объединенный
институт
ядерных
исследований
дубна



E2-87-779

M.Basler

# CANONICAL QUANTIZATION AND COSMOLOGICAL PARTICLE PRODUCTION IN NONABELIAN GAUGE THEORIES

Submitted to "Physics Letters B"

1987

## 1. MOTIVATION

There are two typical conditions, under which quantum fields are considerably influenced by an external, classical gravitational field. First [1], the existence of horizons around black holes may lead to a thermal flux of radiation. At second [2-4], the importance of quantum effects in the very early universe has been worked out with special emphasis on cosmological particle production. With regard to this process estimates show [5], that on the one hand the external field approximation for the gravitational field breaks down for times $t < t_{Planck} \approx 10^{-44}$ s. On the other hand particle production becomes ineffective for times $t > t_{Compton} \approx 10^{-21}$ s (for electrons). In the standard cosmological model these times correspond to energies in the range between $10^9$ GeV and $10^{19}$ GeV and to our present knowledge, some kind of unified nonabelian gauge theory should be responsible for describing matter within this energy range. Thus, it seems to be sensible to study quantized nonabelian gauge theories under the influence of a cosmological space-time. Besides, the quantization of nonabelian gauge theories in curved space is also of general interest, beyond its cosmological application.

In this paper we briefly describe a formalism, suitable for the quantization of nonabelian gauge theories in an external gravitational field and some results of the 1st order calculation of cosmological particle production within these theories.

## 2. CANONICAL QUANTIZATION

Our model will be a SU(2) Yang-Mills-Higgs theory in the unbroken phase, but the method should be applicable to more intricate theories also. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{GF} + \mathcal{L}_{GH} ,$$

$$\mathcal{L}_S = \sqrt{-g}\,\{ -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{2}(D_\mu \phi)^i (D^\mu \phi)^i - \frac{\mu^2}{2}\phi^i \phi^i - \frac{\xi}{2} R \phi^i \phi^i - \frac{\lambda}{4}(\phi^i\phi^i)^2 \} ,$$

$$\mathcal{L}_{GF} = \sqrt{-g}\,\{A^{i\mu}{}_{;\mu} B^i + \frac{1}{2} B^i B^i\}, \tag{1}$$

$$\mathcal{L}_{GH} = -i\sqrt{-g}\,(\partial^\mu \bar{c})^i (D_\mu c)^i,$$

where

$$(D_\mu \phi)^i = \phi^i_{,\mu} + e\epsilon^{ijk} A^j_\mu \phi^k, \tag{2}$$

$$F^i_{\mu\nu} = A^i_{\nu,\mu} - A^i_{\mu,\nu} + e\epsilon^{ijk} A^j_\mu A^k_\nu. \tag{3}$$

Lagrangian (1) is a curved-space generalization of the one, proposed by Kugo and Ojima in Minkowski space ([6], cf. also [7]). The term $-\frac{\xi}{2}\sqrt{-g}R\phi^i\phi^i$ allows for a possible nonminimal coupling of the Higgs field to the gravitational field [12]. Indeed, our whole quantization scheme essentially rests on the flat-space formulation of [6] and the real success is to show that it also works in curved space. The essential points in quantization are 1) that the field $B^i$ is treated as a Lagrange multiplier field, 2) that the Faddeev-Popov-ghosts $c^i$ and $\bar{c}^i$ [11] are taken to be Hermitean and are quantized by anticommutators and 3) that all degrees of freedom, physical as well as unphysical, are quantized on the same footing. This last procedure, of course, demands imposing a subsidiary condition later.

On the basis of these ingredients we have calculated the canonical momenta and have performed the Legendre transform to find the Hamiltonian under proposition of an arbitrary time-orthogonal metric.

$$g^{0a} = g_{0a} = 0.$$

The commutation relations remain the same as in [7] with a suitable generalization of the $\delta$-function.

For calculating physical processes via S-matrix in the interaction picture we do not need the full Hamiltonian, but only the equations of motion without interaction and the vertices in coordinate space. From the Heisenberg equations of motion, applied to the non-interaction Hamiltonian, the following equations of motion can be obtained:

$$\phi^{i}{}_{,\mu}{}^{;\mu} + (\mu^2 + \xi R)\,\phi^i = 0, \tag{4}$$

$$A^i_{\nu;\mu}{}^{;\mu} + R^\mu{}_\nu A^i_\mu = 0, \tag{5}$$



$$c^i{}_{,\mu}{}^{;\mu} = 0, \tag{6}$$

$$\bar{c}^i{}_{,\mu}{}^{;\mu} = 0. \tag{7}$$

One recovers, that the fields are "free" with respect to the interaction ($e = 0$, $\lambda = 0$), but not with respect to the gravitational field, which is taken into account exactly. Note, that the ghost fields $c^i$ and $\bar{c}^i$ are massless, minimally coupled scalar fields, which will be an important proposition for the cancellation of unphysical degrees of freedom.

