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ON THE STABILITY OF THE OPEN STRINGS IN BACKGROUND ELECTROMAGNETIC FIELD

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Recently it has been shown that a consistent theory of the open bosonic string in an external electromagnetic field can be constructed as long as the strength of the external electric field is less than a certain critical value[1-3]. A crucial point was the use of a special block-diagonal form for the electromagnetic strength tensor $F_{\mu\nu}$ *. In the case of four-dimensional space-time a form like that of $F_{\mu\nu}$ corresponds to the parallel electric and magnetic fields. From the physical point of view the other configuration of the background electromagnetic field is the most interesting, when the electric and magnetic fields are equal and perpendicular to each other **.

The constraint on the background electric field was obtained in[1] by two methods. Firstly it was obtained by analysing the boundary conditions on the string coordinates in the orthonormal gauge. It should be noted that this reasoning is valid for arbitrary space-time dependence of $F_{\mu\nu}(x)$. Secondly, this constraint was deduced by determining the mass spectrum of the open bosonic string in a background electromagnetic field. Here the following assumptions have been made. One puts $F_{\mu\nu}(x) = \text{const}$. Only in this case the solution of the equations of motion of the string obeying the corresponding boundary conditions can be obtained in an explicit form. In addition it was supposed that the charges q_1 and q_2 at the string ends satisfy the condition $q_1 + q_2 = 0$, i.e. the net charge of the string is zero. In this case only the total momentum of the string is a conserved quantity, and as a consequence, one can define the mass of the string. Further in[1] the light-like gauge from the free string theory was generalized to the string in a background electromagnetic field. It can be made for the strings with a zero net charge too. As a result one can obtain an expression for the squared mess of the open neutral string M^2 in terms of the independent transverse variables. Further it turns out that the internal string oscillations give no tachyonic contribution to M^2, only if the strength of the background field is less than its critical value. It can be considered as a criterion of stability

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** In reality, one obtains a constraint on some expression constructed from invariants of the tensor* $F_{\mu\nu}$ *which determines the strength of the electric field in a special reference frame.*

*** The canonical form of the ten[illegible] D-dimensional space-time, which corresponds to this configuration, will be [illegible]*

of the string dynamics in an external electromagnetic field[2]. But there arises the problem of interpretation of the tachyonic contribution to M^2 from the motion of the string as a whole in transverse directions.

An analogous expression for the squared mass of the open string placed in an external perpendicular electric and magnetic fields cannot be obtained for arbitrary string charges q_1 and q_2. In the case when $q_1 + q_2 = 0$ the operator M^2 has been found in[3]. It contains also tachyonic contribution from translational motions of the string as a whole. Therefore a more direct method to study the string stability is the analysis of the boundary conditions on the string coordinates in the case of perpendicular external fields, as well. This is the aim of this short communication.

The open charged bosonic string in a background electromagnetic field is described by the equations of motion

$$\ddot{x}^{\mu} - x''^{\mu} = 0, \quad \mu = 0, 1, \ldots, D-1,$$
$$x^{\mu} = x^{\mu}(\tau, \sigma), \quad -\infty < \tau < +\infty, \quad 0 \le \sigma \le \pi, \tag{1}$$

by the orthonormal gauge conditions

$$(\dot{x} \pm x')^2 = 0, \quad \dot{x} = \partial_\tau x, \quad x' = \partial_\sigma x \tag{2}$$

and by the boundary conditions[4,5]

$$T x'_{\mu} + q_1 F_{\mu\nu}(x) \cdot \dot{x}^{\nu} = 0, \quad \sigma = 0,$$
$$T x'_{\mu} - q_2 F_{\mu\nu}(x) \cdot \dot{x}^{\nu} = 0, \quad \sigma = \pi. \tag{3}$$

where T is the string tension, q_1 and q_2 are charges at the string ends, $F_{\mu\nu}(x)$ is the strength tensor of the background electromagnetic field. We assume that the matrix $F_{\mu\nu}$ has the following blockdiagonal form

$$F = \begin{vmatrix} A & 0 \\ 0 & B \end{vmatrix} \tag{4}$$

where A is a (4×4)-matrix of the form

$$A = \begin{vmatrix} 0 & R & 0 & 0 \\ -R & 0 & 0 & R \\ 0 & 0 & 0 & 0 \\ 0 & -R & 0 & 0 \end{vmatrix} \tag{5}$$

and B is the block-diagonal $(D-4) \times (D-4)$-matrix

$$B = \mathrm{diag}\,(B_1, \; B_2, \; \ldots, \; B_{(D/2-2)}). \tag{6}$$

Here B_i, $i = 1,2,\ldots, (D/2) - 2$ are (2×2)-matrices

$$B_i = \begin{vmatrix} 0 & -H_i \\ H_i & 0 \end{vmatrix}. \tag{7}$$

We assume for definiteness that D is an even number.

