measured double differential cross sections take the form in Hand's parametrization [5]

$$\frac{d^2\sigma}{d\Omega' dE'} = \Gamma\left(\sigma_T + \varepsilon\,\sigma_L\right), \qquad (1.39)$$

$$\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega' dE'} - \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega' dE'} = 2\Gamma\left\{\left(1-\varepsilon\frac{E'}{E}\right)A\sigma_T + \varepsilon\frac{\sqrt{Q^2}}{\sqrt{2}\,E}\,\sigma_{1/2\,L}\right\},$$

$$\frac{d^2\sigma^{\uparrow\leftarrow}}{d\Omega' dE'} - \frac{d^2\sigma^{\uparrow\rightarrow}}{d\Omega' dE'} = 2\Gamma\left\{\sqrt{\tfrac{1}{2}\varepsilon(1+\varepsilon)}\,\frac{\sqrt{Q^2}}{E}A\sigma_T - \sqrt{\frac{\varepsilon}{1+\varepsilon}}\left(1-\varepsilon\frac{E'}{E}\right)\sigma_{1/2\,L}\right\},$$

where

$$\Gamma = \frac{\alpha}{4\pi^2}\,\frac{K}{Q^2}\,\frac{E}{E'}\,\frac{2}{(1-\varepsilon)}, \qquad (1.40)$$

$$\frac{1}{\varepsilon} = 1 + 2\left(1+\frac{\nu^2}{Q^2}\right)tg^2\frac{\theta}{2}\,.$$

Note that in the case of real transverse photons

$$Q^2 = 0, \quad K = \nu \qquad (1.36a)$$

$$\sigma_L(\nu,0) = \sigma_{1/2\,L}(\nu,0) = 0,$$

$$\sigma_T(\nu,0) = \frac{4\pi^2\alpha}{\nu}W_1(\nu,0) = \frac{1}{2}\left[\sigma_A(\nu)+\sigma_P(\nu)\right],$$

$$\frac{1}{2}\left[\sigma_{1/2} - \sigma_{3/2}\right] = 4\pi^2\alpha\, G_1(\nu,0) = \frac{1}{2}\left[\sigma_A(\nu) - \sigma_P(\nu)\right],$$

where $\sigma_{A,P}$ are the total cross sections for the absorption of real photons with spin parallel (P) and antiparallel (A) to the proton spin.

The introduced above notions of the total photoabsorption cross sections allow one to present in a transparent form some of the properties of the invariant structure functions entering the hadronic tensor $W_{\mu\nu}$. In particular the before mentioned hermiticity property of this tensor (1.13a) leads to the positivity condition

$$\overset{*}{a}{}^{\mu}\, W_{\mu\nu}\, a^{\nu} \geq 0 \qquad (1.41)$$

for any complex vectors a^{μ}. Making reasonable choice of the vectors $a^{\mu} = \varepsilon^{\mu}_{s}$ (s is the nucleon spin index) it is possible to obtain [8] restrictions on the values of the invariant functions. However their complete proof is rather involved. It is considerably easier to see the origin of these restrictions from the conditions of positivity of the total photoabsorption cross sections. We emphasize, that despite the seeming obviousness of this method it cannot be considered as completely rigorous, since the positivity of the cross sections for unphysical processes involving virtual photons is not so evident. Nevertheless in view of the existence of a more rigorous proof we accept that the following conditions are fulfilled:

$$\sigma_T \geq 0\ ,\ \sigma_L \geq 0\ ,\ \sigma_{1/2} \geq 0\ ,\ \sigma_{3/2} \geq 0$$

$$\sigma_T^2 = \frac{1}{4}(\sigma_{1/2}+\sigma_{3/2})^2 \geq \frac{1}{4}(\sigma_{1/2}-\sigma_{3/2})^2,\ A^2 \leq 1\ . \tag{1.42}$$

Hence, with the account of relations (1.36), the positivity constraints follow

$$\begin{aligned} &W_T \equiv W_1 \geq 0\ ,\\ &W_L \equiv \left(1+\frac{\nu^2}{Q^2}\right)W_2 - W_1 \geq 0\ ,\\ &W_1 \geq |\nu G_1 - Q^2 G_2|\ . \end{aligned} \tag{1.43}$$

One more constraint may be obtained with the help of a Schwarz type inequality:

$$\langle\alpha|T^\dagger T|\alpha\rangle\langle\beta|T^\dagger T|\beta\rangle \geq |\langle\alpha|T^\dagger T|\beta\rangle|^2\ . \tag{1.44}$$

Since, owing to the optical theorem (S-matrix unitarity)

$$\sigma_\alpha \propto \mathrm{Im}\langle\alpha|T|\alpha\rangle = \frac{1}{2}\langle\alpha|T^\dagger T|\alpha\rangle \geq 0$$

$$\sigma_{\alpha\beta} \propto \mathrm{Im}\langle\alpha|T|\beta\rangle = \frac{1}{2}\langle\alpha|T^\dagger T|\beta\rangle\ ,$$

inequality (1.44) can be rewritten like

$$\sigma_\alpha\sigma_\beta \geq \sigma_{\alpha\beta}^2\ . \tag{1.45}$$

As a specific example of inequality (1.45) we have

$$\sigma_{1/2}\,\sigma_L \geq \sigma_{1/2\,L} \tag{1.46}$$

or introducing the ratio (1.37b)

$$\begin{aligned} &\sigma_L = R\,\sigma_T\\ &\sigma_{1/2}\,R\,\sigma_T \geq \sigma_{1/2\,L}^2\ . \end{aligned} \tag{1.46a}$$

Hence using relations (1.36) we obtain another constraint for the structure functions

$$[W_1+\nu G_1 - Q^2 G_2]\,R\,W_1 \geq 2Q^2[G_1+\nu G_2]^2\ . \tag{1.47}$$

For some applications it is more convenient to employ inequality (1.47) in a different form. Since according to the last of the bounds (1.43)

$$W_1 \geq \nu G_1 - Q^2 G_2$$

from the constraint (1.47) it follows

$$R\,W_1^2 \geq Q^2[G_1+\nu G_2]^2\ . \tag{1.48}$$

We shall utilize the latter bound for an estimate of the hadronic structure contribution to the hyperfine splitting of the hydrogen energy levels.

1.3. One-nucleon state contribution

Now consider at more length the contribution of one-nucleon intermediate states in expression (1.12) for the hadronic tensor $W_{\mu\nu}$. Such a contribution corresponds to the elastic electron-nucleon scattering. To this end we make use of the well-known parametrization for the matrix element of the electromagnetic current between one-nucleon states [2]:

$$\langle p', s' | J_\mu(0) | p, s \rangle = \bar{u}^{s'}(p') \, \Gamma_\mu(q) \, u^s(p) ,$$
$$\Gamma_\mu(q) = \gamma_\mu F_1(-q^2) + \frac{i}{2M} \sigma_{\mu\nu} q^\nu F_2(-q^2), \tag{1.49}$$

where

$$\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] , \quad q = p' - p .$$

The form factors $F_{1,2}$ are normalized in the following way (p-proton, n-neutron)

$$\begin{aligned} &F_1^p(0) = 1 , \quad F_1^n(0) = 0 , \\ &F_2^p(0) = \varkappa_p , \quad F_2^n(0) = \varkappa_n . \end{aligned} \tag{1.50}$$

Usually electric and magnetic form factors are also introduced

$$G_E = F_1 - \frac{Q^2}{4M^2} F_2 ,$$
$$G_M = F_1 + F_2 .$$

In terms of these form factors one-nucleon contribution to the invariant structure functions reads as:

$$\begin{aligned} W_1^{el}(\nu, Q^2) &= \frac{Q^2}{4M^2} G_M^2(Q^2) \, \delta\left(\nu - \frac{Q^2}{2M}\right), \\ W_2^{el}(\nu, Q^2) &= \left[F_1^2(Q^2) + \frac{Q^2}{4M^2} F_2^2(Q^2)\right] \delta\left(\nu - \frac{Q^2}{2M}\right) = \\ &= \frac{G_E^2(Q^2) + \frac{Q^2}{4M^2} G_M^2(Q^2)}{1 + \frac{Q^2}{4M^2}} \, \delta\left(\nu - \frac{Q^2}{2M}\right), \end{aligned} \tag{1.51}$$

$$G_1^{el}(\nu, Q^2) = \frac{1}{2M} F_1(Q^2) G_M(Q^2) \, \delta\left(\nu - \frac{Q^2}{2M}\right),$$
$$G_2^{el}(\nu, Q^2) = -\frac{1}{4M^2} F_2(Q^2) G_M(Q^2) \, \delta\left(\nu - \frac{Q^2}{2M}\right).$$

It is straightforward a to check that the following relations are valid

$$\begin{aligned} &\nu G_1^{el} - Q^2 G_2^{el} = W_1^{el} , \\ &G_1^{el} + \nu G_2^{el} = \frac{1}{2M} G_E G_M \, \delta\left(\nu - \frac{Q^2}{2M}\right), \\ &W_L^{el} = \left(1 + \frac{\nu^2}{Q^2}\right) W_2^{el} - W_1^{el} = G_E^2 \, \delta\left(\nu - \frac{Q^2}{2M}\right). \end{aligned} \tag{1.52}$$

In the parton model a parton is defined as a point-like object the mass (four-momentum) of which is some fraction of the nucleon mass (four-momentum) and the parton charge is equal to $e_i e$. In the case if a parton is the point-like Dirac particle (i.e. of spin 1/2 and with $F_1(q^2)=1, F_2(q^2)=0$) we find changing M into xM in eq. (1.51):

$$\begin{aligned} 2MW_1^i &= e_i^2\,\delta\left(x-\frac{1}{\omega}\right), \\ \nu W_2^i &= e_i^2\,x\,\delta\left(x-\frac{1}{\omega}\right) = \frac{2M}{\omega}W_1^i, \\ 2M\nu G_1^i &= e_i^2\,\delta\left(x-\frac{1}{\omega}\right) = 2MW_1^i, \\ G_2^i &= 0, \quad \omega = \frac{2M\nu}{Q^2}. \end{aligned} \tag{1.53}$$

1.4. Scale invariance (automodelity) and sum rules

The peculiar property of deep inelastic electron-nucleon scattering is the scaling (automodel) behaviour of the structure functions $W_{1,2}$. Experimentally it was found that as both ν and Q^2 reach sufficiently high values (compared to M^2) the functions νW_2 and W_1 become nontrivial functions

of the dimensionless ratio $\omega = 2M\nu/Q^2$.

There exist various theoretical models which predict the observed scaling behaviour . Since the hadronic tensor $W_{\mu\nu}$ is expressed through the current commutator, it is possible to show that the asymptotic behaviour of $W_{1,2}$ as $q^2, \nu \to \infty$ is intimately related to the nature of singularities in the vicinity of the light cone $x^2=0$. The most complete investigation of this question on the basis of the Jost-Lehman-Dyson representation for the causal commutator was carried out by Bogolubov, Vladimirov, Tavkhelidze [10] and in the subsequent papers [11] .

Briefly explain why the asymptotic behaviour of structure functions in the Bjorken limit $(\lim_B)$

$$q^2 \to \infty, \nu \to \infty, \quad \omega = -\frac{2M\nu}{q^2} \quad \text{fixed}$$

is connected with the behaviour of the current commutator near the light cone $x^2=0$. To this end recall the Fourier representation (1.14) for the tensor $W_{\mu\nu}$. Choose the reference frame, where $P=(M,\vec{0})$, $q=(\nu,0,0,\sqrt{\nu^2-q^2})$. Then the scalar product ($q\cdot x$) in the exponent can be obviously written down in the form

$$(q\cdot x) = \frac{1}{2}(q_0-q_3)(x_0+x_3)+\frac{1}{2}(q_0+q_3)(x_0-x_3).$$

In the chosen reference frame

$$(q_0 - q_3) = \nu - \sqrt{\nu^2 - q^2} \xrightarrow{\lim_B} \frac{q^2}{2\nu} = -\frac{M}{\omega}$$

$$(q_0 + q_3) = \nu + \sqrt{\nu^2 - q^2} \xrightarrow{\lim_B} 2\nu = -q^2 \frac{\omega}{M} \, .$$

It is well known that the main contribution to the Fourier integral comes from the region of values of $(q \cdot x) \sim 1$, i.e.,

$$|x_0 + x_3| \sim \frac{2}{|q_0 - q_3|} = \frac{2\omega}{M} \, ,$$

$$|x_0 - x_3| \sim \frac{2}{|q_0 + q_3|} = \frac{2M}{|q^2|\omega} \, .$$

Hence, the essential values of x^2 are

$$x^2 = (x_0 - x_3)(x_0 + x_3) - \vec{x}_\perp^{\,2} \leqslant$$

$$\leqslant (x_0 - x_3)(x_0 + x_3) \sim \frac{4}{|q^2|} \xrightarrow{\lim_B} 0 \, .$$

The parton model assumes that a nucleon is composed of quasifree point-like constituents, named partons. This model rests on such an experimental fact that the electroproduction cross section integrated over the energy ν at high fixed q^2 has the same order of magnitude as the Mott cross section on a point-like nucleon. The structure functions $W_{1,2}$ are thereby obtained by integrating over x the parton functions (1.53), multiplied by the distribution functions of the fraction x of longitudinal momentum and summing up over all partons:

$$2MW_1(\nu, Q^2) = \sum_i e_i^2 \int_0^1 dx \, f_i(x) \, \delta(x - \tfrac{1}{\omega}) = F_1(\omega) \tag{1.54}$$

$$\nu W_2(\nu, Q^2) = \sum_i e_i^2 \int_0^1 dx \, x f_i(x) \, \delta(x - \tfrac{1}{\omega}) = F_2(\omega) = \tfrac{1}{\omega} F_1(\omega)$$

$$R \equiv \frac{\sigma_L}{\sigma_T} = \frac{W_2}{W_1}\left(1 + \frac{\nu^2}{Q^2}\right) - 1 = \frac{4M^2}{Q^2\omega^2} \xrightarrow{\lim_B} 0 \, ,$$

where $f_i(x)$ is normalized by the condition

$$\int_0^1 dx \, f_i(x) = \sum_{N_i} N_i \, P(N_i) = \langle N_i \rangle \, ,$$

$\langle N_i \rangle$ is the average number of partons of charge e_i.

