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A.T.Filippov

GAUGE THEORIES OF PARTICLES, STRING AND CORRESPONDING FIELDS

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1. The recent development of the superstring theory revealed a necessity to reconsider some basic ideas of the standard local quantum field theory. It gradually became clear that a relation between the quantum theory of the relativistic particle and the corresponding quantum field theory had been poorly understood. In fact, the transition from the quantum theory of one noninteracting relativistic string (so-called "first" quantized theory) to the interacting string field theory ("second" quantization) proved to be quite a nontrivial problem. For example, in Refs. /1,2/ such a transition is based on a new recipe for going from the relativistic one-particle theory to the relativistic quantum field theory. The most important ingredient of this prescription is a gauge-like formulation of the relativistic particle theory suggested in Ref. /3/ (for a somewhat more accurate treatment of the gauge supergroup in the theory of spinning particles see Ref. /4/, a detailed study of quantization is given in /5/).

"Gauge-like" means that the constraints (such as $p^2+m^2=0$, $p\xi=0$, etc.) are viewed as generators of the reparametrization symmetry and of the local supersymmetry. Accordingly, in /3/ and /5/ the relativistic quantum theory of spinning particles was treated by a (super) generalized Dirac approach to systems with constraints. The modern approach to the quantization of such systems /1/ consists in trading these gauge transformations for the corresponding BRST symmetry. The generator of this symmetry, so-called BRST charge, Ω , plays a prime role in the transition from the one-particle theory to the quantum field theory. Too complicated though it is at first glance this approach exposed new symmetries of the relativistic particle theory /1,3,6/, and what is more important, it can directly be applied to constructing field theories of string /1,2/. Alternatively, one can employ a combination of the BRST and BFV /7/ techniques for constructing perturbation theory diagrams from propagators (see, e.g., recent discussion of such an approach to the relativistic particle theory /8/). In the BRST invariant BFV Lagrangian all variables - original coordinates and momenta, ghost variables, and Lagrange multipliers - are formally treated on equal footing.

This motivates an attempt /9/ to construct the relativistic theory of particles and strings starting from some rudimentary Lagrangian with its rigid symmetries. By gauging some of these symmetries one can generate new, gauge invariant Lagrangians which can be interpreted in terms of relativistic particles and string, respectively.

We begin with constructing the theory of relativistic scalar particles by gauging linear canonical symmetries of the simplest rudimentary Lagrangian

$$L_0 = \frac{1}{2} g_{\mu\nu}\, \dot{q}^{\mu}(t)\, \dot{q}^{\nu}(t)\,, \quad \mu,\nu = 0,1,\dots,D-1\,, \tag{I}$$

where $\dot{q} = dq/dt$ and $g_{\mu\nu}$ is a constant matrix. By linear transformations $q^{\mu} \to L^{\mu}_{\ \nu} q^{\nu}$ we can diagonalize $g_{\mu\nu}$. Neglecting variables corresponding to zero eigenvalues we may regard $g_{\mu\nu}$ in Eq.(I) as a diagonal matrix with ± 1 eigenvalues. The resulting Lagrangian is obviously invariant with respect to rigid Lorentz-like transformations preserving this matrix. This symmetry does not concern us here and is trivially satisfied in what follows.

Consider canonical symmetries of L_0. Defining the canonical momenta $p_{\mu} = \partial L_0 / \partial q^{\mu}$, we rewrite L_0 in the first-order form

$$L_0 = \dot{q} p - \frac{1}{2} p^2 = \frac{1}{2}(\dot{q} p - \dot{p} q) - \frac{1}{2} p^2 + \frac{1}{2}(qp)^{\cdot}. \tag{2}$$

Up to the boundary conditions, which will be discussed below, the last term here can be neglected. The kinetic part of L_0 is invariant with respect to infinitesimal canonical transformations

$$\delta p = -\frac{\partial G}{\partial q}\,, \quad \delta q = \frac{\partial G}{\partial p}\,, \quad G = \sum_i f_i\, G_i(p, q)\,, \tag{3}$$

where f_i are independent of t. Allowing for their dependence on t we obtain $\left(L_0^{(K)} = \frac{1}{2}(\dot{q} p - \dot{p} q) \right)$

$$\delta L_0^{(K)} = \sum_i \dot{f}_i G_i + \frac{d}{dt}\left[\frac{1}{2}(p\partial_p + q\partial_q) G - G \right]. \tag{4}$$

For homogeneous linear transformations the dimension of G is 2 and therefore

$$\delta L_0^{(K)} = \sum_i \dot{f}_i G_i\,. \tag{5}$$

If the algebra of generators G_i is closed with respect to Poisson brackets, the Lagrangian

$$\frac{1}{2}(\dot{q} p - \dot{p} q) - \sum \ell_i G_i$$

is invariant under transformations (3) satisfying (5), provided that the functions $\ell_i(t)$ undergo easily obtainable gauge-like transformations.

However, due to the term $-\frac{1}{2}p^2$ in our rudimentary Lagrangian (2), the choice of the transformations is severely restricted. In this case, the only essentially independent constraint is $p^2 = 0$ (in general, constraints are $G_i = 0$), and this can be obtained by considering the linear canonical transformations

$$G = \frac{1}{2} f_1 p^2 + f_2 (pq) + \frac{1}{2} f_3 q^2. \tag{6}$$

Anyhow, as far as we insist on the standard formulation of the gauging procedure, considering $\Psi \equiv (p, q)$ as a matter field and $\ell_i(t)$ as a gauge potentials, the linearity restriction seems to be unavoidable.

