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FROM THE FIRST TO THE SECOND QUANTIZED STRING THEORY

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Recently covariant string field theories have been intensively investigated. This was initiated by Siegel who performed the covariant second quantization of the free bosonic string using BRST invariance of the first quantized theory[1]. After that, the gauge covariant free string field theories were presented[2-5] and extensions to the interactions were proposed[6-9]. The relations between different kinds of string actions were made clear[10]. The problem of gauge fixing was discussed[11-14] and a set of Feynman rules was derived[12].

Alternatively Polykov[13] proposed an approach to strings in which the perturbation series appears as a sum over two-dimensional Rieman surfaces. An expression for the bosonic string propagator was presented [14] and loop calculations were intensively investigated.

In the present paper, starting from the Hamiltonian formulation of bosonic string we calculate the Feynman amplitude D. Inverting D we get the kinetic operator and thus the free field action. The gauge fixing procedure which leads to this action is discussed. We emphasize that our expression for the propagator closely related to that in[14] but at the same time it is essentially different.

The canonical coordinates of the string are $X^{\mu}(\sigma,\tau)$, where σ parametrizes the position along the string and τ parametrizes its motion in space-time. The action invariant under arbitrary σ, τ reparametrizations, has been written in[15]

$$S = \frac{\alpha}{2}\int_0^T d\tau \int_0^{\pi} d\sigma \sqrt{-g}\, g^{\alpha\beta}\, \partial_\alpha X^{\mu}(\sigma,\tau)\, \partial_\beta X_{\mu}(\sigma,\tau). \tag{1}$$

The $g_{\alpha\beta}(\sigma,\tau)$ is an independent internal metric on the two-dimensional surface described by $X^{\mu}(\sigma,\tau)$, $g = \det g_{\alpha\beta}$ and $\partial_1 = \frac{\partial}{\partial\tau}$, $\partial_2 = \partial/\partial\sigma$.

Let us define the canonical momentum $P_{\mu}(\sigma,\tau)$

$$P_{\mu}(\sigma,\tau) = \frac{\delta L}{\delta \dot{X}^{\mu}}, \qquad \dot{X}^{\mu} = \frac{\partial X^{\mu}}{\partial \tau}. \tag{2}$$

Then, the Lagrangian can be written in the form

$$\mathcal{L} = P^{\mu}\dot{X}_{\mu} - H. \tag{3}$$

where the Hamiltonian H is

$$H = -\frac{g^{01}}{g^{00}} P^{\mu} X'_{\mu} + \frac{1}{\sqrt{-g}\, g^{00}} \frac{1}{2\alpha}\left(P^{\mu}P_{\mu} + \alpha^{2} X'_{\mu} X^{\mu\prime}\right), \qquad X'_{\mu} = \frac{\partial X_{\mu}}{\partial \sigma} \tag{4}$$

The action (1) is also invariant under Weyl transformation of $g_{\alpha\beta}(\sigma,\tau)$

$$g_{\alpha\beta}(\sigma,\tau) \to \Lambda(\sigma,\tau)\, g_{\alpha\beta}(\sigma,\tau). \tag{5}$$

As a result, only two elements of $g_{\alpha\beta}(\sigma,\tau)$ enter in the action (1) and Hamiltonian H independently .

$$\lambda_{1}(\sigma,\tau) = \frac{g^{01}}{g^{00}}, \qquad \lambda_{2}(\sigma,\tau) = -\frac{1}{\sqrt{-g}\, g^{00}}. \tag{6}$$

Varying with respect to them one obtains the constraints

$$\begin{aligned} \varphi_{1}(\sigma,\tau) &\equiv P^{\mu}(\sigma,\tau) X'_{\mu}(\sigma,\tau) = 0 \\ \varphi_{2}(\sigma,\tau) &\equiv \frac{1}{2\alpha}\left[P^{\mu}(\sigma,\tau) P_{\mu}(\sigma,\tau) + \alpha^{2} X'_{\mu}(\sigma,\tau) X^{\mu\prime}(\sigma,\tau)\right]. \end{aligned} \tag{7}$$

