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дубна

E2-87-933

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FIT OF GLUON SELF-COUPLING IN QCD WITH DIFFERENT PARAMETRIZATIONS FOR THE DEEP CROSS SECTION RATIO $R = \sigma_L / \sigma_T$

Submitted to "Ядерная физика"

1987

I. Introduction

In paper /1/ E.Reya suggested a test for the determination of the gluon self-couplings in QCD. Previous tests for QCD together with the deep inelastic scattering data invalidated all strong interaction theories with a fixed point /1,3,4/. However all high statistic experiments were insensitive to the gluon distribution in the nucleon and hence to a three-gluon vertex. Why did this situation occur? E.Reya considered the contribution of the triple-gluon vertex only in the anomalous dimension $\gamma_{GG}^{(0)n}$ whose contribution is very small in Q^2-evolution of the moments of the deep inelastic scattering structure functions /4,5/. However the three-gluon vertex first of all defines the value (more correctly sign) of the first β-function coefficient, that is the vertex is responsible for the asymptotical freedom and it ultimately defines the value of the coupling constant $\alpha_s(Q^2)$ (the constant of the quark and gluon interaction in QCD). Let us qualitatively analyse the reason of the weak dependence of the deep inelastic scattering data on the gluon self-coupling. Q^2-evolution of the moments of the singlet part of the structure function contains the following blocks in the leading order of perturbation theory /5/:

$$\sum\nolimits_n^i (Q_0^2)\left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]^{\frac{\gamma_i^{(0)n}}{2\beta_0}} = \sum\nolimits_n^i (Q_0^2)\left(1-2\beta_0\bar{\alpha}(Q_0^2)\ln\frac{Q^2}{\mu^2}+\dots\right)^{\frac{\gamma_i^{(0)n}}{2\beta_0}} =$$

$$= \sum\nolimits_n^i (Q_0^2)\left(1-\gamma_i^{(0)n}\bar{\alpha}(Q_0^2)\ln\frac{Q^2}{\mu^2}+\dots\right) \qquad (i=+,-) , \tag{1}$$

whose size weakly depends on the value of β_0. In QCD there is the equation $\sum_n^-(Q_0^2) \gg \sum_n^+(Q_0^2)$. After switching-off the gluon interaction the situation is opposite. Hence in QCD the coefficient $\gamma_-^{(0)n} \underset{QCD}{\approx} \gamma_{\psi\psi}^{(0)n}$ dominates in Q^2-evolution of the moment of the structure function and after switching-off the gluon self-coupling the coefficient $\gamma_+^{(0)n} \underset{QCD}{\overset{C_A=0}{\approx}} \gamma_{\psi\psi}^{(0)n}$. To the anomalous dimension $\gamma_{\psi\psi}^{(0)n}$, the triple-gluon vertex does not contribute (for the notation, see below). Q^2-evolution of the moment of the nonsinglet part of the structure function in the leading order does not depend on the gluon interaction. All above properties are preserved on the whole in the next-to-leading the perturbation order. Hence the term $\sim C_A$ both in β_0 and $\gamma_{GG}^{(0)n}$ weakly influences Q^2-evolution of the structure function.

E.Reya suggested to study Q^2-evolution of the very gluon distribution $G(x,Q^2)$ (more exactly its moments $G_n(Q^2)$) which is sensitive to triple-gluon vertex. The qualitative reasoning can be made in a manner like that done for the moments of the structure function. We get that Q^2-evolution of gluon distribution moments weakly depends on the value of β_0 and it is defined by both coefficients $\gamma_-^{(0)n}$ and $\gamma_+^{(0)n}$ and hence it strongly depends on the value of the anomalous dimension $\gamma_{GG}^{(0)n}$. The theoretical Q^2-dependence of the gluon distribution is considered in papers /4,5/. The "experimental" information about the gluon distribution can be got by knowing the longitudinal $F_L(x,Q^2)$ and transverse $F_2(x,Q^2)$ structure function of the deep inelastic scattering /1,4/. The experimental accuracy of the definition of $F_L(x,Q^2)$ (more exactly ratio $R=\sigma_L/\sigma_T$) is small, however we can use the parametrizations suggested in papers /7-9/.

In this paper, we compare the theoretical Q^2-evolution of three moments (n=3,4,5) of the gluon distribution with the experimental data for $F_2(x,Q^2)$ and $F_L(x,Q^2)$ /7,9/. We show that in QCD theoretical and experimental Q^2-dependences of the moments well agree with each other for different parametrizations of $R(x,Q^2)$ (with the exception of the parametrisation in the form of $R=\frac{4\langle p_\perp^2\rangle}{Q^2}$). The agreement is not achieved for the theory in which the self-coupling of gluons is absent. We call this theory QCD_1.

The paper is organized as follows:

In section 2, we show Q^2-evolution of the moments of the gluon distribution in QCD and QCD_1 to leading and next-to-leading orders.

In section 3, we give "experimental" Q^2-dependence of these moments.

In section 4, we present the graphs of theoretical and experimental Q^2-dependence of the moments of the gluon distribution and discuss obtained results.

In appendix we give the next-to-leading corrections to the moments of the longitudinal and transverse structure function found in papers /10-12/.

2. Q^2-evolution of the gluon distribution. Theoretical results.

There are different possibilities of the connection (in the next-to-leading order) of the moments of the deep inelastic scattering and the moments of the parton (quark and gluon) distributions /5/. We choose the parton distributions so that the anomalous dimensions of Wilson operators and β-function are responsible for their evolution and the connection between the structure functions and parton distributions is determined by the Wilson coefficients /5,13/.

Then Q^2-evolution of the moments of the gluon distribution determined by the renormalization group /6/ has the form /5/:

$$\frac{G_n(Q^2)}{G_n(Q_0^2)} = \left(1-\xi_n^{(1)}+\xi_n^{(2)}\frac{\Sigma_n(Q_0^2)}{G_n(Q_0^2)}\right)\left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]^{d_-^n} H_-(Q^2,Q_0^2) + \left(\xi_n^{(1)}-\xi_n^{(2)}\frac{\Sigma_n(Q_0^2)}{G_n(Q_0^2)}\right)\left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]^{d_+^n} H_+(Q^2,Q_0^2), \tag{2}$$

where
$$\xi_n^{(1)}=\frac{\gamma_{\psi\psi}^{(0)n}-\gamma_+^{(0)n}}{\gamma_-^{(0)n}-\gamma_+^{(0)n}}, \quad \xi_n^{(2)}=\frac{\gamma_{G\psi}^{(0)n}}{\gamma_-^{(0)n}-\gamma_+^{(0)n}}, \quad d_\pm^n=\frac{\gamma_\pm^{(0)n}}{2\beta_0}$$

$$\gamma_\pm^{(0)n}=\frac{1}{2}\left(\gamma_{\psi\psi}^{(0)n}+\gamma_{GG}^{(0)n}\pm\left[\left(\gamma_{\psi\psi}^{(0)n}-\gamma_{GG}^{(0)n}\right)^2+4\gamma_{G\psi}^{(0)n}\gamma_{\psi G}^{(0)n}\right]^{\frac{1}{2}}\right)$$

to the leadint order $H_\pm^n(Q^2,Q_0^2)=1$

tb the next-to-leading order

$$H_\pm(Q^2,Q_0^2)=1+\left(\bar{\alpha}(Q^2)-\bar{\alpha}(Q_0^2)\right)Z_n^\pm+\left(\bar{\alpha}(Q_0^2)\left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)}\right]^{(d_\mp^n-d_\pm^n)}-\bar{\alpha}(Q^2)\right)K_{\pm\mp}^n,$$

$$Z_n^\pm=\frac{\gamma_{\pm\pm}^{(1)n}}{2\beta_0}-\frac{\beta_1\gamma_\pm^{(0)n}}{2\beta_0^2}, \quad K_{\pm\mp}^n=\frac{\gamma_{\pm\mp}^{(1)n}}{2\beta_0+\gamma_\pm^{(0)n}-\gamma_\mp^{(0)n}}\cdot\frac{\gamma_{\psi\psi}^{(0)n}-\gamma_\mp^{(0)n}}{\gamma_{\psi\psi}^{(0)n}-\gamma_\pm^{(0)n}}.$$

The coefficients $\gamma^{(1)n}_{\pm\pm}$, $\gamma^{(1)n}_{\pm\mp}$ are expressed through the anomalous dimensions $\gamma^{(m)n}_{ij}$ $(m=1,2;\ i,j=\Psi,G)$ /5/ calculated in paper /14/. The analytical form of $\gamma^{(m)n}_{ij}$ is given in paper /15/. All the notation used hereafter is given in paper /5/.