Now, the transformation (3) with G given by Eq.(6) is, in the matrix form,

$$\delta\Psi = F\Psi\,, \quad F = \begin{pmatrix} -f_2 & -f_3 \\ f_1 & f_2 \end{pmatrix}. \tag{7}$$

The matrices F generate the linear canonical group $SL(2,R) \sim SU(1,1)$. The full Lagrangian L_0 is invariant with respect to its abelian subgroup

$$\delta p = 0\,, \quad \delta q = f_1 p\,; \qquad F_1 = \begin{pmatrix} 0 & 0 \\ f_1 & 0 \end{pmatrix}. \tag{8}$$

To use the standard Yang-Mills procedure of gauging we rewrite $L_0^{(K)}$ as

$$L_0^{(K)} = \frac{1}{4}\left(\Psi^{T} i\sigma_2 \frac{d\Psi}{dt} - \left(\frac{d\Psi}{dt}\right)^{T} i\sigma_2 \Psi \right), \tag{9}$$

where σ_2 is the usual Pauli matrix. The generator F_1 is of the form $F_1 = f_1 \sigma_-$, where $\sigma_- = \frac{1}{2}(\sigma_1 - i\sigma_2)$, and we naturally define the gauge potential $A = \ell_1 \sigma_-$. Introducing it in Eq.(9) we arrive at the gauge-invariant Lagrangian

$$L_1 = \frac{1}{4}\left\{ \Psi^{T} i\sigma_2 \left(\frac{d}{dt} - A\right)\Psi - \left[\left(\frac{d}{dt} - A\right)\Psi\right]^{T} i\sigma_2 \Psi \right\} = p\dot{q} - \frac{\ell_1}{2} p^2. \tag{IO}$$

This is a quite conventional abelian gauge theory. As for all linear canonical transformations, $\Psi' = U\Psi$, the antisymmetric matrix σ_2

is invariant, $\overset{T}{U}\sigma_2 U = \sigma_2$, the Lagrangian (I0) is invariant provided

$$A' = UAU^{-1} + \dot{U}U^{-1}. \qquad \text{(II)}$$

The matrices of the finite transformations corresponding to Eq.(8) are

$$U_1 = 1 + f_1\sigma_-\,, \quad U_1^{-1} = 1 - f_1\sigma_-\,.$$

The physical interpretation of our gauge theory (I0) is obvious. if we choose for $g_{\mu\nu}$ the Minkowski signature $(1, D-1)$, i.e. $g_{\mu\nu}=(-1,+1,\dots,+1)$. In this case the Lagrangian (I0) describes massless, free relativistic particle moving in $(D-1)$ -dimensional space (D -dimensional Minkowski space-time). Of course, in the classical case one might take any other signature $(T, D-T)$ with $T > 1$. However, the quantum theory of the gauge invariant Lagrangian (I0) is consistent only for the Minkowski signature, otherwise the Hilbert space of the system has indefinite metric.

The most adequate approach to the quantization of this model is based on the BRST-BFV technique. To see the reason for this we have to remember the suppressed term $\frac{1}{2}(pq)^{\cdot}$. One can easily prove that this term determines the boundary conditions for the gauge parameter:

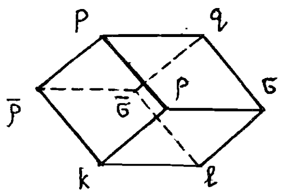
$$\delta S_1 = \delta\int_0^1 dt\,\tfrac{1}{2}(pq)^{\cdot} = \tfrac{1}{2} f_1 p^2\Big|_0^1 = 0 \;\Rightarrow\; f_1(0) = f_1(1) = 0\,, \qquad (12)$$

as far as the symmetry is independent of the equations of motion (i.e. we are not allowed to use the constraint $p^2=0$ in Eq.(12)). This boundary condition determines a nontrivial structure of the gauge group - the zero mode of the gauge potential,

$$\ell_1^{(0)} \equiv \int_0^1 dt\,\ell_1(t) = \int_0^1 dt\,\left[\ell_1(t) + \dot{f}_1(t)\right], \qquad (13)$$

cannot be changed by any admissible gauge transformation. Thus, the gauge transformations are subdivided in the gauge-inequivalent classes enumerated by one real parameter $\ell_1^{(0)}$ (the "Teichmuller parameter") of this group) . An important consequence of this fact is that one cannot choose as a gauge condition simply $\ell_1 \equiv 1$ or $\ell_1 \equiv 0$. The simplest possible gauge choice is that proposed by Fradkin and Vilkovisky [7] : $\dot{\ell}_1 = 0$.

A most algorithmic approach to quantizing our gauge theory uses an extended phase space which we visualize on a "ghost cube" :



$$Q = (q, \ell, \sigma, \bar\sigma); \quad Q_{gh}(\sigma,\rho) = +1$$
$$\mathcal{P} = (p, k, \bar\rho, \rho); \quad Q_{gh}(\bar\sigma,\bar\rho) = -1$$
$$Q_{gh}(p, q, k, \ell) = 0$$

Here the Greek letters are used for Fermi variables, and Latin, for Bose ones. The canonically conjugate pairs are $(pq), (\rho\bar\sigma), (\bar\rho\sigma), (k\ell)$. The standard BRST-generator is

$$\Omega = ik\rho + \sigma p^2. \qquad (14)$$

The simplest BRST-invariant Lagrangian with the gauge condition $\dot{\ell} = 0$ is

$$L = \dot{Q}\mathcal{P} - \mathcal{H} = \dot{q}p + \dot{\ell}k + \dot{\sigma}\bar\rho + \dot{\bar\sigma}\rho - \tfrac{i}{2}\rho\bar\rho - \tfrac{\ell}{2}p^2. \qquad (15)$$

Now the quantization is straightforward both in the operator and path-integral approach; some details may be found in Refs.[6-8] .

