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N=2 SUPERGRAVITY IN SUPERSPACE: DIFFERENT VERSIONS AND MATTER COUPLINGS

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1. Introduction

This paper is a continuation of /1/. There we introduced the framework for the formulation of N=2 SG in harmonic superspace and worked out the example of one of the Einstein versions of the theory. Here we generalize this approach to incorporate conformal SG. Then we study the possibility to compensate the super conformal transformations by coupling conformal SG to a Maxwell and various matter multiplets. In this way we reproduce all previously known versions of N=2 Einstein SG and find a new one. The latter involves an off-shell complex hypermultiplet with its infinite set of auxiliary field. This new version proves to be the only one which does not impose any restrictions on the possible couplings to matter.

Throughout this paper we frequently refer to various results from /1/. The equations in /1/ are numbered by Roman and Arabic numerals (e.g. (II.13)), while here we use only Arabic numerals (e.g. (2.13)).

2. Gauge group and prepotentials of conformal SG

We begin with a brief summary of the realization of the rigid superconformal group $SU(2,2|2)$ in N=2 harmonic superspace /2/. The most significant point is that $SU(2,2|2)$ preserves the structure of the analytic subspace ($x^m_A, \theta^{\hat{\mu}+}_A, u^{\pm}_i$). In particular, the harmonic coordinates have the following peculiar transformation laws

$$\begin{aligned} \delta u^+_i &= \Lambda^{++} u^-_i \\ \delta u^-_i &= 0, \end{aligned} \qquad (2.1)$$

where

$$\Lambda^{++} = u^+_i u^+_j \left(\lambda^{ij} + i k_{\alpha\dot\alpha} \theta^{\alpha i} \cdot \bar\theta^{\dot\alpha j} + i \theta^{\alpha i} \eta^j_\alpha + i \bar\eta^i_{\dot\alpha} \bar\theta^{\dot\alpha j} \right) \qquad (2.2)$$

is a superparameter containing the parameters λ^{ij} of $SU(2)$, $k_{\alpha\dot\alpha}$ of conformal boosts and η^i_α of conformal supersymmetry. Clearly,

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only $\theta^+ = \theta^i u^+_i$ occurs in (2.2), so the transformations (2.1) do not break analyticity (the same is true for δx^m_A and $\delta\theta^{\hat\mu+}_A$ /2/). The transformations (2.1) preserve the defining condition $u^{+i}u^-_i = 1$ for the harmonics but not the complex conjugation relation $\overline{(u^{+i})} = u^-_i$. However, the natural conjugation for the analytic superspace $(x^m_A, \theta^{\hat\mu+}_A, u^\pm_i)$ is the operation $\sim$ ($*$ in /3/), and we see that (2.1) preserves reality of analytic superspace under $\sim$ conjugation ($\widetilde{\Lambda^{++}} = \Lambda^{++}$).

A straightforward implication for (2.1) are the transformation laws for the harmonic derivatives D^{++} , D^{--} , D^{0} (the property $D^{++}\Lambda^{++} = 0$ is used):

$$\delta D^{++} = -\Lambda^{++} D^0 \ , \quad \delta D^{--} = -\left(D^{--}\Lambda^{++}\right) D^{--} ,$$
$$\delta D^0 = 0 . \tag{2.3}$$

The above formulae suggest the following generalization to curved harmonic superspace. We begin with the central basis $\{ Z^M = (x^m, \theta^{\hat\mu i}) \ , \ u^\pm_i \}$. There the suitable general coordinate transformation group is

$$\begin{aligned} \delta x^m &= \tau^m(z) \\ \delta\theta^{\hat\mu i} &= \tau^{\hat\mu i}(z) \\ \delta u^+_i &= \left(u^+_k u^+_\ell \tau^{k\ell}(z)\right) u^-_i \\ \delta u^-_i &= 0 . \end{aligned} \tag{2.4}$$

It is chosen so that the characteristic feature of the central basis, namely the flatness of the harmonic derivatives D^{++}, D^{--}, D^0 is preserved. The transformation laws (2.3) remain unchanged (with $\Lambda^{++} = u^+_k u^+_\ell \tau^{k\ell}(z)$).

The most important feature of N=2 supersymmetry is the existence of analytic superfields depending on $\theta^{\hat\mu+}$ but not on $\theta^{\hat\mu-}$. Therefore we have to define another basis in which a new group acts leaving invariant the analytic subspace ($x^m_A, \theta^{\hat\mu+}_A, u^\pm_i$):

$$AB = \left\{ \zeta^M_A = (x^m_A, \theta^{\hat\mu+}_A) \ , \ u^\pm_{Ai} \ ; \ \theta^{\hat\mu-}_A \right\} \tag{2.5}$$

$$\begin{aligned} \delta x^m_A &= \lambda^m(\zeta_A, u_A) \\ \delta\theta^{\hat\mu+}_A &= \lambda^{\hat\mu+}(\zeta_A, u_A) \\ \delta u^{+i}_A &= \lambda^{++}(\zeta_A, u_A)\cdot u^{-i}_A \ , \quad \delta u^{-i}_A = 0 \\ \delta\theta^{\hat\mu-}_A &= \lambda^{\hat\mu-}(\zeta_A, u_A, \theta_A) . \end{aligned} \tag{2.6}$$

Here $\lambda^m, \lambda^{\hat\mu+}, \lambda^{++}$ are general analytic superfunctions, and $\lambda^{\hat\mu-}$ is a general non-analytic one.