At second, from the interaction part of the Hamiltonian we can read off the Feynman graphs. One finds, that they can be obtained from the Minkowski-space Feynman-gauge graphs in coordinate space by the following rules:

1) Write down the coordinate-space vertex in Minkowski space.
2) Substitute $\eta^{\mu\nu} \to g^{\mu\nu}$.
3) At each vertex attach an additional factor $\sqrt{-g}$.
4) In the lines for $A^i_\mu$, $\phi^i$, $c^i$ and $\bar{c}^i$ substitute the corresponding modes as solutions of (4)-(7).

For details of the interaction picture used, see [9, 10, 13] and chpt. 9 in [5].

## 3. COSMOLOGICAL PARTICLE PRODUCTION

As an interesting application we have applied this theory to the calculation of cosmological Higgs- and gauge boson production in a spatially flat Friedman-Robertson-Walker metric

$$ds^2 = C^2(\eta)\,(d\eta^2 - dx^2 - dy^2 - dz^2). \tag{8}$$

From the calculation of Lotze [10] it is known, that the interaction between electrons and photons does not only lead to an additional production of electron-positron-pairs, but also enables the production of photons, which vanishes to 0th order because of conformal invariance. Thus, a similar effect might occur also here.

As has been shown in [9], starting at conformal time $\eta = -\infty$ with the in-vacuum state $|0, in\rangle$, the number of particles present at $\eta = +\infty$ is for a scalar field

$$N(+\infty) = \langle \psi(+\infty), in |\, \phi^{i\,out}(\vec{k})^+ \phi^{i\,out}(\vec{k})\, |\psi(+\infty), in\rangle. \tag{9}$$

$\phi^{i\,out}(\vec{k})$ and $\phi^{i\,out}(\vec{k})^+$ are the annihilation, resp., creation operators, which appear in the development of $\phi^i(x)$ into modes, which are pure positive frequent at $\eta = +\infty$. Formula, similar to (9), is also applied to the other degrees of freedom.

First, we have to relate the out- operators to in- operators and at second we have to calculate the state $|\Psi(+\infty), in>$ from $|0, in>$ via perturbation theory. The first step is usually done by the Bogoliubov transformation /11/

$$\phi^{i\,out}(\vec{k}) = {}^{\mu}_{\xi}\alpha(k)\,\phi^{i\,in}(\vec{k}) + {}^{\mu}_{\xi}\beta^*(k)\phi^{i\,in}(-\vec{k})^+, \quad k^2 = \vec{k}^2. \tag{10}$$

The same transformation law is valid for the longitudinal and scalar gauge bosons as well as for the ghosts with the only modification, that for them $\mu = 0$ and $\xi = 0$ because of masslessness and minimal coupling. On the other hand, the Bogolubov transformation for the transversal gauge bosons is trivial, $\alpha = 1$, $\beta = 0$. Because the Bogolubov coefficients at least for the simplest expansion laws in (8) are well-known /5/, eq. (10) poses no further problems.

The main difficulty is the calculation of the state $|\psi(+\infty), in>$. Evaluating the S-matrix to first order leads to the following expression $(i + j \geq 2)$

$$\frac{1}{K}|\psi(+\infty), in> = (1 - i\int \mathcal{H}^{int}(x)\,d^4x)\,|0, in> + \mathcal{O}(e^i\lambda^j)$$

$$\begin{array}{lll}
= \kappa|0, in> & +|S\ T\ T> & +|L\ T\ T> \\
+|T\ T\ T> & +|S\ L\ T> + i|\bar{c}\ c\ T> & +|L\ L\ T> \\
+|\phi\ \phi\ T> & +|S\ S\ T> & +|S\ L\ L> + 2i|\bar{c}\ c\ L> \\
+|\phi\ \phi> & +|S\ S\ L> + i|\bar{c}\ c\ S> & +|\phi\ \phi\ L> \\
+|\phi\ \phi\ \phi\ \phi> & +|S\ S\ S> & \\
 & +|\phi\ \phi\ S> &
\end{array} \tag{11}$$

Here, the notation is a symbolic one. $|S\ L\ T>$ for instance denotes a three-fold momentum integral over a coefficient function, consisting of solutions of (4)-(7) times a three-particle state with one scalar, one longitudinal and one transversal gauge boson.

Now we enter the question of finding the physical subspace. Following /8/ this is done in two steps.

1) Find the subspace $\mathcal{C}_{phys} \subset \mathcal{C}$ which obeys the subsidiary condition

$$\xi_B|phys> = 0, \tag{12}$$

where $\xi_B$ is the BRS-charge /15/ in the asymptotic region.



2) After imposing condition (12), there remains a zero-norm subspace $\mathcal{C}_0$ which must be shown not to give any contribution to expectation values of physical observables.