The dependence of X_{μ} and P_{μ} on σ in the open string can be written in the form

$$\begin{aligned} X^{\mu}(\sigma,\tau) &= \frac{1}{2}\sum_{n=-\infty}^{\infty} X^{\mu}_{n}(\tau)\, e^{in\sigma} \qquad & X^{\mu}_{n} = X^{\mu}_{-n} \\ P^{\mu}(\sigma,\tau) &= \frac{1}{2}\sum_{n=-\infty}^{\infty} P^{\mu}_{n}(\tau)\, e^{in\sigma} \qquad & P^{\mu}_{n} = P^{\mu}_{-n}, \end{aligned} \tag{8}$$

where $-\pi \le \sigma \le \pi$ and respectively, the constraints are the following

$$L_{n}(\tau) = \frac{1}{2\pi\alpha}\int_{-\pi}^{\pi} e^{in\sigma}\left[P_{\mu}(\sigma,\tau) + \alpha X'_{\mu}(\sigma,\tau)\right]^{2} d\sigma. \tag{9}$$

Then the Lagrangian (3) admits the representation

$$\mathcal{L}(\tau) = \sum_{n=0}^{\infty} P^{\mu}_{n}(\tau)\dot{X}_{n\mu}(\tau) + \sum_{n=-\infty}^{\infty} \lambda_{n}(\tau) L_{n}(\tau). \tag{10}$$

where $\lambda^{*}_{n}(\tau) = \lambda_{-n}(\tau)$ and $L^{*}_{n}(\tau) = L_{-n}(\tau)$

It is convenient to introduce the real and imaginary parts of L_{n} and λ_{n}

$$\begin{aligned} L_{n} &= L^{1}_{n} + iL^{2}_{n}, \qquad & L_{-n} &= L^{1}_{n} - iL^{2}_{n} \qquad (n \ge 1) \\ \lambda_{n} &= \lambda^{1}_{n} + i\lambda^{2}_{n}, \qquad & \lambda_{-n} &= \lambda^{1}_{n} - i\lambda^{2}_{n} \qquad (n \ge 1) \\ \lambda^{i*}_{n} &= \lambda^{i}_{n}, \qquad & L^{i*}_{n} &= L^{i}_{n}. \end{aligned} \tag{11}$$

Then the conditions that fix the conformal gauge are [14,16,17].

$$\frac{d}{d\tau}\lambda_{0}(\tau) = 0 \tag{12a}$$

$$\lambda^{1}_{n}(\tau) = \lambda^{2}_{n}(\tau) = 0. \tag{12b}$$

From the explicit form of the metric $g_{\alpha\beta}$ in conformal gauge [14] and from Eq.(6) follow that in this gauge $\lambda_{0} = \pm\sqrt{e^{2}}$, where e =const. We choose the sign minus to fulfil the Feynman analytic condition (or in Euclidean space to make the integral over λ_{0} convergent). Then, one can perform the integration over $\lambda_{0}(\tau)$ and $\lambda_{n}(\tau)$ and the result is

$$\begin{aligned} &\int \prod_{n,\tau,i} d\lambda^{i}_{n}(\tau)\, d\lambda_{0}(\tau) \prod_{n,\tau,i}\delta(\lambda^{i}_{n}(\tau)) \prod_{\tau}\delta\!\left(\frac{d\lambda_{0}(\tau)}{d\tau}\right) R(\lambda^{i}_{n}, \lambda_{0}) = \\ &= \int_{0}^{\infty} de\, R(0, -e). \end{aligned} \tag{13}$$