The least obvious part of (2.6) is the choice of δu^+ and δu^-. To a certain extent they resemble the rigid group (2.1): they preserve analyticity, the relation $u^{+i}u^-_i = 1$, $\sim$ reality ($\widetilde{\lambda^{++}} = \lambda^{++}$). However, the rigid case property $D^{++}\Lambda^{++} = 0$ does not hold any more. This, as we shall see shortly, gives rise to a new specific prepotential for conformal supergravity.

The change from central to analytic basis is made with the help of bridges:

$$x^m_A = x^m + v^m(z,u) \ , \quad \theta^{\hat\mu\pm}_A = \theta^{\hat\mu i}u^\pm_i + v^{\hat\mu\pm}(z,u)$$
$$u^{+i}_A = u^{+i} + v^{++}(z,u)\cdot u^{-i} \ , \quad u^{-i}_A = u^{-i} . \tag{2.7}$$

Their transformation laws follow from (2.4) and (2.6)

$$\begin{aligned} \delta v^m &= \lambda^m - \tau^m \ , \quad \delta v^{++} = \lambda^{++} - u^+_i u^+_j \tau^{ij} \\ \delta v^{\hat\mu+} &= \lambda^{\hat\mu+} - \tau^{\hat\mu i}u^+_i - \tau^{k\ell}\theta^{\hat\mu i}u^+_k u^+_\ell u^-_i \\ \delta v^{\hat\mu-} &= \lambda^{\hat\mu-} - \tau^{\hat\mu i}u^-_i . \end{aligned} \tag{2.8}$$

The reader might have noticed that the framework for conformal SG developed so far is rather similar to that for Einstein SG /1/. The differences are the absence of x^5 and its bridge on the one hand, and the presence of local transformations of u^{+i} and of the corresponding bridge v^{++}.

Let us turn out attention to the harmonic covariant derivative $\mathcal{D}^{++}$. In the τ basis it is simply $\mathcal{D}^{++} = \partial^{++}$ and correspondingly transforms as in the rigid case:

$$\delta \mathcal{D}^{++}_{\tau} = -\tau^{++}\mathcal{D}^{0} \tag{2.9}$$

($\mathcal{D}^{0}$ is also flat). Going to the λ basis we not only change the coordinates according to (2.7), we also redefine $\mathcal{D}^{++}$ so that it transforms with λ^{++} rather than τ^{++} :

$$\mathcal{D}^{++} = \mathcal{D}^{++}_{\tau} - v^{++}\mathcal{D}^{0} \Rightarrow$$

$$\Rightarrow \delta\mathcal{D}^{++} = -\lambda^{++}\mathcal{D}^{0}. \tag{2.10}$$

Writing $\mathcal{D}^{++}$ out in detail one finds a number of harmonic vielbeins:

$$\mathcal{D}^{++} = \partial^{++}_{A} + H^{(+4)}\partial^{--}_{A} + H^{++m}\partial^{A}_{m} + H^{++\hat{\mu}\pm}\partial^{A\mp}_{\hat{\mu}}, \tag{2.11}$$

where

$$H^{(+4)} = \mathcal{D}^{++}_{\tau}v^{++} - (v^{++})^{2}$$

$$H^{++m} = \mathcal{D}^{++}_{\tau}v^{m}$$

$$H^{++\hat{\mu}+} = \mathcal{D}^{++}_{\tau}v^{\hat{\mu}+} - v^{++}\theta^{\hat{\mu}+}_{A}$$

$$H^{++\hat{\mu}-} = \mathcal{D}^{++}_{\tau}v^{\hat{\mu}-} - v^{++}v^{\hat{\mu}-} + \theta^{\hat{\mu}i}u^{+}_{Ai}. \tag{2.12}$$

Note that $\mathcal{D}^{0}$ in the λ basis does not differ from its rigid form:

$$\mathcal{D}^{0} = u^{+i}_{A}\frac{\partial}{\partial u^{+i}_{A}} - u^{-i}_{A}\frac{\partial}{\partial u^{-i}_{A}} + \theta^{\hat{\mu}+}_{A}\cdot\frac{\partial}{\partial\theta^{\hat{\mu}+}_{A}} - \theta^{\hat{\mu}-}_{A}\cdot\frac{\partial}{\partial\theta^{\hat{\mu}-}_{A}}. \tag{2.13}$$

Its function is, as always, to count the $U(1)$ charge (recall that all our objects are by definition eigenfunctions of $\mathcal{D}^{0}$).

