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N=2 SUPERGRAVITY IN SUPERSPACE: THE INVARIANT ACTION

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This paper is a continuation of [1]. There we introduced a geometric framework in harmonic superspace consisting of a gauge group with analytic parameters (λ group) and unconstrained analytic prepotentials for N=2 SG. The latter turned out to be the vielbeins H^{++} of the harmonic covariant derivative $\mathcal{D}^{++}$. Inspection of the WZ gauge showed that those prepotentials contain the set of components of off-shell version of N=2 Einstein SG given in [2]. Then we developed the differential geometry formalism for that theory. The vielbeins and connections for the spinor and vector covariant derivatives were expressed in terms of the vielbeins H^{--} for the harmonic covariant derivative $\mathcal{D}^{--}$. The latter were related to the prepotentials H^{--} by a linear differential equation, and we gave the perturbative solution to that equation. We built from H^{--} a number of useful quantities with simple transformation laws. They allowed us to easily construct a density for the full supervolume of harmonic superspace in the analytic basis.

The remaining problem which is solved in this paper is to write down the invariant action for the version of N=2 SG under consideration. We do this in section 1. The action turns out to be covariantization of the action for the Maxwell-like superfield H^{++5} (the latter is the vielbein of $\mathcal{D}^{++}$ responsible for local central charge transformations). The rest of the paper is devoted to the proof of the invariance of this action, which makes use of a new "hybrid" basis in superspace. The appendix contains the proof of some important identities.

In this paper a number of results from [1], we refer to, are numbered by Roman and Arabic numerals (e.g. (III.5)), and those in this paper only by Arabic numerals (e.g. (5)).

1. The action formula

We shall show that the action is the following integral of the correct dimension ($[H^5] = m^{-1}$)

$$S_{SG}^{N=2} = \frac{1}{\kappa^2} \int d^4x_A \, d^4\theta_A^+ \, d^4\theta_A^- \, E^{-1} H^{++5} H^{--5} . \tag{1}$$

We point out that this is nothing but the covariantization of the flat-space action for a Maxwell superfield /3,4/. This is not a coincidence. According to /5/ the version of N=2 SG under consideration can be viewed as the coupling of N=2 conformal SG to an N=2 Maxwell multiplet and an N=2 "non-linear" multiplet. Actually, the Maxwell multiplet is represented by H^{++5}, with its transformation law (II.24) whereas the non-linear one is gauged away in our scheme. More details on conformal SG and the various compensators for it will be given in /6/.

The proof of the invariance of (1) consists of two parts. The easy one involves the transformations of $x^m, \theta^{\hat{\mu}\pm}$ (II.17). Under them H^{++5} and H^{--5} behave as scalars, and E^{-1} compensates the transformations of the volume element (IV.14). The difficult part concerns the x^5 transformation (II.17) (which is in fact an abelian gauge transformation for H^{++5}). In the process we will learn how to integrate by parts the covariant derivatives $\mathcal{D}^{++}$ and $\mathcal{D}^{--}$. A very useful new concept will be introduced. It is a "hybrid" basis in superspace, in which the spinor derivative Δ^+_α (II.33) becomes simply ∂^+_α. We will also make use of several non-trivial identities, for quantities built from the prepotentials. They can be (and have been) proved directly using an identity derived in the Appendix. Instead, we prefer an indirect proof. It is based on showing that the identity under investigation transforms as a tensor, and then checking that there are no fields of the same dimension and index structure in the WZ gauge.

Before plunging in the details of that proof, we would like to demonstrate that (1) contains the right component action /2/.