However, there is one controversial point in applying the path-integral prescription. To calculate the quantum probability for the particle transition from $q^\mu(0)$ to $q^\mu(1)$ one integrates e^{iS} over all dynamical variables. The only essential integration proves to be the one over the Teichmüller parameter $\ell^{(0)}$:

$$P\left(q^\mu(0), q^\mu(1)\right) \sim \int dp \int d\ell^{(0)}\, e^{-ip(q(1)-q(0))}\, e^{-i\frac{\ell^{(0)}}{2}p^2}.$$

Now, if $-\infty < \ell^{(0)} < +\infty$, we obtain the imaginary part of the propagator $(p^2 - i0)^{-1}$. One can construct the full propagator by adding to this the probability of the "time-reversed" transition (in addition, one has to multiply p^2 by $\theta(p^0)$; for details see Ref.[8]). Alternatively, we may insist on having the term p^2 unmodified and use the transition to the Euclidean formulation, with the change $p^2 \to p^2 - i0$. One can see that the corresponding $\ell^{(0)}$ integration is well-defined only for $0 \leqslant \ell^{(0)} < \infty$. It would be nice to find some more direct and rigorous theoretical argument giving this restriction for $\ell^{(0)}$. Note that some authors use even more strong assumption $\ell(t) \geqslant 0$ without any discussion, see e.g. [6]. This condition is not gauge invariant. A more detailed motivation for using the gauge invariant restriction $\ell^{(0)} \geqslant 0$ is presented in

Refs. /10/ where, to my knowledge, the first discussion of the gauge-inequivalent classes in the theory of relativistic particles has been given.

Returning to the Lagrangian (10) one can see that it has an additional rigid symmetry

$$\delta p = -f_2 p\,,\quad \delta q = f_2 q\,,\quad \delta \ell_1 = 2 f_2 \ell_1 \tag{16}$$

which is simply the dilatation invariance. Gauging this symmetry gives a new theory with the gauge potential

$$A = \ell_1 \sigma_- - \ell_2 \sigma_3 = \begin{pmatrix} -\ell_2 & 0 \\ \ell_1 & \ell_2 \end{pmatrix} \tag{17}$$

transforming according to Eq. (11), where

$$U = e^{-f_2 \sigma_3}(1 + a_1 e^{-a_2}\sigma_-) = \begin{pmatrix} e^{-f_2} & 0 \\ f_1 & e^{f_2} \end{pmatrix}. \tag{18}$$

Now the same construction as used to obtain Eq. (10) gives the Lagrangian

$$L_2 = \tfrac{1}{2}(p\dot{q} - q\dot{p}) - \tfrac{1}{2}\ell_1 p^2 - \ell_2 (pq)\,. \tag{19}$$

This theory is non-abelian since

$$\delta \ell_1 = \dot{f}_1 + 2 f_2 \ell_1 - 2 f_1 \ell_2\,,\qquad \delta \ell_2 = \dot{f}_2 \tag{20}$$

(one can easily obtain also the finite transformations). At first sight, this Lagrangian looks translation-noninvariant but in fact there is no restriction on ℓ_2 , and one may choose the gauge $\ell_2 = 0$ in which the translation invariance is manifest.

The BRST-BFV approach to quantizing the Lagrangian (19) is straightforward. Using general rules the operator Ω can be constructed and proved to be nilpotent. To further understand the physical meaning of this apparently new theory of massless scalar particles (dilaton?), the interactions have to be carefully treated. This problem deserves a separate consideration which will be published elsewhere.

Now, we construct the gauge theory of spinning particles. A simplest choice for the rudimentary Lagrangian is

$$L_0 = \tfrac{1}{2}(p\dot{q} - q\dot{p}) - \tfrac{1}{2}p^2 - \tfrac{i}{2}\xi_k \dot{\xi}_k\,, \tag{21}$$

where ξ_k^μ are anticommuting (Grassmann) variables, $k = 1, \ldots, K$. The group of linear canonical transformations in this case is $OSp(1,1/K)$. For gauging the subgroup of this (super)group leaving the Lagrangian (21) invariant we introduce the notation $\Psi \equiv (p, q, \xi_1, \ldots \xi_K)$ and replace $i\sigma_2$ in Eq. (9) by the matrix

$$C = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\mathbb{1} \end{pmatrix}.$$

The matrix of the (super)gauge transformations $\delta\Psi = F\Psi$ and the corresponding matrix of the gauge potential are

$$F = \left(\begin{array}{cc|c} -f_2 & 0 & 0 \;\ldots\; 0 \\ f_1 & f_2 & i\varphi_1 \ldots\; i\varphi_K \\ \hline \varphi_1 & 0 & \\ \vdots & \vdots & f_{ik} \\ \varphi_K & 0 & \end{array}\right) \qquad A = \left(\begin{array}{cc|c} -\ell_2 & 0 & 0 \;\ldots\; 0 \\ \ell_1 & \ell_2 & i\lambda_1 \ldots\; i\lambda_K \\ \hline \lambda_1 & 0 & \\ \vdots & \vdots & \ell_{ik} \\ \lambda_K & 0 & \end{array}\right), \tag{22}$$

where $f_{ik} = -f_{ki}$, $\ell_{ik} = -\ell_{ki}$ (the finite transformation U is easy to obtain). In addition to the bosonic transformations (f_1, f_2) and the local supergauge transformations $(\varphi_1 \ldots \varphi_K)$ we obtain the O(K)-rotations (f_{ik}) . Substituting the gauge potential A into Eq.(10) (with $i\sigma_2 \Rightarrow C$), one can easily calculate the gauge invariant Lagrangian

$$L_1 = p\dot{q} - \tfrac{i}{2}\xi_k \dot{\xi}_k - \tfrac{1}{2}\ell_1 p^2 - \ell_2 (pq) - i\lambda_k (p\xi_k) + \tfrac{i}{2}\xi_i \ell_{ij} \xi_j \tag{23}$$

