сообщения
объединенного
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ядерных
исследований
дубна

E2-87-84

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N=2 SUPERGRAVITY IN SUPERSPACE: SOLUTION TO THE CONSTRAINTS

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1987

I. Introduction

N=2 supergravity (SG) was discovered more than 10 years ago [1]. Since then it has been studied extensively. The majority of the results have been obtained in the component field approach. After the first off-shell version of N=2 Einstein SG was found [2], an elegant general method for constructing such theories has been developed [3,4]. It starts with the conformal version of the theory which is then coupled to a Maxwell and various matter N=2 multiplets in order to compensate the conformal gauge transformations. In this way three off-shell versions of N=2 Einstein SG have been obtained, using a non-linear, a linear and a central charged matter multiplets. Later on the possibilities to couple those theories to supersymmetric Yang-Mills (SYM) and matter multiplets have been investigated [5]. Whereas coupling to SYM presented no particular problems, this was not the case with N=2 matter. The reason was the lack of a proper off-shell description for the principal N=2 matter representation, the Fayet-Sohnius hypermultiplet [6,7]. The central charge formulation of the hypermultiplet [7] which was used in [3] is essentially constrained (in fact, it lies on-shell in 5 dimensions, since $P^2 = Z^2$). Although this framework is good enough for coupling the free hypermultiplet to SG and SYM, it creates severe difficulties when trying to arrange matter self-interactions. On the other hand, there do not exist off-shell versions of the Fayet-Sohnius hypermultiplet without central charge which involve finite sets of auxiliary fields [8].

Some of the above component results have been translated in the language of ordinary superspace differential geometry [9]. As usual, this lead to a number of torsion constraints. The attempts to solve them have succeeded only in the linearized approximation [10].

Adequate formulations of all the N=2 supersymmetric theories became possible after the invention of harmonic superspace [11]. It allowed to give a genuine off-shell theory of the hypermultiplet (as well as all the other matter multiplets), without use of central charges or any restrictions on the hypermultiplet self-couplings [11,12,13]. A geometric unconstrained formulation of the SYM theory was given in [11,14,15]. Finally, the full gauge group and the off-shell unconstrained prepotentials for both conformal and Einstein SG were found[11,16]. It is remarkable that all those theories are based on the fundamental concept of N=2 Grassmann analyticity (much like N=1 supersymmetry, which is based upon chirality[19]).

In this and in two other papers/17,18/, we complete the formulation of N=2 SG in harmonic superspace. In particular, we write down the invariant actions for the various off-shell versions of Einstein SG and discuss their general couplings with N=2 matter. The present paper is devoted to the first off-shell version[2] which is the simplest (in appearance). We begin by studying the torsion constraints of the theory first in ordinary and then in harmonic superspace. The latter allows an interpretation of the principle constraints as integrability conditions for the existence of Grassmann analytic superfields in a curved background. This geometric picture suggests a way of solving the constraints, by defining a new basis in superspace in which analyticity becomes manifest. In this basis the harmonic derivative $\mathcal{D}^{++}$ acquires analytic vielbeins which turn out to be the unconstrained prepotentials of the theory. Their gauge group is the group of general coordinate transformations preserving the analytic subspace. Instead of a systematic study of all the constraints and their consequences we prefer a constructive approach. We build all the necessary elements of the differential geometry formalism from the prepotentials in such a way that they transform properly. The correct Wess-Zumino gauge content of the prepotentials ensures that those geometric objects will automatically satisfy the constraints. This approach supplies a number of useful quantities ("building blocks") which help to find the invariant action of the theory[17] **and are heavily used in the further development of the formalism.**

In the third paper[18] we generalize the framework to incorporate the superconformal gauge group. The new point is the rather unusual realization[16] of the local $SU(2)$ transformations on the harmonic variables. The prepotentials of N=2 conformal SG are also vielbeins in the derivative $\mathcal{D}^{++}$[16] (including a new one related to local $SU(2)$). Next, following the method of[3] we introduce various compensators for the superconformal group in order to obtain different Einstein versions. We observe that the versions using a non-linear (i.e. the one described in the present paper) or a linear[12] compensators put severe restrictions on possible matter couplings. The reason is that all the N=2 matter actions are given by integrals over the analytic subspace. The above versions do not

provide a suitable density to covariantize the supervolume element, so the matter Lagrangians must have non-zero weight. In contrast, the version with a q^+ -hypermultiplet[11] compensator (with its infinite set of auxiliary fields) is free from this problem and allows the most general couplings of SG to matter. We consider it as the natural generalization of N=1 minimal SG which also employs an N=1 analytic (chiral) compensator and does not restrict the matter couplings.

The reader is assumed to be acquainted with the ideas of harmonic superspace, as well as with the notation and conventions of /11,14/.

II. Analytic superspace and prepotentials

In this section we discuss the constraints for N=2 Einstein SG. We show that in a superspace with additional harmonic coordinates these constraints can be interpreted as integrability conditions for the existence of Grassmann analytic superfields. We introduce a new analytic basis in harmonic superspace in which analyticity becomes manifest. In this basis the harmonic derivative $\mathcal{D}^{++}$ acquires vielbeins which are shown to be the prepotentials of the theory.

II.1. <u>N=2 SG constraints as integrability conditions for analyticity.</u>

Traditionally N=2 Einstein SG is formulated in a superspace $\{Z^M = (x^m, \theta^{\mu i}, \bar\theta^{\dot\mu i}) \,;\, x^5\}$ where acts the following general coordinate transformation group:

$$\delta x^m = \tau^m(z) \,, \quad \delta x^5 = \tau^5(z)$$
$$\delta\theta^{\hat\mu i} = \tau^{\hat\mu i}(z) \,, \quad \hat\mu = (\mu, \dot\mu). \qquad (II.1)$$

The fifth space-time coordinate x^5 is needed for the description of N=2 matter with central charge[7]. The general coordinate transformations of x^5 correspond to gauging the central charge and serve as gauge transformations for the graviphoton field in the N=2 SG multiplet. We stress, however, that neither the SG fields nor their gauge parameters depend on this extra coordinate.