Thus deep inelastic electroproduction in the parton model reduces to the sum of elastic scattering processes on point-like partons. We have assumed above that partons are spin 1/2 particles which may be identified with the usual quarks. There is extensive literature dealing with the detailed consideration of this model [12] and, in particular, with the deduction and analysis of various sum rules.

In the case of spin dependent electroproduction one needs in addition to take into account spin degrees of freedom of partons [6]. For spin 1/2 partons two distribution functions emerge, which correspond to the parton helicity directed parallel or opposite to the nucleon helicity:

$$\sum_\lambda f_{i\lambda}(x) \langle i, \lambda | \Lambda | i, \lambda \rangle =$$

$$= \frac{1}{2} \left[f_i^{\uparrow}(x) - f_i^{\downarrow}(x) \right] . \tag{1.55}$$

It is evident, that the function used before

$$f_i(x) = \frac{1}{2}\left[f_i^{\uparrow}(x) + f_i^{\downarrow}(x)\right].$$

As a result on account of the expressions (1.53) for the parton functions $G^i_{1,2}$ we find

$$2M\nu G_1(\nu,Q^2) = g_1(\omega) =$$

$$= \sum_i \frac{1}{2} e_i^2 \int_0^1 dx\, x\left[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)\right]\delta(x-\tfrac{1}{\omega}), \qquad (1.56)$$

$$G_2(\nu,Q^2) \equiv 0 .$$

In the simplest quark-parton model, where the nucleons are composed as

$$p = (uud), \quad n = (udd),$$

$$e_u = \frac{2}{3}, \quad e_d = -\frac{1}{3}.$$

Employing the explicit form of the U(6) wave functions we obtain [13]

$$\frac{1}{2}\left[f_u^{\uparrow}(x) - f_u^{\downarrow}(x)\right] = \left(\frac{5}{6} - \frac{1}{6}\right) f_u(x) = \frac{2}{3} f_u(x)$$

$$\frac{1}{2}\left[f_d^{\uparrow}(x) - f_d^{\downarrow}(x)\right] = \left(\frac{1}{3} - \frac{2}{3}\right) f_d(x) = -\frac{1}{3} f_d(x)$$

Hence, the scaling functions given by eqs. (1.54) and (1.56) take the form:

$$F_1^p(\omega) = \frac{8}{9} f_u(\omega) + \frac{1}{9} f_d(\omega),$$

$$g_1^p(\omega) = \frac{16}{27} f_u(\omega) - \frac{1}{27} f_d(\omega),$$

$$F_1^n(\omega) = \frac{2}{9} f_u(\omega) + \frac{4}{9} f_d(\omega), \qquad (1.57)$$

$$g_1^n(\omega) = \frac{4}{27} f_u(\omega) - \frac{4}{27} f_d(\omega).$$

Relations (1.57) lead to the bounds on the magnitude of the asymmetry (1.37a), namely,

$$A(\omega) = \frac{g_1(\omega)}{F_1(\omega)},$$

$$-\frac{1}{3} \leq A^p(\omega) \leq \frac{2}{3}, \; -\frac{2}{3} \leq A^n(\omega) \leq \frac{1}{3}. \qquad (1.58)$$

If, in addition, we assume that $f_u(\omega) = f_d(\omega)$, then $g_1^n(\omega) = 0$,

$$A^p(\omega) = \frac{5}{9}, \quad A^n(\omega) = 0 \qquad (1.59)$$

and the following sum rule is valid

$$\int_1^\infty \frac{d\omega}{\omega^2} g_1(\omega) = \int_1^\infty \frac{d\omega}{\omega^2} A(\omega) F_1(\omega) = \frac{5}{9}$$

or on account of eq. (1.59)

$$\int_1^\infty \frac{d\omega}{\omega^2} F_1(\omega) = 1. \qquad (1.60)$$

In a more general case on the basis of chiral current algebra Bjorken's famous sum rule can be deduced [6,7]

$$\int_1^\infty \frac{d\omega}{\omega^2}\left[g_1^p(\omega)-g_1^n(\omega)\right] = \frac{1}{3}\left|\frac{G_A}{G_V}\right| , \qquad (1.61)$$

where G_A/G_V is the ratio of the axial and vector weak interaction constants.

The application of the low energy theorem at $Q^2=0$ leads to the Gerasimov sum rule

$$\frac{\varkappa^2}{4M^2} = -\int_{\nu_\pi}^\infty \frac{d\nu}{\nu} G_1(\nu,0) = \frac{1}{8\pi^2\alpha}\int_{\nu_\pi}^\infty \frac{d\nu}{\nu}\left[\sigma_P(\nu)-\sigma_A(\nu)\right].$$

One of the intrinsic parton properties is their point-like nature. It is tempting therefore to abstraot this property and to formulate it in a form of the general hypothesis regarding the absence of any dimensional parameters (except the nucleon mass) fixing the scale of invariant kinematic variables and cross sections for deep inelastic lepton-hadron processes. Under the additional assumption that at high energies and momentum transfers, when

$$|q^2| \gg M^2,\ M\nu = (p\cdot q) \gg M^2,\ \frac{M\nu}{q^2} \text{ fixed},$$

any nontrivial dependence on the hadron masses squared drops out, all physical quantities are expected to become the homogeneous functions of kinematic variables. This actually means that under the scale transformation (dilatation) of the four-momenta

$$p \to \lambda p\ ,\ q \to \lambda q$$

the invariant structure functions, cross section and so on transform according to their physical dimensionalities

$$[F(p,q)] = m^n,$$

$$F(p,q) \to F(\lambda p, \lambda q) = \lambda^n F(p,q),$$

since there is only one dimensional unit, namely, that of mass.

Such a general hypothesis was first formulated by Matveev, Muradyan, Tavkhelidze [14] and is known as the automodelity principle.

In particular, since the basic kinematical invariants have the dimensionality

$$[q^2] = [M\nu] = m^2$$

it is clear that dimensionless invariant functions may depend only on the dimensionless ratio, e.g.,

$$\omega = -\frac{2M\nu}{q^2}$$

and quantities of a type of cross sections having dimensionality

$$[\sigma] = m^{-2}$$

can be represented in the form

$$\sigma(\nu,q^2) = \frac{1}{q^2} F(\omega). \qquad (1.62)$$

Now we apply this simple reasoning to considering the properties of structure functions $W_{1,2}$ and $G_{1,2}$. Then formulae (1.38) with due regard for the definition (1.35) of the virtual photons flux

$$K = \frac{q^2}{2M}(1-\omega) = \nu\left(1-\frac{1}{\omega}\right), \quad \omega \text{ fixed}$$

and eq. (1.62) lead to already familiar relations:

$$\begin{aligned} 2M\,W_1(\nu,Q^2) &= F_1(\omega), \\ \nu\,W_2(\nu,Q^2) &= F_2(\omega), \\ 2M\nu\,G_1(\nu,Q^2) &= g_1(\omega), \\ 2M\nu^2 G_2(\nu,Q^2) &= g_2(\omega). \end{aligned} \tag{1.63}$$

To deduce the latter of relations (1.63) some additional assumption is required, however, we shall not dwell on it [6]. Unlike the analogous relations within the parton model there are no connections between the scale invariant functions $F_{1,2}$ and $g_{1,2}$ and moreover $g_2 \not\equiv 0$. The scaling function satisfies the superconvergent sum rule

$$\int_1^\infty d\omega\, g_2(\omega) = 0 ,$$

which can be most easily deduced with the help of the light cone current algebra [5,6].

Scaling is rather well confirmed experimentally at SLAC [5]. However, recently in experiments carried out at FNAL [15] using

a muon beam some small deviations from scaling were observed. In general it was noticed that scaling sets in earlier and is better fulfilled in the variable

$$\omega' = \omega + \frac{M^2}{Q^2} = \frac{S}{Q} + 1 .$$

In favour of this phenomenon there are some reasons based on the duality idea [16]. Namely, the nucleon and nucleon resonances of mass $\sqrt{S}$ at low energy build up, in the sense of finite energy sum rules, the nondiffractive component of the off-shell forward Compton amplitude on the average. Thus a substantial part of the scaling behaviour of the virtual photon-nucleon amplitude is due to a non-diffractive component which corresponds to the non-Pomeron exchange at high energy.

If the possible deviations from the scaling behaviour of the functions W_1 and νW_2 are parametrized in the form of the factor [5]

$$\left(1 - \frac{2Q^2}{\Lambda_i^2}\right), \quad i = 1,2 ,$$

then for $1.5 < \omega < 3$ the best fit to the data requires

$$\Lambda_1^2 = 62 \pm 9 \ GeV^2 , \quad \Lambda_2^2 = 75 \pm 7 \ GeV^2 .$$

If one uses the variable ω' then the fits with $\Lambda_i^2 = \infty$ are perfectly acceptable and with a 95 % confidence level the lower limits on the Λ_i^2 are

$$\Lambda_1^2 > 84 \ GeV^2 , \quad \Lambda_2^2 > 179 \ GeV^2 .$$

The results of the FNAL experimnet at 150 GeV and 56 GeV can be summarized as follows:

1) Scaling is good to $\sim 10\%$ for $Q^2 > 4.5\ GeV^2$.

2) If the ratios of data at 150 GeV to the data at 56 GeV and to the SLAC data are parametrized in the form

$$R = \frac{N}{(1+Q^2/\Lambda^2)^2}$$

then the fits to both the ratios are consistent with

$$\Lambda > 10 \text{ GeV}$$

with 90% confidence, averaged over a restricted ω range (high Q^2 and low ω).

3) There are indications of low Q^2 or large ω deviations from scaling. An overall fit gives a two standard deviations effect.

The virtual photon's scattering is primarily transverse employing dominantly scattering off spin 1/2 constituents (partons). The experimental value of the ratio

$$R = \frac{\sigma_L}{\sigma_T} = 0.18 \pm 0.10 .$$

The experiments on the spin dependent deep inelastic electroproduction are planned in the near future. For that one needs polarized lepton beams and polarized nucleon targets. Muon beams at FNAL, BNL and CERN automatically possess a longitudinal polarization due to their origin in the weak decays of pions. At SLAC a polarized electron source is being installed.

2. Hyperfine splitting in hydrogen and deep inelastic electroproduction

The magnitude of hyperfine splitting of the singlet and triplet ground-state energy levels of a hydrogen atom is at present the most accurately known physical constant. The hydrogen maser measurements yield the value

$$\Delta\nu_{hfs} = 1\ 420\ 405\ 751.7662\,(3)\ \ \text{Hz} \tag{2.1}$$

with the fantastic accuracy of 10^{-13} .

Theoretical evaluation of this quantity is based on the employment of quantum electrodynamics [19] and relativistic equations for bound states [20] . As such an equation it is the most helpful to utilize the Logunov-Tavkhelidze quasipotential equation [21] in the momentum representation

$$\left(E - \sqrt{\vec{p}^{\,2}+m^2} - \sqrt{\vec{p}^{\,2}+M^2}\right)\Psi(\vec{p}) = \int \frac{d^3p'}{(2\pi)^3} V(\vec{p},\vec{p}\,';E)\,\Psi(\vec{p}\,'), \tag{2.2}$$

where the wave function $\Psi(\vec{p})$ describes the relative motion of an electron of mass m and of a proton of mass M in their centre-of-mass frame with the relative three-momentum $\vec{p}$. The electron-proton interaction is determined by the quasipotential $V(\vec{p},\vec{p}\,';E)$ which is in general a nonlocal (i.e., dependent not only on the difference $(\vec{p}-\vec{p}\,')$) and explicitly energy dependent function. In the nonrelativistic limit

$$p^2 \ll m^2, M^2 \quad ; \quad W = E - m - M \ll m + M$$

the quasipotential equation (2.2) turns into the usual Schrö-dinger equation

$$\left(W - \frac{\vec{p}^{\,2}}{2\mu}\right)\Psi(\vec{p}) = \int \frac{d^3p'}{(2\pi)^3} V(\vec{p},\vec{p}\,')\Psi(\vec{p}\,'), \qquad (2.2a)$$

where the reduced mass

$$\mu = \frac{mM}{m+M} .$$

The quasipotential V is usually given in terms of the scattering amplitude off the mass shell with the help of an operator relation

$$V = T_+ (1 + G_0 T_+)^{-1} = T_+ - T_+ G_0 T_+ + \dots , \qquad (2.3)$$

which can be obtained from the corresponding Lippmann-Schwinger equation for the off-shell scattering amplitude

$$T_+ = V + V G_0 T_+ .$$

The Green's function of free particles

$$G_0(\vec{p},\vec{p}\,';E) = \frac{(2\pi)^3 \delta^3(\vec{p}-\vec{p}\,')}{E - \sqrt{\vec{p}^{\,2}+m^2} - \sqrt{\vec{p}^{\,2}+M^2}} . \qquad (2.4)$$

The scattering amplitude projected onto the positive frequence states

$$T_+(\vec{p},\vec{p}\,';E) = \bar{u}_m(\vec{p}\,')\bar{u}_M(-\vec{p}\,')\, T(\vec{p},\vec{p}\,';E)\, u_m(\vec{p})\, u_M(-\vec{p}) \qquad (2.5)$$

where the amplitude T is given in terms of a sum of Feynman diagrams, parametrized as is shown in Fig.4

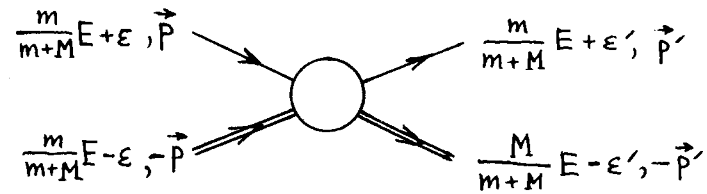


Fig.4

Examples of one-photon and two-photon exchange diagrams of the lowest order in e^2 are displayed in Fig.5

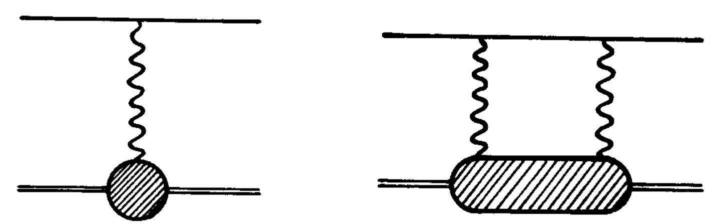


Fig.5

In the initial approximation, as should be expected, we have the purely Coulomb interaction. We are mainly intersted in the contribution to the hyperfone splitting of the two-photon exchange diagram since it includes the amplitude for the Compton scattering of virtual photons off the proton.