2. Checking the component action

To make sure that the invariant (1) coincides with the desired component action, it is sufficient to show that at least one of the **auxiliary** fields enters (1) properly. The remaining fields will then have their correct action terms due to supersymmetry and gauge invariance. The easiest auxiliary component to look for is the field $S^{ij}(x)$. In the WZ gauge (II.28) it appears in the prepotential H^{++5} only. Suppressing all the other fields one finds that E^{-1} in (1), which does not depend on H^{++5} or H^{--5}, reduces to 1. Further, in this case H^{++5} is simply

$$H^{++5} = i(\theta^+)^2 - i(\bar{\theta}^+)^2 + \kappa(\theta^+)^4 S^{ij} u^-_i u^-_j \;;$$

one can also check that H^{--5} becomes (see (III.3))

$$H^{--5} = i\left[(\theta^-)^2 - (\bar{\theta}^-)^2\right] + \frac{\kappa}{3}(\theta^-)^4 S^{ij} u^+_i u^+_j -$$

$$- \frac{2\kappa}{3}\left[\theta^+\theta^-(\bar{\theta}^-)^2 + (\theta^-)^2\bar{\theta}^+\bar{\theta}^-\right] S^{ij} u^+_i u^-_j +$$

$$+ \frac{\kappa}{3}\left[(\theta^-)^2(\bar{\theta}^+)^2 + (\theta^+)^2(\bar{\theta}^-)^2 + 4\theta^+\theta^-\bar{\theta}^+\bar{\theta}^-\right] S^{ij} u^-_i u^-_j \,.$$

Plugging all this into (1) one finds

$$S \to \frac{1}{\kappa^2}\int d^{12}z\, du\, H^{++5} H^{--5} \sim$$

$$\sim \int d^4x\, d^8\theta\, du\, (\theta^+)^4(\theta^-)^4 S^{ij} S^{kl} u^+_i u^+_j u^-_k u^-_l \sim \int d^4x\, S^{ij} S_{ij}$$

which is indeed the right action term for this auxiliary field.

3. Integration by parts for $\mathcal{D}^{++}$ and $\mathcal{D}^{--}$

As explained in sect. 1, the hardest part of the proof of the invariance of the action (1) concerns the transformations $\delta H^{++5} = \mathcal{D}^{++}\lambda^5(z_A, u)$ and $\delta H^{--5} = \mathcal{D}^{--}\lambda^5$ (see (II.24), (III.2)). Varying (1) one obtains

$$\delta S = \frac{1}{\kappa^2}\int dz_A\, du\, E^{-1}\left[\mathcal{D}^{++}(\lambda^5 H^{--5}) + \mathcal{D}^{--}(\lambda^5 H^{++5}) - \right.$$

$$\left. - \lambda^5(\mathcal{D}^{++}H^{--5} + \mathcal{D}^{--}H^{++5})\right]. \qquad (2)$$

Each of the three terms in (2) will be separately shown to vanish. In this subsection we deal with the first two terms. For this we need to prove the following two rules for integration by parts for $\mathcal{D}^{++}$ and $\mathcal{D}^{--}$:

$$\int dz_A\, du\, E^{-1}\mathcal{D}^{++}\phi^{--} = \int dz_A\, du\, (\mathcal{D}^{++} + \mathcal{D}^{++}\ln E)(E^{-1}\phi^{--}) = 0 \qquad (a)$$

$$\int dz_A\, du\, E^{-1}\mathcal{D}^{--}\phi^{++} = \int dz_A\, du\, (\mathcal{D}^{--} + \mathcal{D}^{--}\ln E)(E^{-1}\phi^{++}) = 0. \qquad (b)$$

(3)

In order to calculate $\mathcal{D}^{++}\ln E$ we first consider

$$\mathcal{D}^{++}\ln\det e_\alpha{}^{\hat\mu} = e^{-1}{}_{\hat\mu}{}^{\hat\alpha}\,\partial^+_{\hat\alpha}\,\mathcal{D}^{++}H^{--\hat\mu+} =$$

$$= e^{-1}{}_{\hat\mu}{}^{\hat\alpha}\,\partial^+_{\hat\alpha}\left(\mathcal{D}^{--}H^{++\hat\mu+}+\theta^{\hat\mu+}\right) =$$

$$=\partial^-_{\hat\mu}H^{++\hat\mu+} - \partial_m H^{++\hat\mu+}\, e^{-1}{}_{\hat\mu}{}^{\hat\alpha}\,\partial^+_{\hat\alpha}H^{--m}.$$