In addition to (II.1) one defines a tangent space Lorentz group with parameters

$$\Omega^{\hat\alpha\hat\beta} = \begin{pmatrix} \Omega^{\alpha\beta}(z) & 0 \\ 0 & \bar\Omega^{\dot\alpha\dot\beta}(z) \end{pmatrix}, \quad \Omega^{\alpha}_{\alpha} = \bar\Omega^{\dot\alpha}_{\dot\alpha} = 0. \tag{II.2}$$

In this framework the spinor covariant derivatives $\mathcal{D}^i_\alpha$, $\bar{\mathcal{D}}^i_{\dot\alpha}$ have a local Lorentz index $\alpha, \dot\alpha$ and a rigid $SU(2)$ index i. The essential information about N=2 Einstein SG can be given in the form of the following constraints /9/:

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = \varepsilon_{\alpha\beta}\varepsilon^{ij}\mathcal{D}_5 \quad + \text{curvature (a)}$$
$$\{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}_{\dot\beta j}\} = \delta^i_j \mathcal{D}_{\alpha\dot\beta} \quad + \text{curvature (b)}. \tag{II.3}$$

In fact, only the parts of (II.3) symmetric in i,j are the actual constraints. The traces are just definitions for $\mathcal{D}_{\alpha\dot\beta}$, $\mathcal{D}_5$. Actually, since the gauge parameters τ and Ω do not depend on x^5, one imposes a further constraint:

$$\mathcal{D}_5 = \frac{\partial}{\partial x^5}. \tag{II.4}$$

The geometric meaning of the constraints (II.3) is obscure in the above framework. It can be revealed if one introduces the concept of harmonic superspace and rewrites the constraints there. Harmonic superspace has additional bosonic coordinates $u^\pm_i$ which describe the sphere $SU(2)/U(1)$ /11/

$$\{ Z^M = (x^m, \theta^{\hat\mu i}),\ u^\pm_i;\ x^5 \}. \tag{II.5}$$

In what follows we shall call (II.5) the central basis of harmonic superspace. The new coordinates $u^\pm_i$ do not transform locally:

$$\delta u^\pm_i = 0. \tag{II.6}$$

Now, the essential symmetric parts of the constraints (II.3) can be rewritten in an equivalent form using the notation $\mathcal{D}^+_{\underline{\alpha}} =$

$= u^+_i \mathcal{D}^i_{\underline{\alpha}}$, $\underline{\alpha} = (\alpha, \dot\alpha)$:

$$\{\mathcal{D}^+_{\underline{\alpha}}, \mathcal{D}^+_{\underline{\beta}}\} = \text{curvature} \tag{II.7}$$

One immediately recognizes the integrability conditions for the existence of Grassmann analytic scalar superfields ϕ defined by /11/

$$\mathcal{D}^+_{\underline{\alpha}} \phi = 0 \tag{II.8}$$

in a curved background. The conclusion is that the constraints (II.3) are designed to preserve the analytic representations (II.8) in the curved case. This is very similar to the situation in the N=2 SYM theory /11/.

In the new harmonic framework the spinor covariant derivatives $\mathcal{D}_{\underline{\alpha}}$ (and the conventional ones $\mathcal{D}_a$, $\mathcal{D}_5$) are supplemented by harmonic ones,

$$\mathcal{D}^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} \equiv \partial^{++}, \quad \mathcal{D}^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} \equiv \partial^{--}$$
$$\mathcal{D}^{0} = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} \equiv \partial^{0}. \tag{II.9}$$

Note that (II.9) coincides with the flat-space expressions /11/ because the gauge groups (II.1), (II.6) do not depend on $u^\pm$. Their commutation relations repeat the flat ones as well. In particular, one has

$$[\mathcal{D}^{++}, \mathcal{D}^+_{\underline{\alpha}}] = 0 \quad \text{(a)}$$
$$[\mathcal{D}^{0}, \mathcal{D}^+_{\underline{\alpha}}] = \mathcal{D}^+_{\underline{\alpha}} \quad \text{(b)} \tag{II.10}$$
$$[\mathcal{D}^{0}, \mathcal{D}^{++}] = 2\mathcal{D}^{++} \quad \text{(c)}.$$

Actually, the set of constraints (II.7) and (II.10) are not just consequences of (II.3), they are equivalent. Indeed, assuming the flat-space form (II.9) of the harmonic derivatives (which is allowed by the gauge group), one derives from (II.10b) that $\mathcal{D}^+_{\underline{\alpha}}$ is a harmonic function with $U(1)$ charge $+1$ /11/. Then (II.10a) means that it depends linearly on u^+_i , $\mathcal{D}^+_{\dot\alpha} = u^+_i \mathcal{D}^i_{\dot\alpha}$, which allows to recover the non-conventional parts of (II.3) from (II.7).

II.2. Analytic basis and subspace. Bridges

The new equivalent form of the SG contraints (II.7), (II.10) suggests a way of solving them. As explained above the constraint (II.7) preserves the analytic representations (II.8). In the flat case the analyticity of superfield like (II.8) becomes obvious in a special analytic basis in superspace:

$$\left\{ Z^M_A = (x^m_A, \theta^{\hat\mu\pm}_A),\ u^\pm_i\ ;\ x^5 \right\} \qquad \text{(II.11)}$$

where

$$x^m_A = x^m - i\left(\theta^i \sigma^m \bar\theta^j + \theta^j \sigma^m \bar\theta^i\right) u^+_i u^-_j ,$$
$$x^5_A = x^5 + i\left(\theta^{\alpha i}\theta_\alpha^j - \bar\theta^i_{\dot\alpha}\bar\theta^{\dot\alpha j}\right) u^+_i u^-_j , \qquad \text{(II.12)}$$
$$\theta^{\hat\mu\pm}_A = \theta^{\hat\mu i} u^\pm_i .$$

Remarkably, the superspace (II.11) has a subspace called "analytic"

$$\left\{ \zeta^M_A = (x^m_A, \theta^{\hat\mu +}_A)\ ,\ u^\pm_i\ ;\ x^5_A \right\} \qquad \text{(II.13)}$$

which is invariant under rigid N=2 supersymmetry /11/. Then the solution of (II.8) is just a general function defined in the analytic subspace:

$$D^+_{\dot\alpha} \phi = 0 \ \Rightarrow\ \phi = \phi(\zeta^M_A, u^\pm_i, x^5_A) \qquad \text{(II.14)}$$

(the dependence on x^5_A is optional).