which, for $\ell_2 \equiv 0$, coincides with that derived in Ref. /5/ following the approach to Ref. /3/ . The standard Dirac quantization has been applied to the constrained system (23) in these references. Having formulated the theory as a real gauge theory it is more natural to apply the BRST-BFV approach. This can be done exactly as in the spinless case. Note that the new supergauge-invariant parameters,

$$\lambda_j^{(0)} = \int_0^1 \lambda_j(t)\,dt = \int_0^1 [\lambda_j(t) + \dot{\varphi}_j(t)]\,dt\,,$$

are generated in this case owing to the boundary conditions

$$\varphi_j(0) = \varphi_j(1) = 0\,.$$

To give a gauge formulation for string theories, we consider the simplest rudimentary Lagrangian depending on the variables $x^\mu(s,t)$ $(x'^\mu \equiv \partial x^\mu / \partial s)$:

$$\mathcal{L}_0 = \frac{1}{2}(\dot{x}^2 - x'^2) = p\dot{x} - \frac{1}{2}(p^2 + x'^2). \tag{24}$$

For simplicity we assume $g_{\mu\nu} = (-1, +1, \dots, +1)$ but the signature of $g_{\mu\nu}$ could be fixed exactly as in the particle case. The $(-1, +1)$ metric for the variables t, s is necessary for obtaining a well-defined classical Cauchy problem. Define $\Psi^T = (\Psi_1, \Psi_2) = (p, x)$ and $\partial \equiv \partial/\partial s$. We are to regard the variable s as a continuous index, and accordingly, the transformation matrix $\mathcal{U}$ is a 2x2 matrix with elements depending on powers of ∂ -operator.

By simple calculations one can easily find that the Lagrangian (24) is invariant under transformations

$$\delta\Psi = \partial_+ F \partial_-\,, \quad \partial_+ \equiv \begin{pmatrix} \partial & 0 \\ 0 & 1 \end{pmatrix}, \quad \partial_- \equiv \begin{pmatrix} 1 & 0 \\ 0 & \partial \end{pmatrix}, \quad F = \begin{pmatrix} f_1 & f_2 \\ f_2 & f_1 \end{pmatrix}. \tag{25}$$

For rigid transformations the order of the operators ∂_+, ∂_- is immaterial. However, for f_i depending on s, t the algebra of the transformations (25) is closed if and only if the operators ∂_+ and ∂_- are ordered as specified in Eq. (25). Then, the commutator of any two transformations δ and $\bar{\delta}$ depending on parameters f_i and $\bar{f}_i$ is of the form

$$[\delta, \bar{\delta}] = \partial_+ F_{[\delta,\bar{\delta}]} \partial_-\,, \quad F_{[\delta,\bar{\delta}]} = F_{\bar{\delta}} F'_{\delta} - F_{\delta} F'_{\bar{\delta}}\,, \tag{26}$$

where the matrix $F_{[\delta,\bar{\delta}]}$ is of the same form as in Eq.(25)

$$F_{[\delta,\bar{\delta}]} = \begin{pmatrix} W[\bar{f}_1, f_1] + W[\bar{f}_2, f_2] & W[\bar{f}_1, f_2] + W[\bar{f}_2, f_1] \\ W[\bar{f}_1, f_2] + W[\bar{f}_2, f_1] & W[\bar{f}_1, f_1] + W[\bar{f}_2, f_2] \end{pmatrix} \tag{27}$$

$$W[f, g] \equiv f g' - g f'.$$

It is natural to define the gauge potential

$$\mathcal{A} = \partial_+ A \partial_-\,, \quad A = \begin{pmatrix} \ell_1 & \ell_2 \\ \ell_2 & \ell_1 \end{pmatrix}. \tag{28}$$

Then, the gauge invariant Lagrangian corresponding to the rudimentary Lagrangian (24) is

$$\mathcal{L}_1 = \frac{1}{2} \Psi^T i\sigma_2 (\partial_t + \mathcal{A}) \Psi \tag{29}$$

while the gauge transformations have the usual form

$$\delta\Psi = \mathcal{F}\Psi, \quad \delta\mathcal{A} = \dot{\mathcal{F}} + [\mathcal{A}, \mathcal{F}]\,, \quad \mathcal{F} \equiv \partial_+ F \partial_-\,. \tag{30}$$

In the standard notation

$$\mathcal{L}_1 = p\dot{x} + \ell_1(px') + \frac{1}{2}\ell_2(p^2 + x'^2). \tag{31}$$

Denoting $p^a = (p^0, p^1)$, where $p^0 = p$, $p^1 = \partial\mathcal{L}_1/\partial x'$, the Lagrangian $\mathcal{L}_1$ can be written as

$$\mathcal{L}_{1,s} = p^a \partial_a x - \frac{1}{2} G_{ab} p^a p^b, \quad \partial_a = (\partial_t, \partial_s) \tag{32}$$

with

$$G_{ab} = \frac{1}{\ell_2}\begin{pmatrix} (\ell_1^2 - \ell_2^2) & -\ell_1 \\ -\ell_1 & 1 \end{pmatrix}, \quad \det G_{ab} = -1.$$

Defining the inverse matrix

$$G^{ab} = -\frac{1}{\ell_2}\begin{pmatrix} 1 & \ell_1 \\ \ell_1 & (\ell_1^2 - \ell_2^2) \end{pmatrix}, \quad G^{ab} G_{a'b} = \delta^a_{a'}$$

and introducing the 2-dimensional "metric" tensor g_{ab}

$$g_{ab}/\sqrt{-g} = G_{ab}\,, \quad g^{ab}\sqrt{-g} = G^{ab}\,, \quad g \equiv \det g_{ab}$$

one can rewrite the theory in the standard geometric form

$$\mathcal{L}_g = p^a \partial_a x - \frac{1}{2}\frac{1}{\sqrt{-g}} g_{ab} p^a p^b = \frac{1}{2} g^{ab}\sqrt{-g}\, \partial_a x\, \partial_b x. \tag{33}$$