Note also that to the leading and next-to-leading orders, hereafter different coupling constants $\alpha_{LO}(Q^2)$ and $\alpha_{\overline{MS}}(Q^2)$ are used. They satisfy the equations:

$$\frac{1}{\alpha_{LO}(Q^2)} = \beta_0 \ln \frac{Q^2}{\Lambda^2_{LO}} \tag{3a}$$

$$\frac{1}{\alpha_{\overline{MS}}(Q^2)} + \frac{\beta_1}{\beta_0} \ln \alpha_{\overline{MS}}(Q^2) = \beta_0 \ln \frac{Q^2}{\Lambda^2_{\overline{MS}}}. \tag{3b}$$

In the present paper we use different values of $\Lambda_{\overline{MS}}$ (and the corresponding Λ_{LO}):

$$\Lambda^{(1)}_{\overline{MS}} = 105 \text{ Mev} \quad (\Lambda^{(1)}_{LO} = 90 \text{ Mev})$$

and

$$\Lambda^{(2)}_{\overline{MS}} = 230 \text{ Mev} \quad (\Lambda^{(2)}_{LO} = 200 \text{ Mev})$$

obtained, respectively, by the groups EMC /7/ and BCDMS /9/.

In equations (2) and (3) we used the first two coefficients of the expansion in powers of the coupling constant of the anomalous dimensions of the Wilson operators and β-function:

$$\gamma^n_{ij}(\alpha) = \sum_{m=1} \gamma^{(m)n}_{ij} [\alpha]^{m+1}, \quad \beta(\alpha) = -\sum_{m=1} \beta_m [\alpha]^{m+2}.$$

The parametrizations of the nonsinglet and singlet quarks and gluon distributions are taken at $Q_0^2 = 5\,\text{Gev}^2$ /7/ in the form:

$$x\Delta(x,Q^2) = 0.29\, x^{0.52}(1-x)^{3.26}(1+8.9x)$$

$$x\Sigma(x,Q^2) = 6.3\, x^{0.7}(1-x)^{3.21} + 1.03\,(1-x)^{13.47}$$

$$xG(x,Q^2) = 3.81\,(1-x)^{6.7}.$$

The moments of the distributions are defined as follows:

$$f_n(Q^2) = \int_0^1 dx\, x^{n-1} f(x,Q^2) \qquad (f = \Delta, \Sigma, G).$$

The Casimir operator C_A enters into all quantities of equation (2), which determines the different shape of Q^2-dependence of the gluon distribution with and without the self-coupling of gluons.

Q^2-evolution of the coupling constant (3) is determined by the first coefficient in the expansion of the β-function:

$$\beta_0 = \frac{1}{3}(11C_A - 4T_F).$$

After switching-off the gluon interaction the quantity β_0 changes in sign and the coupling constant grows with Q^2, like in QED:

$$\frac{1}{\tilde{\alpha}_{LO}(Q^2)} = \frac{1}{\tilde{\alpha}_{LO}(Q_1^2)} + \beta_0 \ln \frac{Q^2}{Q_1^2}. \tag{4a}$$

$$\frac{1}{\tilde{\alpha}_{\overline{MS}}(Q^2)} + \frac{\beta_1}{\beta_0} \ln \tilde{\alpha}_{\overline{MS}}(Q^2) = \frac{1}{\tilde{\alpha}_{\overline{MS}}(Q_1^2)} + \frac{\beta_1}{\beta_0} \ln \tilde{\alpha}_{\overline{MS}}(Q_1^2) + \beta_0 \ln \frac{Q^2}{Q_1^2}, \tag{4b}$$

where the symbol $\tilde{\alpha}(Q^2)$ stands for the coupling constant of QCD_1. As follows from equation (4), we must determine the coupling constant at some fixed Q_1^2. In the present paper, the condition is taken;

$$\tilde{\alpha}_{LO}(Q_1^2) = \alpha_{LO}(Q_1^2) \qquad \text{for} \qquad Q_1^2 = 100\,\text{Gev}^2. \tag{5}$$

If we chose the equality of the coupling constants for smaller Q^2, the difference between the quantities $\tilde{\alpha}_{LO}(Q^2)$ and $\alpha_{LO}(Q^2)$ would be larger in the tested region ($Q^2 = 50 \div 1000\ \text{GeV}^2$) and all the effect obtained further would only be increased. For the next-to-leading order we keep the condition (5) in the form:

$$\alpha_{\overline{MS}}(Q_3^2) = \alpha_{LO}(Q_1^2) = \tilde{\alpha}_{LO}(Q_1^2) = \tilde{\alpha}_{\overline{MS}}(Q_2^2),$$

where $(Q_3^2, Q_2^2) \sim 10\ \text{Gev}^2$.

This means that we fix one and the same point of intersection of Q^2-evolution of the coupling constants of QCD and QCD_1 in both the orders of perturbation theory. This situation is neither obligatory, nor unique. For example, we can fix the value Q_4^2 at which the coupling constans of QCD and QCD_1 coincide in both the orders of perturbation theory.

3. "Experimental" Q^2-evolution of the gluon distribution

From the connection of the structure functions and parton distributions we can obtain the longitudinal structure function in the form (the moments of the structure function are defined by $F_{k,n}(Q^2)=\int_0^1 dx\, x^{n-2} F_k(x,Q^2)$ $(k=2,L)$): to the leading order /5/:

$$F_{L,n}(Q^2)=B^{(1)\psi}_{L,n}\,\mathcal{I}(Q^2)\,F_{2,n}(Q^2)+\delta_\psi^2\,B^{(1)G}_{L,n}\,\mathcal{I}(Q^2)\,G_n(Q^2) \tag{6a}$$

to the next-to-leading order /12/

$$F_{L,n}(Q^2)=B^{(1)\psi}_{L,n}\,\mathcal{I}(Q^2)\Big[F_{2,n}(Q^2)\big(1+\mathcal{I}(Q^2)\,\varphi_{2,n}\big)+\delta_{NS}^2\,\mathcal{I}(Q^2)\,\Delta_n(Q^2)\cdot\varphi_{3,n}\Big]+\delta_\psi^2\,B^{(1)G}_{L,n}\,\mathcal{I}(Q^2)\,G_n(Q^2)\big(1+\mathcal{I}(Q^2)\,\varphi_{3,n}\big), \tag{6b}$$

where /10/

$$B^{(1)\psi}_{L,n}=\frac{4C_F}{n+1}\,,\quad B^{(1)G}_{L,n}=\frac{16\,T_F}{(n+1)(n+2)}$$

and the quantities

$$\varphi_{1,n}=R^{(2)G}_{L,n}-B^{(1)G}_{2,n}\,\frac{B^{(1)\psi}_{L,n}}{B^{(1)G}_{L,n}}\,,\quad \varphi_{2,n}=R^{(2)\psi}_{L,n}-B^{(1)\psi}_{2,n}\,,$$

$$\varphi_{3,n}=R^{(2)NS}_{L,n}-R^{(2)\psi}_{L,n}$$

are given in appendix.

Here the coefficients C_A, C_F and T_F are of the form

$$C_A=N\,,\quad C_F=\frac{N^2-1}{2N}\,,\quad T_F=\frac{f}{2}$$

for the SU(N) gauge group and f quark flavours.