In the curved case the above approach is generalized by introducing a curved analytic basis with coordinates

$$x^{m,5}_A = x^{m,5} + v^{m,5}(z,u)$$

$$\theta^{\hat\mu\pm}_A = \theta^{\hat\mu i} u^\pm_i + v^{\hat\mu\pm}(z,u)$$
$$u^\pm_{A i} = u^\pm_i . \qquad \text{(II.15)}$$

The functions $v(z,u)$ defining the change of coordinates generalize the ones in (II.12) and do not depend on x^5 . To preserve the invariance of the analytic subspace under the combined conjugation $\sim$ ($\equiv \star$ from /11/), we demand

$$\widetilde{v^{m,5}} = v^{m,5} ,\quad \widetilde{v^{\mu\pm}} = \bar v^{\dot\mu\pm} ,\quad \widetilde{\bar v^{\dot\mu\pm}} = -v^{\mu\pm}. \qquad \text{(II.16)}$$

We shall call $v(z,u)$ "bridges" from the central (II.5) to the analytic basis in harmonic superspace. One may recall that the bridges from the central to the chiral basis in N=1 SG were the unconstrained prepotentials of that theory /19/.

Here we shall see that the bridges are secondary objects, which can be expressed in terms of the true prepotentials.

The purpose of the coordinate change (II.15) was to make analyticity manifest. In other words, functional dependence on the coordinates of the analytic subspace as in (II.14) should be a covariant notion. To achieve this, we define the following general coordinate transformation group acting in the new analytic basis:

$$\delta x^{m,5}_A = \lambda^{m,5}(\zeta_A, u) ,$$
$$\delta\theta^{\hat\mu +}_A = \lambda^{\hat\mu +}(\zeta_A, u) ,\quad \delta u^\pm_i = 0 ; \qquad \text{(II.17)}$$
$$\delta\theta^{\hat\mu -}_A = \lambda^{\hat\mu -}(\zeta_A, \theta^-_A, u) .$$

One sees that the analytic subspace $(x^{m,5}_A, \theta^{\hat\mu +}, u^\pm_i)$ is invariant (actually, x^5_A can also be put aside since the parameters λ do not depend on it). The remaining coordinates $\theta^{\hat\mu -}$ transform in a general way.

Combining (II.1), (II.15) and (II.17) one finds the transformation laws of the bridges:

$$\delta v^{m,s} \equiv v^{m,s\,\prime}(z',u') - v^{m,s}(z,u) =$$
$$= \lambda^{m,s}(z_A,u) - \tau^{m,s}(z) \quad ,$$
$$\delta v^{\hat{\mu}+} = \lambda^{\hat{\mu}+}(z_A,u) - \tau^{\hat{\mu}i}(z)u^+_i \, ,$$
$$\delta v^{\hat{\mu}-} = \lambda^{\hat{\mu}-}(z_A,u) - \tau^{\hat{\mu}i}(z)u^-_i \, . \qquad \text{(II.18)}$$

It is clear now that the analyticity condition (II.8) can be solved by $\phi = \phi(z_A, u, x^5_A)$ because $\partial/\partial\theta^{\hat{\mu}-}_A$ transforms homogeneously. This suggests the following "almost simple" form for $\mathcal{D}^+_\alpha$:

$$\mathcal{D}^+_\alpha = E_\alpha{}^{\hat{\mu}} \cdot \frac{\partial}{\partial\theta^{\hat{\mu}-}_A} + A^+_\alpha \equiv \nabla^+_\alpha + A^+_\alpha \, . \qquad \text{(II.19)}$$

Here $E_\alpha{}^{\hat{\mu}}(z_A,u)$ are the only remaining vielbeins. They transform under the coordinate group (II.17) and the tangent space Lorentz group (II.2):

$$\delta E_\alpha{}^{\hat{\mu}} = \Omega_\alpha{}^{\hat{\beta}} E_{\hat{\beta}}{}^{\hat{\mu}} + E_\alpha{}^{\hat{\nu}} \cdot \partial^+_{\hat{\nu}} \lambda^{\hat{\mu}-} \qquad \text{(II.20)}$$

(here $\partial^+_{\hat{\nu}} \equiv \partial/\partial\theta^{\hat{\nu}-}_A$). The Lorentz connection A^+_α is the Lie-algebra valued and transforms as follows

$$\delta A^+_{\alpha\,\hat{\beta}\hat{\gamma}} = -\nabla^+_\alpha \Omega_{\hat{\beta}\hat{\gamma}} + \Omega_\alpha{}^{\hat{\delta}} \cdot A^+_{\hat{\delta}\,\hat{\beta}\hat{\gamma}} +$$
$$+ \Omega_{\hat{\beta}}{}^{\hat{\delta}} \cdot A^+_{\alpha\,\hat{\delta}\hat{\gamma}} + \Omega_{\hat{\gamma}}{}^{\hat{\delta}} \cdot A^+_{\alpha\,\hat{\beta}\hat{\delta}} \, . \qquad \text{(II.21)}$$

The quantities E and A in (II.19) are still subject to the constraints (II.7) and (II.10). In what follows we will be able to express then in terms of the unconstrained prepotentials of the theory.

II.3. Prepotentials and Wess-Zumino gauge

Above we have seen that the spinor derivative $\mathcal{D}^+_\alpha$ becomes simple in the analytic basis. Instead the harmonic derivative $\mathcal{D}^{++}$ which was simply ∂^{++} in the central basis, now becomes

$$\mathcal{D}^{++}_{AB} = \mathcal{D}^{++}_{CB}(z_A,u,x^5_A)\frac{\partial}{\partial(z_A,u,x^5_A)} =$$
$$= \partial^{++} + H^{++m,s}\frac{\partial}{\partial x^{m,s}_A} + H^{++\hat{\mu}\pm}\frac{\partial}{\partial\theta^{\hat{\mu}\pm}_A} \, . \qquad \text{(II.22)}$$

The vielbeins H^{++} in (II.22) originate from the coordinate change (II.18)

$$H^{++m,s} = \mathcal{D}^{++} v^{m,s} \qquad \text{(a)}$$
$$H^{++\hat{\mu}+} = \mathcal{D}^{++} v^{\hat{\mu}+} \qquad \text{(b)} \qquad \text{(II.23)}$$
$$H^{++\hat{\mu}-} = \theta^{\hat{\mu}i}u^+_i + \mathcal{D}^{++}v^{\hat{\mu}-} = \theta^{\hat{\mu}+}_A - v^{\hat{\mu}+} + \mathcal{D}^{++}v^{\hat{\mu}-} \qquad \text{(c)} \, .$$

Their transformation laws follow from the fact that $\mathcal{D}^{++}$ does not transform, (see (II.6), and from (II.17):

$$\delta H^{++M} = \mathcal{D}^{++}\lambda^M \quad , \quad M = m,s,\hat{\mu}\pm \quad . \qquad \text{(II.24)}$$

Note that in (II.22) there is no connection since the tangent space group parameters $\Omega(z)$ do not depend on $u^\pm$.