As has been shown in /1/, the constraint on the external electric field E in the case of parallel electric and magneric fields results from the violation of the conditions

$$\dot{x}^2 \geq 0, \quad x'^2 \leq 0 \tag{8}$$

at the string ends when

$$\left(\frac{q_a}{T} E\right)^2 > 1, \quad a = 1, 2. \tag{9}$$

In D-dimensional space-time the metric with signature $(+,-,-,\ldots)$ is used. If $D = 4$, then eq.(9) can be rewritten as

$$\sqrt{I_1^2 + I_2^2} - I_1 > 1, \tag{9'}$$

where

$$I_1 = \left(\frac{q_a}{2T}\right)^2 F_{\mu\nu} F^{\mu\nu}, \quad 2I_2 = \left(\frac{q_a}{T}\right)^2 F_{\mu\nu} \tilde{F}^{\mu\nu}.$$

As is well-known, condition (8) entails the absence of superlight velocities in the string dynamics.

Let us show that in the case of the perpendicular background field with the tensor $F_{\mu\nu}$ of form (4)-(7) the condition (8) at the ends of the string is not violated for the arbitrary values of $R(x)$ and $H_i(x)$. For this purpose the explicit form of the matrix $(F^2)_{\mu\nu} = F_{\mu}^{\cdot\rho} \cdot F_{\rho\nu}$ is required. By a direct calculation we get

$$F^2 = \begin{vmatrix} A^2 & 0 \\ 0 & B^2 \end{vmatrix}, \tag{10}$$

where

$$A^2 = \begin{vmatrix} R^2 & 0 & 0 & -R^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -R^2 & 0 & 0 & R^2 \end{vmatrix}, \tag{11}$$

$$B^2 = \mathrm{diag}\,(H_1^2, H_1^2, \; H_2^2, H_2^2, \ldots, H^2_{\frac{D}{2}-2}, H^2_{\frac{D}{2}-2}).$$

Using the boundary conditions (3) and (10), (11) we obtain

$$x'^2 = -\left(\frac{q_a}{T} R\right)^2 (\dot{x}^0 - \dot{x}^3)^2 -$$

$$-\left(\frac{q_a}{T}\right)^2 \sum_{i=1}^{(D/2)-2} H_i^2 [(\dot{x}^{2i+2})^2 + (\dot{x}^{2i+3})^2] < 0, \tag{12}$$

$a = 1$, $\sigma = 0$; $a = 2$, $\sigma = \pi$.

Thus, in the case of perpendicular fields the condition (8) at the string ends is always satisfied. Hence the velocities of the charges placed at the string ends in this background electromagnetic field do not exceed the velocity of light.

Let us briefly consider the open fermionic string in a background electromagnetic field. The action of this string in the orthonormal gauge is

$$S = -\frac{T}{2} \int d\tau \int_0^{\pi} d\sigma \, (\dot{x}^2 - x'^2 - i\, \bar{\psi}^{\mu} \rho^0 \dot{\psi}_{\mu} - i\, \bar{\psi}^{\mu} \rho^1 \psi'_{\mu}) -$$

$$-\sum_{a=1}^{2} q_a \int d\tau \, [\dot{x}_{\mu} A^{\mu}(x) - \frac{1}{2} \bar{\psi}^{\mu} \psi^{\nu} F_{\mu\nu}(x)], \tag{13}$$

where

$$\psi^{\mu} = \begin{pmatrix} \psi^{\mu}_{+} \\ \psi^{\mu}_{-} \end{pmatrix}, \quad \bar{\psi}^{\mu} = \psi^{+\mu} \rho^0, \quad \rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \tag{14}$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}.$$

For the fermionic variables we have the following boundary conditions

$$(T \psi^{\mu}_{-} + q_a \psi_{+\nu} F^{\nu\mu})\, \delta\psi_{-\mu} = (T \psi^{\mu}_{+} + q_a \psi_{-\nu} F^{\nu\mu})\, \delta\psi_{+\mu}, \tag{15}$$

$a = 1$, $\sigma = 0$, $a = 2$, $\sigma = \pi$.

They are satisfied provided that

$$\psi_+^\mu = \pm \psi_-^\mu, \quad \delta\psi_+^\mu = \pm \delta\psi_-^\mu, \quad \sigma = 0, \pi. \tag{16}$$

Thus for fermionic variables we have the same boundary conditions as for the free fermionic string. Hence, the stability of the fermionic string in a background electromagnetic field is defined by the dynamics of its bosonic degrees of freedom. Therefore in the case of parallel external fields we have in this string model the same constraint on the strength of the background electric field as in the theory of bosonic string /1, 2/.

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Нестеренко В.В. E2-87-805
О стабильности открытых струн
в фоновом электромагнитном поле

Показано, что ограничение на напряженность внешнего электрического поля отсутствует, если открытая бозонная струна помещена в фоновое электромагнитное поле специальной конфигурации. В случае четырехмерного пространства-времени такая конфигурация соответствует взаимно перпендикулярным и равным по величине электрическому и магнитному полям. На фермионные переменные спиновой струны внешнее электромагнитное поле не влияет.

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Nesterenko V.V. E2-87-805
On the Stability of the Open Strings
in Background Electromagnetic Field

It is shown that the constraint on the strength of an external electric field is absent when the open bosonic string is placed in an external electromagnetic field of a special configuration. In the case of four-dimensional space-time it corresponds to the electric and magnetic fields which are equal and perpendicular to each other. The external electromagnetic field does not act on the fermionic variables of the spinning string.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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