In addition to the gauge symmetry this action is invariant under the Weyl transformations

$$g_{ab} \to e^f g_{ab}\,, \quad g^{ab} \to e^{-f} g^{ab}\,, \quad x \to x\,, \quad p \to p.$$

One can make a step further expressing g_{ab} in terms of the "zweibein" : $g_{ab} = e_a^m g^{(0)}_{mn} e_b^n$. Written in terms of e_a^m the Lagrangian has the additional invariance under the two-dimensional Lorentz rotation $e_a^m \to L_a^b e_b^m$, where $L g L^T = g$. Thus, the full two-dimensional symmetry group of the string is

Weyl $\otimes$ 2- Lorentz $\otimes$ Gauge ·

An intriguing relation of the gauge string theory to the gauge theory of the scalar relativistic particle (19) can be observed at this point. Written in terms of e_a^m , the Lagrangian (31) corresponds to the following special choice of the zweibein

$$(e_a^m) = \frac{1}{\sqrt{\ell_2}}\begin{pmatrix} \ell_2 & -\ell_1 \\ 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} e^{a_2} & a_1 \\ 0 & e^{-a_2} \end{pmatrix}$$

which is isomorphic to Eq. (I8) . Thus, the gauge potentials parametrize the group manifold of the gauge symmetry group (I8). Note also that the generators of this group, σ_3 and $\sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$, form the minimal non-abelian subalgebra of the Virasoro algebra as $[\sigma_3, \sigma_+] = \sigma_+$. At this moment, it is hardly possible to judge a real significance of this coincidence. However, the role played by the canonical group $SL(2,R)$ in the gauge string theory seems to be quite remarkable.

Finally, we construct the gauge theory of the spinning string. Introduce the right-moving and left-moving Grassman variables ξ_2^μ, ξ_1^μ and consider the rudimentary Lagrangian ($\xi^T \equiv (\xi_1, \xi_2)$) :

$$\mathcal{L}_0 = p\dot{x} - \frac{1}{2}(p^2 + x'^2) + \frac{i}{2}\xi^T\dot{\xi} - \frac{i}{2}\xi^T\sigma_3\xi' . \tag{34}$$

It is not difficult to find its symmetry

$$\delta\Psi = \hat{\mathfrak{F}}\Psi = \hat{\partial}_+ \hat{F} \hat{\partial}_- \Psi , \quad \Psi^T \equiv (p, x, \xi_1, \xi_2), \tag{35}$$

where

$$\hat{\partial}_\pm \equiv \left(\begin{array}{c|c} \partial_\pm & 0 \\ \hline 0 & \mathbb{1} \end{array}\right), \quad \hat{F} = \left(\begin{array}{c|c} F & \varphi \\ \hline \tilde{\varphi} & \tilde{F} \end{array}\right), \tag{36}$$

$\partial_\pm$ and F are given in Eq. (25), and

$$\varphi = -i\begin{pmatrix} \varphi_1 & -\varphi_2 \\ \varphi_1 & \varphi_2 \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \varphi_1 & \varphi_1 \\ \varphi_2 & -\varphi_2 \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} f_+\partial + \partial f_+ & 0 \\ 0 & f_-\partial + \partial f_- \end{pmatrix}$$

with $f_\pm = \frac{1}{2}(f_1 \pm f_2)$. Here $\varphi, \tilde{\varphi}$ are easily obtained by direct calculations while for finding $\tilde{F}$ one has to use the closure requirement $[\delta, \bar{\delta}] \sim \bar{\bar{\delta}}$. Defining the gauge potential $\hat{\mathcal{A}} = \hat{\partial}_+ \hat{A} \hat{\partial}_-$, where $\hat{A}$ is obtained from A by the change $f_i \to \ell_i$, $\varphi_i \to \lambda_i$, and using the standard procedure we arrive at the gauge-invariant Lagrangian for the spinning string

$$\mathcal{L}_1 = p\dot{x} + \ell_1(px' + \frac{i}{2}\xi^T\xi') + \frac{1}{2}\ell_2(p^2 + x'^2 + i\xi^T\sigma_3\xi') - \tag{37}$$
$$- i\lambda^T\xi\cdot p - i\lambda^T\sigma_3\xi\cdot x' .$$

This string is equivalent to the Neveu-Schwarz-Ramond string [11]. It would be interesting to find a gauge formulation for the heterotic string theory. We also tried to gauge the t-s symmetric theory. There exists the following new symmetry:

$$\delta p^a = \varepsilon^{ab}\partial_b(f_c p^c) , \quad \delta x = \varepsilon^{ba} f_a p_b , \quad \varepsilon^{ab} = -\varepsilon^{ba} , \quad \varepsilon^{01} = 1 ,$$

which, however, is closed only on the equations of motion and thus requires adding some auxiliary fields and symmetries. Apparently, t-s asymmetric approach developed here has a virtue of being extremely simple and transparent.

2. The observations presented above reveal a rather general principle of gauging linear (super)canonical symmetries of bilinear rudimentary Lagrangians. Employing this principle allows one to construct in a transparent and unified manner all known models of relativistic particles as well as gauge formulations of bosonic and fermionic string theory. In addition, quite new theories can be derived. A non-trivial example has been given in Ref.[13] - a relativistic gauge theory of 2 and 3 scalar particles bound by linear (harmonic) forces. As pointed out in [13] the approach can be used to obtain the N-particle theory, however, the identification of the relevant N-particle gauge group, given in [13], is incorrect. Here, a general relativistic theory of N particles bound by harmonic forces is given. It can be applied to hadrons, strings, membranes, etc.