The rigid operator D^{++} has the crucial property of preserving the analyticity of the superfields it acts upon. This allows to write down action formulae for such analytic objects as the q^{+} - hypermultiplet [3,4]. In the curved case the concept of an analytic superfield satisfying the constraint

$$\frac{\partial}{\partial\theta^{\hat{\alpha}-}_{A}}\phi = 0 \Rightarrow \phi = \phi\,(x^{m}_{A}, \theta^{\hat{\mu}+}_{A}, u^{\pm}_{A}) \tag{2.14}$$

is covariant (in the analytic basis). If we wish that $\mathcal{D}^{++}\phi$ remains analytic, we have to demand

$$\partial^{+}_{A\hat{\alpha}}H^{(+4)} = \partial^{+}_{A\hat{\alpha}}H^{++m} = \partial^{+}_{A\hat{\alpha}}H^{++\hat{\mu}+} = 0 \tag{2.15}$$

($H^{++\hat{\mu}-}$ is allowed to be general since it is accompanied by $\partial^{+}_{A\hat{\alpha}}$ in (2.11)). Note that the redefinition (2.10) was also made for consistency with analyticity. We also see that the transformation laws for the vielbeins H^{++} following from (2.10), (2.12) (2.18) are in agreement with the analyticity requirement (2.15):

$$\delta H^{(+4)} = \mathcal{D}^{++}\lambda^{++}$$

$$\delta H^{++m} = \mathcal{D}^{++}\lambda^{m}$$

$$\delta H^{++\hat{\mu}\pm} = \mathcal{D}^{++}\lambda^{\hat{\mu}\pm} \mp \theta^{\hat{\mu}\pm}_{A}\lambda^{++}. \tag{2.16}$$

Thus we have reached the central point in our construction. We postulate that the vielbeins $H^{(+4)}$, H^{++m} , $H^{++\hat{\mu}+}$ and the group (2.6), (2.16) are the unconstrained prepotentials and the gauge group of N=2 conformal SG. This claim is justified by the Wess-Zumino gauge [2]:

$$H^{++m}(\zeta_{A}, u_{A}) = i\theta^{+}\sigma^{a}\bar{\theta}^{+}e^{m}_{a}(x_{A}) + (\bar{\theta}^{+})^{2}\theta^{\mu+}\psi^{m}_{\mu i}(x_{A})\,u^{-i} +$$

$$+ (\theta^{+})^{2}\bar{\theta}^{+}_{\dot{\mu}}\bar{\psi}^{m\dot{\mu}}_{i}(x_{A})\,u^{-i} + (\theta^{+})^{2}(\bar{\theta}^{+})^{2}V^{m}_{ij}(x_{A})\,u^{-i}u^{-j}$$

$$H^{++\mu+}(\zeta_A, u_A) = (\theta^+)^2 \bar{\theta}^+_{\dot\mu} (A^{\mu\dot\mu} + i B^{\mu\dot\mu}) + + (\bar\theta^+)^2 \theta^{\nu+} t_{(\nu}{}^{\mu)} + (\theta^+)^2 (\bar\theta^+)^2 \chi^\mu_i u^i_A ,$$

$$\overline{H}^{++\dot\mu+} = \widetilde{(H^{++\mu+})}, \quad H^{(+4)} = 0, \quad H^{++\hat\mu-} = \theta^{\hat\mu+}_A \tag{2.17}$$

Here one finds the components of the N=2 Weyl multiplet [5]: the graviton e^m_a and gravitino $\psi^m_{\hat\mu i}$, the U(2) gauge fields $V^m_{(ij)}$ and $A^{\mu\dot\mu}$, and the auxiliary fields $t_{(\mu\nu)}, \chi^\mu_i, \mathcal{D} = \partial_m B^m$ (the field B^m undergoes gauge transformations with a divergenceless parameter).

One can see from (2.17) that the prepotentials $H^{(+4)}$ and $H^{++\hat\mu-}$ are pure gauges. In what follows we shall use the gauge

$$H^{(+4)} = 0, \quad H^{++\hat\mu-} = \theta^{\hat\mu+}_A \tag{2.18}$$

It imposes restrictions on the parameters λ^{++} and $\lambda^{\hat\mu-}$

$$\mathcal{D}^{++}\lambda^{++} = 0, \quad \mathcal{D}^{++}\lambda^{\hat\mu-} = \lambda^{\hat\mu+} - \theta^{\hat\mu-}_A \cdot \lambda^{++} \tag{2.19}$$

which make them (and the gauge group structure constants) field-dependent. However, we will gain significant simplifications of the forthcoming expressions.

Passing to the gauges (2.18),(2.17) involves fixing the divergence of a real vector gauge parameter $\ell^m(x)$ in $\lambda^{++}(\zeta_A, u_A) = = \ldots + \theta^+ \sigma^a \bar\theta^+ \ell_a(x) + \ldots$ ($\partial^m \ell_m(x)$ is used to remove the SU(2)-singlet component $\mathcal{D}(x)$ entering $H^{(+4)}$ as a coefficient of $(\theta^+)^2(\bar\theta^+)^2$) The divergenceless part of $\ell_m(x)$ remains unconstrained. Suprisingly, there exists another Wess-Zumino gauge [2] where the ℓ_m - freedom is entirely fixed to gauge away the vector component $B^{\mu\dot\mu}$ in $H^{++\mu+}(\zeta_A, u_A)$, while leaving a non-zero piece in $H^{(+4)}$:

$$H^{++\mu+} = (\theta^+)^2 \bar\theta^+_{\dot\mu} A^{\mu\dot\mu}(x_A) + \ldots, \quad H^{(+4)} = (\theta^+)^2(\bar\theta^+)^2 \mathcal{D}(x_A) \tag{2.20}$$

Note that the gauges (2.17),(2.18) are ill defined globally ($\mathcal{D}(x)$ and its gauge parameter $\partial^m \ell_m(x)$ may have different asymptotic behaviour). No such a defect is inherent to the gauge (2.20) though the latter is less conveneint as regards some technical points.