The combined contribution to the hyperfine splitting of the one- and two-photon exchange diagrams can be written as[22]

$$\Delta E^{\gamma}_{hfs} + \Delta E^{2\gamma}_{hfs} = |\Psi_c(0)|^2 \Big\{ \frac{2\pi\alpha}{3mM}(1+\varkappa)\langle \vec{\sigma}_e \cdot \vec{\sigma}_p \rangle +$$

$$+ \langle T_{2\gamma}(\vec{0},\vec{0}) \rangle - \frac{4\pi}{3mM}(1+\varkappa)\langle \vec{\sigma}_e \cdot \vec{\sigma}_p \rangle \int \frac{d^3q\, V_c(\vec{q})}{(2\pi)^3 \left[W_c - \frac{\vec{q}^{\,2}}{2\mu}\right]} \Big\}, \quad (2.6)$$

where $T_{2\gamma}(\vec{0},\vec{0})$ is the two-photon exchange amplitude taken at zero values of the proton and electron three-momenta, $\varkappa$ is the proton anomalous magnetic moment, $\vec{\sigma}_e$, $\vec{\sigma}_p$ are the usual Pauli spin matrices and symbol $\langle \dots \rangle$ means the matrix element with respect to singlet and triplet states. The Coulomb potential

$$V_c(\vec{q}) = -\frac{e^2}{\vec{q}^{\,2}} = -\frac{4\pi\alpha}{\vec{q}^{\,2}} \qquad (2.6a)$$

the Bohr energy levels

$$W_c = -\frac{\alpha^2\mu}{2n^2}, \quad n = 1, 2, 3, \dots \qquad (2.6b)$$

and the modulus squared of the Coulomb wave function in the coordinate representation at the origin $\vec{r} = 0$ reads as

$$|\Psi_c(0)|^2 = \frac{(\mu\alpha)^3}{\pi n^3}. \qquad (2.6c)$$

In the end, the expression for the triplet-singlet hyperfine splitting in the ground state takes the form [22]

$$\Delta\nu_{hfs} = E(^3S_1) - E(^1S_0) = \Delta\nu_F(1+\delta), \qquad (2.7)$$

where the so-called Fermi splitting

$$\Delta\nu_F = \frac{8\pi\alpha}{3mM}(1+\varkappa)|\Psi_c(0)|^2$$

and the correction

$$\delta = \frac{\alpha m}{\pi(1+\varkappa)M} \Big\{ \frac{3M^2}{4i\pi^2} \int \frac{d^4q}{q^4} N_e^{\mu\nu}(q) N^p_{\mu\nu}(q) - \qquad (2.8)$$

$$- 8M(1+\varkappa) \int_0^\infty \frac{dq}{q^2 - 2\mu W_c} \Big\}$$

$$N^{e,p}_{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{2}(1+\gamma_0)\gamma_5\gamma_3 H^{e,p}_{\mu\nu}\right] = C^{[A]\,e,p}_{\mu\nu}(s_3)$$

$$s_3 = (0, 0, 0, s_3)$$

Thus, tensor $N_{\mu\nu}$ coincides with the spin dependent (anti-symmetric) piece of the off-shell forward Compton amplitude if the spin three-vector $\vec{s}$ in the particle rest frame is directed along the Z-axis. In particular, the electron amplitude

$$N^e_{\mu\nu}(q) = \frac{-4mq^2}{(q^4 - 4m^2\nu^2)} \cdot \frac{1}{m^2} L^{[A]}_{\mu\nu}(\sigma_3).$$

In the integral (2.8) it is possible to perform the Wick rotation of the q^0-axis in the complex plane and thus to integrate over the four-dimensional Euclidean space Q with

$$Q^0 = iq^0 = i\nu \ , \quad Q^2 = -q^2 \ .$$

As a result utilizing the invariant decomposition (1.26) and expression (1.19), we find

$$\delta = \frac{2\alpha m M}{\pi^2(1+\varkappa)} \int \frac{d^4Q}{Q^2(Q^4+4m^2\nu)} \Big[(2Q^2+\nu^2)H_1(i\nu,Q^2) + 3i\nu Q^2 H_2(i\nu,Q^2)\Big] - \frac{8\alpha m}{\pi}\int_0^\infty \frac{dQ}{Q^2-2\mu W_c} \ . \qquad (2.9)$$

For the invariant functions $H_{1,2}(\nu,Q^2)$ with $Q^2>0$ the dispersion relations can be rigorously proved [1] :

$$H_{1,2}(\nu,Q^2) = \int_{\nu_B}^{\infty} \frac{d\nu'^2}{\nu'^2-\nu^2}\, G_{1,2}(\nu',Q^2) \qquad (2.10)$$

where the point $\nu_B = Q^2/(2M)$ corresponds to the position of the proton pole (the Born term), and the cut starts at the pion-nucleon threshold

$$\nu_\pi = \frac{1}{2M}\left(Q^2+m_\pi^2\right) + m_\pi \ .$$

Inserting the dispersion relations (2.10) into expression (2.9) we separate the correction δ into three pieces

$$\delta = \delta_B + \delta_1 + \delta_2 \ , \qquad (2.11)$$

where the "Born piece" corresponds to the contribution of the Feynman diagrams shown in Fig.6 with the real protons form factors at the vertices

Fig.6

This contribution can be easily calculated [19,20,22]

$$\delta_B = -(34.5\pm 2)\,\text{ppm} \ , \quad 1\,\text{ppm} = 10^{-6} \qquad (2.12)$$

The remaining two pieces are expressed directly in terms of the proton spin dependent structure functions

$$\delta_{1,2} = \frac{\alpha m}{2\pi M(1+\varkappa)}\,\Delta_{1,2} \ ,$$

$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2}\left\{\frac{9}{4}F_2^2(Q^2) - 4M^2\int_{\nu_\pi}^\infty \frac{d\nu}{\nu}\,\beta_1\!\left(\frac{\nu^2}{Q^2}\right) G_1(\nu,Q^2)\right\} ,$$

$$\Delta_2 = -12M^2 \int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_\pi}^\infty d\nu\,\beta_2\!\left(\frac{\nu^2}{Q^2}\right) G_2(\nu,Q^2) \ , \qquad (2.13)$$

where

$$\beta_1(\theta) = 3\theta - 2\theta^2 - 2(2-\theta)\sqrt{\theta(\theta+1)} \ ,$$

$$\beta_2(\theta) = 1 + 2\theta - 2\sqrt{\theta(\theta+1)} \ , \quad \theta = \nu^2/Q^2$$

and $F_2(Q^2)$ is the Pauli form factor of the proton. Thus, the two-photon exchange corrections to the Fermi hyperfine splitting can be evaluated with the help of the experimental data on spin dependent inelastic electron proton scattering.

While determining the contribution of the Δ_1 term one may utilize also the data on the spin dependent total photoabsorption cross sections for real photons, since at $Q^2=0$ (see eqs. (1.36a))

$$e^2 G_1(\nu,0) = \frac{1}{2}\left[\sigma_P(\nu)-\sigma_A(\nu)\right]$$

and it satisfies the Gerasimov sum rule. As a result one manages to obtain the estimate

$$|\delta_1| \sim 1 \div 2 \text{ ppm} . \tag{2.14}$$

The situation is more complicated as regards the Δ_2 contribution. In the absence, presently, of any direct experimental information about the structure function G_2, we can make use of those bounds which ensure from the positivity conditions for the proton tensor $W_{\mu\nu}$. Then, from inequalities (1.43) and (1.48) it follows [23] that

$$G_2(\nu,Q^2) \geq -\frac{W_1(\nu,Q^2)}{\nu^2+Q^2}\left(1+R\sqrt{\frac{\nu^2}{Q^2}}\right), \tag{2.15}$$

$$G_2(\nu,Q^2) \leq \frac{W_1(\nu,Q^2)}{\nu^2+Q^2}\cdot\begin{cases}\left(1+\frac{R\nu^2}{8Q^2}\right), & R\frac{\nu^2}{Q^2}\leq 16\\ \left(\sqrt{R\frac{\nu^2}{Q^2}}-1\right), & R\frac{\nu^2}{Q^2}>16\end{cases}$$

$$R=\frac{\sigma_L}{\sigma_T}$$

Employing inequalities (2.15), one can find the corresponding limits for the quantity Δ_2 defined by eqs. (2.13). The existing experimental data for the structure function W_1 and the ratio R permit one to make a numerical estimate [23] of the upper and lower bounds for the δ_2 correction:

$$-2 \text{ ppm} \leq \delta_2 \leq 3 \text{ ppm} . \tag{2.16}$$

The comparison of the theoretical (with radiative corrections) and experimental values of the ground-state hyperfine splitting leads to the relation

$$\frac{\Delta\nu_{exp}-\Delta\nu_{th}}{\Delta\nu_{th}} = (2.5\pm 4.0)\text{ ppm}-\delta_1-\delta_2 ,$$

which is consistent with estimates (2.14) and (2.15) for the structure corrections δ_1 and δ_2.

3. Electron-positron annihilation into hadrons

The process of electron-positron annihilation into hadrons is characterized by the following experimental data [24] :

1) The ratio R of the cross section of hadronic annihilation to the cross section of annihilation into a muon pair is large and is rising from the value $2\div3$ at $\sqrt{q^2} \approx 3$ GeV to about $4\div6$ at $\sqrt{q^2} \approx 5$ GeV.

2) The single-particle inclusive distribution $q^2 d\sigma/d\omega$, where $\omega = x = 2E_\pi/\sqrt{q^2}$ for $e^+e^- \to \pi H$ fails completely to scale and increases rapidly with q^2 for $\omega \lesssim 0.5$. On the contrary for $\omega \gtrsim 0.5$ it is consistent with scaling.

3) The angular distribution $d\sigma/d\Omega$ of charged particles is close to an isotropic one for $3 < \sqrt{q^2} < 5$ GeV and $|\cos\theta| \leq 0.6$.

4) The single-particle inclusive distribution $E_\pi d\sigma/d^3p \propto \exp(-E_\pi/T)$ with $T \approx 170$ MeV for not very high momenta, i.e., it is very similar to the single-particle inclusive distributions found in hadronic reactions.

5) The mean momentum and multiplicity of charged particles rise slowly with $\langle p_c \rangle \approx 400$ MeV and $\langle n_c \rangle \approx 4$.

6) As a result the fraction of the total energy carried by charged particles, evaluated assuming all particles are pions and $d\sigma/d\Omega$ is isotropic, is small and decreases gradually from ≈ 0.6 at $\sqrt{q^2} \approx 3$ GeV to ≈ 0.55 at $\sqrt{q^2} \approx 5$ GeV.

7) In the region $\sqrt{q^2} > 3$ GeV several new vector resonances were discovered with quantum numbers $J^{PC} = 1^{--}$ and properties

	m (GeV)	Γ_{tot} (MeV)	Γ_ℓ (keV)
ψ_1	3.105	0.08	5.2
ψ_2	3.695	0.5	2.2
ψ_3	4.15	250–300	4

where Γ_{tot} is the total width of resonances and Γ_ℓ is the partial width of the decay into a charged lepton pair (e^+e^-), $(\mu^+\mu^-)$

While preparing this section we have used papers [24-30] .