Following the standard procedure [18], we introduce ghosts $C_{0}(\tau), C^{1}_{n}(\tau)$ $C^{2}_{n}(\tau)$ (n $\geq$ 1) and antighosts $\bar{C}_{0}(\tau), \bar{C}^{1}_{n}(\tau), \bar{C}^{2}_{n}(\tau)$ (n $\geq$ 1) . Taking into account Eqs.(12) and (13) we write the gauge fixed action in the form

$$\begin{aligned} S = \int_{0}^{T} d\tau \Big\{ &\sum_{n=0}^{\infty} P_{n}(\tau)\dot{X}_{n}(\tau) - e\Big[\frac{1}{2}P_{0}^{2}(\tau) + \frac{1}{2}\sum_{n=1}^{\infty}\left(P_{n}^{2}(\tau) + n^{2}X_{n}^{2}(\tau)\right)\Big] \\ &- \bar{C}_{0}(\tau)\frac{d^{2}}{d\tau^{2}} C_{0}(\tau) \end{aligned} \tag{14}$$

$$-\sum_{n=1}^{\infty}\left[\bar{C}_n^1(\tau)\left(\frac{d}{d\tau}C_n^1(\tau)+neC_n^2(\tau)\right)+\bar{C}_n^2(\tau)\left(\frac{d}{d\tau}C_n^2(\tau)-neC_n^1(\tau)\right)\right]\Bigg\} \qquad (14)$$

For the sake of convenience we put $\alpha=1$.

Equivalently the action can be written as

$$S=\int_0^T d\tau\Big\{\frac{1}{e}\dot{X}_0^2(\tau)+\frac{1}{2e}\sum_{n=1}^{\infty}\left[\dot{X}_n^2(\tau)-e^2n^2X_n^2(\tau)\right]-$$
$$-\frac{e}{4}\Pi_0(\tau)\Pi_0(\tau)-\frac{e}{2}\sum_{n=1}^{\infty}\Pi_n(\tau)\Pi_n(\tau)-\bar{C}_0(\tau)\frac{d^2}{d\tau^2}C_0(\tau) \qquad (15)$$
$$-\sum_{n=1}^{\infty}\left[\frac{1}{e}\left(\dot{\bar{\theta}}_n(\tau)\dot{\theta}_n(\tau)-e^2n^2\bar{\theta}_n(\tau)\theta_n(\tau)\right)-e\,\bar{\eta}_n(\tau)\eta_n(\tau)\right]\Big\},$$

where the boundary conditions $C_n^2(0)=C_n^2(T)=0$ are used and new variables are introduced

$$\eta_n(\tau)=C_n^2(\tau)+\frac{1}{n}\dot{C}_n^1(\tau)$$

$$\bar{\eta}_n(\tau)=n\bar{C}_n^1(\tau)-\dot{\bar{C}}_n^2(\tau)$$

$$\frac{1}{n}\bar{C}_n^2(\tau)=\bar{\theta}_n(\tau)\,,\quad C_n^1(\tau)=\theta_n(\tau)$$

$$\Pi_n^\mu(\tau)=P_n^\mu(\tau)-\frac{1}{e}\dot{X}_n^\mu(\tau)\,,\quad \Pi_0^\mu(\tau)=P_0^\mu(\tau)-\frac{2}{e}X_0^\mu(\tau). \qquad (16)$$

The transition amplitude from the string state with coordinates $x_i, x_i^n, \theta_i^n, \bar{\theta}_i^n$ at $\tau=0$ to the string state with coordinates $x_f, x_f^n, \theta_f^n, \bar{\theta}_f^n$, at $\tau=T$ has the following path integral representation

$$D(Z_i, Z_f; T)=\int_0^\infty de\int d\mu\, e^{iS}, \qquad (17)$$

where $Z=(x, x^n, \theta^n, \bar{\theta}^n)$ and

$$d\mu=\prod_\tau dx_0(\tau)d\Pi_0(\tau)dC_0(\tau)d\bar{C}_0(\tau)\prod_{n=1}^{\infty}dx_n(\tau)d\Pi_n(\tau)d\theta_n(\tau)d\bar{\theta}_n(\tau)d\eta_n d\bar{\eta}_n$$