Unlike the harmonic derivative $\mathcal{D}^{++}$ above, the derivative $\mathcal{D}^0$ should keep its flat-space form in the analytic basis:

$$\mathcal{D}^0 = \partial^0 + \theta^{\hat{\mu}+}_A \frac{\partial}{\partial\theta^{\hat{\mu}+}_A} - \theta^{\hat{\mu}-}_A \frac{\partial}{\partial\theta^{\hat{\mu}-}_A} \qquad \text{(II.25)}$$

The reason is that all the objects under consideration are taken to be eigenfunctions of this operator, i.e. they carry definite $U(1)$ charges. This means that the constraints (II.10b,c), which were obvious in the central basis, are automatically satisfied in the analytic one as well.

Our next step is to plug the above expressions for $\mathcal{D}^{+}_{\alpha}$ (II.19) and $\mathcal{D}^{++}$ (II.22) into the constraints (II.7) and (II.10a) and see under what conditions on the vielbeins $E_{\alpha}{}^{\hat{\mu}}$, H^{++} and the connection A^{+}_{α} they can be satisfied. We start with the constraint (II.10a) which will lead us to the prepotentials of the theory:

$$-\nabla^{+}_{\alpha} H^{++m,5}\partial_{m,5} - \nabla^{+}_{\alpha} H^{++\hat{\mu}+}\partial^{-}_{\hat{\mu}} + \left(\mathcal{D}^{++}E_{\alpha}{}^{\hat{\mu}} - \nabla^{+}_{\alpha}H^{++\hat{\mu}-}\right)\partial^{+}_{\hat{\mu}} + \mathcal{D}^{++}A^{+}_{\alpha} = 0. \qquad \text{(II.26)}$$

The most important consequence of (II.26) is the analyticity of the vielbenis $H^{++m,5}$ and $H^{++\hat{\mu}+}$:

$$\nabla^{+}_{\alpha} H^{++m,5,\hat{\mu}+} = 0 \quad\Rightarrow$$
$$\Rightarrow\; H^{++m,5,\hat{\mu}+} = H^{++m,5,\hat{\mu}+}(\zeta_{A}, u). \qquad \text{(II.27)}$$

Choosing them to be arbitrary analytic functions and taking into account the transformation laws (II.24), (II.17) one can see that most of the components of these superfields can be gauged away. What remains in the Wess-Zumino gauge turns out [11] to be exactly the set of fields of N=2 Einstein SG in its first off-shell version [2]:

$$\begin{aligned}
H^{++m}(\zeta_{A},u) &= -2i\theta^{+}\sigma^{a}\bar{\theta}^{+}e^{m}_{a}(x_{A}) + k(\bar{\theta}^{+})^{2}\theta^{+}\psi^{mi}(x_{A})u^{-}_{i} + \\
&+ k(\theta^{+})^{2}\bar{\theta}^{+}\bar{\psi}^{mi}(x_{A})u^{-}_{i} + k(\theta^{+})^{2}(\bar{\theta}^{+})^{2}V^{m(ij)}(x_{A})u^{-}_{i}u^{-}_{j}, \\
H^{++5}(\zeta_{A},u) &= i\left[(\theta^{+})^{2}-(\bar{\theta}^{+})^{2}\right]\det e + ik\,\theta^{+}\sigma^{a}\bar{\theta}^{+}B_{a} + \\
&+ k(\bar{\theta}^{+})^{2}\theta^{+}\rho^{i}u^{-}_{i} + k(\theta^{+})^{2}\bar{\theta}^{+}\bar{\rho}^{i}u^{-}_{i} + k(\theta^{+})^{2}(\bar{\theta}^{+})^{2}S^{(ij)}u^{-}_{i}u^{-}_{j}, \\
H^{++\mu+}(\zeta_{A},u) &= k(\theta^{+})^{2}\bar{\theta}^{+}_{\dot{\mu}}(A^{\mu\dot{\mu}} + iV^{\mu\dot{\mu}}) + k(\bar{\theta}^{+})^{2}\theta^{\nu+}\left[\delta_{\nu}{}^{\mu}(M+iN) + T_{(\nu}{}^{\mu)}\right] + \\
&+ k(\theta^{+})^{2}(\bar{\theta}^{+})^{2}\xi^{\mu i}u^{-}_{i}\,; \quad \bar{H}^{++\dot{\mu}+} = \widetilde{H^{++\mu+}}.
\end{aligned} \qquad \text{(II.28)}$$

So, we conclude that the analytic superfields $H^{++m,5,\hat{\mu}+}$ are the unconstrained prepotentials of the theory. The remainder of this paper will be devoted to the construction of all the other objects (e.g., the vielbeins $E_{\alpha}{}^{\hat{\mu}}$, the connections A^{+}_{α}, etc.) in terms of these.

The careful reader may ask the question: what about eqs. (II.23) which seem to express H^{++} in terms of the bridges v? The answer is: the bridges are secondary objects in our scheme. This means that given the arbitrary superfields H^{++} one can

solve (II.23) for v. These are highly non-linear differential equations which can be solved perturbatively. The general procedure is described in the context of N=2 SYM theory in [14]. However, we emphasize that the analytic basis is self-contained, i.e. everything can be formulated in it. In particular, all the geometric objects of N=2 SG including the action, as well as N=2 matter and SYM are most naturally formulated in that basis. So in practice we shall not use the explicit form of the bridges v for going back to the central basis, it is sufficient to know that they exist.

II.4. We continue the examination of the consequences of (II.26).

The vielbein $H^{++\hat{\mu}-}$ can be gauged away since the parameter $\lambda^{\hat{\mu}-}$ is general. The suitable gauge for $H^{++\hat{\mu}-}$ coincides with its flat limit [11]:

$$H^{++\hat{\mu}-} = \theta^{\hat{\mu}+}_{A}. \qquad \text{(II.29)}$$

Then $\lambda^{\hat{\mu}-}$ is fixed by the relation

$$\mathcal{D}^{++}\lambda^{\hat{\mu}-} = \lambda^{\hat{\mu}+}. \qquad \text{(II.30)}$$

Note that is gauge choice is not obligatory and is made for convenience.