First we present a rather general formulation of our approach to gauging canonical symmetries. Extending the ideas of Refs.[15,16,3,9] , consider the following rudimentary Lagrangian

$$L_0 = g_{\mu\nu} p_i^\mu \dot{q}_i^\nu - \frac{i}{2} h_{\alpha\beta}\xi^\alpha\dot{\xi}^\beta - \mathcal{H}_0(p, q, \xi), \tag{38}$$

where the index $i=1,\dots,N$ enumerates the particles. The constant matrices $g_{\mu\nu}$ and $h_{\alpha\beta}$ can be diagonalized by suitable linear transformations of canonical variables. As stated above (see /9,12-14/), the quantum interpretation of the gauge invariant theories of free relativistic particles and strings is consistent only for Minkowsky metric $g_{\mu\nu}=(-1,+1,\dots,+1)$, otherwise the Hilbert space of the system has indefinite metric. In what follows we use the Minkowski metric $g_{\mu\nu}$ and suppress all contracted space-time indices μ,ν. Lorentz invariance is trivially satisfied everywhere. The anticommuting variables ξ may be chosen, to some extent, arbitrarily, and this allows one to describe spin and internal degrees of freedom (e.g., adding to (I) the term $-\frac{i}{2}\xi^\mu\dot{\xi}^\mu$ gives the spin 1/2 massless particle Lagrangian, adding to that $-\frac{i}{2}\xi^D\dot{\xi}^D$ gives the theory of the Dirac particle). The Lagrangian (38) can easily to written in the standard form

$$L_0=\frac{1}{2}\overset{T}{\Psi}C(\partial_t-H_0)\Psi+\Delta_B, \tag{39}$$

where the boundary term is a total derivative in t, and it influences only boundary conditions for the gauge transformation functions $f(t)$, $\varphi(t)$; usually we leave it aside. The rigid supercanonical symmetries of this Lagrangian, $\delta\Psi=F(f,\varphi)\Psi$, satisfy the conditions

$$\overset{T}{F}C+CF=0,\quad [F,H_0]\equiv FH_0-H_0F=0 \tag{40}$$

(remember that the transposed supermatrix $\overset{T}{F}$ is defined so as to preserve the relation $(F\Psi)^T=\overset{T}{\Psi}\overset{T}{F}$, with due respect to anticommutativity). Now the gauged Lagrangian L_1, that is invariant under the local transformations, $\delta\Psi=F(f(t),\varphi(t))\Psi$, can be presented in the form

$$L_1=\frac{1}{2}\overset{T}{\Psi}C(\partial_t-A)\Psi, \tag{41}$$

where the supermatrix $A(\ell,\lambda)$ is obtained from $F(f,\varphi)$ simply by substituting $f\to\ell$, $\varphi\to\lambda$. The gauge transformations of A are defined by the standard formula

$$\delta A=\dot{F}+[F,A]\equiv\dot{F}+(FA-AF),$$
$$\delta\Psi=F(f(t),\varphi(t))\Psi. \tag{42}$$

To derive the boundary conditions, for the gauge parameters $f(t),\varphi(t)$ one has to calculate the variation of the boundary term Δ_B. This completes formulating our gauge construction.

A more practical approach to determining the rigid symmetry group of the rudimentary Lagrangians as well as to constructing the corresponding gauge theory is based on using, instead of the supermatrices F, the generating function of the supercanonical transformations

$$\delta X=[G,X]_{P.B.},\quad G(p,q,\xi)=\sum_a f_a g_a+\sum_\alpha \varphi_\alpha\gamma_\alpha. \tag{43}$$

$$\delta p=-\frac{\partial G}{\partial q},\quad \delta q=\frac{\partial G}{\partial p},\quad \delta\xi=i\frac{\partial^L G}{\partial\xi}.$$

Under local symmetry transformations, $[G,H_0]_{P.B}=0$ and H_0 is unchanged, while the variation of Lagrangian (38) is

$$\delta L_0=\frac{d}{dt}\left[p\frac{\partial G}{\partial p}+\frac{1}{2}\xi\frac{\partial^L G}{\partial\xi}-G\right]+\dot{f}\frac{\partial G}{\partial f}+\dot{\varphi}\frac{\partial^L G}{\partial\varphi}.$$

The first term defines the boundary conditions for $f(t),\varphi(t)$ and other terms are cancelled by adding to (38) the obvious compensating terms

$$-\frac{1}{2}\overset{T}{\Psi}CA\Psi=-\sum_a \ell_a(t)g_a(p,q,\xi)-\sum_\alpha\lambda_\alpha(t)\gamma_\alpha(p,q,\xi),$$

where $\overset{T}{\Psi}=(p_i,q_i,\xi_\alpha)$. The transformation law for the gauge potentials ℓ,λ can be derived either from eq. (42) or directly by applying to the new Lagrangian the requirement of gauge invariance (remember that the superalgebra of the generators g,γ is closed with respect to the Poisson brackets, due to the condition $[G,H_0]_{P.B}=0$.