$$\delta^D(x_0(0)-\tfrac{1}{2}x_i)\delta^D(x_0(T)-\tfrac{1}{2}x_f)\delta^D(\Pi_0(0))\delta^D(\Pi_0(T))\delta(C_0(0))\delta(C_0(T))\delta(\bar{C}_0(0))$$
$$\delta(\bar{C}_0(T))\prod_{n=1}^{\infty}\delta^D(x_n(0)-x_i^n)\delta^D(x_n(T)-x_f^n)\,\delta(\theta_n(0)-\theta_i^n)\delta(\theta_n(T)-\theta_f^n)$$
$$\delta(\bar{\theta}_n(0)-\bar{\theta}_i^n)\delta(\bar{\theta}_n(T)-\bar{\theta}_f^n)\delta(\eta_n(0))\delta(\eta_n(T))\delta(\bar{\eta}_n(0))\delta(\bar{\eta}_n(T)).$$

To perform the integration we make a change of variables

$$X_n(\tau)=y_n(\tau)+X_n^0(\tau) \qquad n\geq 0$$
$$\theta_n(\tau)=\xi_n(\tau)+\theta_n^0(\tau) \qquad n\geq 1$$
$$\bar{\theta}_n(\tau)=\bar{\xi}_n(\tau)+\bar{\theta}_n^0(\tau) \qquad n\geq 1, \qquad (18)$$

where $X_n^c(\tau)$, $\theta_n^0(\tau)$ and $\bar{\theta}_n^0(\tau)$ are the solutions of the classical equations of motion with the following boundary values

$$X_n^0(0)=X_i^n\,,\quad X_n^c(T)=X_f^n\,,\quad X_0^c(0)=\tfrac{1}{2}X_i\,,\quad X_0^c(T)=\tfrac{1}{2}X_f$$

$$\theta_n^c(0)=\theta_i^n\,,\quad \theta_n^0(T)=\theta_f^n\,,\quad \bar{\theta}_n^0(0)=\bar{\theta}_i^n\,,\quad \bar{\theta}_n^0(T)=\bar{\theta}_f^n.$$

The result is

$$D(Z_i, Z_f, T)=\int_0^\infty de\; e^{iS_0(Z_i, Z_f, e, T)}\,D(e,T), \qquad (19)$$

where

$$S_0(Z_i, Z_f, e, T)=\frac{1}{4eT}(x_i-x_f)^2+$$
$$+\frac{1}{2}\sum_{n=1}^{\infty}\frac{n}{\sin eTn}\left[(x_i^{n2}+x_f^{n2})\cos eTn-2x_i^n x_f^n\right]- \qquad (20)$$
$$-i\sum_{n=1}^{\infty}\frac{n}{\sin eTn}\left[(\bar{\theta}_i^n\theta_i^n+\bar{\theta}_f^n\theta_f^n)\cos eTn-(\bar{\theta}_i^n\theta_f^n+\bar{\theta}_f^n\theta_i^n)\right].$$

The function D(T,e) can be represented by the functional integral

$$D(T,e) = \int e^{i\hat{S}}\, d\hat{\mu}\,, \tag{21}$$

where $\hat{S}$ is obtained from S replacing $X_n(\tau) \to y_n(\tau)$, $\theta_n(\tau) \to \xi_n(\tau)$, $\bar{\theta}_n(\tau) \to \bar{\xi}_n(\tau)$ and the integration is over the functions which are zero at the parameter interval boundary.

The integral measure for the matter fields $\varphi(\sigma,\tau)$ ($\bar{\varphi}(\sigma,\tau)$) is generally defined using a complete set of basis fields $\{\varphi_k(\sigma,\tau)\}$ $(\{\bar{\varphi}_k(\sigma,\tau)\})$. To define a measure invariant under the general coordinate transformation we use weight 1/2 field variables [19] subjected to the conditions

$$\int \bar{\varphi}_k(\sigma,\tau)\,\varphi_{k'}(\sigma,\tau)\sqrt{-g}\, d\tau\, d\sigma = \delta_{k,k'}. \tag{22a}$$