In the above gauge the coefficient of $\partial^{+}_{\hat{\mu}}$ in (II.26) yields the equation

$$\mathcal{D}^{++}E_{\alpha}{}^{\hat{\mu}} = 0 \qquad \text{(II.31)}$$

which means that $E_{\alpha}{}^{\hat{\mu}}$ is covariantly independent of $u^{\pm}$ (back in the central basis it would be simply independent of $u^{\pm}$). This allows us to make a further gauge choice. The (covariantly) $u^{\pm}$-independent Lorentz parameter $\Omega_{\hat{\alpha}\hat{\beta}}$ can be used to gauge away parts of $E_{\alpha}{}^{\hat{\mu}}$ (see (II.20))

$$E_{\alpha}{}^{\hat{\mu}} = \begin{pmatrix} F\,\delta_{\alpha}{}^{\mu} & F\cdot F_{\alpha}{}^{\dot{\mu}} \\ \tilde{F}\cdot\tilde{F}_{\dot{\alpha}}{}^{\mu} & \tilde{F}\,\delta_{\dot{\alpha}}{}^{\dot{\mu}} \end{pmatrix}. \qquad \text{(II.32)}$$

Then the Lorentz transformations are induced by the superspace ones:

$$\Omega_{\alpha\beta} = - \Delta^+_{(\alpha} \lambda^-_{\beta)}$$

$$\Delta^+_\alpha \equiv \frac{1}{F} \nabla^+_\alpha = \partial^+_\alpha + F_\alpha{}^{\dot\mu} \bar\partial^+_{\dot\mu} \ . \tag{II.33}$$

The quantities F, $F_\alpha{}^{\dot\mu}$ will then transform as follows:

$$\delta \ln F = \frac{1}{2} \Delta^+_\alpha \lambda^{\alpha -} = \frac{1}{2} \partial^+_\alpha \lambda^{\alpha -} + \frac{1}{2} F_\alpha{}^{\dot\beta} \bar\partial^+_{\dot\beta} \lambda^{\alpha -} \ ,$$

$$\delta F_\alpha{}^{\dot\mu} = \Delta^+_\alpha \bar\lambda^{\dot\mu -} - \Delta^+_\alpha \lambda^{\beta -} F_\beta{}^{\dot\mu} =$$

$$= \partial^+_\alpha \bar\lambda^{\dot\mu -} + F_\alpha{}^{\dot\beta} \bar\partial^+_{\dot\beta} \bar\lambda^{\dot\mu -} - \partial^+_\alpha \lambda^{\beta -} F_\beta{}^{\dot\mu} - F_\alpha{}^{\dot\beta} \bar\partial^+_{\dot\beta} \lambda^{\beta -} F_\beta{}^{\dot\mu} \ . \tag{II.34}$$

Finally, (II.26) yields

$$\mathcal{D}^{++} A^+_{\hat\alpha} = 0 \ . \tag{II.35}$$

Now we turn our attention to the constraint (II.7). The vanishing of the torsion in that anticommutator allows to express the connection $A^+_{\hat\alpha}$:

$$A^+_{\alpha\beta\gamma} = 2 \varepsilon_{\alpha(\beta} \Delta^+_{\gamma)} F \tag{II.36}$$

$$A^+_{\alpha\dot\beta\dot\gamma} = -2 \left(\nabla^+_\alpha E_{(\dot\beta}{}^{\hat\mu}\right) E^{-1}{}_{\hat\mu \dot\gamma)} - 2 \left(\nabla^+_{(\dot\beta} E_\alpha{}^{\hat\mu}\right) E^{-1}{}_{\hat\mu \dot\gamma)} \ . \tag{II.37}$$

Note that $[\mathcal{D}^{++}, \nabla^+_{\hat\alpha}] = 0$ and $\mathcal{D}^{++} E_{\hat\alpha}{}^{\hat\mu} = 0$ imply (II.35). Another consequence of (II.7) is a futher restriction on $F_\alpha{}^{\dot\mu}$:

$$\Delta^+_{(\alpha} F_{\beta)}{}^{\dot\mu} = 0 \quad \Rightarrow \quad \{ \Delta^+_\alpha, \Delta^+_\beta \} = 0 \ . \tag{II.38}$$

Concluding this section we formulate the remaining problem. It is to find suitable expressions, in terms of the prepotantials for F, $F_\alpha{}^{\dot\mu}$ satisfying (II.31), (II.38). For this purpose we shall make use of the vielbeins of the harmonic derivative $\mathcal{D}^{--}$ which we are going to introduce in the next section.

III. Covariant derivatives $\mathcal{D}^{--}$ and $\mathcal{D}^-_{\hat\alpha}$

In the preceding section we considered part of the covariant derivatives in harmonic superspace $\mathcal{D}^+_{\hat\alpha}$, $\mathcal{D}^{++}$, $\mathcal{D}^0$. To complete the set of derivatives we need also $\mathcal{D}^{--}$, $\mathcal{D}^-_{\hat\alpha}$, $\mathcal{D}_{\alpha\dot\alpha}$ and $\mathcal{D}_5$. As we shall see, $\mathcal{D}^{--}$ is the main one, the others are simply defined by conventional constraints.

In the central basis $\mathcal{D}^{--}$ is flat (II.9) and obviously satisfies the relation

$$[\mathcal{D}^{++}, \mathcal{D}^{--}] = \mathcal{D}^0 \ . \tag{III.1}$$

Switching from central to analytic basis one creates new vielbeins (see (II.22)):

$$\mathcal{D}^{--}_{AB} = \partial^{--} + H^{--m,5} \partial_{m,5} + H^{--\hat\mu\pm} \partial^{\mp}_{\hat\mu}$$

$$\delta H^{--m,5,\hat\mu\pm} = \mathcal{D}^{--} \lambda^{m,5,\hat\mu\pm} \ . \tag{III.2}$$

They can be obtained from the bridges $\mathcal{V}$ (cf. (II.23)). However, the constraint (III.1) relates the vielbeins H^{--} directly to the prepotentials H^{++} (see (III.25))

$$\mathcal{D}^{++} H^{--m,5} - \mathcal{D}^{--} H^{++m,5} = 0$$

$$\mathcal{D}^{++} H^{--\hat\mu\pm} - \mathcal{D}^{--} H^{++\hat\mu\pm} = \pm \theta^{\hat\mu\pm}_A \ . \tag{III.3}$$

These are <u>linear</u> differential equations for the unknown H^{--} . Moreover, their solution is unique since the homogeneous equations $\mathcal{D}^{++} H^{--} = 0$ implies $H^{--} = 0$ /11/. Recalling the analogous equations (II.23) for the bridges $\mathcal{V}$ one sees that the latter are non-linear (the arguments of H^{++} are $Z_A = Z + \mathcal{V}(z,u)$) and their solutions contain an ambiguity corresponding to the gauge freedom (II.18). All this makes the equations (II.23) very difficult to solve.