In principle, having introduced the prepotentials one could go on and develop the full differential geometry formalism for N=2 conformal SG. This includes vielbeins and connection for $\mathcal{D}^{--}$, the spinot covariant derivatives $\mathcal{D}^\pm_\alpha$ etc. Our prime interest in this paper is in studying the coupling of conformal SG to N=2 Maxwell and matter multiplets. For this purpose we shall need only two new objects: $\mathcal{D}^{--}$ and a density for the full harmonic superspace integral.

3. Conformal properties of the building blocks and supervolume density

In [6,1] it is shown that the vielbeins of the covariant derivative $\mathcal{D}^{--}$ can be expressed in terms of the prepotentials H^{++} and can be subsequently used for constructing the superspace integral density and for the Maxwell action. That procedure can be repeated in the conformal case with minor modifications.

In the central basis (2.3) $\mathcal{D}^{--}$ is simply ∂^{--} and transforms as follows

$$\delta \mathcal{D}^{--}_\tau = -(\mathcal{D}^{--}_\tau \tau^{++}) \mathcal{D}^{--}_\tau . \tag{3.1}$$

To make $\mathcal{D}^{--}$ fit in our analytic frame with parameters λ (2.6), we redefine it with the help of the bridge v^{++} (2.7):

$$\mathcal{D}^{--} = \frac{1}{1 + \mathcal{D}^{--}_\tau v^{++}} \cdot \mathcal{D}^{--}_\tau . \tag{3.2}$$

The new transformation law is

$$\delta \mathcal{D}^{--} = -(\mathcal{D}^{--}\lambda^{++}) \cdot \mathcal{D}^{--} . \tag{3.3}$$

Writing out $\mathcal{D}^{--}$ in the analytic basis we define the vielbeins H^{--}:

$$\mathcal{D}^{--} = \partial^{--}_A + H^{--m} \partial^A_m + H^{--\hat\mu\pm} \partial^{\mp A}_{\hat\mu} . \tag{3.4}$$

They can be expressed in terms of the bridges (2.7) but we shall not need this. Instead, we can relate them to the prepotentials H^{++} by imposing the conventional constraint

$$[\mathcal{D}^{++}, \mathcal{D}^{--}] = \mathcal{D}^0 . \tag{3.5}$$

It is easy to check the gauge invariance of (3.5) (see (2.10),(3.3) and the gauge condition (2.19)):

$$\delta\,[\mathcal{D}^{++},\mathcal{D}^{--}] = [-\lambda^{++}\mathcal{D}^{0},\mathcal{D}^{--}] + [\mathcal{D}^{++}, -(\mathcal{D}^{--}\lambda^{++})\mathcal{D}^{--}] =$$
$$= 2\lambda^{++}\mathcal{D}^{--} - (\mathcal{D}^{++}\mathcal{D}^{--}\lambda^{++})\,\mathcal{D}^{--} = 0\,.$$

Plugging the expressions (2.11) for $\mathcal{D}^{++}$ (in the gauge (2.18)) and (3.4) for $\mathcal{D}^{--}$ into (3.5) one obtains a set of linear differential equations for H^{--} which exactly coincides with the analogous one (III.3) discussed in /1/. Therefore we refer to /1/ for details of the solution. Note that the gauge choice (2.18) greatly simplifies the equation (3.5). With the λ^{++} parameter unconstrained we would have to use H^{+4} to covariantize (3.5):

$$[\mathcal{D}^{++} - H^{(+4)}\mathcal{D}^{--},\, \mathcal{D}^{--}] = \mathcal{D}^{0}\,. \tag{3.6}$$

This equation is quadratic in H^{--}, and cannot be solved as easily as (3.5).

From (3.4), (3.3) and (2.6) one derives the transformation laws for H^{--M}

$$\delta H^{--M} = -(\mathcal{D}^{--}\lambda^{++})\,H^{--M} + \mathcal{D}^{--}\lambda^{M}\,. \tag{3.7}$$

The only new term in (3.7) compared with (II.2) of /1/ is the weight transformation with parameter $\mathcal{D}^{--}\lambda^{++}$. In /1/ we constructed building blocks $e^{m}_{\hat\alpha\hat\beta}$ (IV.5) and $e_{\hat\mu}^{\hat\nu}$ (IV.6) from H^{--} and later used them to find the supervolume density E (IV.15). Here we shall show that precisely the same expression for E serves as a density under the conformal gauge group including the new parameters λ^{++}. The second term in (3.7) leads to the same coordinate transformations of the building blocks and E as in /1/.