Now we apply the general approach to constructing relativistic gauge models for N particles bound by harmonic forces. To simplify the presentation we only treat here the spinless particles. Then, the rudimentary Lagrangian is

$$L_0=p_i\dot{q}_i-\frac{1}{2}p_ip_i-\frac{1}{4}v_{ij}(q_i-q_j)^2,\quad v_{ij}=v_{ji},\quad v_{ii}=0. \tag{44}$$

The most general linear canonical transformation is defined by the generating function

$$G = \frac{1}{2} a_{ij} p_i p_j + b_{ij} p_i q_j + \frac{1}{2} c_{ij} q_i q_j \equiv \frac{1}{2} \Psi^T \begin{pmatrix} a & b \\ b^T & c \end{pmatrix} \Psi, \tag{45}$$

where $a_{ij} = a_{ji}$, $c_{ij} = c_{ji}$ (remind that we are not considering the Lorentz transformations and all indices μ, ν are contracted). The Lagrangian (44) is invariant under the transformations (43), or $\delta\Psi = C^{-1} \partial^L G / \partial \Psi$, if and only if

$$[V, a] = [V, b] = 0, \quad b^T = -b, \quad c = -Va, \tag{46}$$

where V is the following NxN matrix

$$V_{ii} = -\sum_{j=1}^{N} v_{ji}, \quad V_{ij} = v_{ij}, \quad i \neq j. \tag{47}$$

Equations (46) leave in G not less than N independent commuting generators which are some linear combinations of the bilinear Lorentz invariants $p_i p_j$, $p_i q_j$, $q_i q_j$. Therefore, the time components of the coordinates and momenta can always be excluded by solving $\geqslant$ N constraints together with the same number of gauge fixing conditions.

The physics content of the gauge Lagrangian corresponding to the rudimentary Lagrangian (44) crucially depends on the coupling parameters v_{ij}. If $v_{ij} = v_0$ for all i, j, the Lagrangian describes the system of N identical particles with pair harmonic coupling. The gauge group in that case is $T_1 \otimes U_1 \otimes SU_{N-1}$. This can be shown with the aid of the general formulae (15)-(17). To see this more directly we introduce new canonical coordinates. Define center-of-mass coordinates and momenta

$$Q = \frac{1}{\sqrt{N}} \sum q_i, \quad \mathcal{P} = \frac{1}{\sqrt{N}} \sum p_i$$

and choose other coordinates y_i and momenta z_i $(i = 1, \ldots, N-1)$ so as to diagonalize the Lagrangian:

$$L_0 = \mathcal{P}\dot{Q} - \frac{1}{2}\mathcal{P}^2 + z_i \dot{y}_i - \frac{1}{2} z_i z_i - \frac{1}{2} y_i y_i \tag{48}$$

(the parameter v_0 is absorbed in coordinates, with due rescaling of t). Applying our general construction we arrive at the gauge Lagrangian

$$L_1 = \mathcal{P}\dot{Q} + z_i \dot{y}_i - \frac{1}{2} l_0 (\mathcal{P}^2 + \mathcal{M}^2) - \frac{1}{2} \bar{l}_0 (z_i z_i + y_i y_i - \mathcal{P}^2 - \mathcal{M}^2)$$
$$- \frac{1}{2} l_i (z_i^2 + y_i^2) - \frac{1}{2} l_{ij}^s (z_i z_j + y_i y_j) - \frac{1}{2} l_{ij}^a (z_i y_j - z_j y_i), \tag{49}$$

where

$$l_{ij}^s = l_{ji}^s, \quad \sum_i l_{ii}^s = 0, \quad l_{ij}^a = - l_{ji}^a. \tag{50}$$

Here the constraint coupled to l_0 generates the translations T_1, the one coupled to $\bar{l}_0$ generates U_1, and the others give the algebra of SU_{N-1} (the constraints coupled to l_{ii}^s generate its Cartan subalgebra). In writing eq. (49) we have used the abelian nature of the T and U_1 generators which allows one to add the mass parameters $\mathcal{M}^2, m^2$ without destroying the gauge symmetry (likewise, the term $-\mathcal{P}^2$ in the U_1 generator commuting with all generators can be removed or multiplied by an arbitrary number). If the pair couplings are not identical, i.e. v_{ij} depend on i, j, the SU_{N-1} group will be broken. Note that the gauge group for N=2 is $T_1 \otimes U_1$. To obtain the corresponding Lagrangian from eq. (49) one simply has to set $z_i = y_i = 0$, $i \geqslant 2$, and to keep the first two constraints.

A most natural approach to quantizing this theory is that described in the first part of this report. Hopefully, the application of BRST-BFV modern methods /1/,/6-7/ will allow one to develop both a relativistic quantum theory of free composite particles and an effective quantum field theory describing their interactions.

To obtain a theory of discrete strings, i.e. of linear chains of particles bound by harmonic forces, we choose $v_{ij} = \delta_{|i-j|,1}$ for open strings and $v_{ij} = \delta_{|i-j|,1} + \delta_{iN}\delta_{j1} + \delta_{i1}\delta_{jN}$ for closed ones, and employ the general formulae (46),(47). The detailed derivation will be presented elsewhere and here we only calculate the number of the gauge parameters. The equations for b_{ij} are easy to solve. For the open string $b_{ij} \equiv 0$ and for the closed one the condition $[V, b] = 0$ is equivalent to the relations

$$b_{ij} = b_{j-i}, \quad i < j; \quad b_{ij} = b_{N-|i-j|}, \quad i > j.$$

Together with $b_{ij} = -b_{ji}$ this leaves $[(N-1)/2]$ independent parameters $b_1, \dots, b_{[(N-1)/2]}$ where the square brackets denote the integer part of the enclosed number. The most difficult to solve are the equations $[V, a] = 0$. For the open string there are N independent parameters a_{ij}, as

$$a_{ij} = \sum_{\ell=j-i+1}^{j+i-1} a_{1\ell} (-1)^{\ell-j+i-1}, \quad i \le j \le N-i+1 ;$$

$$a_{ij} = a_{N-j+1, N-i+1}, \quad i+j > N+1 .$$

For the closed string the equations $[V, a] = 0$ are rather complicated due to periodicity conditions. However, the number of independent parameters a_{ij} is easy to calculate, it is $[(3N-1)/2]$. The total number of independent gauge parameters for the closed string is

$(2N-2)$ if N is even; $(2N-1)$ if N is odd.

