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N.Salié

THE VARIATIONAL PRINCIPLE OF NEUGEBAUER AND THE PROBLEM OF A CAUSAL HEAT CONDUCTION EQUATION

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1. Introduction

By the variational principle of Neugebauer (1974, 1977) a beautiful, comprehensive and practicable method is given to derive the equations of irreversible thermodynamics and continuum mechanics as well as the field equations of gravitation and electromagnetism from a first principle. It is also applicable to media with internal degrees of freedom and order parameters (Meier, Salié 1979, 1980) and other additional fields. Further, Wulfert , Zimdahl and Salié (1983,1984) could show that the Neugebauer principle forms the natural base of a general relativistic fluctuation theory in the sense of Landau and Lifshitz (1957, 1966, 1968).

But as many simple approaches to relativistic thermodynamics, also this principle has a serious disadvantage providing noncausal transport equations; e.g. for heat conduction one receives a covariant but parabolic equation as in classical physics. In the case of superconducting media this problem could be avoided (Meier, Salié 1979, 1980). The general relativistic Ginzburg-Landau equation resulting from the variational principle is a hyperbolic one. But a too simple choice of the superconductivity contribution to the free energy would also lead to a parabolic equation.

To remove these difficulties, one can change the free energy, the Onsager relations or the thermodynamics itself. Here we consider the first two possibilities. But before, a brief survey of the Neugebauer principle is given.

In the first part a causal heat conduction equation is derived by using a free energy depending on the temperature and its first derivatives. For simplicity we consider a fluid though the method works also in the case of more complicated media. The result is a third order heat conduction equation. The third order terms form a hyperbolic expression in $\dot{T} = T_{,k}u^k$ (T - temperature, u^k - four velocity). These terms determine the causal structure of the equation and guarantee a finite heat flow velocity. But the coefficients themselves will be very small. Additional to these third order terms the normal heat equation with the well-known coefficients appear. This part of the equation dominates in the usual laboratory experiments. Because of the low accuracy of thermodynamical measurements it was impossible up to now to determine experimentally any additional conductivity coefficients being relevant in this connection.

In the second part the usual unchanged free energy and modified Onsager relations are considered. Also here one can get a causal third order equation again.

A certain change of variation prescription would lead to a second order telegraph equation often discussed in this connection, f.i. by Cattaneo (1948), Vernotte (1958), Kranyš (1966), Müller (1969), Israel and Stewart (1979). Also higher order equations were treated, e.g. by Müller (1967). In this connection it is remarkable that Cattaneo (1948) derived first an expression for the heat current from statistics providing a fourth order equation. For small coefficients and near the equilibrium, nevertheless, he could approximately transform it into a telegraph equation again. In our case, however, one cannot prove the entropy production density to be the divergence of the entropy current. Therefore this equation is not further treated here.

Since the Neugebauer principle is a phenomenological one, the investigated modifications provide only the functional structure but not the actual values and functions. It is not the subject of this paper to determine these quantities in the sense of a relativistic kinetic theory as it has been done by Chernikov (1963), (1964), Israel (1963), Israel and Stewart (1979) for gases. It would also be very difficult to extend calculations like that to solids and liquids.

2. Neugebauer's variational principle

The system is characterized by a set of field functions V_Θ. For a fluid we choose $(V_\Theta) = (g_{ik}, \rho, \xi^i)$. Here g_{ik} is the metric, ρ the particle number density and $\xi^i = u^i/T$ the temperature vector (u^i - four velocity, T - temperature). The variational prescription is the following: Along ξ^i the V_Θ are changed to virtual functions $\bar{V}_\Theta$. $\bar{V}_\Theta = V_\Theta + \delta V_\Theta$

$$\delta V_\Theta = \pounds_\xi V_\Theta \cdot \delta\omega . \tag{1}$$

$\pounds_\xi V_\Theta$ is the Lie derivative, $\delta\omega$ is an infinitesimal function with

$$\delta\omega \geq 0 \quad in \ \mathcal{V}_4 \ , \qquad \delta\omega\Big|_{[\mathcal{V}_4]} = 0, \ (\delta\omega)_{,\ell}\Big|_{[\mathcal{V}_4]} = 0. \tag{2}$$