From equations (6), we obtain for the gluon distribution: to the leading order

$$G_n(Q^2)=\frac{\dfrac{F_{L,n}(Q^2)}{\mathcal{I}(Q^2)}-B^{(1)\psi}_{L,n}\,F_{2,n}(Q^2)}{\delta_\psi^2\,B^{(1)G}_{L,n}} \tag{7a}$$

to the next-to-leading order

$$G_n(Q^2)=\frac{\dfrac{F_{L,n}(Q^2)}{\mathcal{I}(Q^2)}-B^{(1)\psi}_{L,n}\,F_{2,n}(Q^2)\big(1+\mathcal{I}(Q^2)\varphi_{2,n}\big)-\delta_{NS}^2\,\mathcal{I}(Q^2)\,B^{(1)\psi}_{L,n}\,\Delta_n(Q^2)\,\varphi_{3,n}}{\delta_\psi^2\,B^{(1)G}_{L,n}\big(1+\mathcal{I}(Q^2)\,\varphi_{1,n}\big)} \tag{7b}$$

In equations (7) we will use the quantity $R=\sigma_L/\sigma_T$ in place of the longitudinal structure function. They are related as follows:

$$F_L(x,Q^2)=\frac{R(x,Q^2)}{1+R(x,Q^2)}\,F_2(x,Q^2)\equiv\bar{R}(x,Q^2)\,F_2(x,Q^2).$$

The experimental data of the $R(x,Q^2)$ have large uncertainties, therefore the mean values with errors which are valid in some interval of x and Q^2 are given as a rule. The groups /7,9/ present, respectively, the values:

$$R=-0.010\pm0.037 \tag{8a}$$

$$R=0.015\pm0.013\,. \tag{8b}$$

Here we reported only the statistical errors.

In paper /8/ the data of EMC group /7/ were analysed for the purpose of defining the region of values of the quantity $R(x,Q^2)$. The dependence on x was not singled out:

$$R(x,Q^2)\equiv R(Q^2)$$

and Q^2-evolution was given in the forms:

$$R=const\,,\quad R=\frac{4\langle p_\perp^2\rangle}{Q^2}\,,\quad R=\frac{c}{\ln\frac{Q^2}{\Lambda^2}}\,.$$

The quantities R, $\langle p_\perp^2 \rangle$, c and the QCD parameter $\Lambda \equiv \Lambda_{LO}$ were found to equal:

$$R = 0.000 \pm 0.021 , \qquad \Lambda = 83 \text{ Mev} \tag{9a}$$

$$\langle p_\perp^2 \rangle = (0.000 \pm 0.119) \text{ Gev}^2, \qquad \Lambda = 80 \text{ Mev} \tag{9b}$$

$$c = 0.000 \pm 0.175 , \qquad \Lambda = 82 \text{ Mev} . \tag{9c}$$

In the present paper, we made use of all the five parametrizations (8,9).

As the parametrizations for the quantity R do not depend on the variable x , there follows the simple relation for the moments of the structure functions:

$$F_{L,n}(Q^2) = \bar{R}(Q^2)\, F_{2,n}(Q^2).$$

Hence, equations (7) can be rewritten in a more simple form: to the leading order

$$G_n(Q^2) = \frac{\dfrac{\bar{R}(Q^2)}{\mathcal{I}(Q^2)} - B^{(1)\psi}_{L,n}}{\delta_\psi^2\, B^{(1)G}_{L,n}}\, F_{2,n}(Q^2) \tag{10a}$$

to the next-to-leading order

$$G_n(Q^2) = \frac{\dfrac{\bar{R}(Q^2)}{\mathcal{I}(Q^2)} - B^{(1)\psi}_{L,n}\left(1 + \mathcal{I}(Q^2)\,\bar{\varphi}_{2,n}(Q^2)\right)}{\delta_\psi^2\, B^{(1)G}_{L,n}\left(1 + \mathcal{I}(Q^2)\,\varphi_{1,n}\right)}\, F_{2,n}(Q^2) , \tag{10 B}$$

where

$$\bar{\varphi}_{2,n}(Q^2) = \varphi_{2,n} - \delta^2_{NS}\, \varphi_{3,n}\, \frac{\Delta_n(Q^2)}{F_{2,n}(Q^2)} .$$

In paper /7,9,16/ the problem of x -dependence of the

parametrizations of $R(x,Q^2)$ was discussed, as in the experiments (see /7,9/) the decrease of the ratio was observed with the increase of the variable x (for the constant Q^2). In paper /17/ the parametrization of $R(x,Q^2)$ obtained on the basis of QCD had the behaviour $(1-x)$ in the region of large x .

In the present paper, besides the parametrizations (8,9) we use also parametrizations in the form:

$$R(x,Q^2) = a\, R(Q^2)(1-x) . \tag{11}$$

For the parametrizations (9) we took a =2.5. With this meaning value of a , the region restricted by the new parametrization covers the previous region of values up to x =0.6. For the parametrizations (8) we took $a = 2$ so that the regions restricted by new parametrizations obtained from (8a) and (9a) coincide, respectively.

Note, that

$$R(x,Q^2) \ll 1 \quad \text{or} \quad \bar{R}(x,Q^2) \approx R(x,Q^2)$$

and, hence, equation (11) can be presented in the form

$$\bar{R}(x,Q^2) = a\, \bar{R}(Q^2)(1-x) .$$

Taking into consideration equation (12) we get a new expression for the gluon distribution, $\bar{G}(x,Q^2)$:

to the leading order

$$\bar{G}_n(Q^2) = G_n(Q^2) + \frac{\bar{R}(Q^2)\left[(a-1)F_{2,n}(Q^2) - a\, F_{2,n+1}(Q^2)\right]}{\delta_\psi^2\, \mathcal{I}(Q^2)\, B^{(1)G}_{L,n}} .$$

to the next-to-leading order

$$\bar{G}_n(Q^2) = G_n(Q^2) + \frac{\bar{R}(Q^2)\left[(a-1)F_{2,n}(Q^2) - a\, F_{2,n+1}(Q^2)\right]}{\delta_\psi^2\, \mathcal{I}(Q^2)\, B^{(1)G}_{L,n}\left(1 + \mathcal{I}(Q^2)\varphi_{1,n}\right)} .$$

For the moments of $F_{2,n}(Q^2)$ we use the parametrization of the transverse structure function in the form /7/:

$$F_2(x,Q^2) = \left(c_1 x^{c_2}(1-x)^{c_3} + c_4(1-x)^{c_5}\right)\left[1 + \left(c_6(1-x)^{c_7} + c_8\right)\ln\frac{Q^2}{3}\right],$$

where

$C_1 = 3.373$; $C_2 = 0.985$; $C_3 = 3.688$; $C_4 = 0.276$;

$C_5 = 10.629$; $C_6 = 0.282$; $C_7 = 8.995$; $C_8 = -0.078$.

There is also another possibility. We can use the parametrization of the structure function a certain value of Q_0^2 ($Q_0^2 = 5\,GeV^2$), then we can decompose it in the moments and reproduce the moments for other Q^2 using known QCD equation to the leading order

$$F_{2,n}(Q^2) = F_{2,n}(Q_0^2)\left[\frac{\mathcal{I}(Q^2)}{\mathcal{I}(Q_0^2)}\right]^{d^n_{NS}} , \qquad (15)$$

$$F_2(x, Q_0^2 = 5\,GeV^2) = 1.75\cdot x^{0.59}(1-x)^{2.45}(1-0.76x) \qquad (\text{for } x > 0.3) \qquad (16)$$

This procedure is possible for the following reasons:

First, to determine Q^2-evolution of the moments (for $n \geq 3$) of the gluon distribution, we can use the parametrization of the structure function for $x > 0.3$ (that is we can neglect the sea quarks) (see /1/);

Second, the predictions of QCD (both to the leading and next-to-leading order) well describe the experimental data, that is, this approach is as good as a simpe expansion in the moments of the expression (14);

Third, the values of the expressions (15) do not change practically when the self-coupling of the gluon is switched off, that is under the change in eq.(15): $\mathcal{I}(Q^2) \to \tilde{\mathcal{I}}(Q^2)$ (see the paper /1/ and Introduction).

Hence, it may be said that in this approach we get the model-independent (of QCD) expression for the moments of the structure function at any Q^2: Moreover the parametrization (14) for $Q^2 \gtrsim 10^6$ gets negative values. The quantity $F_{2,n}(Q^2)$ in the expression (15) is positive at any Q^2.