In conformal gauge $\sqrt{-g} = e\sqrt{g(\sigma,\tau)}$, where $g(\sigma,\tau)>0$ is the third independent element of the metric $g_{\alpha\beta}(\sigma,\tau)$. One can make a transformation to the basis $\{\hat{\varphi}_k\}$ $(\{\hat{\bar{\varphi}}_k\})$

$$\int \hat{\bar{\varphi}}_k(\sigma,\tau)\,\hat{\varphi}_{k'}(\sigma,\tau)\, e\, d\tau\, d\sigma = \delta_{k,k'}. \tag{22b}$$

The Jacobian of this transformation is nontrivial and depends on the field $g(\sigma,\tau)$ [13,20]. When the space-time dimension is d=26 the Jacobian of the transformation of bose variables (string coordinates) and that of the transformation of the ghosts cancel each other and one can use the basis $\{\hat{\varphi}_k\}$, $(\{\hat{\bar{\varphi}}_k\})$ to define the invariant measure. We expand the variables $y_n(\tau)$, $\Pi_n(\tau)$, $\theta_n(\tau)$, $\bar{\theta}_n(\tau)$, η, $\bar{\eta}$ in terms of basis function $\{\sqrt{\frac{2}{eT}}\sin\frac{k\pi}{T}\tau\}$. The only exceptions are the zero modes $C_0(\tau)$ and $\bar{C}_0(\tau)$. We expand them in terms of functions $\{C_k^0(\tau)\}$ and $\{\bar{C}_k^0(\tau)\}$ with

$$\int_0^T \bar{C}_k^0(\tau)\, C_{k'}^0(\tau)\, d\tau = \delta_{k,k'}\,, \quad \overset{(-)}{C}{}_k^0(0) = \overset{(-)}{C}{}_k^0(T) = 0. \tag{23}$$

The necessity of this follows from the gauge fixing (12a) that has been imposed.

To perform the integration in (21) means to integrate over the coefficients in the expansions, which are c-numbers for the bose variables and Grassmann values for the ghosts. The result is

$$D(T,e) = \Big[\prod_{k=1}^{\infty}\Big(\frac{\pi k}{eT}\Big)^2\Big]^{-\frac{d}{2}} \prod_{n=1}^{\infty}\Big[\prod_{k=1}^{\infty}\Big(\frac{\pi^2k^2}{e^2T^2} - n^2\Big)\Big]^{-\frac{d}{2}} \prod_{k=1}^{\infty}\Big(\frac{\pi k}{T}\Big)^2 \prod_{n=1}^{\infty}\prod_{k=1}^{\infty}\Big(\frac{k^2\pi^2}{e^2T^2} - n^2\Big).$$

Using ζ-function renormalization we obtain

$$\prod_{k=1}^{\infty}\Big(\frac{\pi k}{X}\Big)^2 = \mathrm{const}\, X\,, \quad \prod_{k=1}^{\infty}\Big(\frac{\pi^2k^2}{X^2} - n^2\Big) = \mathrm{const}\,\frac{\sin Xn}{n}\,, \tag{24}$$

where the constants are independent of X and n and can be determined for the sake of convenience. The final expression for the transition amplitude does not depend on the length of the parameter interval T and the result is as follows

$$D(Z_i, Z_f) = \int_0^{\infty} d\beta\, \frac{1}{(4\pi i\beta)^{\frac{D}{2}}}\, e^{i\frac{(X_i - X_f)^2}{4\beta}} \cdot$$

$$\prod_{n=1}^{\infty}\Big[\Big(\frac{n}{2\pi i \sin\beta n}\Big)^{\frac{D-2}{2}} \exp\Big\{i\frac{n}{\sin\beta n}\Big[(X_i^{n\,2} + X_f^{n\,2})\cos\beta n - 2X_i^n\cdot X_f^n\Big] +$$

$$+ \frac{n}{\sin\beta n}\Big[(\bar{\theta}_i^n\theta_i^n + \bar{\theta}_f^n\theta_f^n)\cos\beta n - (\bar{\theta}_i^n\theta_f^n + \bar{\theta}_f^n\theta_i^n)\Big]\Big]\Big\} \tag{25}$$