Here we used the fact that $\partial^+_{\hat\alpha}$ and $\mathcal{D}^{++}$ commute (in the gauge (II.29)), (IV.6), (III.3) and the analyticity of $H^{++\hat\mu+}$.
In a similar way one finds

$$\mathcal{D}^{++}e_{\alpha\hat\beta}{}^{m} = e_{\alpha\hat\beta}{}^{n}\,H_n^{++m},$$

$$H_n^{++m}=\partial_n H^{++m}-\partial_n H^{++\hat\mu+}\,e^{-1}{}_{\hat\mu}{}^{\hat\nu}\,\partial^+_{\hat\nu}H^{--m}. \qquad (4)$$

As a consequence of (4) one gets

$$\mathcal{D}^{++}f^{\alpha\dot\alpha}=\mathcal{D}^{++}\left(e^m e_m^{\alpha\dot\alpha}\right)=$$

$$=e^n H_n^{++m}e_m^{\alpha\dot\alpha} - e^m e_m^{\beta\dot\beta}\,e^k_{\beta\dot\beta}\,H_k^{++l}\,e_l^{\alpha\dot\alpha}=0, \qquad (5)$$

therefore (see (IV.12), (IV.13))

$$\mathcal{D}^{++}F=\mathcal{D}^{++}F_\alpha{}^{\dot\mu}=0 \qquad (6)$$

in agreement with (II.31), (II.32).
The net result for $\mathcal{D}^{++}\ln E$ is

$$\mathcal{D}^{++}\ln E=\partial_m H^{++m}-\partial^-_{\hat\mu}H^{++\hat\mu+}\equiv$$
$$\equiv(-1)^M\,\partial_M^A\,H^{++M},\quad M=(m,\hat\mu\pm). \qquad (7)$$

Finally,

$$\mathcal{D}^{++}+\mathcal{D}^{++}\ln E = \partial^{++}+H^{++M}\partial_M+(-1)^M\left(\partial_M H^{++M}\right)=$$
$$=\partial^{++}+(-1)^M\partial_M\left(H^{++M}\dots\right. \qquad (8)$$

which proves (3a).
For the proof of (3b) we shall use the existence of central basis. We start by calculating

$$\mathcal{D}^{++}\ln \mathrm{Ber}\,\frac{\partial z_A^M}{\partial z^N} = \mathcal{D}^{++}\ln\mathrm{Ber}\left(\delta_N^M+\partial_N v^M\right), \qquad (9)$$

where $v^M(z,u)$ are the bridges from central to analytic basis (II.15). In the central basis $\mathcal{D}^{++}_{CB}=\partial^{++}$, so it commutes with $\partial_N=\partial/\partial z^N$. Using (II.23) we find

$$\mathcal{D}^{++}\ln\mathrm{Ber}\,\frac{\partial z_A}{\partial z}=(1+\partial v)^{-1}{}_M{}^N\,\partial_N H^{++M}\cdot(-1)^M=$$
$$=\frac{\partial}{\partial z_A^M}H^{++M}\cdot(-1)^M=\mathcal{D}^{++}\ln E. \qquad (10)$$

This means that

$$\mathcal{D}^{++}\ln\left[E^{-1}\,\mathrm{Ber}\left(\partial z_A/\partial z\right)\right]=0$$

which implies

$$\mathcal{D}^{--}\ln\left[E^{-1}\,\mathrm{Ber}\left(\partial z_A/\partial z\right)\right]=0. \qquad (11)$$

Repeating the steps (9), (10) with $\mathcal{D}^{--}$ we obtain

$$\mathcal{D}^{--}\ln E=(-1)^M\,\partial_M{}^A\,H^{--M}. \qquad (12)$$

4. Hybrid basis in superspace

The last term in (2) is the trickiest one. Using (III.3) one can rewrite it as $-2\lambda^5 E^{-1}\mathcal{D}^{--}H^{++5}$. Thus, one has to show that the integral

$$I = \int d^4x_A\, d^4\theta^+_A\, d^4\theta^-_A\, du\; \lambda^5 E^{-1}\mathcal{D}^{--}H^{++5} \tag{13}$$

vanishes. The idea of the proof can be traced back to the flat case. There the corresponding integral is (D^{--} is the flat value of $\mathcal{D}^{--}$)