The generators correspond to the Virasoro generators for the closed string. The detailed derivation of the discrete string "Virasoro algebra" will be presented elsewhere. Note that the closed "string" with N=3 is described by eq.(49).

In conclusion we mention some possible extension and applications of our results. By adding suitable Grassmann variables one can describe the bound states of N spinning particles having internal degrees of freedom. Similarly one can construct discrete strings of different sorts, e.g. compactified on tori or orbifolds. The theory of N- particle bound states can be applied to the quark model of hadrons; while the theory of discrete strings, to an approximate description of massless string states in realistic models. A quantum field theory of discrete strings is possibly simpler than that of continual ones. The scheme discussed here can in principle be applied to constructing other relativistic discrete theories, e.g., membranes (i.e. two-dimensional lattices of particles with nearest-neighbour harmonic couplings). To find the gauge group in that case is a more complicated technical problem.

3. Finally, we will make several general remarks and list some problems for future investigations. The observed interrelation between linear canonical transformations and special relativity looks, at first sight, somewhat mysterious, and it certainly requires further considerations. The practical advantages of our gauge approach over the usual one, based on reparametrisation invariance, are indisputable.

For free particles and strings the gauge symmetry is more or less equivalent to reparametrization symmetry, but, even in this case, our method gives the final answer in a much more direct and clear manner. Using the standard approach one has either to guess a reparametrization invariant Lagrangian (nonlinear and unrelated to the nonrelativistic one) or choose some a priori constraints (also having nothing to do with the nonrelativistic Lagrangian). Our principle of gauging linear canonical symmetries gives, in a straightforward way, the one and only relativistic theory corresponding to a given nonrelativistic Lagrangian (up to now we have used only bilinear Lagrangians). One may view this principle as a device for transforming simple nonrelativistic theories into the corresponding relativistic ones. Applying this device to N particles bound by harmonic forces immediately produces the well-defined classical relativistic Lagrangian (49), which can easily be quantized. It is terribly difficult to find an equivalent reparametrization invariant theory of this Lagrangian, and moreover such an equivalent theory would be practically impossible to quantize.

Another important remark is related to the "No-interaction theorem"/17/. For some time, it has been known that a gauge-like approach to interacting relativistic particles allows one to bypass the restrictions of this theorem (see, e.g. /18/, /19/ and a detailed recent discussion in Ref. /20/). The price to be paid for avoiding this theorem is that the phase-space coordinates p_i^μ, q_i^μ are, in general, not observable, as they are not gauge-invariant. This is not a defect of the theory but a direct consequence of the need of having some auxiliary variables in the relativistic description. The constraints (i.e. the generators of the gauge transformations), together with the corresponding gauge conditions, completely fix the independent physical observables. One may find in the current literature the incorrect statement that different gauge choices might correspond to different physical systems (see, e.g. /20/). This opinion is probably based on neglecting the boundary conditions for the gauge transformation functions and Teichmüller parameters that are defined using gauge fields (e.g. $\ell_1^{(0)} = \int_0^1 dt\, \ell_1(t)$, etc.). As it has been stressed above, different values of these parameters correspond to different gauge orbits of the same physical system. This simple fact has been obscured by the use of rather complicated and usually not Lorentz-invariant gauge fixing conditions. A more detailed discussion of the gauge fixing conditions will be given elsewhere.

We finish this report with an incomplete list of problems: 1) The problem of giving a complete quantum theory of particles with spin and internal quantum numbers. Of special interest is the massless particle theory with the dilatation invariance. 2) The problem of constructing the corresponding quantum field theories, following the BRST-BFV-Parisi-Sourlas approach. 3) The problem of finding all possible continual strings. Of special interest is the Green-Schwarz superstring (in the standard approach it contains second-class constraints while the gauge approach can only generate first-class ones). 4) The problem of formulating a first-quantized theory of N bound particles and trying to find a corresponding effective field theory (second quantization). An interesting application would be a construction of some new basis for the Bethe-Salpeter description of confined quarks. 5) The problem of generalizing the N-particle model to treat realistic bound states of quarks and gluons (i.e. including the spin and internal degrees of freedom). 6) A very interesting problem is to study gauge theories of discrete strings.

The first part of this report represents the invited talk of the author given at the Moscow seminar "Quantum Gravity", May 1987. A bulk of it has been published in Ref. /12/ (in Russian). The second part is based on Refs. /13,14/ . The third part is an attempt to answer numerous questions raised in discussions of the ideas presented in these papers.

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Филиппов А.Т. E2-87-806
Калибровочные теории частиц, струн
и связанных с ними полей

В докладе приведен обзор работ по калибровочному подходу к теориям релятивистских частиц и струн. Релятивистские теории систематически строятся по нерелятивистским с помощью процедуры локализации линейных /супер/ канонических симметрий простейших билинейных лагранжианов. Весьма простым и последовательным методом получены известные теории спиновых частиц, бозонных и фермионных струн. Построены новые калибровочные модели частиц, связанных гармоническими силами, в том числе калибровочные модели дискретных струн. Все эти калибровочные теории частиц можно квантовать методами БРСТ-БФВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Filippov A.T. E2-87-806
Gauge Theories of Particles, String
and Corresponding Fields

A review of the gauge approach to relativistic particles and strings is given. Relativistic theories are systematically produced from nonrelativistic ones by gauging the linear (super) canonical symmetries of simplest bilinear Lagrangians. The known theories of spinning particles, bosonic and fermionic strings are derived in a simple and transparent manner. New gauge models for N relativistic particles bound by harmonic forces, including gauge models for discrete strings, are proposed. All these gauge theories of particles can be quantized by BRST-BFV methods.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1987