In contrast, the equations (III.3) for H^{--} can be easily solved, as shown by B.M.Zupnik /20/. For this purpose one defines a new "quasi-flat" basis:

$$x_0^m = x_A^m + i(\theta_A^+\sigma^m\bar\theta_A^- + \theta_A^-\sigma^m\bar\theta_A^+),$$

$$x_0^5 = x_A^5 - i(\theta_A^{\mu+}\theta^-_{A\mu} - \bar\theta^+_{A\dot\mu}\bar\theta_A^{\dot\mu -}), \qquad \text{(III.4)}$$

$$\theta_0^{\hat\mu i} = u^{+i}\theta_A^{\hat\mu -} - u^{-i}\theta_A^{\hat\mu +}.$$

The covariant derivatives $\mathcal{D}^{++}$ and $\mathcal{D}^{--}$ become

$$\mathcal{D}^{++} = \partial^{++} + h^{++M}\partial^0_M,$$

$$\mathcal{D}^{--} = \partial^{--} + h^{--M}\partial^0_M. \qquad \text{(III.5)}$$

Here

$$h^{\pm\pm m}(z_0,u) = H^{\pm\pm m} + 2i\,\theta_0^\pm\sigma^m\bar\theta_0^\pm, \qquad \text{(III.6)}$$

$$h^{\pm\pm 5}(z_0,u) = H^{\pm\pm 5} - i(\theta_0^\pm\theta_0^\pm - \bar\theta_0^\pm\bar\theta_0^\pm),$$

$$h^{\pm\pm\hat\mu i}(z_0,u) = u^{+i}H^{\pm\pm\hat\mu -} - u^{-i}H^{\pm\pm\hat\mu +} \mp u^{\pm i}\theta_0^{\hat\mu\pm}$$

are the deviations of the vielbeins H^{++} from their flat-space values. In this notation the equations (III.3) take the following form:

$$\partial^{++}h^{--} - \partial^{--}h^{++} + [h^{++},h^{--}] = 0, \qquad \text{(III.7)}$$

where $h^{\pm\pm} = h^{\pm\pm M}\partial^0_M$. Equation (III.7) is very similar to the analogous equation in N=2 SYM theory and has the following unique solution /20/

$$h^{--}(z_0,u) = \sum_{n=1}^{\infty}\int du_1 \dots du_n \frac{(-1)^n\, h^{++}(z_0,u_1)\dots h^{++}(z_0,u_n)}{(u^+u_1^+)(u_1^+u_2^+)\dots(u_n^+u^+)}. \qquad \text{(III.8)}$$

The harmonic distributions $(u^+u_1^+)^{-1}$, $(u^+u_1^+)^{-2}$ were defined in /14/. They have the following properties:

$$\partial^{++}(u^+u_1^+)^{-2} = \partial^{--}\delta^{(2,-2)}(u,u_1),$$

$$\partial^{++}(u^+u_1^+)^{-1} = \delta^{1,-1}(u,u_1), \quad (u^+u_1^+) = -(u_1^+u^+),$$

which help to check that (III.8) is the solution of (III.7). Finally, one puts (III.8) into (III.6) and goes back to the coordinates (Z_A, u), thus obtaining the solution of (III.3).

Having constructed the harmonic covariant derivative $\mathcal{D}^{--}$ we can proceed to the definition of $\mathcal{D}^-_{\dot\alpha}$:

$$\mathcal{D}^-_{\dot\alpha} = [\mathcal{D}^{--},\mathcal{D}^+_{\dot\alpha}]. \qquad \text{(III.9)}$$

This relation is obvious in the central basis. To evaluate the commutator (III.9) in the analytic basis one has to use (II.19) and (III.2). Further, since $\mathcal{D}^{++}E_{\dot\alpha}^{\hat\mu} = 0$ (II.31) in the gauge (II.29), one can conclude that in the central basis $E_{\dot\alpha}^{\hat\mu}$ does not depend on $u^\pm$, so $\mathcal{D}^{--}E_{\dot\alpha}^{\hat\mu} = 0$ as well (which is of course true in any basis). Thus one finds

$$\mathcal{D}^-_{\dot\alpha} = -\nabla^+_{\dot\alpha}H^{--\hat\mu\pm}\partial^\mp_{\hat\mu} - \nabla^+_{\dot\alpha}H^{--m,5}\partial_{m,5} + A^-_{\dot\alpha},$$

$$A^-_{\dot\alpha} = \mathcal{D}^{--}A^+_{\dot\alpha}. \qquad \text{(III.10)}$$

Finally, the vector covariant derivative is defined as usual (see (II.3b)):

$$\mathcal{D}_{\alpha\dot\alpha} = \{\mathcal{D}^+_\alpha,\bar{\mathcal{D}}^-_{\dot\alpha}\} + \{\mathcal{D}^-_\alpha,\bar{\mathcal{D}}^+_{\dot\alpha}\}. \qquad \text{(III.11)}$$

For the derivative $\mathcal{D}_5$ we keep the expression (II.4), which generates further restrictions of F and $F_\alpha^{\hat\mu}$ (II.32) via the constraint (II.3a). In the next section we will present explicit expressions for F, $F_\alpha^{\hat\mu}$ in terms of the derivatives of H^{--}. By construction they will transform properly (II.34), which will guarantee that they satisfy all the restrictions. This will complete the differential geometry formalism.

IV. Building blocks. Superspace density

There are two ways to find the quantities F, $F_\alpha{}^{\dot\mu}$ which are needed to complete the definition of the covariant derivatives $\mathcal{D}^{\pm}_{\hat\alpha}$. One is to study the various consequences of the constraints (such as (II.31), (II.38) and others following from (II.4) which we have not written out explicitly) and try to solve them. Experience with N=1 SG /21/ has shown that it is much easier and more constructive to first find some "building blocks" with simple transformation laws built out of the prepotentials. Then it becomes an easy task to form the unknown quantities out of them. The guarantee that these will be the right objects will be their transformation laws. Indeed, if there were two different quantities with the same transformation laws and of dimension zero (as is the case of F, $F_\alpha{}^{\dot\mu}$) one could form a dimensionless tensor out of them, but we know from the WZ-gauge that there are no such tensors.