$$I_0 = \int d^{12}z_A\, du\; \lambda^5 D^{--}H^{++5} =$$

$$= \int d^4x_A\, d^4\theta^+_A\, d^2\bar\theta^-_A\, (D^+)^2(\lambda^5 D^{--}H^{++5}) = 0 \tag{14}$$

since $D^+_\alpha\lambda^5 = 0$, $(D^+)^2 D^{--}H^{++5} = -2D^{+\alpha}D^-_\alpha H^{++5} = 0$ (H^{++5} is analytic). The covariantization of this procedure is not so easy. First of all, the Berezin integration rule

$$\int d^2\theta^-_A \;\rightarrow\; \partial^{+\alpha}_A\,\partial^+_{A\alpha} \tag{15}$$

produces a non-covariant operator in our case. Indeed, $\partial^+_{A\alpha} = \frac{\partial}{\partial\theta^{-\alpha}_A}$ transforms as follows

$$\delta\partial^+_{A\alpha} = -\partial^+_{A\alpha}\lambda^{\beta-}\partial^+_{A\beta} - \partial^+_{A\alpha}\bar\lambda^{\dot\beta-}\bar\partial^+_{A\dot\beta}\,. \tag{16}$$

In order to perform the trick (14) covariantly, one should be able to go to a special basis in which $\partial/\partial\theta^-_\alpha$ transforms homogeneously. Fortunately, such a basis exists. To see this let us consider (II.38)*). Its solution is

$$F_\alpha{}^{\dot\mu} = \Delta^+_\alpha\,\bar\psi^{\dot\mu-}, \tag{17}$$

where $\bar\psi^{\dot\mu-}$ is a new bridge with the following transformation law:

*) The easiest way to prove (II.38) is to show that $\Delta^+_{(\alpha}F_{\beta)}{}^{\dot\mu}$ transforms homogeneously (using (II.33) and (II.34)). However, there is not any suitable component of this type in the WZ gauge (II.28) of the prepotentials, so $\Delta^+_{(\alpha}F_{\beta)}{}^{\dot\mu}$ must vanish.

$$\delta\bar\psi^{\dot\mu-} = \bar\lambda^{\dot\mu-} - \bar\rho^{\dot\mu-}, \quad \Delta^+_\alpha\bar\rho^{\dot\mu-} = 0. \tag{18}$$

The λ-term in (18) corresponds to the inhomogeneous term in $\delta F_\alpha{}^{\dot\mu}$ (II.34), and the $\bar\rho$ term is a "pregauge" transformation. Further, (17) is an algebraic equation for $F_\alpha{}^{\dot\mu}$ which can be solved:

$$F_\alpha{}^{\dot\mu} = \partial^+_\alpha\bar\psi^{\dot\rho-}\left(1-\bar\partial^+\bar\psi^-\right)^{-1}{}_{\dot\rho}{}^{\dot\mu}\,. \tag{19}$$

Then one sees that after the following change of coordinates:

$$\bar\theta^{\dot\alpha-}_H = \bar\theta^{\dot\alpha-}_A - \bar\psi^{\dot\alpha-},$$
$$(x^{m,5}_H, \theta^{\hat\alpha+}_H, \theta^{\alpha-}_H) = (x^{m,5}_A, \theta^{\hat\alpha+}_A, \theta^{\alpha-}_A) \tag{20}$$

the operator $\Delta^+_\alpha = \partial^+_{A\alpha} + F_\alpha{}^{\dot\mu}\bar\partial^+_{A\dot\mu}$ becomes simply

$$(\Delta^+_\alpha)_H = \frac{\partial}{\partial\theta^{\alpha-}_H}\,. \tag{21}$$