The $\mathcal{D}^{--}\lambda^{++}$ term in (3.7) yields the following new transformations of $e^{m}_{\hat\alpha\hat\beta}$ and $e_{\hat\mu}^{\hat\nu}$

$$\delta_{\lambda^{++}} e^{m}_{\hat\alpha\hat\beta} = -(\mathcal{D}^{--}\lambda^{++})\,e^{m}_{\hat\alpha\hat\beta} - e^{n}_{\hat\alpha\hat\beta}\left[\partial_n\lambda^{++}\left(H^{--m} - H^{--\hat\mu+}\,e^{-1}{}_{\hat\mu}{}^{\hat\nu}\,\partial^{+}_{\hat\nu} H^{--m}\right)\right],$$

$$\delta_{\lambda^{++}} e_{\hat\mu}{}^{\hat\nu} = -(\mathcal{D}^{--}\lambda^{++})\,e_{\hat\mu}{}^{\hat\nu} - \left(\partial^{+}_{\hat\mu} H^{--m}\,\partial_m\lambda^{++} + e_{\hat\mu}{}^{\hat\varsigma}\,\partial^{-}_{\hat\varsigma}\lambda^{++}\right) H^{--\hat\nu+}\,. \tag{3.8}$$

To check this one makes use of (3.4) and the analyticity of λ^{++} ($\partial^{+}_{\hat\alpha}\lambda^{++} = 0$). The transformation of $e^{m}_{\hat\alpha\hat\beta}$ looks like a world vector rotation of the index m, so the quantity $f^{\alpha\dot\alpha} = e^{m} e_{m}^{\alpha\dot\alpha}$ (IV.10) remains invariant. Taking all this into account one obtains

$$\delta_{\lambda^{++}} E = \delta_{\lambda^{++}}\left[\det(e^{m}_{\alpha\dot\alpha})\cdot \det{}^{-1}(e_{\hat\mu}{}^{\hat\nu})\cdot\sqrt{(1-f\bar f)^2 - f^2\bar f^2}\,\right] =$$
$$= (\partial^{--}_{A}\lambda^{++} - \mathcal{D}^{--}\lambda^{++})\,.\,E\,. \tag{3.9}$$

On the other hand, the coordinate transformations (2.6) of u^{+i}_{A} contribute to the volume element transformation the following term

$$\frac{\partial}{\partial u^{+i}_{A}}\,\delta u^{+i}_{A} = \partial^{--}_{A}\lambda^{++}. \tag{3.10}$$

Consequently, the conformally covariant volume element of superspace can be formed with the help of E^{-1} :

$$\delta(d^{12}z_A\, du_A\, E^{-1}) = (\mathcal{D}^{--}\lambda^{++})\cdot(d^{12}z_A\, du_A\, E^{-1})\,. \tag{3.11}$$

We conclude that the integrand in an invariant integral must have weight -1 under the λ^{++} transformations. In the next section we apply these results to the construction of the N=2 Maxwell action in a superconformal background.

4. Coupling of conformal SG to Maxwell and non-linear multiplets

Our main aim in this paper is to construct actions for different off-shell versions of N=2 Einstein SG. Following /5/ we started with the conformal SG multiplet with its full gauge group. Next we have to compensate the Weyl, γ^5 and $SU(2)$ gauge transformations by coupling the conformal supergravity multiplet to a Maxwell multiplet (for Weyl and γ^5) and to various matter multiplets (for $SU(2)$). In superspace there is a natural way to describe the Maxwell multiplet. It consists in introducing an extra space-time coordinate x^5_A . The gauge group is also extended by the transformations of x^5_A ,

$$\delta x^5_A = \lambda^5(z_A, u) \tag{4.1}$$

which we choose so that they preserve the analytic structure of harmonic superspace. Correspondingly, the harmonic derivatives $\mathcal{D}^{++}$ (2.11) and $\mathcal{D}^{--}$ (3.4) acquire new vielbeins:

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + H^{++5}\partial_5 , \quad \mathcal{D}^{--} \to \mathcal{D}^{--} + H^{--5}\partial_5 . \tag{4.2}$$

Note that neither the gauge parameter λ^5 nor the vielbeins H^{++5}, H^{--5} depend on the new coordinate x^5_A . It is only matter superfields with central charge which are allowed to depend on x^5_A ; the SG multiplet itself has no central charge. In order that $\mathcal{D}^{++}$ preserves analyticity, the vielbein H^{++5} must be analytic (see (2.15)):

$$\partial^+_{A\hat{\alpha}} H^{++5} = 0 . \tag{4.3}$$

The transformation law for $\mathcal{D}^{++}$ (2.10) does not change, so H^{++5} transforms as follows:

$$\delta H^{++5} = \mathcal{D}^{++}\lambda^5 . \tag{4.4}$$

The analytic superfield H^{++5} with the transformation law (4.4) is indeed the prepotential for an N=2 Maxwell multiplet[3]. The difference from an ordinary Maxwell superfield is that we postulate that H^{++5} has a non-vanishing flat limit

$$H^{++5} = i(\theta^+)^2 - i(\bar{\theta}^+)^2 . \tag{4.5}$$

This makes it a compensator for the Weyl and γ^5 gauge transformations in the group (2.6).

The vielbein H^{--5} is obtained as the solution of the ∂_5 torsion constraint following from (3.5) and (4.2). The explicit expression was given in [1] eq. (III.8). With the help of H^{--5} one can construct the quantity F (IV.12) which transforms as shown in (II.34):

$$\delta(\ln F) = \frac{1}{2}\Delta^+_\alpha \lambda^{\alpha -} .$$

This is precisely the compensator for the Weyl and γ^5 gauge parameters contained in $\lambda^{\alpha -}$.