3.1. Kinematics of the process

Consider the inclusive process of electron-positron annihilation in the one-photon approximation with a single detected hadron in the final state

$$e^- + e^+ \to \gamma \to h + \underbrace{\sum_n \text{hadrons}}_{\text{unobserved}} . \tag{3.1}$$

The diagram of this process is shown in Fig.7, where $k_\pm$ are the momenta of the colliding electron and positron with mass m, $q = k_+ + k_-$ is the four-momentum of the virtual photon with $q^2 > 0$ and $\bar{p}$ is the momentum of the detected hadron with mass M. The rest of the hadrons (unobserved) in the final state owing to conservation laws carry the momentum

$$p_n = q - \bar{p}$$

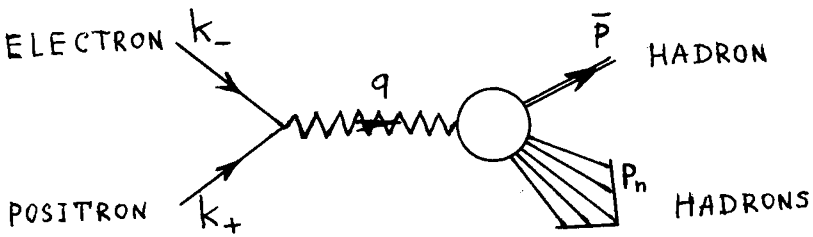


Fig.7

with the invariant mass squared

$$p_n^2 = s = (q-\bar{p})^2 = q^2 + M^2 - 2(\bar{p}\cdot q). \qquad (3.2)$$

Introduce, also the invariant variable

$$\nu = -\frac{(\bar{p}\cdot q)}{M} < 0 \quad , \quad \bar{p}^2 = M^2 , \qquad (3.3)$$

which in the centre-of-mass frame of the colliding leptons ($\vec{q}=0$) is proportional to the detected hadron energy E

$$M\nu = -E\sqrt{q^2} \ , \qquad (3.3a)$$

and the hadron three-momentum squared

$$\vec{\bar{p}}^2 = E^2 - M^2 = M\left(\frac{\nu^2}{q^2} - 1\right). \qquad (3.3b)$$

The scattering angle in the same frame of reference is defined relative to the direction of the lepton beams

$$(\vec{k}\cdot\vec{\bar{p}}) = |\vec{k}|\,|\vec{\bar{p}}|\cos\theta \ , \qquad (3.4)$$

and four-momentum squared of the virtual photon

$$q^2 = 4\vec{k}^2 + 4m^2 > 0 . \qquad (3.4a)$$

In terms of the invariant variables q^2 and ν the effective mass squared of the unobserved hadrons

$$s = M^2 + q^2 + 2M\nu . \qquad (3.5)$$

Since the lowest final state is the hadron-antihadron pair, it is clear, that

$$s \geqslant M^2 , \qquad (3.6)$$

and hence from eq. (3.5) we obtain the boundary of the physical region

$$q^2 \geqslant -2M\nu , \qquad (3.7)$$

Introducing as before the dimensionless variable

$$\omega = -\frac{2M\nu}{q^2} = x = \frac{2E}{\sqrt{q^2}} = \frac{s-M^2}{-q^2} + 1 , \qquad (3.8)$$

we rewrite inequality (3.7) in the form

$$0 \leqslant \omega \leqslant 1 . \qquad (3.7a)$$

The other boundary we can find from the positivity condition for the three-momentum squared of the detected hadron in the lepton c.m. frame ($\vec{q}=0$)

$$\vec{\bar{p}}^2 \geqslant 0 \ , \quad E = \sqrt{\vec{\bar{p}}^2 + M^2} \geqslant M .$$

Then from eqs. (3.3a), (3.3b) the inequalities follow

$$\nu^2 \geq q^2 \quad , \quad \frac{E}{M} = -\frac{\nu}{\sqrt{q^2}} \geq 1 \tag{3.9}$$

which determine the second boundary of the physical region. The domain singled out by inequalities (3.7) and (3.9) is displayed in Fig.8

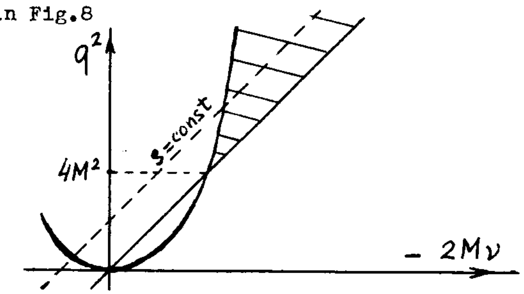


Fig.8

According to the Feynman rules the amplitude of the process (3.1) is of the form

$$T_{fi} = \frac{4\pi\alpha}{q^2+i0}\, \bar{v}(k_+)\gamma^\mu u(k_-) \langle \bar{p}, p_n | j_\mu(0) | 0 \rangle , \tag{3.10}$$

where $v(k)$ is the negative frequency spinor normalized as $\bar{v}v = -1$, and the invariant differential cross section, averaged over the polarizations of the detected hadron with spin S, is equal to

$$(2\pi)^3 E \frac{d\sigma}{d^3\vec{\bar{p}}} = \frac{2m^2 M(2S+1)}{\sqrt{q^2(q^2-4m^2)}} \sum_{n,av} |T_{fi}|^2 (2\pi)^4 \delta^4(q-\bar{p}-p_n) . \tag{3.11}$$

The phase space element with the help of eq. (3.3b) may be written as

$$\frac{d^3\bar{p}}{E} = |\vec{\bar{p}}|\, dE\, d\Omega = M\left[\frac{\nu^2}{q^2} - 1\right] dE\, d\Omega \ ,$$
$$d\Omega = 2\pi\, d\cos\theta . \tag{3.12}$$

Neglecting the lepton mass, compared to q^2 and on account of eq. (3.12), we represent the differential cross section (3.11) in the form

$$\frac{d\sigma}{d\Omega\, dE} = \frac{4\alpha^2}{q^4} \frac{(2S+1)}{q^2} M |\vec{\bar{p}}|\, \bar{L}^{\mu\nu} \bar{W}_{\mu\nu} \ , \tag{3.13}$$

where averaging over the spins of the initial leptons gives the tensor

$$\frac{1}{2}\bar{L}^{\mu\nu} = \frac{m^2}{4} \sum_{\sigma_+,\sigma_-} \bar{u}^{\sigma_-}(k_-)\gamma^\mu v^{\sigma_+}(k_+) \bar{v}^{\sigma_+}(k_+) \gamma^\nu u^{\sigma_-}(k_-) =$$
$$= \frac{1}{16} \mathrm{Tr}\left[(\hat{k}_- + m)\gamma^\mu (\hat{k}_+ - m)\gamma^\nu\right] = \tag{3.14a}$$
$$= \frac{1}{4}\left[k_+^\mu k_-^\nu + k_-^\mu k_+^\nu - \frac{1}{2} q^2 g^{\mu\nu}\right] ,$$
$$\hat{k} = \gamma \cdot k ,$$

and the hadronic tensor

$$\overline{W}_{\mu\nu}=\frac{(2\pi)^3}{(2S+1)}\sum_{c,n}\langle 0|J_\mu(0)|\bar{p},p_n\rangle\langle\bar{p},p_n|J_\nu(0)|0\rangle\,\delta^4(q-\bar{p}-p_n)=$$

$$=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\overline{W}_1(\nu,q^2)+\frac{1}{M^2}\tilde{P}_\mu\tilde{P}_\nu\overline{W}_2(\nu,q^2), \quad (3.14b)$$

$$\tilde{P}=\bar{P}-\frac{(\bar{p}\cdot q)}{q^2}q\ ,\quad (\tilde{P}\cdot q)=0\ ,$$

where the spin of the detected hadron is averaged over, and c denotes the connected part.

As a result, making necessary transformations we obtain the expression for the cross section (3.13) in terms of the structure functions $W_{1,2}$

$$\frac{d^2\sigma}{d\Omega dE}=\frac{\alpha^2}{q^4}(2S+1)\frac{M^2\nu}{\sqrt{q^2}}\left[1-\frac{q^2}{\nu^2}\right]^{1/2}\left\{2\overline{W}_1(\nu,q^2)+\right. \quad (3.15)$$

$$\left.+\left(\frac{\nu^2}{q^2}-1\right)\overline{W}_2(\nu,q^2)\sin^2\theta\right\},$$

from which it, also, follows, that

$$\frac{d\sigma}{d\omega}=\frac{\pi\alpha^2}{q^2}(2S+1)\omega\left[1-\frac{4M^2}{q^2\omega^2}\right]^{1/2}\left\{2M\overline{W}_1(\nu,q^2)+\right. \quad (3.15a)$$

$$\left.+\frac{\omega}{3}\left[1-\frac{4M^2}{q^2\omega^2}\right]\nu\overline{W}_2(\nu,q^2)\right\},$$

$$q^2\frac{d\sigma}{d\omega}\approx\pi\alpha^2(2S+1)\omega\left\{2M\overline{W}_1(\nu,q^2)+\frac{\omega}{3}\nu\overline{W}_2(\nu,q^2)\right\}$$

$$q^2\to\infty\ ,\quad \omega\ \text{fixed}\ .$$

In close analogy with the case of electroproduction we may introduce the "longitudinal" structure function (note the change of signs compared to eq. (1.36))

$$\overline{W}_L=\overline{W}_1+\left(\frac{\nu^2}{q^2}-1\right)\overline{W}_2\ .$$

Then the positivity conditions are of the form

$$\overline{W}_T=W_1\geqslant 0\ ,\quad \overline{W}_L\geqslant 0\ ,$$

and the differential cross section (3.15) may be rewritten as

$$\frac{d^2\sigma}{d\Omega dE}=\frac{\alpha^2}{q^4}(2S+1)M|\vec{\bar{p}}|\left\{(1+\cos^2\theta)\overline{W}_1(\nu,q^2)+\right.$$

$$\left.+\ (1-\cos^2\theta)\ \overline{W}_L(\nu,q^2)\right\}. \quad (3.15b)$$

Consider the process of "elastic" annihilation when in the final state only particle and its antiparticle are produced, i.e., $s=p_n^2=M^2$, $\vec{\bar{p}}^2=\frac{q^2}{4}\left(1-\frac{4M^2}{q^2}\right)$. Then the structure functions $\overline{W}_{1,2}$ are expressed through the elastic form factors of a hadron as follows:

a) case of a spin 0 particle

$$\langle p,\bar{p}|J_\mu(0)|0\rangle=\frac{1}{2M}(p-\bar{p})_\mu F(q^2)\ ,\quad q=p+\bar{p},$$

$$\overline{W}_1=0\ ,\quad \overline{W}_2=|F(q^2)|^2\,\delta\!\left(\nu-\frac{q^2}{2M}\right),$$

b) case of a spin 1/2 particle

$$\overline{W}_1(\nu,q^2)=\frac{q^2}{4M^2}|G_M(q^2)|^2\,\delta\!\left(\nu-\frac{q^2}{2M}\right),$$

$$\overline{W}_L(\nu,q^2) = |G_E(q^2)|^2 \delta\left(\nu - \frac{q^2}{2M}\right).$$

In particular, for a point-like spin 1/2 particle (e.g., muon) we find the cross section [26]

$$\frac{d\sigma_{1/2}}{d\Omega} = \frac{\alpha^2}{4q^2}\left[1-\frac{4m_\mu^2}{q^2}\right]^{1/2}\left\{1+\frac{4m_\mu^2}{q^2}+\left[1-\frac{4m_\mu^2}{q^2}\right]\cos^2\theta\right\}$$

which in the limit $m_\mu^2 \ll q^2$ turns into the "parton" cross section

$$\frac{d\sigma_{1/2}}{d\Omega} = \frac{\alpha^2}{4q^2}(1+\cos^2\theta),\ \sigma_{1/2}^{tot} = \frac{4\pi\alpha^2}{3q^2}. \qquad (3.16a)$$

In the case of point-like scalar particles (spin 0 partons)

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{8q^2}\left[1-\frac{4M^2}{q^2}\right]^{3/2}(1-\cos^2\theta) \qquad (3.16b)$$

and the total cross section in the limit $M^2 \ll q^2$

$$\sigma_0^{tot} = \frac{\pi\alpha^2}{3q^2} = \frac{1}{4}\sigma_{1/2}^{tot}. \qquad (3.16c)$$

In the general case of spin S particles the "elastic" differential cross section reads as follows [26]

a) for integer spins

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(2S+1)}{8q^2}\left(1-\frac{4M^2}{q^2}\right)^{3/2}\Big\{(1-\cos^2\theta)E(q^2) + + (1+\cos^2\theta)\frac{q^2}{4M^2}M(q^2)\Big\},$$

b) for half-integer spins

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(2S+1)}{2q^4}M^2\left(1-\frac{4M^2}{q^2}\right)^{1/2}\Big\{(1-\cos^2\theta)E(q^2) + + (1+\cos^2\theta)\frac{q^2}{4M^2}M(q^2)\Big\}$$

where $E(q^2)$ is the sum of the moduli squared of the electric form factors with $E(0) = Q^2$ the square of electric charge and $M(q^2)$ is the sum of the moduli squared of the magnetic form factors with

$$M(0) = \frac{S+1}{3S}\mu^2$$

the square of the dipole magnetic moment. The total cross section for point-like particles ($Q=\mu=1$) in the limit $q^2 \gg M^2$ is of form

a) integer spin $S \geqslant 1$

$$\sigma_S^{tot} = \frac{\pi\alpha^2}{3M^2}\,\frac{(2S+1)(S+1)}{6S} = const,$$

b) half-integer spin S

$$\sigma_S^{tot} = \frac{4\pi\alpha^2}{3q^2}\,\frac{(2S+1)(S+1)}{6S} \propto \frac{const}{q^2}.$$

Note that for point-like particles of integer spin $S \geqslant 1$ the total cross section tends to a constant value as $q^2 \to \infty$.

3.2. Inclusive annihilation and electroproduction

Thus we see that the description of the inclusive annihilation proceeds quite in parallels to the description of the inclusive electroproduction. Accordingly, if the scale invariance is still valid, then on the basis of eq. (3.16) and dimensional analysis (the automodelity principle) we should expect, that

$$2M\,\overline{W}_1(\nu,q^2) = \overline{F}_1(\omega), \qquad (3.17)$$
$$-\nu\overline{W}_2(\nu,q^2) = \overline{F}_2(\omega)$$

in the limit $\nu, q^2 \to \infty$, ω- *fixed*.