The meaning of the change (20) is that a new basis is defined where the constraint

$$\partial^+_{H\alpha}\phi = 0 \;\rightarrow\; \phi = \phi(x^{m,5}_H, \theta^{\hat\alpha+}_H, \bar\theta^{\dot\alpha-}_H) \tag{22}$$

is covariant. Indeed, in the basis (20) $\delta\partial^+_{H\alpha} = -\partial^+_{H\alpha}\lambda^{\beta-}\cdot\partial^+_{H\beta}$. The new kind of analyticity (22) is a hybrid of the analyticity $\partial^+_\alpha\phi = \bar\partial^+_{\dot\alpha}\phi = 0$ and of chirality $\partial^+_\alpha\phi = \partial^-_\alpha\phi = 0$, therefore we call the basis (20) the "hybrid basis". We point out that the bridge $\bar\psi^{\dot\mu-}$ is not an independent object. One can show that the following expression (see the Appendix)

$$\bar\psi^{\dot\mu-} = \bar\theta^{\dot\mu-}_A - \bar H^{--\dot\mu+} - H^{--\mu+}(AB^{-1})_\mu{}^{\dot\mu}, \tag{23}$$
$$A_\mu{}^{\dot\mu} = e^{-1}_\mu{}^{\dot\mu} - e^{-1}_\mu{}^{\nu}F_\nu{}^{\dot\mu},$$
$$B_{\dot\mu}{}^{\dot\nu} = e^{-1}_{\dot\mu}{}^{\dot\nu} - e^{-1}_{\dot\mu}{}^{\nu}F_\nu{}^{\dot\nu}$$

satisfies (17) identically.

Armed with the new tool, the hybrid basis, we can start the covariantization of the procedure (14). The change of variables (20) in the integral (13) produces the following Jacobian:

$$\left(\mathrm{Ber}\,\frac{\partial z_H}{\partial z_A}\right)^{-1} = \det\frac{\partial\bar\theta_H}{\partial\bar\theta_A} = \det\left(\delta_{\dot\mu}{}^{\dot\nu} - \partial^+_{A\dot\mu}\,\bar\Psi^{\dot\nu -}\right) \equiv J. \qquad (24)$$

Then (13) becomes (λ^5 is analytic, i.e. $\partial^+_{H\alpha}\lambda^5=0$):

$$I = \int d^4x_H\, d^4\theta^+_H\, d^4\theta^-_H\, du\; \lambda^5\, J E^{-1}\, \mathcal{D}^{--} H^{++5} =$$

$$= \int d^4x_H\, d^4\theta^+_H\, d^2\bar\theta^-_H\, du\; \lambda^5 (\partial_H^{+\alpha}\partial^+_{H\alpha})(J E^{-1}\mathcal{D}^{--}H^{++5}) =$$

$$= \int d^4x_H\, d^4\theta^+_H\, d^2\bar\theta^-_H\, du\; \lambda^5 \{(\partial^+_H)^2(JE^{-1}) . \mathcal{D}^{--}H^{++5} \qquad (25)$$

$$+ JE^{-1}[(\partial^+_H)^2\mathcal{D}^{--}H^{++5} + 2\partial_H^{+\alpha}\ln(JE^{-1}) . \partial^+_{H\alpha}\mathcal{D}^{--}H^{++5}]\}.$$

The two terms in the last integral in (25) vanish separately. First consider the term

$$(\partial^+_H)^2(JE^{-1}) = JE^{-1}[(\partial^+_H)^2\ln(JE^{-1}) +$$

$$+ \partial_H^{+\alpha}\ln(JE^{-1}) . \partial^+_{H\alpha}\ln(JE^{-1})].$$

The quantity $\varphi^+_\alpha = \partial^+_{H\alpha}\ln(JE^{-1}) = \Delta^+_\alpha\ln(JE^{-1})$ (26) has a simple transformation law (see (IV.14), (18), (19), (24)):

$$\delta\varphi^+_\alpha = -\Delta^+_\alpha\lambda^{\beta-}\varphi^+_\beta + \Delta^+_\alpha[-\partial_M\lambda^M(-1)^M +$$

$$+ (1-\bar\partial^+_A\bar\Psi^-)^{-1}{}_{\dot\mu}{}^{\dot\nu}(\bar\partial^+_{A\dot\nu}\bar\rho^{\dot\mu-} - \bar\partial^+_{A\dot\nu}\bar\lambda^{\dot\mu-} + \bar\partial^+_{A\dot\nu}\lambda^{\hat\mu-}\partial^+_{A\hat\mu}\bar\Psi^{\dot\mu-})] =$$

$$= -\Delta^+_\alpha\lambda^{\beta-}\varphi^+_\beta + \Delta^+_\alpha[-\partial_m\lambda^m + \partial^-_{\hat\mu}\lambda^{\hat\mu+} + \partial^+_\mu\lambda^{\mu-} +$$

$$+ \bar\partial^+_{\dot\mu}\lambda^{\mu-}F_\mu{}^{\dot\mu} + \bar\partial^+_{H\dot\mu}\bar\rho^{\dot\mu-}] =$$

$$= -\Delta^+_\alpha\lambda^{\beta-}\varphi^+_\beta + \frac{1}{2}\Delta^{+\beta}\Delta^+_\beta\lambda^-_\alpha . \qquad (27)$$