$$D(Z_i, Z_f) = \int_0^{\infty}\frac{d\beta}{(4\pi i\beta)^{\frac{D}{2}}}\, e^{i\frac{(X_i - X_f)^2}{4\beta}} \prod_{n=1}^{\infty} K_n(X_i^n, \theta_i^n, \bar{\theta}_i^n; X_f^n, \theta_f^n, \bar{\theta}_f^n; \beta), \tag{25a}$$

where d=26 and the new variable β =eT is introduced.

Inverting D(Z_i, Z_f) one gets the kinetic operator in the string field theory. To obtain the free field Lagrangian let us introduce bosonic (fermionic) raising and lowering operators $a_{n\mu}^+$, $a_{\mu n}$ (C_n^+, C_n, $\bar{C}_n^+$, C_n) which satisfy

$$[a_n^{\mu}, a_m^{+\nu}] = \delta_{n,m}\,\eta^{\mu\nu}\,, \quad \{\bar{C}_n^+, C_m\} = \{C_n^+, \bar{C}_m\} = \delta_{n,m} \tag{26}$$

(all the others are zero).

For given n we introduce the "vacuum" function

$$\varphi_0^{(n)}(x_n^\mu, \theta_n, \bar\theta_n) = \left(\frac{n}{\pi}\right)^{\frac{D-2}{2}} \exp\left\{-\frac{n}{2}x_n^2 - i n \bar\theta_n \theta_n\right\} \tag{27a}$$

and determine the following functions

$$\varphi^{(n)\mu_1\cdots\mu_s}(x_n, \theta_n, \bar\theta_n) = c\, a_n^{\mu_1 +}\cdots a_n^{\mu_s +}\, \varphi_0^{(n)}$$

$$\varphi_1^{(n)\mu_1\cdots\mu_s}(x_n, \theta_n, \bar\theta_n) = c\, a_n^{\mu_1 +}\cdots a_n^{\mu_s +}\, c_n^+ \bar c_n^+\, \varphi_0^{(n)} \tag{27b}$$

$$\psi_1^{(n)\mu_1\cdots\mu_s}(x_n, \theta_n, \bar\theta_n) = c\, a_n^{\mu_1 +}\cdots a_n^{\mu_s +}\, \bar c_n^+\, \varphi_0^{(n)}$$

$$\psi_2^{(n)\mu_1\cdots\mu_s}(x_n, \theta_n, \bar\theta_n) = c\, a_n^{\mu_1 +}\cdots a_n^{\mu_s +}\, c_n^+\, \varphi_0^{(n)},$$

where C are normalization constante. They are eigenfunctions of the "Hamiltonian" H

$$H_n = n\, a_n^{\mu +} a_{n\mu} + n(\bar c_n^+ c_n + c_n^+ \bar c_n) + \frac{D-2}{2} n \tag{28}$$

with the eigenvalues n(d/2-1+s), n(d/2+1+s), n(d/2+s) and n(d/2+s).
The following relation holds

$$\begin{aligned} K_n(Z_i^n, Z_f^n; \beta) = \\ = \sum_{\substack{s \\ \mu = 0,1\cdots D-1}} \Big\{ & \varphi^{(n)\mu_1\cdots\mu_s}(Z_i^n)\, e^{-in\left(\frac{D-2}{2}+s\right)\beta}\, \varphi^{(n)}_{\mu_1\cdots\mu_s}(Z_f^n) - \\ & - \varphi_1^{(n)\mu_1\cdots\mu_s}(Z_i^n)\, e^{-in\left(\frac{D+2}{2}+s\right)\beta}\, \varphi^{(n)}_{1\,\mu_1\cdots\mu_s}(Z_f^n) + \\ & + i\, \psi_1^{(n)\mu_1\cdots\mu_s}(Z_i^n)\, e^{-in\left(\frac{D}{2}+s\right)\beta}\, \psi^{(n)}_{2\,\mu_1\cdots\mu_s}(Z_f^n) - \\ & - i\, \psi_2^{(n)\mu_1\cdots\mu_s}(Z_i^n)\, e^{-in\left(\frac{D}{2}+s\right)\beta}\, \psi^{(n)}_{1\,\mu_1\cdots\mu_s}(Z_f^n), \end{aligned} \tag{29}$$