$\mathcal{V}_4$ is an arbitrary four-dimensional volume. The Neugebauer variational principle reads as follows: For any thermodynamical system there exists a time-like vector field $\xi^i = u^i/T$ such that the action W in the virtual states $\bar{V}_\Theta$ corresponding to this field is never smaller than the action in the actual state V_Θ

$$\delta W = \delta \int_{v_4} L \sqrt{-g}\, d^4x \geq 0 . \tag{3}$$

The Lagrange density L consists of a field-theoretical and a thermodynamical part

$$L(V_\Theta ; V_{\Theta,k} ; V_{\Theta,k,l}) = -\frac{R}{2\varkappa_0} - \rho f \tag{4}$$

(R - curvature invariant, $g = \det|g_{ik}|$, $\varkappa_0$ - Einstein's gravitational constant, f - molar free energy).

Performing the variation in (3) one gets

$$\sigma = \sum_\Theta \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta V_\Theta} \cdot \pounds_\xi V_\Theta \geq 0 . \tag{5}$$

By the Noether theorem Neugebauer (1977) (1980) showed

$$\sum_\Theta \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta V_\Theta} \pounds_\xi V_\Theta \equiv S^i{}_{;i} = \sigma \geq 0 \tag{6}$$

with S^i and σ being the densities of entropy flux and production. The sum (6) takes the form

$$\sum_\Theta J^\Theta X_\Theta = \sigma \geq 0 \tag{7}$$

with the thermodynamical currents J_Θ and forces X_Θ

$$J^\Theta = \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L)}{\delta V_\Theta} , \tag{8}$$

$$X_\Theta = \pounds_\xi V_\Theta \tag{9}$$

These quantities are connected by the Onsager relations

$$J^\Theta = \sum_\Omega L^{\Theta\Omega} X_\Omega . \tag{10}$$

The symmetric coefficients $L^{\Theta\Omega}$ form a positive definite quadratic expression guaranteeing $\sigma \geq 0$ in (7). On the other hand (10) with (8) and (9) represent the field equations

$$\frac{1}{\sqrt{-g}} \cdot \frac{\delta(L\sqrt{-g})}{\delta V_\Theta} = \sum_\Omega L^{\Theta\Omega} \pounds_\xi V_\Omega . \tag{11}$$

For a medium with the Lagrangian (4) one gets (Neugebauer 1977)

$$\frac{R^{ik} - \frac{1}{2} g^{ik} R}{\varkappa_0} + T^{ik}_{(rev)} = - T^{ik}_{(irr)} , \tag{12}$$

$$T^{ik}_{(rev)} = \rho e u^i u^k / c^2 - s^{ik} . \tag{13}$$

The left-hand side is given by variation of L with the use of the particle number conservation $(\rho u^i)_{;i} = 0$ and the relations for the thermodynamical potentials ($e = f - T\partial f/\partial T$ - internal energy, s^{ik} - stress tensor). $T_{(irr)}$ is given by the Onsager relations

$$T^{ik}_{(irr)} = - s^{ik}_{(visc)} + (q^i u^k + q^k u^i)/c^2 , \tag{14}$$

$$s^{ik}_{(visc)} = 2\eta_I h^{ir} h^{ks} u_{(r,s)} + \eta_{II} h^{ik} u^r{}_{;r} , \tag{15}$$

$$q^i = -\varkappa h^{ir} (T_{,r} + \dot{u}_r T/c^2) \tag{16}$$

(q^i - heat current; $h_i^k = g_i^k + u_i u^k/c^2$; $\eta_I, \eta_{II}, \varkappa$ - coefficients of viscosity and heat conductivity).

3. Modification of the Lagrangian of the Neugebauer principle

3.1. Free energy

In the usual thermodynamics and fluid mechanics the molar free energy is a function of ρ and T

$$f = f(\rho, T) . \tag{17}$$

In this section we postulate a weak dependence of f on $T_{,k}$. In the Appendix some ideas are discussed which possibly could explain a structure of this kind. Here we investigate the consequences of that ansatz. In the theory of elasticity such structures are investigated concerning the derivatives of the deformation variables (Bressan 1978).