Here we used the relations $\Delta^+_\alpha = \partial^+_{H\alpha}$ and $\partial^+_{H\alpha}\bar\rho^{\dot\mu-}=0$ (18) and $\Delta^+_\alpha\lambda^{m,\hat\mu+}=0$. Note that (27) is similar to the

transformation law for the quantity $\Delta^+_\alpha F$ in the expression (II.36) for the Lorentz connection $A^+_{\alpha\beta\mu}$. In fact one can show that $\varphi^+_\alpha = 2\Delta^+_\alpha\ln F$.

From (27) it follows that the expression in the brackets in (26), $\Delta^{+\alpha}\varphi^+_\alpha + \varphi^{+\alpha}\varphi^+_\alpha$, is a tensor. So, it must correspond to a Lorentz scalar isotriplet (charge +2) field of dimension 1. The only such field in the WZ gauge (II.28) is in the prepotential H^{++5} but the ingredients of φ^+_α do not involve H^{++5}. The conclusion is that the above term vanishes. This result means that the full invariant volume of superspace vanishes as well:

$$\int dz_A\, du\, E^{-1} = \int dz_H\, du\, JE^{-1} =$$

$$= \int d^4x_H\, d^4\theta^+_H\, d^2\bar\theta^-_H\, (\partial^+_H)^2(JE^{-1}) = 0.$$

This fact was first established in a different approach in /7/.

Finally, we turn to the second term in (25). Using (III.2) and the analyticity of H^{++5} we find

$$\partial^+_{H\alpha}\mathcal{D}^{--}H^{++5} = \Delta^+_\alpha H^{--m,\hat\mu+}\,\partial^A_{m,\hat\mu+}H^{++5},$$

so it is sufficient to show that

$$(\Delta^+)^2 H^{--m,\hat\mu+} + 2\varphi^{+\alpha}\Delta^+_\alpha H^{--m,\hat\mu+} \equiv W^{m,\hat\mu+} = 0. \qquad (28)$$

The quantity W transforms as follows (see (III.2), (27) and use the analyticity of $\lambda^{m,\hat\mu+}$):

$$\delta W^{m,\hat\mu+} = (\Delta^{+\alpha}\lambda^-_\alpha)W^{m,\hat\mu+} + W^{n,\hat\nu+}\partial^A_{n,\hat\nu+}\lambda^{m,\hat\mu+}.$$

Once again, we deal with a covariant object of dimension 0 (for m) or $1/2$ (for $\hat\mu+$). Inspection of the WZ gauge (II.28) shows that no such fields are contained in the prepotentials $H^{++m,\hat\mu+}$ involved in (28).

This concludes the rather lengthy proof of the invariance of the action /1/. All these results will be used in the accompanying paper /6/ for the development of the conformal SG formalism and its applications.

Appendix

Here we shall prove that the expression (V.23) for $\overline{\Psi}^{\dot\mu -}$ satisfies (V.17). Since $\Delta^+_\alpha \overline{\theta}^{\dot\mu -}_A = F_\alpha{}^{\dot\mu}$, one has to show that

$$\Delta^+_\alpha \left[\overline{H}^{--\dot\mu +} + H^{--\mu +}(AB^{-1})_\mu{}^{\dot\mu} \right] = 0. \qquad (A.1)$$