At this point we have a framework which is very similar to the one described in[1]. The only difference is that now the gauge group is bigger, it includes the λ^{++} parameters (2.6), (2.19) (containing local $SU(2)$). Therefore the same set of prepotentials describes a smaller set of fields, the so-called "minimal representation" of N=2 SG[5]. It is well known that one cannot write down a correct action for this multiplet, although there exists an action-like invariant integral. It is the action for the Maxwell superfield H^{++5} in a conformal SG background:

$$S_{Maxw} = \frac{1}{\kappa^2}\int d^{12}z_A\, du_A\, E^{-1} H^{++5} H^{--5} . \tag{4.6}$$

Its form coincides with the action (1) for the off-shell version of N=2 Einstein SG considered in [1] but it has a larger gauge symmetry. Its invariance under the $\lambda^{m,5}$, $\lambda^{\hat{\mu}\pm}$ transformations has already been proved in[1]. Here we have to check if it is invariant under λ^{++} transformations as well. Indeed, from (3.3),(4.2) and (4.4) we find

$$\delta_{\lambda^{++}} H^{++5} = 0 , \quad \delta_{\lambda^{++}} H^{--5} = -(\mathcal{D}^{--}\lambda^{++})\cdot H^{--5} . \tag{4.7}$$

Comparing this with (3.11) we conclude that the action (4.6) is superconformally invariant.

In order to obtain a full N=2 Einstein SG action from (4.6) one has to compensate the λ^{++} transformations by coupling certain matter multiplets to the conformal SG multiplet. We discuss here how to get the action given in[1]. To this end, note that (4.6) is invariant under superconformal gauge group before imposing any gauge, i.e. with $H^{(+4)} \neq 0$ and λ^{++} still unrestricted. Then a careful inspection of eq. (4.6) shows the presence of a troublesome term in (4.6) linear in the component $\mathcal{D}(x)$ of $H^{(+4)}$, namely $\sim \int d^4x\, \mathcal{D}(x)$. Passing to gauges (2.17),(2.18) involves a gauge transformation $\delta \mathcal{D}(x) = \partial^m b_m(x) + \ldots$. If one assumes as usual that the integrals of full derivatives vanish this transformation cannot be used to remove the above D-term. Thus we conclude that the gauges (2.17),(2.18) are not implementable in eq.(4.6) (while the gauge (2.20) still is). On the other hand, one easily observes that putting $H^{(+4)} = 0$ in eq.(4.6) and in the equations which define H^{--M} in terms of H^{++M}, simultaneously with the restriction of original gauge group to $\lambda^{++} = 0$, yield just the N=2 Einstein SG action considered in /1/. This reduction $H^{(+4)} = 0$, $\lambda^{++} = 0$ can be performed consistently with the whole superconformal group (2.6),(2.16) by means of the trick exploiting the so-called "non-linear" multiplet /5/. In flat harmonic superspace it is described by a real analytic superfield N^{++} satisfying the nonlinear constraint

$$\mathcal{D}^{++} N^{++} + (N^{++})^2 = 0 \tag{4.8}$$

(Cf. the constraint $\mathcal{D}^{++} L^{++} = 0$ for the linear multiplet /7/).
It can easily be put in a conformal SG background by replacing (4.8) by

$$\mathfrak{D}^{++} N^{++} + (N^{++})^2 - H^{(+4)} = 0 \tag{4.9}$$

assuming that N^{++} transforms as follows:

$$\delta N^{++} = \lambda^{++} \tag{4.10}$$

Now we may view eq. (4.9) as a superconformally covariant definition of $H^{(+4)}$ in terms of N^{++}. (This involves a noncanonical redefinition of $\mathcal{D}(x)$ which starts now with the divergence of a vector component of N^{++}). Then we substitute the expression for $H^{(+4)}$ in eq. (4.6) and in the equations for H^{--M} and finally choose the gauge

$$N^{++} = 0 \;\Rightarrow\; \lambda^{++} = 0 \tag{4.11}$$

thereby eliminating the entire gauge freedom in λ^{++} including local SU(2). At the same time $H^{(+4)} = 0$ as a consequence of (4.9), so the action (4.6) involves $H^{m,5}$, $H^{\hat{\mu}}$ and is invariant under the $\lambda^{m,5}$, $\lambda^{\hat{\mu}}$ gauge group only. Thus one recovers the off-shell version of Einstein SG /8/ as it was described in /1/.

Concluding this section we briefly mention that it is possible to use the so-called "linear" multiplet as a compensator /9/. Its formulation in harmonic superspace and coupling to conformal SG have been discussed in /7/ .

5. A new Einstein SG version and general matter couplings

We claim that the most natural and least restrictive (in matter couplings) version of N=2 Einstein SG is obtained when using a q^+ hypermultiplet /3,4/ as a compensator for the λ^{++}($SU(2)$) transformations. The Fayet-Sohnius hypermultiplet is described by an <u>unconstrained</u> analytic superfield q^{+i} forming a pseudoreal $SU(2)$ doublet, $\widetilde{q^{+i}} = \varepsilon_{ij} q^{+j}$. Note that this $SU(2)$ is an external (Pauli-Gursey) group, unrelated to the inner supersymmetry automorphism $SU(2)$. The flat space free action is given by the <u>analytic</u> superspace integral

$$S_{q^+} = -\frac{1}{2k^2} \int d\zeta_A^{(-4)} du\, q^{+i} D^{++} q^+_i \,. \tag{5.1}$$