Note, that with the parametrization (15) for $F_2(x,Q^2)$, the quantity $\bar{\Phi}_{2,n}$ (see equation (10)) does not depend on Q^2:

$$\bar{\Phi}_{2,n}(Q^2) \to \bar{\Phi}_{2,n} = \Phi_{2,n} - \delta^2_{NS}\,\Phi_{3,n}\,\frac{\Delta_n(Q_0^2)}{F_{2,n}(Q_0^2)} .$$

In the present paper both the parametrizations (14) and (15) were used. In the tested region they have very close values.

For finding of the "experimental" Q^2-dependence of the moments of the gluon distribution in QCD_1 we must make further substitution $\mathcal{I}(Q^2) \to \tilde{\mathcal{I}}(Q^2)$ in the expressions (6), (7), (10) and (13).

4. The discussion of the results

E.Reya has suggested to consider the Q^2-evolution of the gluon distribution only for the first (n=3,4,5) moments /1/ as the values of higher moments are very small and they cannot be obtained from the combination of the moments of the structure function (which are much larger magnitude and which have some errors). We have also restricted our consideration to $n \leq 5$.

The graphs of the theoretical and "experimental" Q^2-evolution of the moments of the gluon distribution for different parametrizations of $R = \sigma_L/\sigma_T$ (see (8), (9) and (11)) are given in Fig.1.

Let us make common conclusions concerning the fit of the gluon self-coupling and the parametrizations $R = \sigma_L/\sigma_T$.

1. First of all note that the results weakly depend on the order of perturbation theory and on the choice of the parametrization for $F_2(x,Q^2)$ in the form (14), (15), therefore the graphs are given only for the parametrization (15) to the leading order.

2. As follows from Fig.1c, the parametrization of $R = \sigma_L/\sigma_T$ in the form $4\langle p_\perp^2\rangle/Q^2$ is nonacceptable: for $Q^2 > 200\,GeV^2$ the region of the moments of the gluon distribution degenerates into the curve that does not coincide with the theoretical curves of QCD and QCD_1. With the change of the interval of errors for $\langle p_\perp^2\rangle$ the situation does not change with the exception of some displacement of the degeneration point. All this is valid for the fourth and fifth moments, therefore the graphs for them (with this parametrization for $R = \sigma_L/\sigma_T$) are not given. This disagreement of theory and experiment has important consequences. The parametrization $4\langle p_\perp^2\rangle/Q^2$ corresponds to the highest-twist contribution to the ratio R. There are attempts to explain the data for $R = \sigma_L/\sigma_T$ only by the highest-twist contribution /18/, which, according to the Fig.1c is not correct.

3. (For the moments n=4 and n=5) . The region of experimental data on Q^2-evolution of the gluon distribution in QCD contains the theoretical curve of QCD. Thus QCD agrees with the experiment. For the x -dependent parametrization of $R = \sigma_L/\sigma_T$ (11) the agreement of the theory with experiment is more obvious.

Fig. 1. Graphs of the theoretical and "experimental" Q^2-evolution of the moments (n=3,4,5) of the gluon distribution to the leading order of perturbation theory. The dashed curve corresponds to the theoretical Q^2-dependence. The solid (with primes) curves denote the upper (lower) boundary of the "experimental" Q^2-evolution of the gluon distribution for different parametrizations of $R=\sigma_L/\sigma_T$:
a), f) and j) - for (8a); b), g) and k) for (9a); c) - for (9b); d), h) and l) - for (9c); e), i) and m) - for (8b). The dotted (with incision) curve corresponds to the upper (lower) boundary of the "experimental" Q^2-evolution of the gluon distribution after the introduction of x-dependence (11) into the parametrization for $R=\sigma_L/\sigma_T$. The indices 1 and 2 denote the curves for QCD and QCD_1, respectively.