where $Z^n = (x^n, \theta^n, \bar\theta^n)$. Then

$$H_n K_n(Z_i^n, Z_f^n; \beta) = i\frac{\partial}{\partial\beta} K_n(Z_i^n, Z_f^n; \beta) \tag{30a}$$

and therefore

$$\mathcal{H} \prod_{n=1}^{\infty} K_n(Z_i^n, Z_f^n; \beta) = i\frac{\partial}{\partial\beta} \prod_{n=1}^{\infty} K_n(Z_i^n, Z_f^n; \beta). \tag{30b}$$

where $\mathcal{H} = \sum_{n=1}^{\infty} H_n$. From (25a) and (30b) follow that

$$(-p^2 - \mathcal{H}) D(z, z') = \delta^D(x^\mu - x'^\mu) \prod_{n=1}^{\infty} \delta^D(x_n^\mu - x_n'^\mu)\, \delta(\theta_n - \theta_n')\, \delta(\bar\theta_n - \bar\theta_n'). \tag{31}$$

Let us introduce the vacuum $\varphi_0 = \prod_{n=1}^{\infty} \varphi_0^{(n)}$ and the functions

$$\varphi^{m_1\cdots m_j;\, \bar m_1 \cdots \bar m_k}_{e_1\mu_1, \cdots e_i\mu_i} = a_{e_1}^{\mu_1 +}\cdots a_{e_i}^{\mu_i +}\, c_{m_1}^+ \cdots c_{m_j}^+\, \bar c_{\bar m_1}^+ \cdots \bar c_{\bar m_j}^+\, \varphi_0 . \tag{32}$$

They are eigenfunctions of the Hamiltonian $\mathcal{H}$, of the ghost number operator

$$G = -i \sum_{n=1}^{\infty} (c_n^+ \bar c_n - \bar c_n^+ c_n) \tag{33}$$

and form the orthonormal basis. The integral

$$\int \varphi^{(\cdots)}_{(\cdots)}(\{x_n\}, \{\theta_n\}, \{\bar\theta_n\})\, \varphi'^{(\cdots)}_{(\cdots)}(\{x_n\}, \{\theta_n\}, \{\bar\theta_n\})\, d\mu . \tag{34}$$

where $d\mu = \prod_{n=1}^{\infty} d^D x_n\, i\frac{d\theta_n d\bar\theta_n}{2\pi}$ is nonzero provided that the sum of the ghost numbers of the functions is zero.

Let us define the string field $\Phi(x, \{x_n\}, \{\theta_n\}, \{\bar\theta_n\})$ as a linear combination of the functions (32) with coefficients, which are c-number or Grassman values functions of x, chosen so that the string field to be Grassman even. Then, the Green's function (the propagator) of the string field Φ in a field theory with an action

$$S = \frac{1}{2}\int \varphi L \varphi d^D x\, d\mu, \tag{35}$$

where $L = -P^2 - \mathcal{H}$, is equal to $D(z, z')$. This action is the Siegel's gauge fixed action [1], and the string field φ contains all the physical relevant fields in the theory (gauge fields ghost and antighost fields).

Appendix We shall clarify the gauge fixing and quantization [12] which lead to the Siegel action and respectively to the propagator (25).