It can be coupled to conformal SG /7/ by replacing the rigid D^{++} by the covariant one and ascribing the following transformation law to q^{+i} :

$$\delta q^{+i} = -\frac{1}{2} \Lambda q^{+i} \,, \tag{5.2}$$

where Λ is the infinitesimal transformation of the analytic supervolume element,

$$\Lambda = \partial_{A\,m} \lambda^m - \partial^-_{A\,\hat{\mu}} \lambda^{\hat{\mu}+} + \partial^{--}_A \lambda^{++} \,. \tag{5.3}$$

Then the action

$$S^{\text{curved}}_{q^+} = -\frac{1}{2k^2} \int d\zeta_A^{-4} du_A\, q^{+i} \mathfrak{D}^{++} q^+_i \tag{5.4}$$

is invariant, because the term $q^{+i} (\mathfrak{D}^{++} \Lambda) \cdot q^+_i$ vanishes identically.

To see that q^{+i} does compensate the remaining freedom of the λ^{++} transformations, we assign a non-vanishing flat space limit to $u^-_i q^{+i}$ (e.g. $(u^-_i q^{+i})_0 = 1$). Then we find from (2.6), (5.2) that

$$\delta \left(\frac{u^+_i q^{+i}}{u^-_j q^{+j}} \right) = \lambda^{++} \tag{5.5}$$

so the parameter λ^{++} is indeed compensated.

It is important to realize that the compensator q^{+i} adds infinitely many new fields to the minimal $(32+32)$ set described by $H^{++m,5}, \hat{\mu}^{+}$. The reason is that the off-shell description of the complex hypermultiplet necessarily requires an infinite set of auxiliary fields/10/. Note that in/5/ a "short" version of the complex hypermultiplet was used as a compensator. It involved a central charge satisfying the off-shell constraint $P^2 = Z^2$. In our version this constraint is removed. Thus, we can have a non-zero central charge (e.g., letting q^{+i} depend on x^5_A like $e^{imx^5_A} q^{+i}(\zeta_A, u)$) without tying it down to the off-shell value of P^2.

The most remarkable feature of this version is the existence of a scalar dimensionless and chargeless density. This is the quantity

$$(u^-_i q^{+i})^2 \;, \quad \delta (u^- q^+)^2 = -\Lambda (u^- q^+)^2. \tag{5.6}$$

Since it is analytic, one can use it to construct an invariant volume element for the analytic superspace:

$$d\zeta^{(-4)}_A du_A \cdot (u^- q^+)^2. \tag{5.7}$$

This allows us to couple the q^+ version to any kind of matter. Indeed, all types of matter can be described by analytic superfields (e.g., hypermultiplets Q^+ or ω, linear multiplets L^{++}, etc./3,4/). Their Lagrangians are analytic too, so it is sufficient to simply covariantize $\mathcal{L}$ matter and integrate it with the measure (5.7). The matter superfields are not required to transform as densities, so there are no restrictions on their self-couplings. Thus, one concludes that the q^+ version allows for the most general matter couplings.

Actually, in/13/ we have shown that the most general N=2 matter self-interactions can be described in terms of just the Q^+ hypermultiplet*). This includes the general 4n-dimensional hyper-Kahler

*)As shown in/4/, all the other matter multiplet (linear, relaxed hypermultiplets, etc.) self-couplings can be reduced by means of duality transformations to subclasses of Q^+ self-interactions.

sigma-model

$$S_Q = \int d\zeta^{(-4)}_A du \left[H^{+a}(Q,u)\, D^{++} Q^+_a + \right. \\ \left. + \mathcal{L}^{(+4)}(Q,u) \right]. \tag{5.8}$$

Here $a = 1, \ldots, 2n$; H^{+a} and $\mathcal{L}^{(+4)}$ are arbitrary functions of Q^+_a and the harmonic variables, which can be regarded as the prepotentials of hyper-Kahler geometry. The rigid action (5.8) can be coupled to the q^+-version of N=2 SG by replacing D^{++} by the following combination

$$\nabla^{++} = \mathcal{D}^{++} + \left(\frac{u^+_i q^{+i}}{u^-_j q^{+j}} \right) \mathcal{D}^{0}. \tag{5.9}$$

Using (2.10), (5.5) amd assuming that Q^+_a transforms as a weightless scalar, one can check that $\nabla^{++} Q^+_a$ is a scalar as well.

Further, one should replace the harmonic variables $u^{\pm}$ appearing explicitly in (5.8) by the following variables:

$$w^+_i = \frac{q^+_i}{u^-_j q^{+j}}$$
$$w^-_i = u^-_i \tag{5.10}$$

which are inert under the SG group. Finally, covariantizing the supervolume as shown in (5.7), one obtains

$$S^{curved}_Q = \int d\zeta^{(-4)}_A du_A (u^- q^+)^2 \cdot \left[H^{+a}(Q,w) \nabla^{++} Q^+_a + \right. \\ \left. + \mathcal{L}^{(+4)}(Q,w) \right]. \tag{5.11}$$

A further generalization of (5.11) could be achieved by letting Q^+ depend on the central charge coordinate x^5 and coupling it to the Maxwell gauge superfield H^{++5}. In this way one can obtain the most general SG-matter coupling.