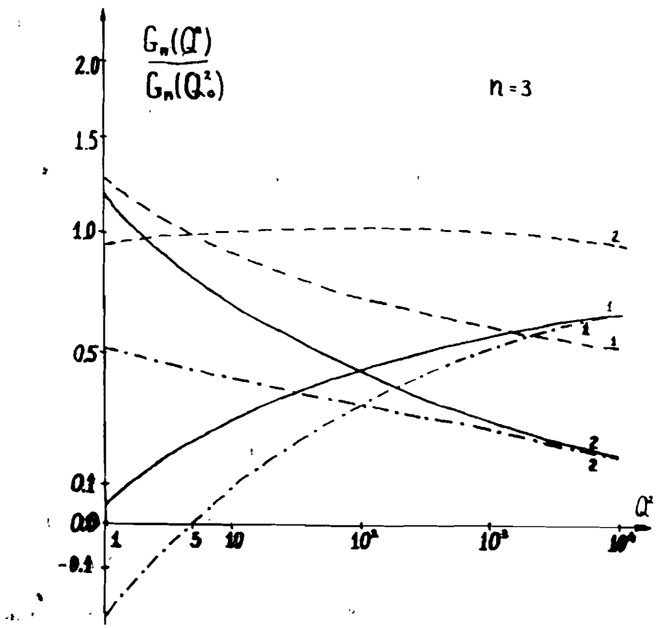

Fig.1a

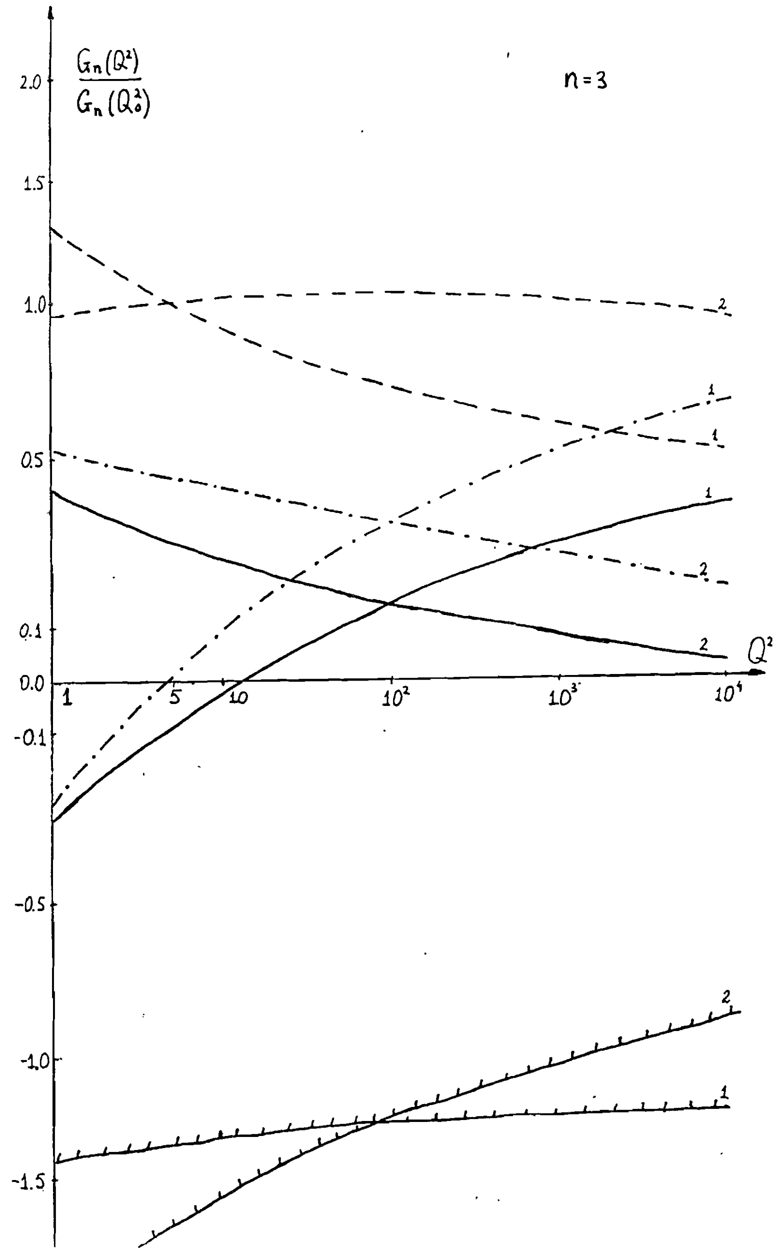

Fig.1b

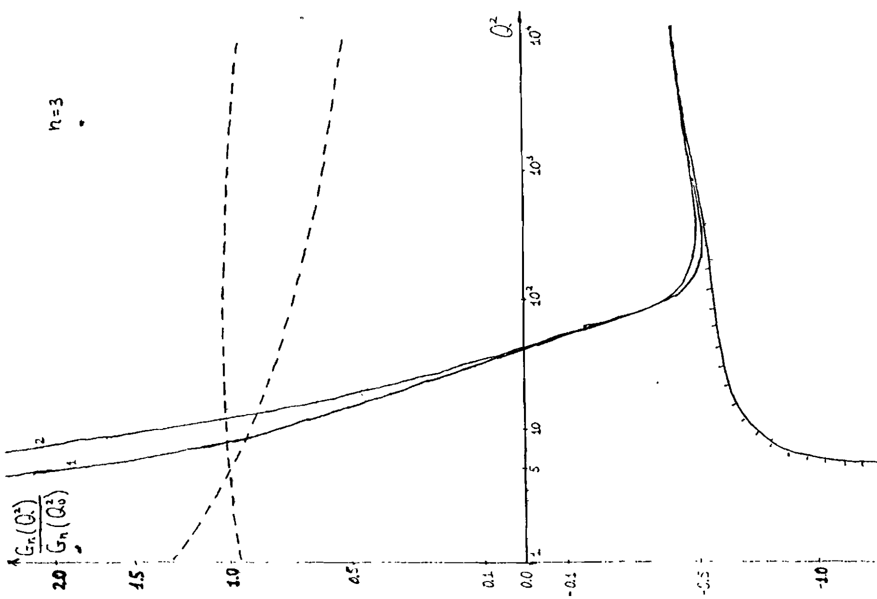

Fig.1c

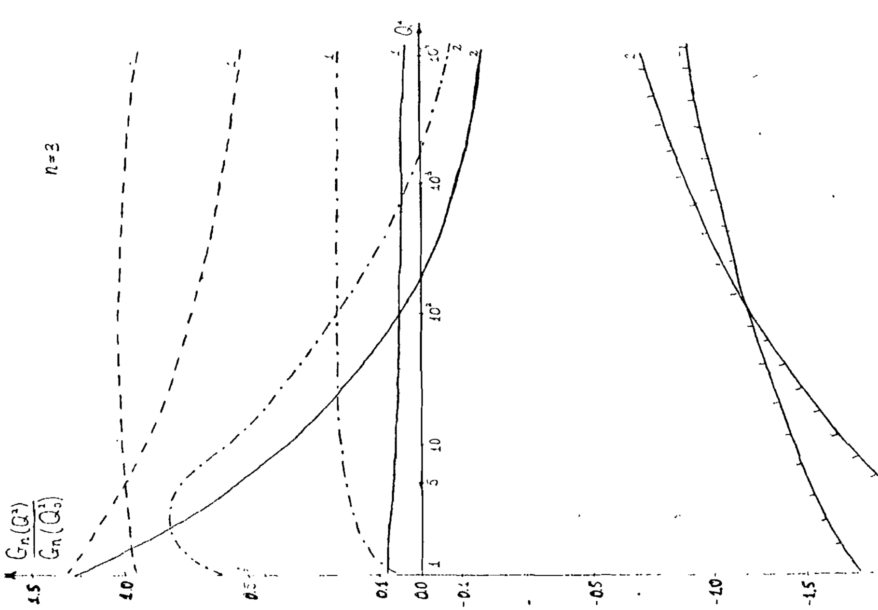

Fig.1d

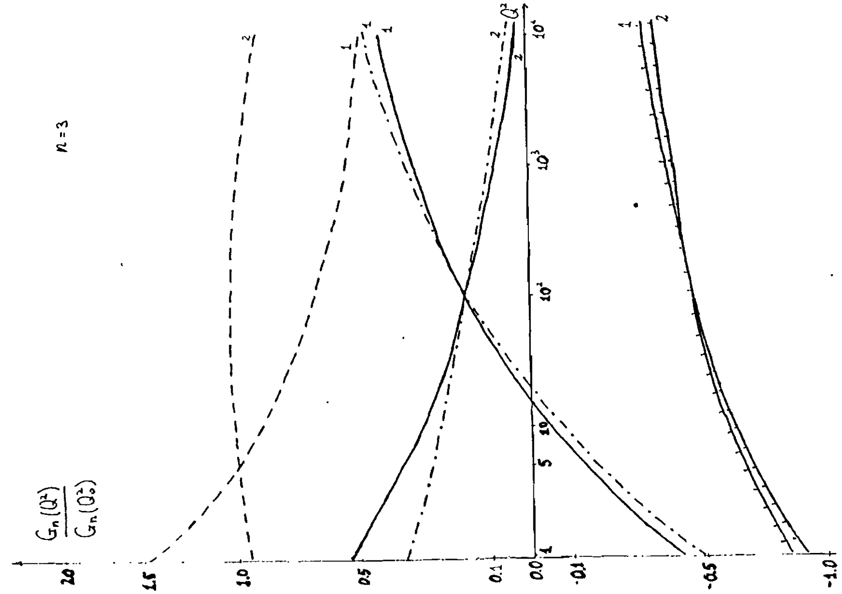

Fig.1e

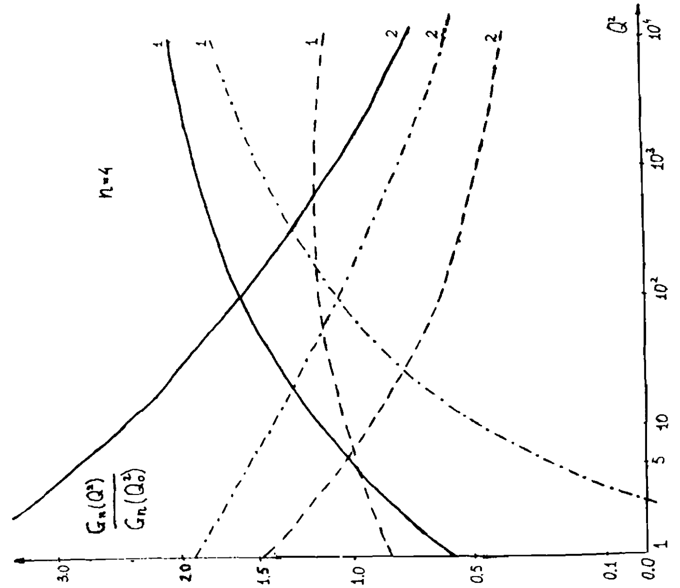

Fig.1f

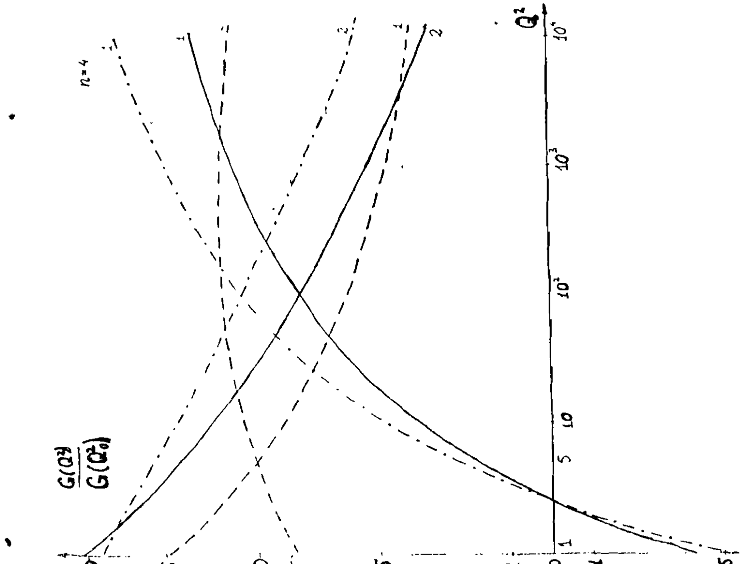

Fig.1g

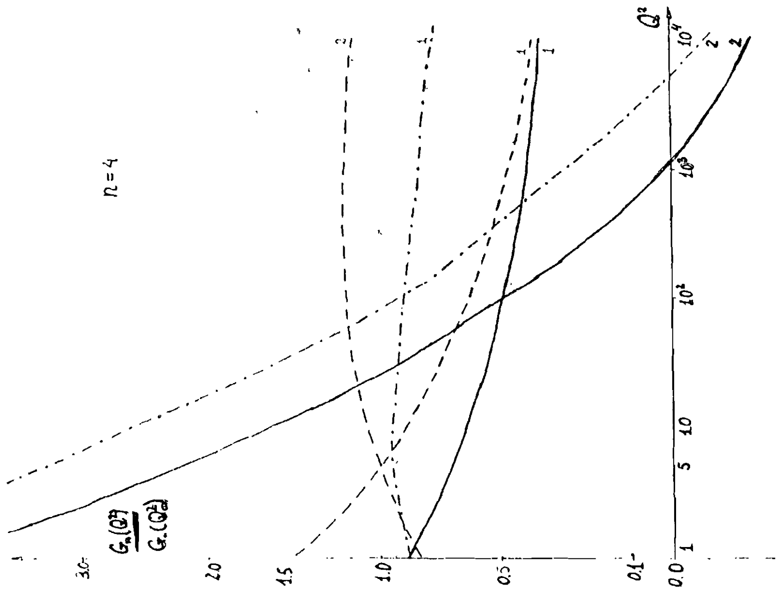

Fig.1h

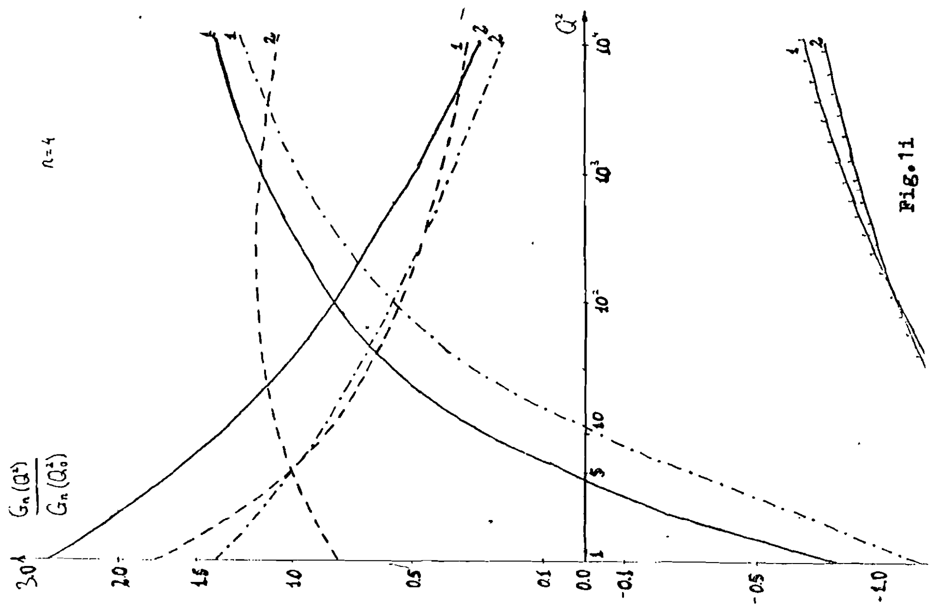

Fig.1i