However, the experimental data up to now show, that $q^2 d\bar{\sigma}/d\omega$ fails to be a scale invariant function (i.e., a function depending on ω only). So the question arises, whether the straightforward connection (of the type of crossing symmetry) exists between the structure functions of inclusive annihilation and electroproduction.

For the beginning note, that in distinction from the case of electroproduction the hadronic annihilation tensor cannot be represented in the form of a matrix element of the

current commutator (or product)[31] and consequently, does not coincide with the "imaginary" part of the forward Compton amplitude for the virtual photons with $q^2 > 0$ and $\nu \leqslant 0$.

In fact, when $q^2 > 0$ the Fourier transform of a matrix element of the current commutator

$$(2\pi) W_{\mu\nu}(p,q) = \int d^4x\, e^{iq\cdot x} \langle p | [J_\mu(x), J_\nu(0)] | p \rangle , \qquad (3.18)$$

besides the familiar contribution displayed in Fig.3 contains the contributions of different types of intermediate states, shown in Fig.9

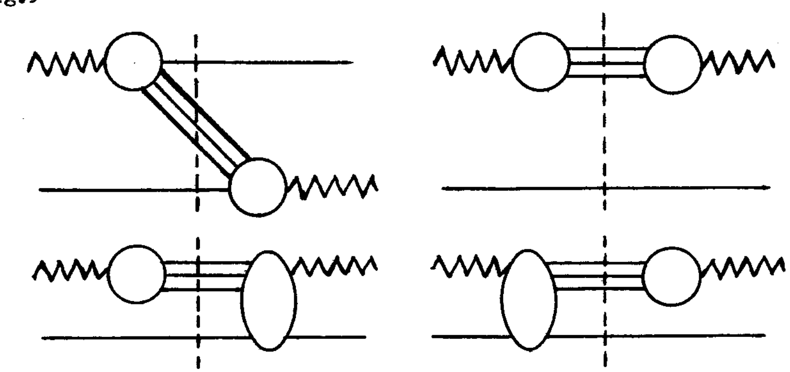


Fig.9

Only Z-type diagrams in Fig.9a are relevant to the process of single-particle conclusive annihilation. In order to avoid the appearance of the graphs in Fig. 9b,c it is necessary to consider the discontinuity ("imaginary" part) of the nonforward Compton scattering amplitude for the virtual photons with

$q_1^2 \neq q_2^2$. Denote it by

$$C_{\mu\nu}(s,t,q_1^2,q_2^2), \quad t=(q_1-q_2)^2.$$

The substitution rule tells us that under the interchange of the initial and final hadrons with $-p_1 \leftrightarrow p_2$, hadron $\to$ antihadron $C_{\mu\nu}$ behaves as follows (depending on the hadron statistics and assuming invariance under charge conjugation).

$$C_{\mu\nu}(s,t,q_1^2,q_2^2)=\pm C_{\mu\nu}(u,t,q_1^2,q_2^2),$$

$$s\to u, \quad \nu\to-\nu$$

where

$$u= s-4M\nu = (q_1-p_2)^2,$$

$$s= (q_1+p_1)^2,$$

and the upper (lower) sign refers to the interchange of bosons (fermions).

Recalling eq. (I.17) we have in the case of electroproduction:

$$W_{\mu\nu}(p,q) \propto [C_{\mu\nu}(s+i0,0,q^2,q^2)-C_{\mu\nu}(s-i0,0,q^2,q^2)]= \\ = \mathrm{disc}_s\, C_{\mu\nu}(s,0,q^2,q^2). \qquad (3.18a)$$

Since here $q^2<0$ we are not in a region where $C_{\mu\nu}$ has cuts in q^2, and, therefore, it is not necessary to shift the values of q^2 from the real axis.

Similarly, in the case of inclusive annihilation the hadronic tensor $\overline{W}_{\mu\nu}$ is given by the discontinuity

$$\overline{W}_{\mu\nu}(\overline{p},q) \propto [C_{\mu\nu}(s+i0,0,q^2+i0,q^2-i0)-C_{\mu\nu}(s-i0,0,q^2+i0,q^2-i0)]= \\ = \mathrm{disc}_s\, C_{\mu\nu}(s,0,q^2+i0,q^2-i0), \qquad (3.18b)$$

which is shown graphically in Fig.10

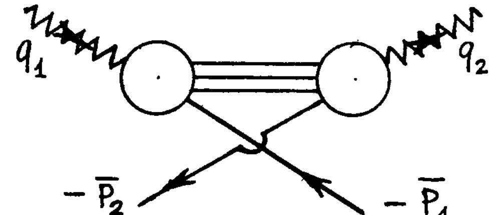


Fig.10

Note that now $q^2>0$ and we are in the region of cuts (physical thresholds) in the variables $q_{1,2}^2$.

As it follows from eqs. (3.10), (3.11) one must take q_1^2 above its cut ($+i0$) and q_2^2 below its cut $(-i0)$, to insure the correct selection of the hermitian tensor $\overline{W}_{\mu\nu}$ and consequently, the real structure functions $\overline{W}_{1,2}$.

We illustrate the aforesaid by a simple example [28]. Let the electroproduction structure functions be of the form

$$W(\nu,q^2)=\lim_{q_1^2\to q_2^2}\varphi(q_1^2)\varphi(q_2^2)f(\nu)=\varphi^2(q^2)f(\nu).$$

Then the annihilation structure functions are evidently equal to

$$\overline{W}(\nu,q^2)=|\varphi(q^2)|^2 f(\nu), \quad q^2>0,$$

and, generally speaking, cannot be obtained from the function without additional assumptions.

Now assume that from field theory we know the Compton amplitude $C_{\mu\nu}$ for $q_1^2 \neq q_2^2$ and in particular its analytic properties. Namely, one expects that $C_{\mu\nu}$ has (for $t<0$) the right- and left-hand cuts in S and right-hand cuts in q^2 Then discontinuity of $C_{\mu\nu}$ in S while not being an analytic function in S , may still have simple analytic properties in $q_{1,2}^2$ variables. Since the lines of fixed $S > M^2$ pass through the physical regions both of electroproduction and annihilation, we may get from one to the other by continuing $\mathrm{disc}_S\, C_{\mu\nu}$ in $q_{1,2}^2$ at fixed values of S (Fig.11)

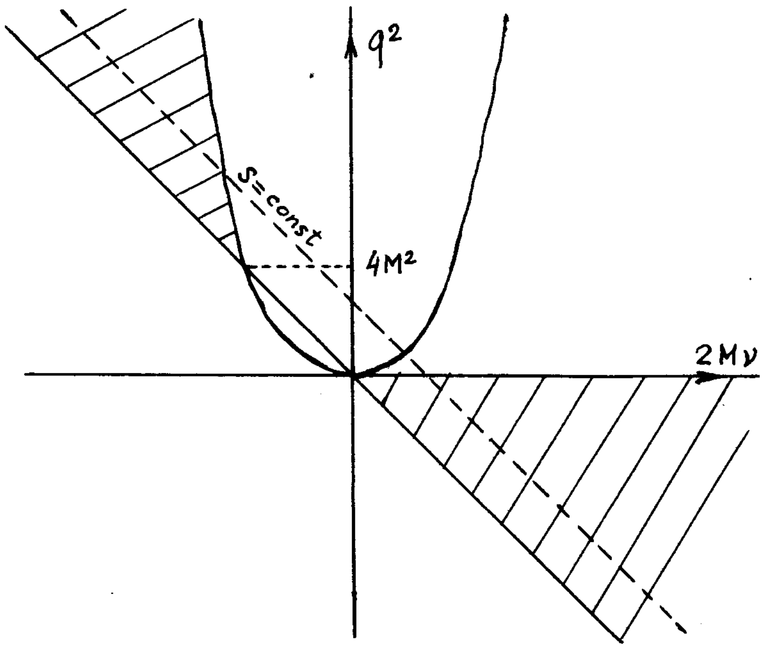


Fig. 11

It is obvious that in the scaling limit one must introduce a pair of scaling variables

$$\omega_{1,2} = \frac{s - M^2}{-q_{1,2}^2} + 1$$

and consider the generalized Bjorken limit $(\lim_B)\ s, q_{1,2}^2 \to \infty$ with t , $\omega_{1,2}$ fixed. Then the correct analytic continuation leads to the statement

$$-\bar{F}_1(\omega) = F_1(\omega + i0,\ \omega - i0), \qquad (3.19)$$

$$\bar{F}_2(\omega) = F_2(\omega + i0,\ \omega - i0),$$

where

$$F_1(\omega_1, \omega_2) = \lim_B 2MW_1(s, t=0, q_1^2, q_2^2),$$

$$F_2(\omega_1, \omega_2) = \lim_B \nu W_2(s, t=0, q_1^2, q_2^2),$$

$$F_{1,2}(\omega) = F_{1,2}(\omega, \omega).$$

In various models more simple relations were obtained. For instance, if the Compton amplitude is taken to be a sum of ladder diagrams in the field theory with a cut-off of transverse momenta [32] , then naive continuation relations hold

$$\bar{F}_1(\omega) = \mp F_1(\omega), \qquad (3.20)$$

$$\bar{F}_2(\omega) = \pm F_2(\omega),$$

where the upper (lower) sing refers to fermions (bosons).

Summing up leading logarithmic terms $g^2 \ln q^2$ in the scaling region in some (neutral γ_5 and γ_μ) renormalizable field theories, Gribov and Lipatov [33] have found reciprocal relations

$$\overline{W}(\omega, \ln q^2) = -\frac{1}{\omega} W\left(\frac{1}{\omega}, \ln(-q^2)\right),$$
$$W = 2M W_1 = \omega \nu W_2 . \qquad (3.21)$$

In this case scaling is violated by terms in $\ln q^2$. Relations (3.21) are interesting because unlike analytic continuation (e.g., naive formulae (3.20)) they relate values of W in its physical region $(\omega \geq 1)$ to the values of $\overline{W}$ also in its physical region ($0 \leq \omega \leq 1$).

There is an important theorem [34] pertaining to the threshold region near the point $\omega = 1$. The theorem is true under rather general circumstances and states:

If as $\omega \to 1+$

$$F(\omega) \to A(\omega - 1)^{\alpha}, \qquad (3.22)$$

and as $\omega \to 1-$

$$\overline{F}(\omega) \to \overline{A}(1-\omega)^{\overline{\alpha}}$$

then

$$\overline{A} = A, \quad \overline{\alpha} = \alpha .$$

The proof is based on the assumption that the Bjorken limit is controlled by generalized ladder diagrams with exact propagators and vertices. This assumption is valid for all simple models. At the same time in some models [28,16] it is possible to relate the index α in eq. (3.22) to the power of asymptotic decreasing of the elastic hadron from factors:

$$\alpha = 2n - 1$$
$$G^{el}(q^2) = (q^2)^{-n}, \quad q^2 \to \infty . \qquad (3.23)$$

Presently one is used to take $n = 2$, then $\alpha = 3$, which seems to be consistent with experimental data..

Gilman [5c] made a comparison of reciprocal relations (3.21) with the available experimental data. Under some simplifying assumptions he finds that the reciprocal relation between $e^+e^- \to \bar{p} H$ and $e^- p \to e^- H$ is in rough agreement with the data, while analogous relation between $e^+e^- \to \pi H$ and $e^- \pi \to e^- H$ would predict, using annihilation data at $\omega = 0.5$, for $e\pi$- electroproduction at $\omega = 2$ the result by an order of magnitude larger than that measured for $e^- p \to e^- H$, which seems quite unreasonable.

3.3. Total cross section of e^+e^-–annihilation and hadronic vacuum polarization

Now we proceed to considering the inclusive process of electron-positron annihilation into hadrons with no hadron singled out:

$$e^+ + e^- \to \gamma \to \sum_n \text{hadrons} \qquad (3.24)$$

The one-photon Feynman diagram for this reaction, is shown in Fig.12

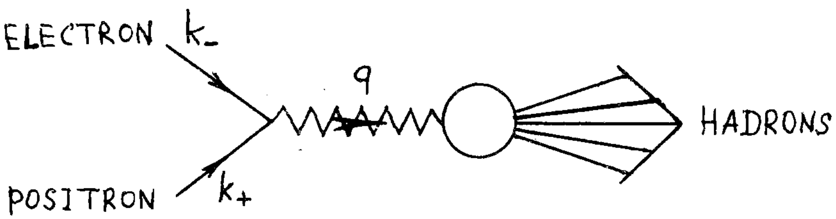


Fig.12

with the notations already adopted. The amplitude of the process (3.24) in the e^2– approximation has the form evidently (Cf.eq. (3.10))

$$T_{fi} = \frac{4\pi\alpha}{q^2}\, \bar{v}(k_+)\gamma^\mu u(k_-) \langle p_n | J_\mu(0) | 0 \rangle , \qquad (3.25)$$

and the total cross section according to eq. (3.11) is equal to

$$\sigma_h(q^2) = \frac{32\pi^3\alpha^2}{\sqrt{q^2(q^2-4m^2)}}\, L^{\mu\nu} \rho^h_{\mu\nu} , \qquad (3.26)$$

where the leptonic tensor $L^{\mu\nu}$ is given by eq. (3.14a) and the hadronic tensor

$$\begin{aligned} \rho^h_{\mu\nu}(q) &= (2\pi)^3 \sum_n \langle 0 | J_\mu(0) | n \rangle \langle n | J_\nu(0) | 0 \rangle \delta^4(q - p_n) = \\ &= \frac{1}{2\pi} \int d^4x\, e^{iq\cdot x} \langle 0 | [J_\mu(x), J_\nu(0)] | 0 \rangle \Big|_{q^0>0} = \\ &= (-g_{\mu\nu} q^2 + q_\mu q_\nu)\, \rho^h(q^2) \end{aligned} \qquad (3.27)$$

satisfies the requirements of gauge invariance $q^\mu \rho^h_{\mu\nu} = q^\nu \rho^h_{\mu\nu} = 0$ Neglecting the lepton mass compared to q^2 we finally obtain

$$\sigma_h(q^2) = \frac{16\pi^3\alpha^2}{q^2} \rho^h(q^2) . \qquad (3.28)$$

The hadronic spectral function $\rho^h(q^2)$ is closely connected with the hadronic vacuum polarization, namely, it is just the imaginary part of the hadronic polarization operator

$$\begin{aligned} \Pi^h_{\mu\nu}(q) &= \int d^4x\, e^{iq\cdot x} \langle 0 | T J_\mu(x) J_\nu(0) | 0 \rangle = \\ &= (-g_{\mu\nu} q^2 + q_\mu q_\nu)\, \Pi^h(q^2) \end{aligned} \qquad (3.29)$$