Being an invariant, f should also depend on invariant expressions. The simplest invariant is $\dot{T} = T_{,k} u^k$. But because $\pounds_\xi \xi^i = 0$, for convenience we use ξ^i as variables instead of u^i. In (11) the whole variational term is vanishing therefore. Forming the simplest invariants out of g^{ik}, ξ^i and $T_{,k}$ one gets

$$A = T_{,k}\xi^k = \dot{T}/T , \tag{18}$$

$$B = g^{ik} T_{,i} T_{,k} . \tag{19}$$

Then the amount of the spatial gradient $h_i^k T_{,k}$ is given by

$$h^{ik} T_{,i} T_{,k} = B + T^2 A^2/c^2 \tag{19'}$$

and f has the shape

$$f = F(\rho, T, A, B). \tag{20}$$

Since $\xi^i = u^i/T$, T is expressed through the variables ξ^i and g_{ik}

$$T = \frac{c}{\sqrt{-g_{rs}\xi^r\xi^s}} . \tag{21}$$

3.2. Variational principle and field equations

With the variables (g_{ik}, ρ, ξ^i) the equations (3) and (5) take the form

$$\delta \int d^4x \sqrt{-g}\left\{ -\frac{R}{2\varkappa_0} - \rho F(\rho, T, A, B)\right\} \geqslant 0 , \tag{22}$$

$$\frac{1}{\sqrt{-g}}\left(\frac{\delta\sqrt{-g}L}{\delta g_{ik}} \pounds_\xi g_{ik} + \frac{\delta\sqrt{-g}L}{\delta\rho}\pounds_\xi \rho \right) = \sigma \geqslant 0 \tag{23}$$

with the Lie derivations

$$\begin{aligned} \pounds_\xi g_{ik} = \left(\frac{u_i}{T}\right)_{;k} + \left(\frac{u_k}{T}\right)_{;i} &= h_i^r h_k^s \frac{u_{r;s} + u_{s;r}}{T} \\ &- \frac{1}{T^2}(h_i^r u_k + h_k^r u_i)\left(T_{,r} + \frac{\dot{u}_r}{c^2} T\right) \\ &+ \frac{2u_i u_k}{c^2} \cdot \frac{\dot{T}}{T} , \end{aligned} \tag{24}$$

$$\begin{aligned} \pounds_\xi \rho = \rho_{,k}\frac{u^k}{T} &= \frac{1}{T}(\rho u^k)_{;k} - \frac{\rho}{T} u^k_{;k} \\ &= \frac{1}{T}(\rho u^k)_{;k} - \frac{1}{2}\rho h^{ik}\pounds_\xi g_{ik} . \end{aligned} \tag{25}$$

Because of the particle number conservation

$$(\rho u^i)_{;i} = 0 \tag{26}$$

$\pounds_\xi \rho$ is expressed in terms of $\pounds_\xi g_{ik}$ and (23) reads

$$\frac{1}{\sqrt{-g}}\left(\frac{\delta\sqrt{-g}L}{\delta g_{ik}} - \frac{1}{2}\rho h^{ik}\frac{\delta\sqrt{-g}L}{\delta\rho}\right) \cdot \pounds_\xi g_{ik} \geqslant 0 \tag{27}$$

with the field equations

$$\frac{1}{\sqrt{-g}}\left(\frac{\delta\sqrt{-g}L}{\delta g_{ik}} - \frac{1}{2}\rho h^{ik}\frac{\delta\sqrt{-g}L}{\delta\rho}\right) = L^{ikmn}\pounds_\xi g_{mn} . \tag{28}$$

A longer straightforward calculation provides

$$\frac{R^{ik}-\frac{1}{2}g^{ik}R}{\varkappa_0}+\rho\Big\{e+T\Big(\frac{1}{T}\frac{\partial F}{\partial A}\Big)^{\cdot}+$$

$$2\frac{T}{\rho}\Big(\rho\frac{\partial F}{\partial B}T^{|l}\Big)_{;l}\Big\}\frac{u^i u^k}{c^2}+p\,h^{ik}+2\rho\frac{\partial F}{\partial B}T^{|i}T^{|k}$$

$$=2\eta_I\,h^{ir}h^{ks}u_{(r;s)}+\eta_{II}\,h^{ik}u^r{}_{;r}$$

$$+\frac{\varkappa}{c^2}\big(h^{ir}u^k+h^{kr}u^i\big)\Big(T_{|r}+\frac{\dot u_r}{c^2}T\Big) \qquad (29)$$

The left-hand side follows from (28) by variational derivation; the right-hand side, by the Onsager relations using (14), (15) and (16).

$$e = F - T\,\partial F/\partial T\,, \qquad (30)$$

$$p = \rho^2\,\partial F/\partial\rho\,. \qquad (31)$$

For our problem it is not necessary to define new expressions for e and p.