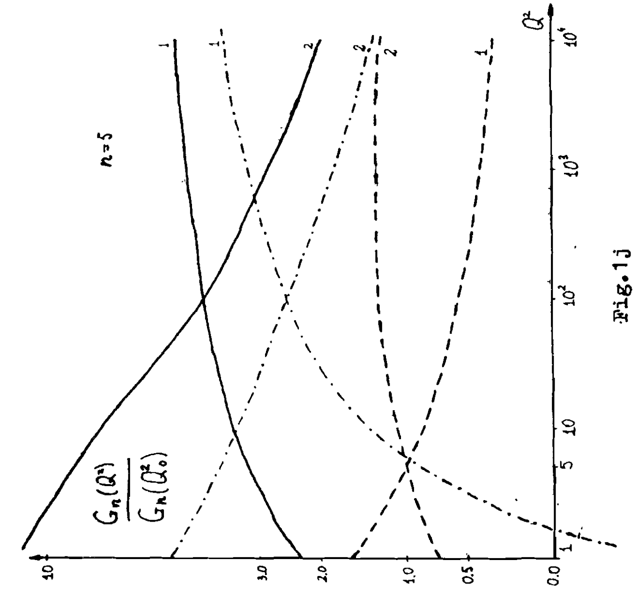

Fig.1j

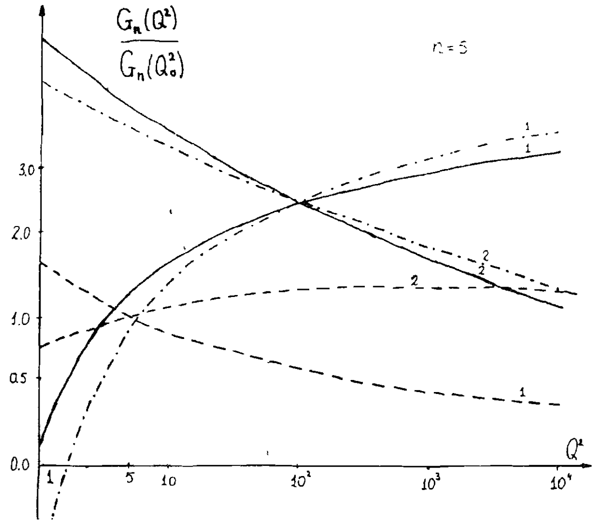

Fig.1k

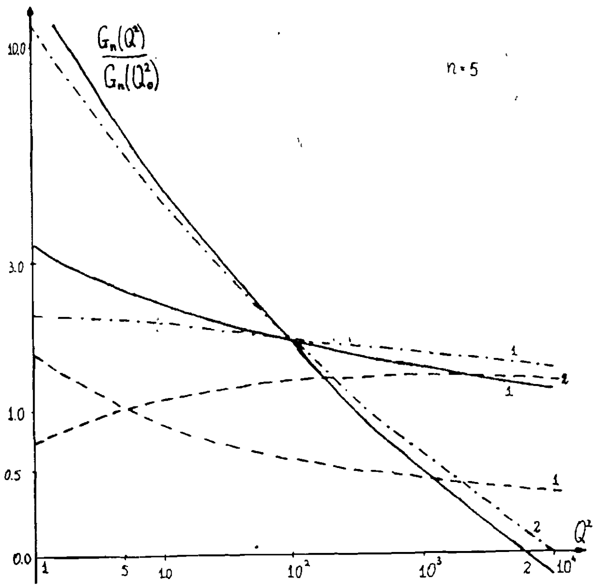

Fig.1l

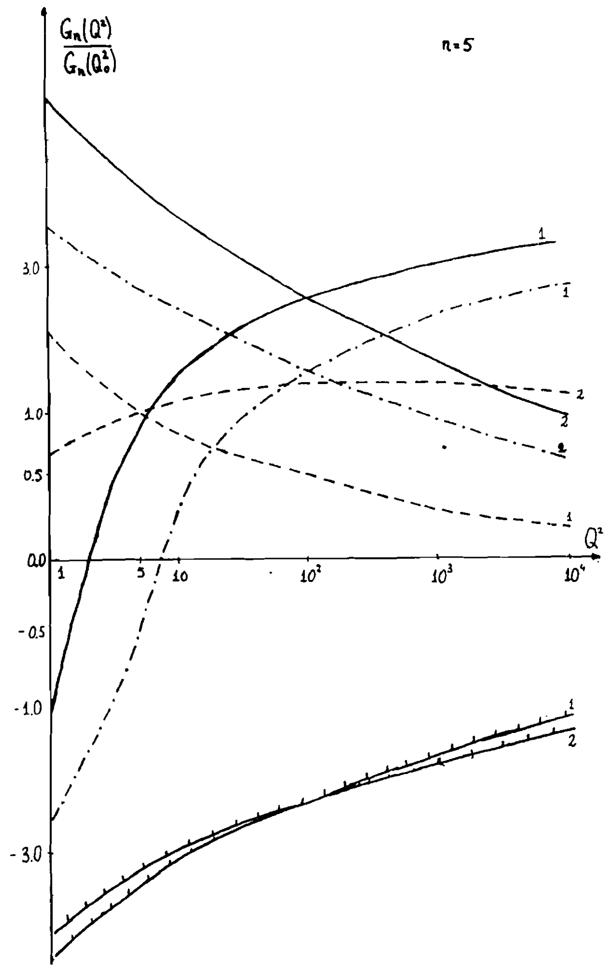

Fig.1m

4. The experimental data of Q^2-evolution of the gluon distribution in QCD_1(with the exception of the parametrization R =const, n=5) do not contain the theoretical curve of QCD_1 beginning from some Q^2. Hence, does not agree with the experiment.