The other versions of N=2 SG are much more restrictive due to the absence of a proper analytic density. In fact, in all versions there is a density coinciding with the Berezinian of the vielbeins

$E_A{}^M$, where $A=(a,\hat{\alpha}-)$, $M=(m,\hat{\mu}+)$. The explicit expression for it is (see (II.32), (IV.12), (IV.15))

$$\gamma = E^{-1}\det{}^{-1}(E_{\alpha}{}^{\hat{\mu}}) = (F\tilde{F})^{-2} X \; ; \; \delta\gamma = -\Lambda\gamma . \tag{5.12}$$

where X does not depend on H^5. However, this density is not analytic. Indeed, $\mathcal{D}^+_{\alpha}\ln\gamma$ is a tensor (the parameter Λ in (5.12) is analytic). In the WZ gauge the only suitable component of the same type is contained in H^5, and close inspection of (5.12) shows that it appears in $\mathcal{D}^+_{\alpha}\ln\gamma$, so γ cannot be analytic. Further, if one is able to construct another (analytic) density with the help of a compensator superfield, the ratio of the two densities must be a dimensionles invariant scalar. The only compensator which contains such a scalar is q^+.

The discussion above showed that the different versions of N=2 off-shell Einstein SG are not equivalent in the presence of matter. However, when there is no matter, one is able to perform duality transformations from the q^+ version to the other ones. Dual equivalence to the version with compensation by linear multiplet was already proven in [7]. Here we consider the transformation to the version with nonlinear compensator N. One makes the following change of variables:

$$q^+_i = (u^+_i - N^{++}u^-_i)\,\omega \tag{5.13}$$

or vice versa,

$$\omega = u^-_i q^{+i} \; , \quad N^{++} = \frac{u^+q^+}{u^-q^+} . \tag{5.14}$$

Putting this into the q^+ action (5.4) one obtains

$$S_{\omega,N^{++}} = \frac{1}{2k^2}\int d\zeta_A^{(-4)}du_A\, \omega^2\left[\mathcal{D}^{++}N^{++} + (N^{++})^2 - H^{(+4)}\right] . \tag{5.15}$$

Varying (5.15) with respect to ω reproduces the non-linear multiplet constraint (4.9). Clearly, in the presence of matter the constraint (4.9) will be modified by matter terms, since the density ω will appear in the matter action. We have observed a similar phenomenon in the so-called "flexible" version of N=1 SG[11].

We find a deep analogy between the q^+ version of N=2 SG and the minimal version of N=1 SG[12]. In both cases the matter compensators are unconstrained analytic superfields ($q^+(\zeta_A,u_A)$ in the N=2 case, and a chiral superfield $\varphi(\zeta_L)$ in the N=1 case). Both compensators can be used as densities for the corresponding analytic superspace integrals, which allows one to couple SG to matter in the most general way. All the other off-shell versions of those theories are classically equivalent to the former (by means of duality transformations), but only in the absence of matter.

6. Conclusions

In [1] and in the present paper we have developed the unconstrained off-shell formalism for N=2 SG. We have shown how to construct the most general N=2 SG matter couplings. This can be achieved only in the version with a q^+ hypermultiplet compensator. According to [14] such couplings give rise to a class of quaternionic manifolds, so we can claim to have found the prepotentials for such manifolds. This subject will be studied in a separate paper.

Another possibile applications of the formalism developed is a manifestly supersymmetric quantization scheme for N=2 SG. It is also interesting to try to formulate SG in 6 dimensions in a similar manner.

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Гальперин А.С. и др. E2-87-86
N = 2 супергравитация в суперпространстве: различные версии и материальные связи

В данной работе завершается построение N = 2 супергравитации в гармоническом суперпространстве. Ранее развитый подход обобщается на случай конформной супергравитации. Затем суперконформная группа компенсируется путем включения максвелловского и различных материальных мультиплетов. На этом пути могут быть воспроизведены все ранее известные версии эйнштейновской N = 2 супергравитации. Мы даем один явный пример /с нелинейным мультиплетом в качестве компенсатора/. Наш главный результат - новая версия эйнштейновской N = 2 супергравитации, содержащая комплексный гипермультиплет с бесконечным числом вспомогательных полей. Эта версия представляется наиболее фундаментальной. Только в ее рамках существует инвариантный аналитический суперобъем, что позволяет построить наиболее общие взаимодействия материи. В других версиях отсутствие подходящей аналитической плотности налагает сильные ограничения на вид материальных связей.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1987

Galperin A.S. et al. E2-87-86
N = 2 Supergravity in Superspace: Different Versions and Matter Couplings

This paper concludes the formulation of N = 2 supergravity in harmonic superspace. We generalize the approach developed earlier to include conformal supergravity. The superconformal group is then compensated by coupling to a Maxwell and various matter multiplets. All the previously known versions of N = 2 Einstein supergravity are reproducible in this way. We give explicitly one example (with the nonlinear multiplet as a compensator). Our main result is a new version of N = 2 Einstein supergravity which involves an off-shell complex hypermultiplet with its infinitely many auxiliary fields. We believe this version to be most fundamental. It is the only one in which the analytic supervolume can be made invariant. This property allows us to write down the most general matter couplings. In contrast, the absence of a proper analytic density in the other versions imposes severe restrictions on matter couplings.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987