<u>Remark</u>: Using, besides ρ, ξ^i and g_{ik}, T as an additional variable with the subsidiary condition (21) in the form $T^2 g_{ik}\xi^i\xi^k + c^2 = 0$, one can simplify the calculation. Also (26) $(\sqrt{-g}\,\rho T\xi^i)_{,i}=0$ can be taken as subsidiary condition with a Lagrangian multiplier.

3.3. Heat conduction equation and characteristics

The heat conduction equation follows from $u_i T^{ik}{}_{;k}=0$. Using (29) one gets

$$\rho\Big[e+T\Big(\frac{1}{T}\frac{\partial F}{\partial A}\Big)^{\cdot}+\frac{2T}{\rho}\Big(\rho\frac{\partial F}{\partial B}T^{|l}\Big)_{;l}\Big]^{\cdot}$$

$$+\,p\,u^k{}_{;k}-u_i\Big(2\rho\frac{\partial F}{\partial B}T^{|i}T^{|k}\Big)_{;k}$$

$$-\,2\eta_I\,h^{ir}h^{ks}u_{(i;k)}u_{(r,s)}-\eta_{II}\,(u^r{}_{;r})$$

$$-\,\varkappa\Big(h^{kl}\big(T_{|l}+\dot u_l T/c^2\big)\Big)_{;k}-\varkappa\,\dot u^r\big(T_{|r}+\dot u_r T/c^2\big)=0\,. \qquad (32)$$

The largest contributions are given by $\rho\,(\partial e/\partial T)\cdot\dot T\equiv\rho c_v\dot T$ and $-\varkappa\,h^{kl}T_{|k;l}$, the terms of the usual heat equation. The propagation velocity of the first signal of any event, however, is given by the characteristics. To form the characteristics, one needs only the terms of the highest order. With (18) and (19) one gets

$$\frac{1}{\rho}\big(\rho c_v\dot T-\varkappa\,h^{kl}T_{,k,l}+\dots\big)+\dots$$

$$+\frac{\partial^2 F}{\partial A^2}\cdot\frac{\dddot T}{T}+4\frac{\partial^2 F}{\partial A\,\partial B}\,g^{rs}T_{|s}\ddot T_{|r}$$

$$+\,2T\cdot\frac{\partial F}{\partial B}\cdot g^{kl}\dot T_{,k,l}+4T\frac{\partial^2 F}{\partial B^2}T^{|k}T^{|l}\dot T_{,k,l}$$

$$+\dots=0\,. \qquad (33)$$

From (33) the equations for the characteristics follow

$$\Big(\frac{1}{T}\frac{\partial^2 F}{\partial A^2}\cdot\dot\varphi^2+4\frac{\partial^2 F}{\partial A\,\partial B}\,g^{rs}T_{|s}\varphi_{|r}\dot\varphi$$

$$+\,2T\frac{\partial F}{\partial B}\,g^{kl}\varphi_{|k}\varphi_{|l}+4T\frac{\partial^2 F}{\partial B^2}T^{|k}T^{|l}\varphi_{|k}\varphi_{|l}\Big)\cdot\dot\varphi=0\,. \qquad (34)$$

The second factor

$$\dot\varphi\equiv u^i\frac{\partial\varphi}{\partial x^i}=u^4\Big\{\frac{\partial\varphi}{\partial x^4}+\frac{u^\alpha}{u^4}(x^\nu,x^4)\frac{\partial\varphi}{\partial x^\alpha}\Big\}=0 \qquad (35)$$

defines a characteristic $\varphi_{(1)}(x^k) = 0$. The velocities $u^i(x^k)$ are assumed to be limited and continuous differentiable in a certain region. Then by $u_i u^i = -c^2$ at least one u^i is non-zero (e.g. u^4) and a solution of (35) exists (Kamke 1979). For the propagation velocity given by the characteristic $\varphi_{(1)}(x^k) = 0$

$$d\varphi_{(1)} = \frac{\partial \varphi_{(1)}}{\partial x^k} dx^k = 0, \tag{36}$$

through the solution procedure (Kamke (1979), Courant Hilbert (1962)) one receives the same velocity u^i as in (35). Being a time-like vector inside the light cone, u^i can only provide a propagation slower than that of light.