As is well known /1/, the invariant function $\Pi(q^2)$ satisfies the Kallen-Lehman spectral representation with one subtraction

$$\Pi(q^2) = e^2 q^2 \int_{4M^2}^{\infty} \frac{ds\, \rho(s)}{s(s - q^2 - i0)} \qquad (3.30)$$

under the condition of convergence of the integral. From eq. (3.30) it immediately follows that (see Fig.13)

$$\mathrm{Im}\,\Pi(s) = 4\pi^2\alpha\,\rho(s). \tag{3.30a}$$

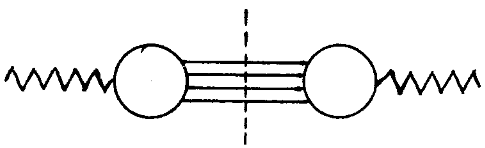
$\mathrm{Im}\,\Pi =$ 

Fig.13

The photon propagator then takes on the form (see Fig.14)

$$D^{\mu\nu}(q) = -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) D(q^2)$$
$$D(s) = \frac{1}{s\,[1+\Pi(s)]} \tag{3.31}$$

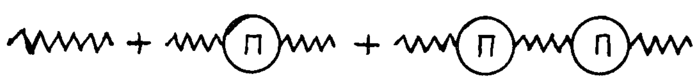
$D(s) =$ 

Fig.14

Now applying the Cauchy theorem to the function is is possible to obtain finite energy sum rule (FESR) [35]

$$\frac{1}{2\pi}\oint_C ds\,[\Pi(s) - \Pi^{>}(s)] = \int_0^{s_0} ds\,\mathrm{Im}[\Pi(s) - \Pi^{>}(s)] = 0\,.$$

$$\int_0^{s_0} ds\,\mathrm{Im}\,\Pi(s) = \int_0^{s_0} ds\,\mathrm{Im}\,\Pi^{>}(s)\,, \tag{3.32}$$

where $\Pi^{>}(s)$ at sufficiently high $s \geqslant s_0$ behaves like $\Pi(s)$, i.e.,

$$\theta(s-s_0)\,[\Pi(s) - \Pi^{>}(s)] = 0\,.$$

It is interesting, also, to note that the integral of can be directly related to the so-called Schwinger term in the equal time current commutator [28] :

$$\int_{-\infty}^{\infty} dq^0\, q^0 \rho(q^2) = \frac{1}{(2\pi)} \int_{-\infty}^{\infty} dq^0 \frac{\partial}{\partial q_i} \int d^4x\, e^{iq\cdot x} \langle 0|[J_0(x), J_i(0)]|0\rangle\Big|_{\vec{q}=0} =$$
$$= -i \int dx^0\, \delta(x^0) \int d^3x\, x_i \langle 0|[J_0(x), J_i(0)]|0\rangle\,,$$
$$\int_0^{\infty} dq^2\, \rho(q^2) = -i \int d^3x\, x_i \langle 0|[J_0(0,\vec{x}), J_i(0)]|0\rangle\,.$$

Now if the equal-time commutator contains the c-number Schwinger term of the form

$$[J_0(0,\vec{x}), J_i(0)] = i c \frac{\partial}{\partial x_i} \delta^3(\vec{x})\,,$$

then integrating by parts we easily find

$$C = \int_0^\infty \rho(s)\,ds .$$

Usually C is a diverging quantity.

It is customary to relate the total cross section of annihilation into hadrons, to the cross section of annihilation into a muon pair. In the lowest e^2– approximation the leptonic spectral function is given by [1] (m_ℓ is the lepton mass)

$$\rho^\ell(s) = \frac{1}{12\pi^2}\left(1+\frac{2m_\ell^2}{s}\right)\sqrt{1-\frac{4m_\ell^2}{s}} \underset{s\gg m_\ell^2}{\approx} \frac{1}{12\pi^2} . \tag{3.33}$$

Thus in the e^2– approximation the total cross section of this reaction according to eq. (3.28) is equal to

$$\sigma_\mu^0(s) = \frac{4\pi\alpha^2}{3s} , \quad s \gg m_\mu^2 \tag{3.34}$$

and the corresponding diagram is presented in Fig.15.

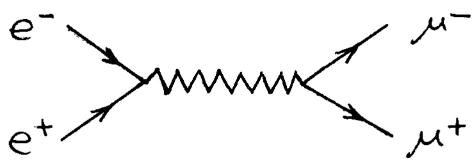


Fig.15

Then the quoted ratio (in the e^2– approximation)

$$R(s) = \frac{\sigma_h^0(s)}{\sigma_\mu^0(s)} = 12\pi^2\rho^h(s) . \tag{3.37a}$$

Replacing in eq. (3.25) the photon propagator by its modified expression (3.32) we find the cross section σ_μ with the account of corrections on the hadronic vaouum polarization

$$\sigma_\mu(s) = \frac{4\pi\alpha^2}{3s}\left|s\,D(s)\right|^2 = \frac{4\pi\alpha^2}{3s\left|1+\Pi^h(s)\right|^2} = \frac{\sigma_\mu^0(s)}{\left|1+\Pi^h(s)\right|^2} . \tag{3.35}$$

Similar corrections to the total cross section of annihilation into hadrons eq. (3.28) lead to the expression (on account of relation (3.30c))

$$\sigma_h(s) = \frac{16\pi^3\alpha^2}{s}\rho(s)\left|s\,D(s)\right|^2 = \frac{4\pi\alpha\,\mathrm{Im}\,\Pi^h(s)}{s\left|1+\Pi^h(s)\right|^2} = 4\pi\alpha\,\mathrm{Im}\,D(s). \tag{3.36}$$

As a result we obtain [25] that the ratio (3.37a) remains unmodified

$$R(s) = \frac{\sigma_h(s)}{\sigma_\mu(s)} = 12\pi^2\rho^h(s) = \frac{3}{\alpha}\,\mathrm{Im}\,\Pi^h(s) . \tag{3.37}$$

If, besides, we take into account the contribution to vacuum polarization from lepton (e and μ) pairs given by eq. (3.33) as well, we find, on using eq. (3.37)

$$\operatorname{Im}\Pi = \operatorname{Im}\Pi^h + 2\operatorname{Im}\Pi^\ell = \frac{\alpha}{3}\left[R+2\right]. \qquad (3.38)$$

This relation allows one to deduce rather a strong bound for the one-photon contribution to the total annihilation cross section into hadrons [36]. According to eqs. (3.35) and (3.37) we have

$$\sigma_h = R\,\sigma_\mu = \frac{4\pi\alpha^2 R}{3s\left[(1+\operatorname{Re}\Pi)^2+(\operatorname{Im}\Pi)^2\right]} \leqslant \frac{4\pi\alpha^2 R}{3s(\operatorname{Im}\Pi)^2}.$$

Substituting there expression (3.38) we, finally, come to the important inequality

$$\sigma_h \leqslant \frac{12\pi R}{s\,(R+2)^2}. \qquad (3.39)$$

This bound can be obtained in a different way using unitarity condition [36] under the following approximations and assumptions:

a) The whole process of annihilation occurs from the $J^{PC} = 1^{--}$ state. Only the state with opposite lepton (e and μ) helicities enters in the reactions (in the limit

of zero lepton mass):

$$|\ell^+,\ell^-\rangle \equiv |\ell\rangle = \frac{1}{\sqrt{2}}\left[|+1,-1\rangle + |-1,+1\rangle\right], \; J=1.$$

This is connected with the presence of only the single--photon state in the S-channel and conservation of the helicity by the electromagnetic vertex γ_μ.

b) Final states with photons are neglected. Hard photon contribution to $\sigma_{h,\mu}$ is of order α higher than the terms which are kept. Both real and virtual soft photons should be neglected in order to preserve unitarity. Thus, the bound applies to the cross sections, obtained from the experimental data after subtracting radiative corrections due to both real and virtual soft photons. The cross sections so defined should be independent of lepton mass.

c) Final states of $J^{PC} = 1^{--}$ with lepton pairs and hadrons are neglected, which means the neglect of processes like

$$\ell^+ + \ell^- \to \ell^+ + \ell^- + \text{hadrons}.$$

The cross section of this process is of order α^2 relative to $\sigma_{h,\mu}$ and in the state 1^{--} is bounded by const $/s$.

d) μ-e universality is assumed.

Now consider the S-matrix in the subspace of $J^{PC}=1^{--}$ states $|e\rangle$, $|\mu\rangle$ and $|n\rangle$, where $|e\rangle$, $|\mu\rangle$ are the relevant states of e^+e^-, $\mu^+\mu^-$ and $|n\rangle$ is the set of hadronic states distinguished by the index n .

The following relations for the S-matrix elements hold:

$$\langle e|S|e\rangle = \langle \mu|S|\mu\rangle = \eta e^{2i\delta},$$
$$\langle e|S|\mu\rangle = \langle \mu|S|e\rangle = if, \tag{3.40}$$
$$\langle e|S|n\rangle = \langle \mu|S|n\rangle = if_n .$$

The total cross section for the process $a+b \to F$ with a fixed value of the total angular momentum is expressed through the helicity amplitudes as follows [2]

$$\sigma^J(\lambda_a,\lambda_b,F) = \frac{\pi(2J+1)}{k^2}\left|\langle F|T_J|\lambda_a,\lambda_b\rangle\right|^2, \tag{3.41}$$

where k is the relative momentum in the centre-of-mass frame. In the case of inelastic reaction it is obvious that

$$i\langle F|T_J|I\rangle = \langle F|S_J|I\rangle$$

where $S_J = 1 + iT_J$. Averaging over the initial electron and positron polarizations, we find that the cross section for the reaction $e^+e^- \to \mu^+\mu^-$

$$\sigma_\mu = \frac{1}{4}\,\frac{3\pi}{k^2}|f|^2 \approx \frac{3\pi}{s}|f|^2 \tag{3.42}$$

and the cross section for the reaction $e^+e^- \to$ hadrons

$$\sigma_h = \frac{3\pi}{s}\sum_n |f_n|^2 . \tag{3.43}$$

The S-matrix can be written as

$$S = \begin{pmatrix} \eta e^{2i\delta} & if & if_n \\ if & \eta e^{2i\delta} & if_n \\ \cdots & \cdots & \cdots \end{pmatrix} \tag{3.44}$$

Then the unitarity of the S-matrix

$$S_J^\dagger S_J = 1$$

yields the relations

$$\sum_F \langle \ell'|S^\dagger|F\rangle\langle F|S|\ell\rangle = \delta_{\ell'\ell} , \tag{3.45}$$

or more explicitly

$$\sum_F |\langle F|S|e\rangle|^2 = \eta^2 + |f|^2 + \sum_n |f_n|^2 = 1 \tag{3.45a}$$

$$\sum_F \langle \mu|S^\dagger|F\rangle\langle F|S|e\rangle = = -2\eta\,\mathrm{Im}\left(f e^{-2i\delta}\right) + \sum_n |f_n|^2 = 0 . \tag{3.45b}$$

Since $\eta^2 \geq 0$, relation (3.45a) on account of eqs. (3.43) and (3.42) directly leads to the simple but weak unitarity limit [25]

$$\sigma_h + \sigma_\mu \leq \frac{3\pi}{s} . \tag{3.46}$$

Relation (3.45) with the account of eq. (3.45a) gives rise to the following inequality

$$\Big[\sum_n |f_n|^2\Big]^2 = 4\eta^2 \big[\mathrm{Im}(f e^{-2i\delta})\big]^2 \leqslant$$

$$\leqslant 4\eta^2 |f|^2 = 4|f|^2\Big(1-|f|^2-\sum_n |f_n|^2\Big),$$

which after substituting eqs. (3.42) and (3.43) gives

$$(\sigma_h + 2\sigma_\mu)^2 \leqslant \frac{12\pi}{s}\sigma_\mu\,, \qquad (3.47)$$

$$\sigma_h^2 < \frac{12\pi}{s}\sigma_\mu\,.$$

Using ratio (3.37) we rewrite inequality (3.47) as

$$\sigma_h\Big(1+\frac{2}{R}\Big)^2 \leqslant \frac{12\pi}{sR}\,,$$

which immediately leads to the strong bound given by eq.(.3.39).