The left-hand side of (A.1) can be rewritten as $-H^{--\mu+}\Delta^+_\alpha (AB^{-1})_\mu{}^{\dot\mu}$, so the problem is to prove the identity

$$\Delta^+_\alpha (AB^{-1})_\mu{}^{\dot\mu} = 0. \qquad (A.2)$$

We introduce the notation

$$C_{\hat\mu}{}^{\dot\alpha} = \begin{pmatrix} A_\mu{}^{\dot\alpha} \\ B_{\dot\mu}{}^{\dot\alpha}\end{pmatrix} = e^{-1}{}_{\hat\mu}{}^{\dot\alpha} - e^{-1}{}_{\hat\mu}{}^{\alpha} F_\alpha{}^{\dot\alpha} \qquad (A.3)$$

and calculate

$$\Delta^+_\alpha C_{\hat\mu}{}^{\dot\alpha} = -e^{-1}{}_{\hat\mu}{}^{\hat\lambda}\Delta^+_\alpha e_{\hat\lambda}{}^{\hat\rho} C_{\hat\rho}{}^{\dot\alpha} - e^{-1}{}_{\hat\mu}{}^{\beta}\Delta^+_\alpha F_\beta{}^{\dot\alpha}. \qquad (A.4)$$

With the help of the identity (which will be proved later on)

$$\Delta^+_\alpha F_\beta{}^{\dot\alpha} = -\frac12 \varepsilon_{\alpha\beta}\left(\partial^{+\gamma} e_\gamma{}^{\hat\rho} + A\bar\partial^{+\dot\gamma} e_{\dot\gamma}{}^{\hat\rho} - 2F^{\gamma\dot\gamma}\partial^+_\gamma e_{\dot\gamma}{}^{\hat\rho}\right) C_{\hat\rho}{}^{\dot\alpha} \qquad (A.5)$$

and $F^{\alpha\dot\alpha}F_{\alpha\dot\alpha} = 2A$ (see (IV.12,13)) we find

$$\Delta^+_\alpha C_{\hat\mu}{}^{\dot\alpha} = -C_{\hat\mu}{}^{\dot\beta}\Delta^+_\alpha e_{\dot\beta}{}^{\hat\rho} C_{\hat\rho}{}^{\dot\alpha}. \qquad (A.6)$$

Using this result it is very easy to check (A.2).

Finally, we have to prove (A.5). From (IV.12) and $A = \frac12 F^{\alpha\dot\alpha}F_{\alpha\dot\alpha}$ one derives the identity

$$\Delta^+_\alpha F^{\beta\dot\beta} = \left(\Delta^+_\alpha f^{\gamma\dot\gamma} + A\,\Delta^+_\alpha \tilde f^{\gamma\dot\gamma}\right)\left(\delta_\gamma^\beta \delta_{\dot\gamma}^{\dot\beta} + F_{\gamma\dot\gamma}\tilde f^{\beta\dot\beta}(1 - f\tilde f - A\tilde f^{2})^{-1}\right). \qquad (A.7)$$

This reduces the problem to calculating $\Delta^+_\alpha f^{\gamma\dot\gamma}$ and $\Delta^+_\alpha \tilde f^{\gamma\dot\gamma}$ (see (IV.10a)):

$$\Delta^+_\alpha f^{\gamma\dot\gamma} = \Delta^+_\alpha e^m \cdot e_m^{\gamma\dot\gamma} - f^{\rho\dot\rho}\,(\Delta^+_\alpha e^m_{\rho\dot\rho})\, e_m^{\gamma\dot\gamma},$$

$$\Delta^+_\alpha \tilde f^{\gamma\dot\gamma} = \Delta^+_\alpha \tilde e^m \cdot e_m^{\gamma\dot\gamma} - \tilde f^{\rho\dot\rho}\,(\Delta^+_\alpha e^m_{\rho\dot\rho})\, e_m^{\gamma\dot\gamma}. \qquad (A.8)$$