The first factor in (34) provides obviously a quadratic form which is indefinite (hyperbolic) if the coefficients have appropriate values. While the old parabolic heat conduction equation was in contradiction with the principle of relativity, it can here be considered to be valid, providing only some restrictions on the coefficients.

If only a dependence of F on A exists, one gets $\dot{\varphi} = 0$ as in the case considered first. If all second derivatives of F are sufficiently small as compared to $2T\partial F/\partial B$, the (hyperbolic) term $g^{kl}\varphi_{,k}\varphi_{,l}$ dominates. Its only presence would provide c as the heat propagation velocity.

4. Modifications of the Onsager relations

There was never any doubt that the Onsager relations (10) are linear approximations of a general physical law connecting thermodynamical currents $\mathcal{J}^\Theta$ and forces X_Θ

$$F_\Gamma(\mathcal{J}^\Theta, X_\Omega) = 0, \tag{37}$$

where the F_Γ could also be functionals containing derivatives $X_{\Omega;k}$ as well (see the Appendix).

Following the procedure of the preceding sections the variational principle provides (6) and (7) rsp. (23) and (27). By (6) the second law of thermodynamics is fulfilled. The shape of (37) has to guarantee $\sigma \geqslant 0$, generating a quadratic form in (7). Using an expansion of $\mathcal{J}^\Theta$ with respect to X_Ω and $X_{\Omega;k}$ (instead of (10)) one can discuss several simple ansatzes (providing $\sigma \geqslant 0$):

$$\mathcal{J}^\Theta = L^{\Theta\Omega} X_\Omega + \overset{*}{L}{}^{\Theta\Omega}(T; T_{,p}) X_\Omega , \tag{38a}$$

$$\mathcal{J}^\Theta = L^{\Theta\Omega} X_\Omega + \hat{L}^{\Theta\Omega}(T) X_\Omega \cdot \exp\{\gamma^{\Delta l} X_{\Delta;l}\} =$$

$$= L^{\Theta\Omega} X_\Omega + \hat{L}^{\Theta\Omega}(T) X_\Omega \cdot \{1 + \gamma^{\Delta l} X_{\Delta;l} + \dots\}, \tag{38b}$$

$$\mathcal{J}^\Theta = L^{\Theta\Omega} X_\Omega + \hat{L}^{\Theta\Omega(\Gamma p)(\Delta q)} X_\Omega (X_\Gamma)_{;p} (X_\Delta)_{;q} . \tag{38c}$$

The coefficients $\gamma^{\Delta l}$, $L^{*\Theta\Omega}$, $\hat{L}^{\Theta\Omega}_{\;\;\;\;\;;\cdot}$ have to be very small. The forms of the latter are positive definite (at least together with $L^{\Theta\Omega}X_\Theta X_\Omega$). (38a) leads to a second order equation. By the factor X_Ω $(= \pounds_\xi g_{ik})$ the signs in the quadratic form, however, depend on the variables and its derivatives (e.g. on $\dot{T}$, $\Theta = u^i_{;i}$). For special processes (special initial values) one can generate elliptic forms. Thus one cannot use (38a). (38b) and (38c) provide similar third order equations. We consider (38b) more in detail. Here (11) rsp. (28) takes the form

$$\frac{R^{ik} - \frac{1}{2} g^{ik} R}{\varkappa_0} + \frac{\varrho e}{c^2} u^i u^k + p h^{ik} =$$

$$= L^{ik\cdot mn} \mathcal{L}_\xi g_{mn} + \hat{L}^{ik\cdot mn}(\mathcal{L}_\xi g_{mn}) \cdot \exp\{\gamma^{rs\cdot \ell}(\mathcal{L}_\xi g_{rs})_{;\ell}\} \tag{39}$$

For the usual coefficients $L^{ik\cdot mn} \equiv - T^{ik}_{(irr)}$. and the ansatz

$$\gamma^{rs\,\ell} = c_1 h^{rs} u^\ell + c_2 (h^{r\ell} u^s + h^{s\ell} u^r) - c_3 u^r u^s u^\ell \tag{40}$$

formed covariantly out of the variables u^i and h^{pq}, one gets with

$$\mathcal{L}_\xi g_{rs} = (u_r/T)_{;s} + (u_s/T)_{;r} = 2\, u_{(r;s)}/T - 2\, u_{(r}T_{,s)}/T^2 : \tag{41}$$