3.4. Scale invariance and some models

We make use of the automodelity principle [4,14] over again. Then naive dimensional analysis of the type presented by eq. (1.62) leads to a simple result that the total annihilation cross section into hadrons eq. (3.28) as $q^2 \to \infty$ behaves like

$$\sigma_h(q^2) = \frac{const}{q^2}\,, \quad \rho^h(q^2) = const\,, \quad q^2 \to \infty \qquad (3.48)$$

and the ratio defined by eq. (3.37) is equal to

$$R(q^2) = 12\pi^2\rho^h(q^2) = \frac{3}{\alpha}\mathrm{Im}\,\Pi^h(q^2) = const. \qquad (3.49)$$

The same result can be also obtained from the analysis of the Wilson's operator product expansion in the vicinity of the light cone under the assumption of normal (canonical) dimensionality of the current operator and from asymptotically free field theories.

In the parton model [29,30] the operator of hadronic vacuum polarization takes on a simple form, shown in Fig.16

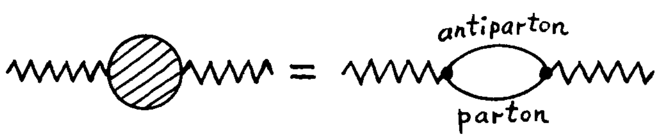


Fig.16

Hence, the imaginary part of this operator in the case of spin 1/2 partons is given by eq. (3.33) as for leptons. In this way we get

$$\mathrm{Im}\,\Pi_i^h(s) = 4\pi^2\alpha\,\rho_i^h(s) = \frac{\alpha}{3}Q_i^2\,, \qquad (3.50)$$

where Q_i is the i-th parton charge and the total annihilation cross section into hadrons eq. (3.28) is equal to (cf. eq. (3.16a))

$$\sigma_h(s) = \frac{4\pi\alpha^2}{3S}\sum_i Q_i^2 = \sigma_\mu \sum_i Q_i^2 . \tag{3.51}$$

If spin 0 partons also exist, then expression (3.51) is modified according to eq. (3.16c) as follows

$$\sigma_h(s) = \frac{4\pi\alpha^2}{3S}\left\{\sum_{\text{spin}1/2} Q_i^2 + \frac{1}{4}\sum_{\text{spin}0} Q_i^2\right\} . \tag{3.51a}$$

The ratio R takes on especially simple form in the parton model

$$R = \sum_{\text{Spin}1/2} Q_i^2 + \frac{1}{4}\sum_{\text{spin}0} Q_i^2 . \tag{3.52}$$

Specific realizations of the parton model with the help of spin 1/2 quarks yield the following values of R :

$R = \frac{2}{3}$ the usual fractionally charged triplet: $\left(Q = \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

$R = \frac{2}{3} + \frac{4}{9} = \frac{10}{9}$ the fractionally charged quartet with the charge of a charmed quark $Q_c = \frac{2}{3}$

$R = 3\cdot\frac{2}{3} = 2$ three coloured fractionally charged triplets

$R = 2 + \frac{4}{3} = \frac{10}{3}$ three coloured fractionally charged quartets

$R = 2+2 = 4$ three integrally charged triplets

$$Q = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$R = 2+4 = 6$ three integrally charged quartets

$$Q = \begin{pmatrix} 0 & -1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

The separation of R into two terms refers to a separation of the electromagnetic current into

$$\mathcal{J} = \mathcal{J}^N + \mathcal{J}^c$$

where the "normal" part $\mathcal{J}^N$ has the usually assumed transformation property of an octet under SU(3) and singlet under charm, colour,etc. groups that is

$$\mathcal{J}^N = \mathcal{J}(8,1) = \mathcal{J}^3 + \frac{1}{\sqrt{3}}\mathcal{J}^8 .$$

The part $\mathcal{J}^c$ is associated with new hadronic degrees of freedom (new quantum numbers such as charm, colour and soon). And transforms differently (in particular as a singlet under usual SU(3) in the models considered above). Thus, one may think that at relatively low energies ($\sqrt{q^2} < 3$ GeV) these additional degrees of freedom are frozen out and $R = 2$. However, after the "thaw" at higher energies the value of R increases to 4÷6 (Note, however, that it may pose serious problems for electroproduction).

For single-particle inclusive cross sections, naive parton model gives the following predictions (see eqs. (3.16a,b))

$$\frac{d\sigma}{d\Omega} \propto \sum_{spin\,1/2} Q_i^2 (1+\cos^2\theta) + \frac{1}{2}\sum_{spin\,0} Q_i^2 (1-\cos^2\theta) \tag{3.53}$$

$$q^2 \frac{d\sigma}{d\omega} = f(\omega) + O\left(\frac{1}{q^2\omega^2}\right)$$

and leads to the two-jet structure of the energetic hadron emission (a rapidity plateau) while experimentally the angular distribution is consistent with isotropy, scaling fails at $\omega < 0.5$ in contradiction with eqs. (3.53). Spin 0 partons cannot compensate the spin 1/2 partons, since such a large contribution of spin 0 partons is not seen, in electroproduction.

The isotropy of angular distribution, the absence of scaling at $\omega < 0.5$ and the exponential fall of the inclusive single particle momentum spectrum with the specific cut-off parameter $T \approx 170$ MeV are well explained within the framework of various thermodynamic and statistical models of hadron interactions. However, such models fail to explain scaling at $\omega > 0.5$, energy crisis and they can say nothing about the absolute value of the total annihilation cross

section and apparently fail to describe the spectrum of highly energetic hadrons.

The ratio R, found experimentally at $\sqrt{q^2} \approx 5$ GeV, reached the values of order 4–6 compared to 2÷3 at $\sqrt{q^2} \approx 3$ GeV. A linear rise of $R \propto q^2$ corresponds to a constant total cross section of annihilation into hadrons, which contradicts scale invariance (automodelity principle), parton model and above all rigorous unitary bound (3.39). With the existing experimental data ($\sigma_h \approx 20$ nb) this bound is saturated at $\sqrt{q^2} \approx 30$ GeV where the rise is expected to stop in any case. A linear rise of $R(s)$ leads also to the diverging integral in representation (3.30) and therefore a new subtraction constant is required.

One of the possible explanations of the observed rise of $R(s)$ is given by the model of giant resonance in a quark system of the type which occurs in nucleon system (nucleus) [37].

3.5. Implications for quantum electrodynamics (QED)

According to eqs. (3.32) and (3.37) the photon propagator can be written in the form

$$D(q^2) = \frac{1}{q^2+i0}\left[1 + \frac{\alpha}{3\pi} q^2 \int_{4m_\pi^2}^{\infty} \frac{ds\; R(s)}{s(s-q^2-i0)}\right]^{-1} \tag{3.54}$$

At low momentum squared $q^2 \ll 4m_\pi^2$ it follows from eq. (3.54) that

$$D(q^2) \approx \frac{1}{q^2}\left[1 - \frac{\alpha}{3\pi}\,\frac{q^2}{m_h^2}\right], \tag{3.55}$$

where

$$m_h^{-2} = \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} R(s)$$

is the effective hadronic inverse mass squared required for the lepton anomalous magnetic moment and atomic physics applications.

It is also helpful to note that the Coulomb potential ($-Z\alpha/r$) becomes modified to

$$V(r) = 4\pi Z\alpha \int \frac{d^3q}{(2\pi)^3} D(-\vec{q}^{\,2}) e^{i\vec{q}\cdot\vec{r}} = \tag{3.56}$$

$$= -\frac{Z\alpha}{r}\left[1 + \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s} R(s) e^{-\sqrt{s}\,r}\right],$$

which is a sum of Yukawa type interactions. As follows from eq. (3.55) at large $r^2 \gg 4m_\pi^{-2}$ the hadronic modification is effectively given by

$$V(r) \approx -Z\alpha\left[\frac{1}{r} + \frac{\alpha}{3\pi}\,\frac{\delta^3(\vec{r})}{m_h^2}\right]. \tag{3.56a}$$

In general for q^2 spacelike the electromagnetic interaotion is increased in strength by hadronic vaouum polarization.

For estimates we set

$$R(s) = R\left(\frac{s}{s_0}\right)^n, \quad s \le \Lambda^2, \quad 0 \le n \le 1 \tag{3.57}$$

up to some large $s_{max} = \Lambda^2$ and

$$R(s) = R\left(\frac{\Lambda^2}{s_0}\right)^n, \quad s > \Lambda^2$$

for higher values of s with $s_0 \approx 25\ \mathrm{GeV}^2$ and $R \approx 5$. Then from eq. (3.54) for $q^2 \ll \Lambda^2$ we have

a) $0 < n < 1$

$$D(q^2) = \frac{1}{q^2}\left[1 - \frac{\alpha R}{3\sin\pi n}\left(-\frac{q^2}{s_0}\right)^n\right]^{-1} \tag{3.58a}$$

b) $n = 1$

$$D(q^2) = \frac{1}{q^2}\left[1 + \frac{\alpha}{3\pi} R \frac{q^2}{s_0} \ln\left(-\frac{\Lambda^2}{q^2}\right)\right]^{-1} \tag{3.58b}$$

Note that for timelike $q^2 > 0$ one must take in eq. (3.58a) according to the prescription $(q^2 + i0)$

$$(-1)^n = e^{-i\pi n} = \cos\pi n - i\sin\pi n .$$

In a special case of $n = 0$, $R(s) = R = \mathrm{const}$ which correspondes to normal QED of point-like particles we obtain a well-known result [1]

$$D(q^2) = \frac{1}{q^2}\left[1 - \frac{\alpha}{3\pi} R \ln\left(-\frac{q^2}{m^2}\right)\right]^{-1}. \tag{3.58c}$$

It shows, in particular, that the effective charge squared

$$e^2_{eff} = e^2 q^2 D(q^2)$$

grows as $|q^2|$ gets large. In contrast to that in non-abelian gauge theories we have an opposite sign in front of log in the denominator of eq. (3.58c).

In such a case the effective charge at large q^2 becomes equal to

$$g^2_{eff} \approx \frac{3\pi}{N \ln(q^2/m^2)} \to 0, \quad q^2 \to \infty$$

and asymptotically vanishes at small distances (large q^2). Thus the theory is asymptotically free since there is no interaction and implies a constant value of R at high q^2.

The real part of the photon propagator is of special experimental interest since only it interferes with the lowest order [28]. From eq. (3.58a) we find:

$$\mathrm{Re}\, D(q^2) = \frac{1}{q^2}\left[1 + \frac{\alpha}{3}\,\mathrm{ctg}\,\pi n\, R(q^2)\right]^{-1}, \quad 0<n<1. \qquad (3.59)$$

Note that for $n = 1/2$ the hadronic modification of $\mathrm{Re}\, D(q^2)$ vanishes identically and hence there is no hadronic correction to the process $e^+e^- \to \mu^+\mu^-$ since only the annihilation diagram enters there. For the process $e^+e^- \to e^+e^-$ only spacelike $(q^2 = t < 0)$ modifications are important if $n = 1/2$.

The modifications of the photon propagator due to hadronic vacuum polarization yield corrections of order several percent

$(\sim \alpha R)$ to the lowest order cross sections of lepton processes $e^+e^- \to e^+e^-$ and $e^+e^- \to \mu^+\mu^-$ for values of q^2 of order several tenths GeV^2 and $\theta = 90°$.

In general the perturbation theory for QED would breakdown completely, when $|\Pi(s)| \sim 1$, i.e., at $R(s) \le \frac{3}{\alpha} = 411$ which is reached for $R(s) \propto s$ at $\sqrt{s} = 40$ GeV.

The contribution of hadronic vacuum polarization to muon anomalous magnetic moment can be expressed directly in terms of the annihilation cross section (the corresponding diagram is shown in Fig.17)

$$\Delta_h a_\mu = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, \sigma_h(s) \int_0^1 \frac{dz\ z^2(1-z)}{z^2 + (1-z)(s/m_\mu^2)} . \qquad (3.60)$$

For

$$s > s_0 \gg 4m_\pi^2 \qquad (3.60a)$$

$$\Delta_h a_\mu = \frac{\alpha^2}{9\pi^2} m_\mu^2 \int_{s_0}^{\infty} \frac{ds}{s^2} R(s) = \frac{\alpha^2}{9\pi^2} \frac{m_\mu^2}{\tilde{m}_h^2} = \frac{6.7 \times 10^{-9}}{\tilde{m}_h^2 (\mathrm{GeV}^2)} .$$

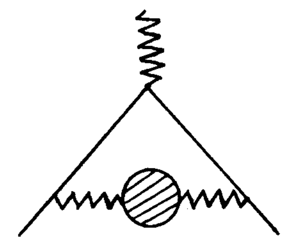


Fig. 17

A detailed analysis of hadronic contributions to $\Delta_h a_\mu$ leads to the following results [39,40]

$$\Delta_h a_\mu (4m_\pi^2 < s < 4\,GeV^2) = (6.6 \pm 0.9) \times 10^{-8}$$

$$\Delta_h a_\mu (s > 4\,GeV^2) \approx 0.5 \times 10^{-8}$$

which gives in total

$$\Delta_h a_\mu \approx (7.1 \pm 0.9) \times 10^{-8}. \qquad (3.61)$$

The comparison of theoretical and experimental values of a_μ leads to

$$a_\mu^{exp} - a_\mu^{th}(QED + had) = (27 \pm 31) \times 10^{-8},$$

where the quoted error is taken from the experimental value of a_μ. Thus, one needs to rise the accuracy of measurements by a factor of 10 in order to reveal the contribution of hadronic vacuum polarization. Such experiments are in preparation at CERN and Los-Alamos.

3.6. New vector particles and e^+e^- annihilation

Recent discovery of new vector mesons $\psi_{1,2,3,\ldots}$ at SLAC and BNL apparently change desicively the whole situation in hadronic annihilation. In fact, since presently the experimental data are obtained in the region, where several resonances are found, so in analogy with the deep inelastic electroproduction one should expect scaling only on the average in the sense of a "new duality" with respect to vector mesons[35,38].