A straightforward calculation produces the following results (see (IV.5,6,8)):

$$\Delta^+_\alpha e^m \cdot e_m^{\gamma\dot\gamma} = \frac12 F_\alpha{}^{\dot\rho} e_m^{\gamma\dot\gamma} K_{\dot\rho}^{m+} + \frac12 \partial^{\rho+} e_\rho{}^{\hat\mu}\cdot M_{\hat\mu\alpha}^{\gamma\dot\gamma},$$

$$\Delta^+_\alpha \tilde e^m \cdot e_m^{\gamma\dot\gamma} = \frac12 e_m^{\gamma\dot\gamma} L_\alpha^{m+} + \frac12 \bar\partial^{\dot\rho+} e_{\dot\rho}{}^{\hat\mu}\cdot M_{\hat\mu\alpha}^{\gamma\dot\gamma},$$

$$f^{\rho\dot\rho}(\Delta^+_\alpha e^m_{\rho\dot\rho})e_m^{\gamma\dot\gamma} = \frac12 f_\alpha{}^{\dot\rho} e_m^{\gamma\dot\gamma} K_{\dot\rho}^{m+} + \frac12 f^{\rho\dot\rho}F_{\alpha\dot\rho}\, e_m^{\gamma\dot\gamma} L_\rho^{m+} + f^{\rho\dot\rho}\,\partial^+_\rho e_{\dot\rho}{}^{\hat\mu}\, M_{\hat\mu\alpha}^{\gamma\dot\gamma},$$

$$\tilde f^{\rho\dot\rho}(\Delta^+_\alpha e^m_{\rho\dot\rho})e_m^{\gamma\dot\gamma} = \frac12 \tilde f_\alpha{}^{\dot\rho} e_m^{\gamma\dot\gamma} K_{\dot\rho}^{m+} + \frac12 \tilde f^{\rho\dot\rho}F_{\alpha\dot\rho}\, e_m^{\gamma\dot\gamma} L_\rho^{m+} + \tilde f^{\rho\dot\rho}\,\partial^+_\rho e_{\dot\rho}{}^{\hat\mu}\, M_{\hat\mu\alpha}^{\gamma\dot\gamma},$$

where

$$K_{\dot\rho}^{m+} = \bar\partial^+_{\dot\rho}(\partial^+)^2 H^{--m} - \bar\partial^+_{\dot\rho}\partial^{\gamma+} e_\gamma{}^{\hat\mu}\cdot e^{-1}{}_{\hat\mu}{}^{\hat\nu}\,\partial^+_{\hat\nu} H^{--m},$$

$$L_\alpha^{m+} = \partial^+_\alpha(\bar\partial^+)^2 H^{--m} - \partial^+_\alpha\bar\partial^{\dot\gamma+} e_{\dot\gamma}{}^{\hat\mu}\cdot e^{-1}{}_{\hat\mu}{}^{\hat\nu}\,\partial^+_{\hat\nu}H^{--m},$$

$$M_{\hat\mu\alpha}^{\gamma\dot\gamma} = e^{-1}{}_{\hat\mu}{}^{\dot\gamma}\delta_\alpha^\gamma - e^{-1}{}_{\hat\mu}{}^{\dot\nu}F_{\alpha\dot\nu}\tilde f^{\gamma\dot\gamma} + e^{-1}{}_{\hat\mu\alpha} f^{\gamma\dot\gamma} - e^{-1}{}_{\hat\mu}{}^{\gamma}F_\alpha{}^{\dot\gamma}.$$

Putting all this in (A.8) and then in (A.7) one observes that the terms containing K and L cancel and one easily derives (A.5).

The identity (A.5) can be rewritten in the equivalent form

$$\Delta^{+\alpha}\Delta^+_\alpha H^{--\hat\mu+}\cdot C_{\hat\mu}{}^{\dot\beta} = 0. \qquad (A.9)$$

With the help of (A.9) one can prove a number of relations which were established in sections 4,5 using tensor arguments.

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N=2 супергравитация в суперпространстве: инвариантное действие

В данной работе продолжено построение супергравитации в гармоническом суперпространстве. Дается инвариантное действие для первой внемассовой версии теории. Доказательство инвариантности основывается на существовании нового "гибридного" базиса в гармоническом суперпространстве, в котором наряду с аналитичностью наполовину явной становится киральность.

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N=2 Supergravity in Superspace: the Invariant Action

This paper continues the formulation of harmonic superspace supergravity. We write down the invariant action for the first off-shell version of the theory. The proof of the invariance relies on the existence of a new "hybrid" basis in harmonic superspace in which semi-chirality combined with analyticity are manifest.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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