5. (For the third moment). The curves corresponding to the upper boundary of the "èxperimental" Q^2-dependence of the moments of the gluon distribution for QCD barely overlap (see figure 1a) with the theoretical curve or do not overlap at all (see Fig.'s 1b), d) and e)). This disperancy can be attributed for several reasons:

1) The parametrization of $R(x,Q^2)$ must depend on x.

2) To the third moment of $F_2(x,Q^2)$ the sea quarks give essential contribution, which is not taken into account in the expression (16).

3) The parametrizations for $R(x,Q^2)$ are correct not for the whole interval of x, but beginning from some $x \simeq 0.2$. For $x < 0.2$ the parametrizations for R must have larger values (see [12]).

Introduction of the x-dependence into the parametrizations of $R(x,Q^2)$ (see (11)) gives a more satisfactory behaviour of the curves in Figs. 1a), b) and e). In Fig.1d) the situation is better although the theoretical curve and the upper boundary of the "experimental" region do not overlap as before. Hence the x-dependent parametrizations (11) solve this problem only partially.

As the result slightly changes when using the parametrization (14 for $F_2(x,Q^2)$, the essential contribution of the sea quarks seems improbable.

If the discrepancy is accounted for the third reason the agreement between the theoretical and "experimental" Q^2-evolutions of the gluon distribution in QCD is poor for the third moment (n=3) and it will be improved with increasing n. As is seen, this situation takes place in reality. Hence the poor agreement of the theory with experiment for the third moment does not out in doubt the self-consistency of QCD obtained at larger values of n.

6. Out of the used parametrizations of R, $R \sim (1-x)$ and $R \sim (1-x)/\ln\frac{Q^2}{\Lambda^2}$, the second is more successful, where the theoretical and "experimental" Q^2-evolutions of the gluon distribution in QCD not only coincide for a large interval of Q^2 but also have the same tendency as $Q^2 \to \infty$. However, even in this parametrization there is no such coincidence for small Q^2 where to $R = \sigma_L/\sigma_T$ the higher-twist contribution is essential. In the whole interval of Q^2 the most successful parametrization is (see Fig.2):

$$R = \frac{0.50\cdot(1-x)}{\ln\frac{Q^2}{\Lambda^2}} + 0.05\ \frac{4\langle p_\perp^2\rangle}{Q^2}.$$

Certain x-dependence of the second part of the parametrization (17) is possible.

In Fig. 2, the theoretical and "experimental" curves are obtained with the help of expressions (2) and (13) to the next-to-leading order, respectively. Generally speaking, a correct analysis of the behaviour of the gluon distribution in the region of small values

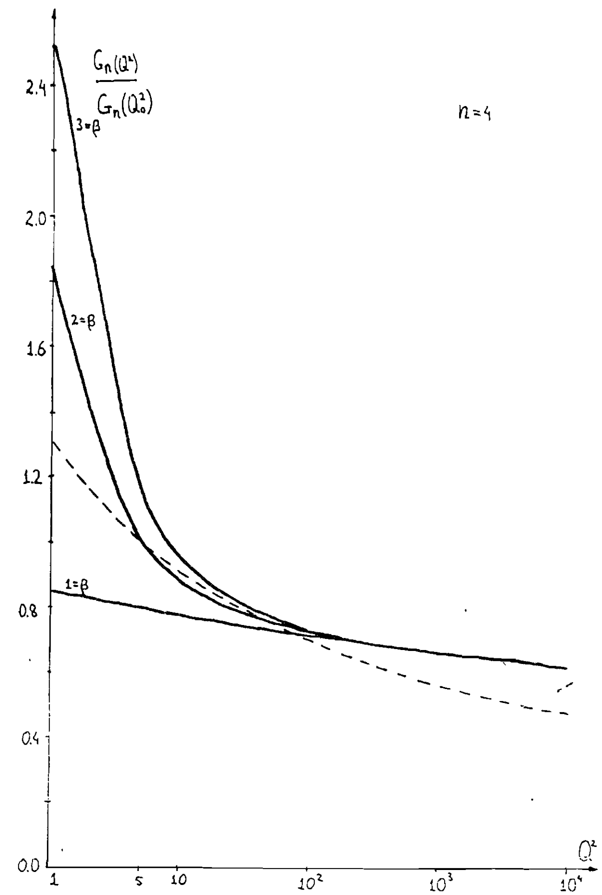

Fig. 2. The graph of the theoretical (the dashed curve) and upper boundary of the "experimental" Q^2-evolution of the gluon distribution in QCD to the next-to-leading order for the parametrization $R = \frac{(1-x)}{\ln\frac{Q^2}{\Lambda^2}} + \beta\ \frac{4\langle p_\perp^2\rangle}{Q^2}$.

of Q^2 requires to calculate the higher-twist contribution in the anomalous dimensions and the coefficient function of the Wilson operator. Unfortunately, this calculation has not yet been carried out.

7. Let us compare what is the agreement between the theoretical and "experimental" Q^2-evolutions of the gluon distribution at values of $R(x,Q^2)$ obtained by the groups EMC [7] and BCDMS [9]. For

the BCDMS parametrization at n=4 and n=5 at $Q^2 = 10^3 \div 10^4$ GeV^2 the theoretical curve is placed approximately in the centre of the experimental data (at n=3 this is not so, see subsection 6). For the data of the EMC group (and the data obtained in paper /8/) the theoretical curve is placed near by the upper boundary of the experimental data as the mean value for $R(x,Q^2)$ is negative (equal to zero). Note that the logarithmic parametrization in the form (9c) is desirable for the BCDMS data.

5. Conclusion

Using the experimental data obtained in ref.'s /7,9/ we have carried out the fit of the gluon self-coupling in QCD. We have also show that only the power violation of Kallan-Gross correlation /2/ is impossible. However, it is necessary to have more accurate values for $F_2(x,Q^2)$ and particularly for the ratio $R(x,Q^2)$. So, even the decrease of the interval of errors in $2 \div 3$ times would allow us to single out in per cent the logarithmic and power violation of the Kallan-Gross correlation and to obtain an x-dependence of the ratio $R = \sigma_L/\sigma_T$.

The author is grateful to Kazakov D.I. for permanent interest in the work and numerous discussions and also to Efremov A.V., Radyushkin A.V. and Vladimirov A.A. for useful remarks.