IV.1. A trip to 6 dimensions

A useful trick to find the building blocks we need will be a temporary extension of the dimension of space-time to 6. Then we can define "almost covariant" derivatives by taking (II.19) and (III.10) and replacing $F \to 1,\ F_\alpha{}^{\dot\mu} \to 0$:

$$\mathbb{D}^{\pm}_{\hat\alpha} = \mathbb{E}^{\pm}_{\hat\alpha}{}^{M}\partial_M + \mathbb{A}^{\pm}_{\hat\alpha}\ ; \qquad \text{(IV.1)}$$

$$\mathbb{E}^{+}_{\hat\alpha}{}^{\hat\mu -} = \delta_{\hat\alpha}{}^{\hat\mu},\quad \mathbb{E}^{+}_{\hat\alpha}{}^{\hat m,\hat\mu +} = 0,\quad \mathbb{E}^{-}_{\hat\alpha}{}^{\hat m,\hat\mu\pm} = -\partial^{+}_{\hat\alpha} H^{--\hat m,\hat\mu\pm}.$$

Here $\hat m = (m, 5, 6)$. They transform as follows

$$\delta\, \mathbb{D}^{\pm}_{\hat\alpha} = -\partial^{+}_{\hat\alpha}\lambda^{\hat\beta -}\,\mathbb{D}^{\pm}_{\hat\beta}\ , \qquad \text{(IV.2)}$$

and the connections $\mathbb{A}$ transform correspondingly (we shall not need $\mathbb{A}$ explicitly). Next we define and almost covariant 6-vector derivative:

$$\mathbb{D}_{[\hat\alpha\hat\beta]} = \{\mathbb{D}^{+}_{[\hat\alpha},\ \mathbb{D}^{-}_{\hat\beta]}\} = \mathbb{E}_{[\hat\alpha\hat\beta]}{}^{M}\,\partial_M + \mathbb{A}_{[\hat\alpha\hat\beta]}\ , \qquad \text{(IV.3)}$$

where

$$\mathbb{E}_{[\hat\alpha\hat\beta]}{}^{\hat m,\hat\mu +} = -\partial^{+}_{\hat\alpha}\partial^{+}_{\hat\beta} H^{--\hat m,\hat\mu +} - \mathbb{A}^{+}_{[\hat\alpha\hat\beta]}{}^{\hat\mu}\,\partial^{+}_{\hat\mu} H^{--\hat m,\hat\mu +} \qquad \text{(IV.4)}$$

(the expression for $\mathbb{E}_{[\hat\alpha\hat\beta]}{}^{\hat\mu -}$ will not be needed).
The trick now is to consider the 6x6 matrix which is used to calculate the Berezinian Ber $(\mathbb{E}_A{}^M)$:

$$e_{[\hat\alpha\hat\beta]}{}^{\hat m} = -\mathbb{E}_{[\hat\alpha\hat\beta]}{}^{\hat m} + \mathbb{E}_{[\hat\alpha\hat\beta]}{}^{M}\,(\mathbb{E}^{-1})_M{}^A\,\mathbb{E}_A{}^{\hat m} =$$
$$= \partial^{+}_{\hat\alpha}\partial^{+}_{\hat\beta} H^{--\hat m} - \partial^{+}_{\hat\alpha}\, e_{\hat\beta}{}^{\hat\mu}\cdot e^{-1}_{\hat\mu}{}^{\hat\nu}\cdot\partial^{+}_{\hat\nu} H^{--\hat m}, \qquad \text{(IV.5)}$$

where

$$e_{\hat\alpha}{}^{\hat\mu} = \partial^{+}_{\hat\alpha} H^{--\hat\mu +}. \qquad \text{(IV.6)}$$

This matrix is built out of known quantities (the derivatives of H^{--}) and is expected to transform homogeneously. Indeed, using (II.17), (III.2) and the analyticity of the parameters $\lambda^{\hat m,\hat\mu +}$ one can check that

$$\delta e_{[\hat\alpha\hat\beta]}{}^{\hat m} = e_{[\hat\alpha\hat\beta]}{}^{\hat n}\,\lambda_{\hat n}{}^{\hat m} - \partial^{+}_{\hat\alpha}\lambda^{\hat\mu -}\, e_{[\hat\mu\hat\beta]}{}^{\hat m} + \partial^{+}_{\hat\beta}\lambda^{\hat\mu -}\, e_{[\hat\alpha\hat\mu]}{}^{\hat m};$$

$$\lambda_{\hat n}{}^{\hat m} = \partial_{\hat n}\lambda^{\hat m} - \partial_{\hat n}\lambda^{\hat\mu +}\, e^{-1}_{\hat\mu}{}^{\hat\nu}\,\partial^{+}_{\hat\nu} H^{--\hat m}. \qquad \text{(IV.7)}$$

We would like to note that the 6-dimensional construction above was purely auxiliary. Nevertheless, it may prove relevant in a future attempt to formulate 6-dimensional SG in superspace.

IV.2. Expressions for F and $F_\alpha{}^{\dot\mu}$

Returning to our 4 (+1) dimensional world we can split the matrix $e_{[\hat\alpha\hat\beta]}{}^{\hat m}$ in the following way:

$$e_{[\hat\alpha\hat\beta]}{}^{\hat m} = \begin{pmatrix} e_{\alpha\dot\beta}{}^{m} & e_{\alpha\dot\beta}{}^{5} & 0 \\ e^{m}\varepsilon_{\alpha\beta} & e^{5}\varepsilon_{\alpha\beta} & 0 \\ \tilde e^{m}\varepsilon_{\dot\alpha\dot\beta} & \tilde e^{5}\varepsilon_{\dot\alpha\dot\beta} & 0 \end{pmatrix} \qquad \text{(IV.8)}$$

(the zeros stand for the sixth components of $\hat m$). From (IV.7) we find the transformation laws of the entries in (IV.8):

$$\delta e^{m,5}_{\alpha\dot\beta} = e^{n}_{\alpha\dot\beta}\,\lambda_n^{m,5} - \partial^+_\alpha \lambda^{\mu -} e^{m,5}_{\mu\dot\beta} - \bar\partial^+_{\dot\beta}\bar\lambda^{\dot\mu -} e^{m,5}_{\alpha\dot\mu} - \\ -\bar\partial^+_{\dot\beta}\lambda^-_\alpha e^{m,5} + \partial^+_\alpha \bar\lambda^-_{\dot\beta}\,\tilde e^{m,5}\,; \qquad (a)$$

$$\delta e^{m,5} = e^{n}\lambda_n^{m,5} - \partial^+_\alpha \lambda^{\alpha -} e^{m,5} + \partial^{+\alpha}\bar\lambda^{\dot\beta -} e^{m,5}_{\alpha\dot\beta}\,. \; (b) \qquad \text{(IV.9)}$$