The gauge string field A is define as a Grassman even field with ghost number zero. It is convenient to introduce an auxiliary gauge field ξ which is a Grassman odd and has a ghost number-1. Then, the action can be written in the form [2,5,6]

$$S_{inv} = \int d^D x\, d\mu \{\frac{1}{2} A L A + \xi Q A - \frac{1}{2}\xi M \xi\}, \tag{36}$$

where Q is ghost number one and M ghost number two operators defined by Kato and Ogawa [20]. The action is invariant under the gauge transformations

$$\begin{aligned} &A \to A' = A + Q\Lambda_{-1} + M\mathcal{E}_{-2} \\ &\xi \to \xi' = \xi - L\Lambda_{-1} + Q\mathcal{E}_{-2}, \end{aligned} \tag{37}$$

where the gauge parameters fulfil

$$iG\Lambda_{-1} = -\Lambda_{-1}, \quad iG\mathcal{E}_{-2} = -2\mathcal{E}_{-2} \tag{38}$$

and are Grassman odd and even, respectively. The generators R_1, R_2 of the transformations (37) ($R_1^A = Q$, $R_2^A = M$; $R_1^\xi = -L$, $R_2^\xi = Q$) are linear dependent. One can find the vector (Y_1, Y_2) which satisfies

$$R_1 Y_1 + R_2 Y_2 = 0. \tag{39}$$

As a result, the number of conditions required to fix the gauge is smaller than those of the gauge parameters. It is evident from (36) that the condition

$$\xi = 0 \tag{40}$$

which corresponds to the parameter Λ_{-1} fixes the gauge. Following the standard procedure [18] we introduce Nakanishi-Lautrup (NL) field B_1 which is Grassman odd and has ghost number 1. Correspondingly, we introduce the antighost which is Grassman even and has the same ghost number. The ghost C_{-1} and η_{-2} correspond to the gauge parameters. They fulfil (38) and are Grassman even and odd. The gauge fixing and Faddeev-Popov terms are

$$\mathcal{L}_{GF+FP} = B_1 \xi + \bar{C}_1 L C_{-1} + \bar{C}_1 Q \eta_{-2}. \tag{41}$$

As a result of (39) the Lagrangian (41) is invariant under the gauge transformations

$$\begin{aligned} &C_{-1} \to C'_{-1} = C_{-1} + Q\Lambda_{-2} + M\mathcal{E}_{-2} \\ &\eta_{-2} \to \eta'_{-2} = \eta_{-2} - L\Lambda_{-2} + Q\mathcal{E}_{-2}. \end{aligned} \tag{42}$$

The above transformations are the same as those in (37), so that the scheme of quantization must be repeated. Reiterating the procedure to infinity we obtain the gauge fixing Lagrangian

$$\mathcal{L} = \mathcal{L}_{inv} + \sum_{n=1}^{\infty} [B_n \eta_{-n} + \bar{C}_n L C_{-n} + \bar{C}_n Q \eta_{-(n+1)}], \tag{43}$$

where $\eta_{-1} = \xi$.

Finally, integrating over the NL fields B_n and over the auxiliary fields η_{-n} one gets the action

$$S = \int d^D x\, d\mu \{\frac{1}{2} A L A + \sum_{n=1}^{\infty} \bar{C}_n L C_{-n}\}. \tag{44}$$

Introducing the string field

$$\varphi = A + \sum_{n=1}^{\infty} \bar{C}_n + \sum_{n=1}^{\infty} C_{-n}$$

we obtain the action (35) from (44).

After this paper was submitted for publication we received the work[22] in which an analogous representation for the string propagator is obtained by a different approach.

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От первично квантовой ко вторично квантовой теории бозе-струны

Исходя из гамильтоновой формулировки бозе-струны методом континуального интеграла вычисляется фейнмановская амплитуда перехода $D(Z_i, Z_f)$. Получен обратный к D оператор, а тем самым и свободное действие в полевой теории струны. Обсуждается процедура фиксации калибровки во вторично квантованной теории, приводящая к этому действию и тем самым к полученному пропагатору бозе струны.

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From the First to the Second Quantized String Theory

Starting from the Hamiltonian formulation of bosonic string we calculated the Feynman transition amplitude D. Inverting D we get the kinetic operator and thus the free field action. The gauge fixing procedure which leads to this action is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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