Appendix

1. The one-loop corrections to the coefficients in the Wilson expansion for the transverse structure function are of the form /10,12/:

$$B^{(1)\Psi}_{2,n} = 2C_F\left[S_1^2(n) - S_2(n) + S_1(n)\left(\frac{3}{2} - \frac{1}{n(n+1)}\right) - \frac{9}{2} + \frac{3}{2}\frac{1}{n} + \frac{2}{n+1} + \frac{1}{n^2}\right]$$

$$B^{(1)G}_{2,n} = \frac{2^2 T_F}{n+1}\left[S_1(n)\left(-1 - \frac{1}{n} + \frac{2}{n+1}\right) + \frac{1}{n} + \frac{4}{n+2} + \frac{1}{n^2}\right],$$

where

$$S_i(n) = \sum_{K=1}^{n} \frac{1}{K^i}.$$

2. The two-loop corrections to the coefficients in the Wilson expansion for the longitudinal structure function are of the form /11,12/:

$$R^{(2)NS}_{L,n} = (2C_F - C_A)\Big[8K_2(n)S_1(n) - 8Q(n) + 4K_3(n) - 4S_3(n) + 12\zeta(3) -$$
$$-\frac{6}{5}\left\{\frac{1-\delta_n^2}{n-2}(4K_2(n)-3) + \delta_n^2(6\zeta(3)-7)\right\} - 8K_2(n)\left(1 + \frac{1}{n} - \frac{1}{n+1} - \frac{3}{5}\cdot\frac{1}{n+3}\right) -$$
$$-\frac{23}{3}S_1(n) - \frac{215}{18} + \frac{11}{3}\frac{1}{n} + \frac{11}{3}\frac{1}{n+1} - \frac{18}{5}\frac{1}{n+3}\Big] - \frac{4}{3}T_F\left[S_1(n) + \frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right]$$
$$+2C_F\left[S_1^2(n) - S_2(n) + S_1(n)\left(\frac{19}{6} - \frac{1}{n} - \frac{1}{n+1}\right) + \frac{277}{36} - \frac{7}{6}\frac{1}{n} - \frac{19}{6}\frac{1}{n+1} + \frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$R^{(2)\Psi}_{L,n} = R^{(2)NS}_{L,n} + \frac{2^3 T_F}{(n+1)(n+2)}\left[S_1(n)\left(-1 - \frac{4}{n-1} + \frac{2}{n}\right) - 1 + \frac{5}{3}\frac{1}{n-1} - \frac{3}{n} + \frac{1}{n+1} + \frac{4}{3}\frac{1}{n+2} - \frac{2}{n^2}\right]$$

$$R^{(2)G}_{L,n} = -2C_F\Big[\frac{2}{5}\left\{\frac{1-\delta_n^2}{n-2}(4K_2(n)-3) + \delta_n^2(6\zeta(3)-7)\right\} + 2K_2(n)\left(1 - \frac{4}{5}\frac{1}{n+3}\right)$$
$$+S_1(n)\left(\frac{3}{2} + \frac{1}{n(n+1)}\right) + \frac{14}{5} - \frac{3}{2}\frac{1}{n} + \frac{1}{2}\frac{1}{n+1} + \frac{6}{5}\frac{1}{n+3} - \frac{1}{n^2} + \frac{1}{(n+1)^2}\Big] +$$
$$+2C_A\Big[S_1^2(n) - S_2(n) + 2K_2(n) + 2S_1(n)\left(2 - \frac{1}{n-1} + \frac{1}{n} - \frac{2}{n+1}\right) + \frac{3}{2} +$$
$$+\frac{17}{6}\frac{1}{n-1} - \frac{3}{n} + \frac{1}{2}\frac{1}{n+1} + \frac{11}{24}\frac{1}{n+2} - \frac{2}{n^2} + \frac{2}{(n+1)^2} - \frac{4}{(n+2)^2}\Big],$$

where

$$K_2(n) = (-1)^n \sum_{K=1}^{n} \frac{(-1)^{K+1}}{K^2} + \zeta(2)\frac{1-(-1)^n}{2}$$

$$K_3(n) = (-1)^n \sum_{K=1}^{n} \frac{(-1)^{K+1}}{K^3} + \frac{3}{2}\zeta(3)\frac{1-(-1)^n}{2}$$

$$Q(n) = (-1)^n \sum_{K=1}^{n} \frac{(-1)^{K+1} S_1(K)}{K^2} + \frac{5}{4}\zeta(3)\frac{1-(-1)^n}{2}.$$

All the above results are given in the $\overline{MS}$ scheme.

References

1. Reya E. Phys.Rev.Lett., 1979, 43, 8.
2. Callan G.G., Gross D.J. Phys.Rev.Lett., 1969, 22, 156.
3. Glück M., Reya E. Phys. Rev., 1977, D16, 3242;
   Glück M., Reya E. DESY Report No. DESY 79/13, 1979.
4. Reya E. Phys.Rep. 1981, 69, 195.
5. Buras A.J. Rev.Mod.Phys., 1980, 52, 199.
6. Gell-Mann M., Low F. Phys.Rev., 1954, 95, 1300;
   Bogoliubov N.N., Shirkov D.V. Nuovo Cim., 1956, 3, 845;
   Vladimirov A.A., Shirkov D.V. UFN, 1979, 129, 407.
7. Aubert J.J. and all. Nucl.Phys., 1985, B259, 189.
8. Bilenkaya S.C., Stamenov D.B. JINR preprint, P2-86-65, Dubna, 1986.
9. Benventi A.C. et.al. CERN preprint CERN EP/87-DRAFT.
10. Bargeen W.A., Buras A.J., Duke D.W. Muta T. Phys.Rev., 1978, D18, 3998;
    Altarelli G., Ellis R.K., Martinelli G. Nucl.Phys., 1979, B157, 481.
11. Kazakov D.I., Kotikov A.V. Jad.Fiz. 1987, 46, 1767.
12. Kazakov D.I., Kotikov A.V. JINR preprints, P2-87-656, P2-87-657, Dubna, 1987.
13. Baulieu L., Kounnas C. Nucl. Phys., 1978, B141, 423;
    Kodaira J., Uematsu T. Nucl.Phys., 1978, B141, 497.
14. Floratos E.G., Ross D.A. Sachrajda C.T. Nucl.Phys. 1977, B129, 66 (E., 1978, B139, 545);
    Floratos E.G., Ross D.A., Sachrajda C.T. Nucl.Phys., 1979, B152, 493.
15. Lopez C., Yndurain F.J. Nucl. Phys. 1981, B183, 157.
16. Savin I.A. Proc. 22nd Intern.Conf. on High Energy Physics (Leipzig,1984).
17. Couzaler-Arroyo A., Lopez C. Yndurain F.J. Phys.Lett. 1981, 98B, 215.
18. Ekelin S., Fredriksson S. Phys.Lett., 1985, 162B, 373;
    Abbott L.F., Berger E.L., Blankenbecler R., Kane G.L. Phys.Lett., 1979, 88B, 157.
    Miramontes J.L., Sanchez Guillen J. CERN preprint CERN-TH.4565/86.

Received by Publyshing Department
on December 30, 1987.

Котиков А.В. E2-87-933

Фитсамодействия глюонов в КХД при различных параметризациях соотношения $R = \sigma_L/\sigma_T$ в глубоконеупругом рассеянии лептонов на адронах

Проведен фит самодействия глюонов в квантовой хромодинамике для трех /n = 3,4,5/ моментов глюонного распределения в области $Q \sim 50 \div 1000$ ГэВ^2. Результат сильно зависит от параметризации отношения $R(x, Q^2) = \sigma_L/\sigma_T$, где σ_T и σ_L - сечения взаимодействия виртуальных поперечных и продольных фотонов с нуклоном. Показана ведущая роль логарифмического /а не степенного/ нарушения соотношения Каллана - Гросса. Введена также x-зависимость в парметризацию для отношения R, что улучшает результат фита.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Kotikov A.V. E2-87-933

Fit of Gluon Self-Coupling for Different Parametrizations of $R = \sigma_L/\sigma_T$ Ratio in Deep Inelastic Lepton Scattering on Hadrons

Fit of gluon self-coupling is made in QCD for three (n = 3, 4, 5) moments of gluon distribution in the region of $Q^2 \sim 50 \div 1000$ GeV^2. The result depends strongly on the parametrization of the ratio $R(x, Q^2) = \sigma_L/\sigma_T$ where σ_L and σ_T are cross sections of the longitudinal and transverse photon scattering on a nucleon. The necessity is shown mainly of the logarithnic (not power) violation of the Callan-Gross correlation. x-dependence is also introduced into the parametrization of the ration $R(x, Q^2)$, which improves the result of the fit.

The investigation has been performed at the Laboratory of High Energies, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987.