From the other point of view if the new vector particles bear, in some form, a new quantum number (of the type of "charm" "colour", etc), then the rise of $R(s)$ between $\sqrt{s} = 3$ GeV and 5 GeV may be explained as a threshold effect of opening (thaw) of a new channel (of the type of charmed quark production). Scaling in the single-particle inclusive distributions may also fail in this region.

We illustrate the above considerations by a simple example of enlarged (generalized) vector dominance model [35,38]. The main assumption of this model is the existence of an infinite linear rising spectrum of vector meson masses squared:

$$m_n^2 = m_0^2\,(1 + an). \qquad (3.62)$$

Such a spectrum is characteristic for dual models. The total annihilation cross section in this case is equal to an infinite sum of the Breit-Wigner terms

$$\sigma_h(s) = \frac{12\pi}{s} \sum_n \frac{m_n^2\, \Gamma_n\, \Gamma_n^\ell}{(s - m_n^2)^2 + m_n^2 \Gamma_n^2} \qquad (3.63)$$

where Γ_n is the total meson width and the partial decay width into a lepton pair reads as

$$\Gamma_n^\ell = \frac{4\pi\alpha^2 m_n}{3 f_n^2}.$$

In the infinitely narrow width approximation eq. (3.55) takes the form

$$\sigma_h(s) = \frac{12\pi^2}{s} \sum_n m_n \Gamma_n^\ell \delta(s - m_n^2) = \tag{3.64}$$

$$= \sigma_\mu \, 12\pi^2 \sum_n \frac{m_n^2}{f_n^2} \delta(s - m_n^2).$$

Now it is quite, obvious, that in order to obtain scaling behaviour of $\sigma_h \propto 1/s$ one should require

$$m_n \Gamma_n = const \, , \quad \frac{m_n^2}{f_n^2} = const$$

so that

$$\Gamma_n = \frac{\Gamma_0 m_0}{m_n} = \frac{4\pi\alpha^2 m_0^2}{3 m_n f_0^2} .$$

This condition directly leads to the result

$$\sigma_h = \frac{12\pi^2}{s} \frac{\Gamma_0^\ell m_0}{a m_0^2} = \sigma_\mu \frac{12\pi^2}{a f_0^2} \, , \tag{3.65a}$$

$$R \equiv \frac{\sigma_h}{\sigma_\mu} = \frac{12\pi^2}{a f_0^2} . \tag{3.65b}$$

For usual vector mesons ρ, ω, φ one should take

$$m_0^2 = m_\rho^2 \, , \quad a = 2 \, , \quad \frac{f_0^2}{4\pi} = \frac{f_\rho^2}{4\pi} = 2.56$$

and multiply eqs. (3.57) by a factor of $\frac{4}{3} = 1 + \left(\frac{1}{\sqrt{3}}\right)^2$ which accounts for both the isovector and isoscalar components. Thus

$$R_{\rho,\omega,\varphi} = \frac{2\pi}{f_\rho^2/4\pi} \approx 2.5 . \tag{3.66a}$$

For the family of new vector mesons assuming that Ψ_2 is an excited state of Ψ_1 we have

$$\Delta m_\Psi^2 = a_\Psi m_{\Psi_1}^2 = m_{\Psi_2}^2 - m_{\Psi_1}^2 \cong 4.1 \text{ GeV}^2,$$

$$\Gamma_0^\ell = \Gamma_{\Psi_1}^\ell = 5.2 \text{ keV}, \quad \frac{f_{\Psi_1}^2}{4\pi} \approx 10.5 ,$$

$$R_\Psi = \frac{3\pi}{a_\Psi f_\Psi^2/4\pi} = \frac{12\pi^2 m_{\Psi_1}^2}{\Delta m_\Psi^2 f_\Psi^2} \approx 2.1 .$$

In the end, the total value of

$$R = R_{\rho,\omega,\varphi} + R_{\psi} \approx 2.5 + 2.1 = 4.6 \qquad (3.66)$$

which is consistent with the present experimental data.

The same result may be obtained with the help of the sum rule (3.32). Really, recalling relation (3.37) and choosing

$$R^{>}(s) = \frac{3}{\alpha}\,\mathrm{Im}\,\Pi^{>}(s) = R\left(\frac{s}{s_0}\right)^n$$

we write the sum rule (3.32) in the form

$$\int_0^{s_{max}} ds\, R(s) = \frac{R\, s_{max}}{n+1}\left(\frac{s_{max}}{s_0}\right)^n, \quad 0 \leq n \leq 1. \qquad (3.67)$$

Saturating the **left-hand** part of eq. (3.67) by contributions (3.56) of vector mesons

$$R_V(s) = 12\pi^2 \frac{m_V^2}{f_V^2}\, \delta(s - m_V^2),$$

we find

$$12\pi^2 \sum_V \frac{m_V^2}{f_V^2} = \frac{R\, s_{max}}{n+1}\left(\frac{s_{max}}{s_0}\right)^n. \qquad (3.68)$$

In the case of scaling behaviour we have $R(s) = const$, $n=0$ hence

$$R = \frac{12\pi^2}{s_{max}} \sum_V \frac{m_V^2}{f_V^2}, \qquad (3.68a)$$

and the sum rule is also valid locally.

Now assuming the usual SU(3) ratios for the constants m_V^2/f_V^2 $V = \rho, \omega, \varphi$ that is 9:1:12 and taking as before $s_{max} = 2m_\rho^2$ we come to a familiar result (Cf. eq. (3.66a), note that again $1 + \frac{1}{9} + \frac{2}{9} = \frac{4}{3}$)

$$R_{\rho,\omega,\varphi} = \frac{8\pi^2}{f_\rho^2}.$$

Similarly for the ψ-meson contribution we find, choosing $s_{max} = \Delta m_\psi^2$

$$R_\psi = \frac{12\pi^2 m_\psi^2}{\Delta m_\psi^2\, f_\psi^2}$$

in accordance with eq. (3.66b).

In the case if a ψ meson is the pure $c\bar{c}$-state (c is the charmed quark) the SU(4) symmetry predicts the ratio

$$\frac{m_\omega^2}{f_\omega^2} : \frac{m_\psi^2}{f_\psi^2} = 1:8,$$

which is well satisfied by the experimental values.

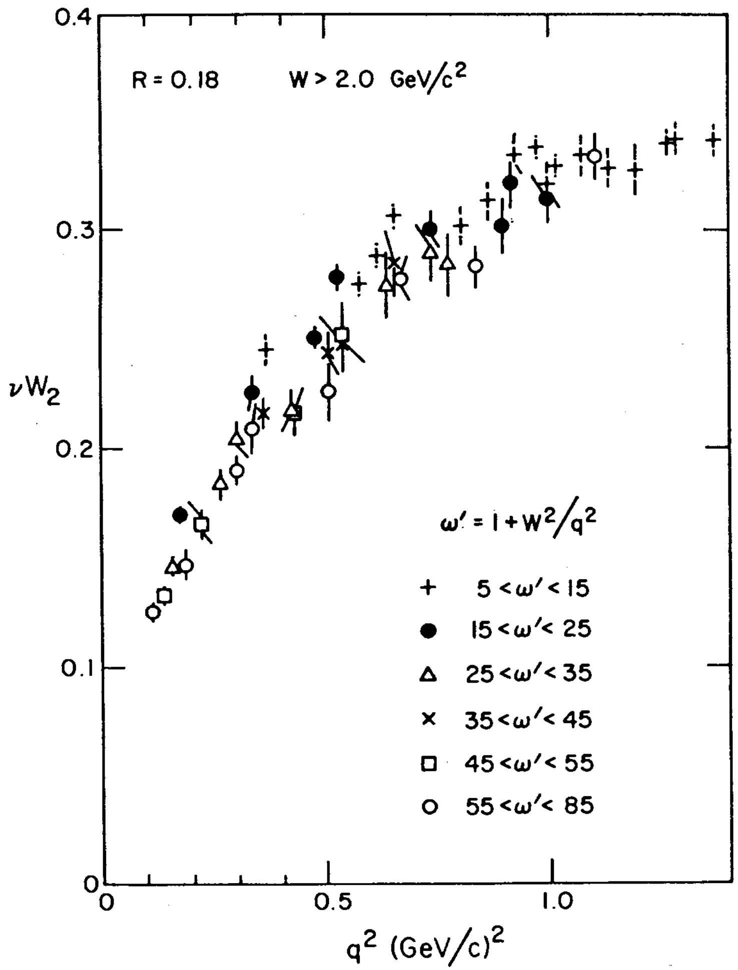


Fig. 18. Approach to Scaling for e-p scattering.

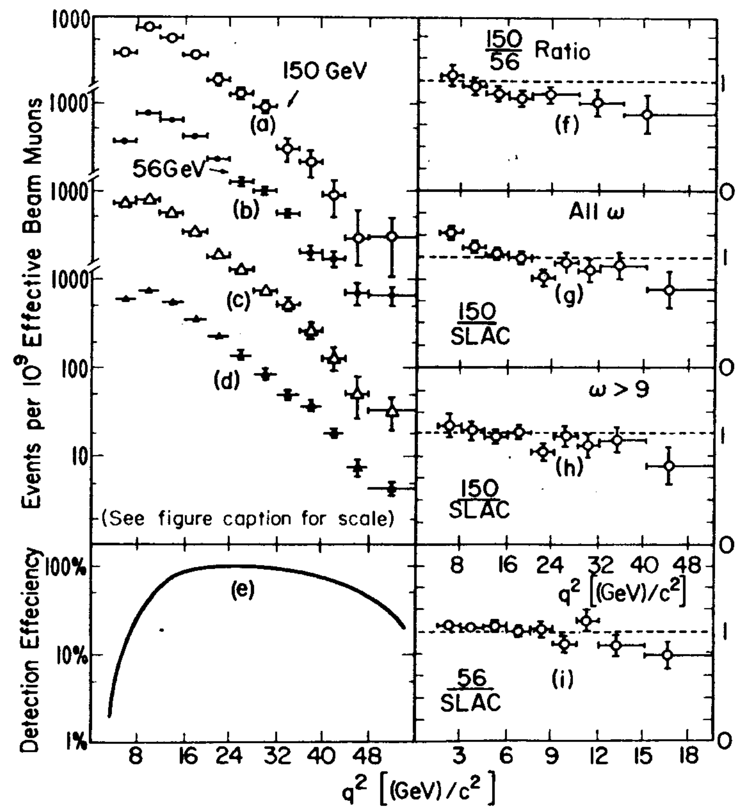


Fig. 19. μ-p scaling results.

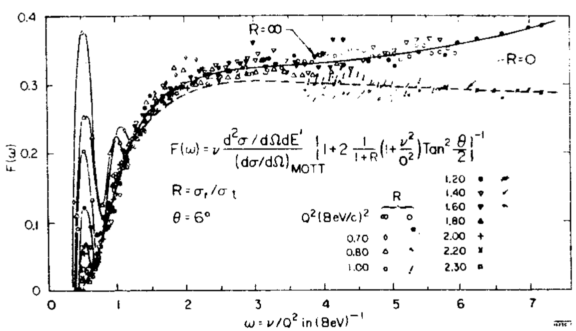


Fig. 20

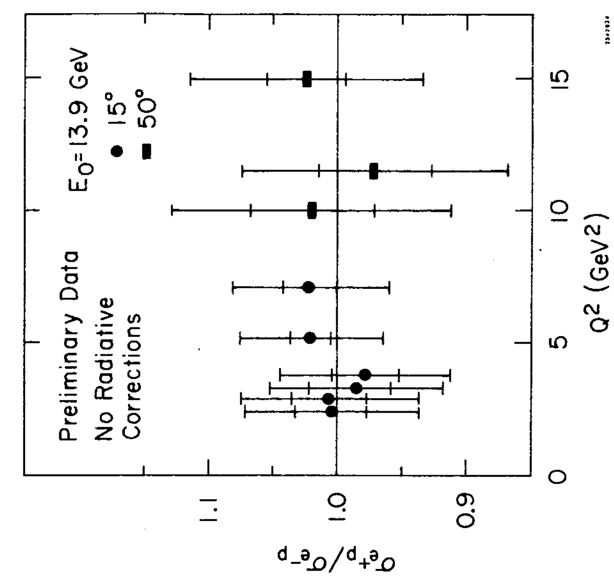


Fig. 22

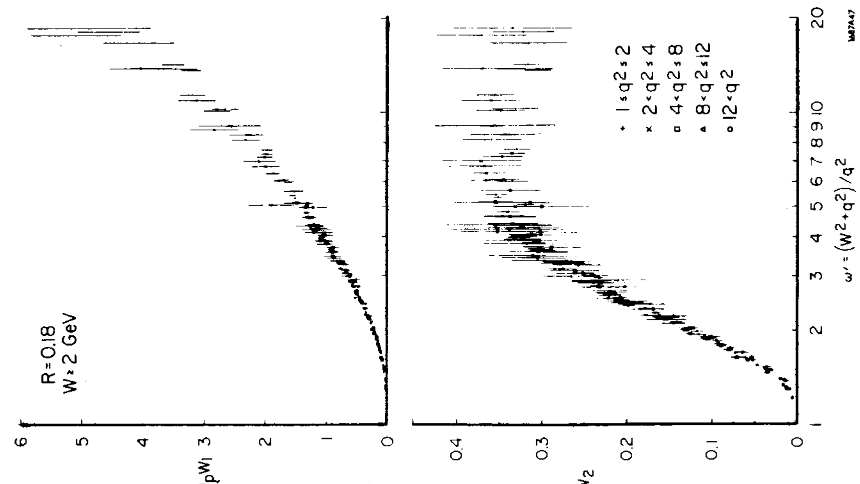


Fig. 21

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