Exploiting Poincare invariance in Minkowski space it has been proved in /6/ that for a theory of the type considered here the quotient space $H_{phys} = \mathcal{C}_{phys}/\mathcal{C}_0$ is indeed a Hilbert space and that the S-matrix restricted to $H_{phys}$ is unitary. Because of lack of Poincare-invariance this is far from being obvious here.

At first, we find that the third group of states does not fulfill $\xi_B| \quad > = 0$, thus they must not be contained in $|\psi(+\infty), in>$. A careful inspection of the coefficient functions indeed shows, that in all these cases, after a finite number of partial integrations, their contributions to $|\psi(+\infty), in>$ vanish. Thus we can cancel them in (10). Evaluating the scalar products of all residual states shows, that the zero-norm-subspace $\mathcal{C}_0$ is spanned by the second group of states. As a result, only the first group of states belongs to $H_{phys}$.

Now, inserting (11) into (9) and the similar formulas for the other degrees of freedom leads to the following results:

1) The combined number of longitudinal and scalar gauge bosons plus both kinds of ghosts, produced by the gravitational field is equal to zero. No net unphysical particles are produced by the gravitational field.
2) Evaluating the number of transverse gauge bosons, as well as the number of Higgs bosons produced one recovers that indeed the three-particle states belonging to $\mathcal{C}_0$ give no contribution.
3) Concerning the Higgs sector, the formulas given in /9/ for a pure $\phi^4$-theory are reproduced with modifications by group factors of the order 3 to 8.
4) There is an additional contribution of Higgs production, arising from the interaction with the gauge field and which, contrary to the terms obtained already in /9/, is proportional to the gauge coupling squared.
5) The interaction with the Higgs field enables the production of transverse gauge bosons also, which is absent to 0th order because of conformal invariance in the gauge sector. This contribution is found to be proportional to the gauge coupling squared, too.

The full formula obtained together with details of the calculation will be published elsewhere.

We should mention, that a complimentary investigation on the production of massive gauge bosons in the broken phase, neglecting interactions, has been carried out before /14/.

I would like to thank K.H.Lotze for numerous discussions on quantum fields in curved space and A.Hädicke for the discussion of special points. Further, I thank Profs. N.A.Chernikov and E.Kapuscik for kind hospitality at JINR, Dubna, where part of this work was done.

REFERENCES


1. Hawking S.W. - Nature, 1974, 248, p.30.
   Hawking S.W. - Comm. Math.Phys., 1975, 43, p.199.
2. Parker L. - Phys.Rev.Lett., 1968, 21, p.562.
   Parker L. - Phys.Rev., 1969, 183, p.1057.
3. Sexl R.U., Urbantke H.K. - Acta Phys.Austr., 1967, 26, p.339; Sexl R.U., Urbantke H.K. - Phys.Rev., 1969, 179, p.1247.
4. Zel'dovich Ya.B. - Pis'ma Zh. Eksp. Teor. Fiz., 1970, 12, p.443.
5. Birrell N.D., Davies P.C. Quantum Fields in Curved Space, Cambridge Unversity Press, Cambridge (UK), 1982.
6. Kugo T., Ojima I. - Suppl. Progr. Theor.Phys., 1979, 66, p.1.
7. Haller K., Brickner R.G. - Nuov. Cim., 1983, A73, pp.89, 117.
8. Bogolubov N.N. - Zh. Eksp.Teor.Fiz., 1958, 34, p.58.
9. Birrell N.D., Ford L.H. - Ann.Phys. (NY), 1979, 122, p.1.
10. Lotze K.H. - Class. Quant. Grav., 1985, 2, pp.351,363.
11. Faddeev L.D., Popov V.N. - Phys.Lett., 1967, B25, p.29.
12. Chernikov N.A., Tagirov E.A. - Ann. Inst. Henri Poincaré, 1968, 9, p.109; Chernikov N.A., Shavokhina N.S. In: Probl. Grav. i Elem. Chast., Moskva, Atomisdat, No.5, 1974.
13. Audretsch J., Spangehl P. - Class. Quant. Grav., 1985, 2, p.733; Audretsch J.,Spangehl P. - Phys.Rev., 1986, D33, p.997.
14. Grib A.A., Nesteruk A.V. - Jad. Fiz. 1982, 35, p.216.


Received by Publishing Department
on October 28, 1987.



Базлер М. E2-87-779

Каноническое квантование и космологическое рождение частиц в неабелевых калибровочных теориях

Сформулирована схема канонического квантования неабелевых калибровочных полей во внешнем гравитационном поле. Потом на основе этого формализма исследовано космологическое рождение хиггсовых и калибровочных бозонов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1987

Basler M. E2-87-779

Canonical Quantization and Cosmological Particle Production in Nonabelian Gauge Theories

A canonical quantization scheme for nonabelian gauge fields in an external, classical gravitational field is formulated and applied to the problem of cosmological Higgs and gauge boson production. Via interaction, the mass of the Higgs field not only leads to additional Higgs production, but also enables the production of massless gauge bosons.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1987