There are two sorts of terms in (IV.9): vector and spinor rotations. Since the quantities F, $F_\alpha{}^{\dot\mu}$ that we are looking for have only terms of the second kind in their laws (II.34), we shall get rid of the $\lambda_n{}^{m,5}$ terms in (IV.9). Define the following objects:

$$f^{\alpha\dot\alpha} = e^m e_m^{\alpha\dot\alpha}\,, \quad e^m_{\alpha\dot\alpha}\, e_m^{\beta\dot\beta} = \delta_\alpha{}^\beta \delta_{\dot\alpha}{}^{\dot\beta}\,; \qquad (a)$$

$$f^5 = e^5 - e^m e_m^{\alpha\dot\alpha}\, e^5_{\alpha\dot\alpha}\,. \qquad (b) \qquad \text{(IV.10)}$$

Their transformation laws are

$$\delta f^{\alpha\dot\alpha} = \partial^{+\alpha}\bar\lambda^{\dot\alpha -} + f^{\beta\dot\alpha}\partial^{+\alpha}\lambda^-_\beta + f^{\alpha\dot\beta}\bar\partial^+_{\dot\beta}\bar\lambda^{\dot\alpha -} - \\ - f^{\beta\dot\beta}\partial^+_\beta \bar\lambda^-_{\dot\beta}\,\tilde f^{\alpha\dot\alpha} + \tilde f^{\beta\dot\beta}\bar\partial^+_{\dot\beta}\lambda^-_\beta\, f^{\alpha\dot\alpha}\,; \qquad (a)$$

$$\delta f^5 = f^5(-\partial^+_\alpha\lambda^{\alpha -} + f^{\alpha\dot\alpha}\bar\partial^+_{\dot\alpha}\lambda^-_\alpha) - \tilde f^5 f^{\alpha\dot\alpha}\partial^+_\alpha\bar\lambda^-_{\dot\alpha}\,. \; (b) \qquad \text{(IV.11)}$$

Finally, combining the above results, after some work, we arrive at the following expressions for $F_\alpha{}^{\dot\mu}$, F :

$$F_\alpha{}^{\dot\mu} = f_\alpha{}^{\dot\mu} + \tilde f_\alpha{}^{\dot\mu}\cdot A\,,$$

$$F = (f^5 + \tilde f^5\cdot A)^{-\frac{1}{2}}\,, \qquad \text{(IV.12)}$$

where

$$A = f^2\left[1 - f\tilde f + \sqrt{(1-f\tilde f)^2 - f^2\tilde f^2}\,\right]^{-1},$$

$$f^2 \equiv f_{\alpha\dot\alpha} f^{\alpha\dot\alpha}\,, \quad f\tilde f \equiv f_{\alpha\dot\alpha}\tilde f^{\alpha\dot\alpha}\,. \qquad \text{(IV.13)}$$

One can check that they have the desired transformation properties (II.34).

IV.3. Superspace density

At this point we already have all the ingredients of the differential geometry formalism expressed in terms of the prepotentials. This allows us to calculate another very important object, the Berezinian of the vielbein $E_A{}^M$ entering the spinor and vector covariant derivatives (II.19), (III.10), (III.11). It is needed as a density which compensates the transformations of the volume element $d^4x_A\, d^4\theta^+_A\, d^4\theta^-_A\, du$:

$$E = \mathrm{Ber}\, E_A{}^M\,,$$

$$\delta E = (\partial_m \lambda^m - \partial^\pm_\mu \lambda^{\mu\mp})\, E\,. \qquad \text{(IV.14)}$$

Once again, instead of calculating E directly it is easier to compile it from the building blocks considered above. Indeed, one can check that the following expression

$$E = \det(e^m_{\alpha\dot\alpha})\,\det{}^{-1}(e_{\hat\alpha}{}^{\hat\mu})\sqrt{(1-f\tilde f)^2 - f^2\tilde f^2} \qquad \text{(IV.15)}$$

transforms as required by (IV.14). As intermediary steps one should show that

$$\delta e_{\hat\alpha}{}^{\hat\mu} = -\partial^+_{\hat\alpha}\lambda^{\hat\nu -} e_{\hat\nu}{}^{\hat\mu} + e_{\hat\alpha}{}^{\hat\nu}\partial^-_{\hat\nu}\lambda^{\hat\mu +} + \\ + \partial^+_{\hat\alpha} H^{--m}\partial_m \lambda^{\hat\mu +}\,, \qquad \text{(IV.16)}$$

$$\delta \ln\sqrt{(1-f\tilde f)^2 - f^2\tilde f^2} = f^{\alpha\dot\alpha}\bar\partial^+_{\dot\alpha}\lambda^-_\alpha - \tilde f^{\alpha\dot\alpha}\partial^+_\alpha\bar\lambda^-_{\dot\alpha}\,.$$

An interesting property of the density (IV.15) is that it depends only on H^{--m} and $H^{--\hat\mu +}$, but does not involve H^{--5}. The reason (which will become clear in [18]) is that E is essentially a conformally covariant object. On the other hand H^5 will serve as a compensator for some conformal transformations in the action for N=2 Einstein SG, which will be given in [17].

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Received by Publishing Department
on February 12, 1987.

Гальперин А.С., Нгуен Ан Ки, Сокачев Е. E2-87-84

N=2 супергравитация в суперпространстве: решение связей

Это первая из серии работ, в которых развивается внемассовая формулировка без связей N=2 супергравитации в гармоническом суперпространстве. Здесь мы построим из препотенциалов /аналитических реперов гармонической ковариантной производной D^{++}/ элементы дифференциальной геометрии. Тем самым найдено решение связей первой внемассовой версии эйнштейновской N=2 супергравитации. Из препотенциалов построен также ряд полезных "кубиков" с простыми трансформационными свойствами. Из них составлена плотность для полного суперобъема гармонического супрепространства.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1987

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N=2 Supergravity in Superspace: Solution to the Constraints

This is the first of a series of papers in which we develop the off-shell unconstrained formulation of N=2 supergravity in harmonic superspace. Here we construct the elements of differential geometry in terms of the prepotentials (the analytic vielbeins of the harmonic covariant derivative D^{++}). Thus, we find the solution to the constraints of the first off-shell version of Einstein N=2 supergravity. A number of useful "building blocks" with simple transformation laws are constructed from the prepotentials. They are used to write down a density for the full supervolume of harmonic superspace.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987