$$\begin{aligned} \gamma^{rs\cdot\ell}(\mathcal{L}_\xi g_{rs})_{;\ell} &= 2 c_2 c^2 T^{-2} h^{rs} T_{,r;s} - 2 c_3 c^2 T^{-2} \ddot{T} + . \\ &= 2 c_2 c^2 T^{-2} [\, \Delta T - (c_3/c_2) \ddot{T} \,] + \dots \\ &\equiv 2 c_2 c^2 T^{-2} \Box_v T + \dots \end{aligned} \tag{42}$$

where only the terms with the highest derivatives are written down. These are also the highest derivatives in the whole equation (39), which provides the heat conduction equation with $u_i T^{ik}{}_{;k} = 0$:

$$(\varrho c_v \dot{T} - \Delta T) + . = \quad +$$

$$- u_i (\hat{L}^{ik\cdot mn} \mathcal{L}_\xi g_{mn}) \exp\{\gamma^{rs\,\ell}(\mathcal{L}_\xi g_{rs})_{;\ell}\} \cdot 2 c_2 c^2 T^{-2} \cdot \Box_v T_{,k} . \tag{43}$$

The factor $u_i \hat{L}^{ik\cdot mn} \mathcal{L}_\xi g_{mn}$ is a vector and can be guessed as a covariant expression formed out of h_{ij}, u^k

$$u_i \hat{L}^{ik\cdot mn} \mathcal{L}_\xi g_{mn} = A \cdot u^k \tag{44}$$

provided by $\hat{L}^{ik\,mn} = a\, u^i u^k u^m u^n$. This leads to

$$(\varrho c_v \dot{T} - h^{rs} T_{,r;s}) + \dots = \dots - 2 c_2 c^2 (A/T^2) \exp\{\gamma^{rs\cdot\ell}(\mathcal{L}_\xi g_{rs})_{;\ell}\} \Box_v \dot{T} \tag{45}$$

with the same structure of characteristics as in (34).

Appendix

In a small region of the medium the thermodynamical variables V_Θ (e.g. the temperature) will be nearly constant but in fact they are space-time functions. One can define a mean value with a certain distribution function $K(x,x')$ being relevant only for small distances from the considered point

$$\overline{V}_\Theta(x) = \int K(x,x')\, V_\Theta(x+x')\, d^4x', \tag{A.1}$$

$$\int K(x,x')\, d^4x' = 1. \tag{A.2}$$

Such expressions could also be provided by statistics. Developing V_Θ in the neighbourhood of x one gets by a well-known technic

$$\begin{aligned} \overline{V}_\Theta(x) &\doteq V_\Theta(x) + \int K(x,x') \cdot x'^k d^4x' \cdot V_{\Theta,k}(x) \\ &\quad + \tfrac{1}{2} \int K(x,x') x'^k x'^\ell d^4x' \cdot V_{\Theta,k,\ell}(x) + \dots \end{aligned} \tag{A.3}$$

$$\overline{V}_\Theta(x) = V_\Theta(x) + c^k(x) \cdot V_{\Theta,k}(x) + c^{k\ell}(x) V_{\Theta,k,\ell}(x) + \dots \tag{A.4}$$

with (small) coefficients $c^k, c^{k\ell} \dots$. If the free energy is formed with this mean value one gets an expression containing the gradient of T in the first approximation

$$F(\overline{T}; \varrho) = F(T + c^k(x) T_{,k} + \cdot\, ; \varrho). \tag{A.5}$$

Another possibility would be to consider F as a functional formed in the spirit of (A.1). Then similar to (A.4) one would receive

$$\bar{F} = F(T;\varrho) + c^{\ell}(x)\, F(T;\varrho)_{,\ell} + \dots$$
$$= F(T,\varrho) + c^{\ell}(x)\,\frac{\partial F}{\partial T}\cdot T_{,\ell} + \tag{A.6}$$

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Залие Н. E2-87-886
Вариационный принцип Нойгебауера и проблема причинного уравнения теплопроводимости

Показано, что вариационный принцип Нойгебауера не ведет с необходимостью к параболическому уравнению теплопроводимости; при небольших изменениях свободной энергии или феноменологических уравнений Онсагера получается причинная структура.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Salié N. E2-87-886
The Variational Principle of Neugebauer and the Problem of a Causal Heat Conduction Equation

It is shown that Neugebauer's variational principle does not necessary lead to a parabolic heat conduction equation. By slight modifications either of the free energy or of the Onsager relations a causal structure is produced.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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