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HOW TO SATISFY THE ENERGY-MOMENTUM CONSERVATION LAW AND TO TAKE INTO ACCOUNT THE FERMI MOTION OF THE CONSTITUENTS IN SIMULATION OF COMPOSITE SYSTEM INTERACTIONS

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Simulation of hadron-hadron, hadron-nucleus and nucleus-nucleus interactions with the help of the Monte-Carlo methods is of wider use in experimental and theoretical physics of high energies. The results of simulations are quite often used when experimental devices are designed and background conditions are evaluated.

Therefore it is important that the characteristics of artificial "events" are as close as possible to those of natural events. First of all they must satisfy the energy-momentum conservation law. This can be achieved, for example, at simulation of hadron-hadron interactions within the framework of the quark-gluon string model if one neglects quark transverse momenta /1,2/. This approximation is quite justified at energies up to 500 GeV. At higher energies, however, the theory /3/ predicts a more important role of transverse momenta of constituents. Hence, a question of elaborating algorithms for generation of artificial "events" satisfying the law of energy-momentum conservation arises again.

Simulation of hadron-nucleus and nucleus-nucleus interactions put forward a problem of allowance for the Fermi motion of nuclear nucleons. This problem is most often solved under the assumption /4,5/ that the projectile momentum does not change and all the nucleons are on the mass shell and have momenta distributed according to a law defined by the nonrelativistic nuclear wave function. It is clear that these assumptions lead to the violation of energy-momentum conservation law.

To take into account the Fermi motion correctly, let us consider a simple model task: disintegration of deuterium under the action of a projectile hadron h. Within the framework of Glauber's approximation the cross section of the process $h+d \to h+p+n$ is expressed as /6,7/

$$\frac{d\sigma}{d^2q\, d^3K_p} = \frac{1}{16\pi^2 p_0^2} \left| f_{hp}(\vec{q})\,\Psi(\vec{K}_p) + f_{hn}(\vec{q})\,\Psi(\vec{q}-\vec{K}_p) + \ldots \right|^2, \quad (1)$$

where $\vec{q}$ is the transverse transfer, $\vec{K}_p$ is the proton momentum, $f_{hp}\,(f_{hn})$ is the amplitude of elastic hadron-proton(hadron-neutron) scattering, Ψ is the deuteron wave function. Neglecting double collisions and interference terms in (1) we have

$$\frac{d\sigma}{d^2q\,d^3k_p}\Big|_{\vec{q}=0} \sim |f_{hp}(0)|^2 |\Psi(\vec{k}_p)|^2 . \qquad (2)$$

As is seen, the momentum distribution of protons is defined in this case by a wave function. Let us suppose that expression (2) is true in any frame, in particular in the center of mass system (CMS). In this frame the law of energy-momentum conservation reads as follows

$$\sqrt{m_p^2+k_p^2}+\sqrt{m_n^2+k_n^2}+\sqrt{m_h^2+p_h^2}=\sqrt{s} \qquad (3a)$$

$$k_p+k_n=p_h . \qquad (3b)$$

Here m_p, m_n, m_h are the masses of the proton, neutron and hadron, respectively, $\sqrt{s}$ is the total energy, k_i are the longitudinal momenta.

According to the system (3) an exclusive hadron state is completely defined by the value of one kinematic variable. Let us take $x=k_p/(k_p+k_n)$ for this variable. Then system (3) degenerates into one equation in an unknown variable p_h

$$\sqrt{m_p^2+x^2p_h^2}+\sqrt{m_n^2+(1-x)^2p_h^2}+\sqrt{m_h^2+p_h^2} . \qquad (4)$$

Supposing $p_h=p_h(x,\sqrt{s})$, $k_p=k_p(x,\sqrt{s})$ and $k_n=k_n(x,\sqrt{s})$ we get $d\sigma/d^2q\,dk_p \sim |f_{hN}(0)|^2|\Psi(x,\sqrt{s})|^2$. Since limit fragmentation takes place at high energy, the dependence of Ψ on $\sqrt{s}$ is rather weak. Neglecting this dependence and supposing that in a multiple production process the decay of the nucleus is first to occur, and then generation of new particle takes place, an algorithm to allow for the Fermi motion of nuclear nucleons in simulation of hadron-nucleus interactions can be formulated as follows:

1. From the distribution $|\Psi(x_1,x_2,\dots,x_A)|^2$ we sample x_1, $x_2,\dots,x_A$ satisfying the condition $\sum_{i=1}^{A} x_i=1$ (A is the mass number of the nucleus);

2. From equation
$$\sum_{i=1}^{A}\sqrt{m_i^2+x_i^2p_h^2}+\sqrt{m_h^2+p_h^2}=\sqrt{s}$$
we get the value of the hadron momentum in the CMS;

3. We determine nucleon momenta $k_i=-x_ip_h$.

Then one can use conventional, e.g. cascade-type, models of hadron-nucleus interaction.

Generalization to the case of interaction of two composite systems with A and B constituents is trivial. One should take two functions $|\Psi_A(x_1,\vec{k}_{\perp 1},\dots,x_A,\vec{k}_{\perp A})|^2$ and $|\Psi_B(\xi_1,\vec{p}_{\perp 1},\dots,\xi_B,\vec{p}_{\perp B})|^2$.

At a given set of $\{x_i\},\{\vec{k}_{\perp i}\},\{\xi_i\},\{\vec{p}_{\perp i}\}$ satisfying the conditions

$$\sum_{i=1}^{A} x_i=1 , \quad \sum_{i=1}^{A}\vec{k}_{\perp i}=0$$
$$\sum_{i=1}^{B}\xi_i=1 , \quad \sum_{i=1}^{B}\vec{p}_{\perp i}=0 \qquad (5)$$

one should solve the equation for variable p

$$\sum_{i=1}^{A}\sqrt{m_{A_i}^2+\vec{k}_{\perp i}^2+x_i^2p^2}+\sum_{i=1}^{B}\sqrt{m_{B_i}^2+\vec{p}_{\perp i}^2+\xi_i^2p^2}=\sqrt{s} . \qquad (6)$$

The longitudinal momenta are determined as $p_{\parallel i}=\xi_i p$ and $k_{\parallel i}=-x_ip$.

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Как удовлетворить закону сохранения энергии-импульса и учесть ферми-движение конституентов при моделировании взаимодействий составных систем

Предложен метод, позволяющий при монте-карловском моделировании эксклюзивных состояний в адрон-ядерных и ядро-ядерных взаимодействиях учесть ферми-движение ядерных нуклонов, а при моделировании адрон-адронных взаимодействий учесть поперечный импульс кварков без нарушения закона сохранения энергии-импульса.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Shmakov S.Yu., Uzhinskii V.V. E2-87-780

How to Satisfy the Energy-Momentum Conservation Law and to Take into Account the Fermi Motion of the Constituents in Simulation of Composite System Interactions

A method which allows taking into account the Fermi motion of nuclear nucleons at Monte-Carlo simulation of exclusive states in hadron-nucleus and nucleus-nucleus interactions is suggested. In the case of hadron-hadron interactions the method allows taking into account quark transverse momenta and ensuring energy-momentum conservation.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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